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Definitions of Statistics, Probability, and Key Terms

The science of **statistics** deals with the collection, analysis, interpretation, and presentation of **data**. We see and use data in our everyday lives.

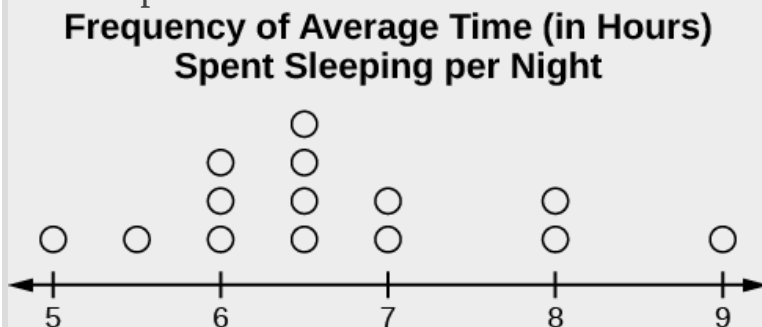
Note:

Collaborative Exercise

In your classroom, try this exercise. Have class members write down the average time (in hours, to the nearest half-hour) they sleep per night. Your instructor will record the data. Then create a simple graph (called a **dot plot**) of the data. A dot plot consists of a number line and dots (or points) positioned above the number line. For example, consider the following data:

5 5.5 6 6 6 6.5 6.5 6.5 6.5 7 7 8 8 9

The dot plot for this data would be as follows:



Does your dot plot look the same as or different from the example? Why? If you did the same example in an English class with the same number of students, do you think the results would be the same? Why or why not? Where do your data appear to cluster? How might you interpret the clustering?

The questions above ask you to analyze and interpret your data. With this example, you have begun your study of statistics.

In this course, you will learn how to organize and summarize data.

Organizing and summarizing data is called **descriptive statistics**. Two ways to summarize data are by graphing and by using numbers (for example,

finding an average). After you have studied probability and probability distributions, you will use formal methods for drawing conclusions from "good" data. The formal methods are called **inferential statistics**. Statistical inference uses probability to determine how confident we can be that our conclusions are correct.

Effective interpretation of data (inference) is based on good procedures for producing data and thoughtful examination of the data. You will encounter what will seem to be too many mathematical formulas for interpreting data. The goal of statistics is not to perform numerous calculations using the formulas, but to gain an understanding of your data. The calculations can be done using a calculator or a computer. The understanding must come from you. If you can thoroughly grasp the basics of statistics, you can be more confident in the decisions you make in life.

Probability

Probability is a mathematical tool used to study randomness. It deals with the chance (the likelihood) of an event occurring. For example, if you toss a **fair** coin four times, the outcomes may not be two heads and two tails. However, if you toss the same coin 4,000 times, the outcomes will be close to half heads and half tails. The expected theoretical probability of heads in any one toss is $\frac{1}{2}$ or 0.5. Even though the outcomes of a few repetitions are uncertain, there is a regular pattern of outcomes when there are many repetitions. After reading about the English statistician Karl **Pearson** who tossed a coin 24,000 times with a result of 12,012 heads, one of the authors tossed a coin 2,000 times. The results were 996 heads. The fraction $\frac{996}{2000}$ is equal to 0.498 which is very close to 0.5, the expected probability.

The theory of probability began with the study of games of chance such as poker. Predictions take the form of probabilities. To predict the likelihood of an earthquake, of rain, or whether you will get an A in this course, we use probabilities. Doctors use probability to determine the chance of a vaccination causing the disease the vaccination is supposed to prevent. A stockbroker uses probability to determine the rate of return on a client's investments. You might use probability to decide to buy a lottery ticket or

not. In your study of statistics, you will use the power of mathematics through probability calculations to analyze and interpret your data.

Key Terms

In statistics, we generally want to study a **population**. You can think of a population as a collection of persons, things, or objects under study. To study the population, we select a **sample**. The idea of **sampling** is to select a portion (or subset) of the larger population and study that portion (the sample) to gain information about the population. Data are the result of sampling from a population.

Because it takes a lot of time and money to examine an entire population, sampling is a very practical technique. If you wished to compute the overall grade point average at your school, it would make sense to select a sample of students who attend the school. The data collected from the sample would be the students' grade point averages. In presidential elections, opinion poll samples of 1,000–2,000 people are taken. The opinion poll is supposed to represent the views of the people in the entire country. Manufacturers of canned carbonated drinks take samples to determine if a 16 ounce can contains 16 ounces of carbonated drink.

From the sample data, we can calculate a statistic. A **statistic** is a number that represents a property of the sample. For example, if we consider one math class to be a sample of the population of all math classes, then the average number of points earned by students in that one math class at the end of the term is an example of a statistic. The statistic is an estimate of a population parameter. A **parameter** is a numerical characteristic of the whole population that can be estimated by a statistic. Since we considered all math classes to be the population, then the average number of points earned per student over all the math classes is an example of a parameter.

One of the main concerns in the field of statistics is how accurately a statistic estimates a parameter. The accuracy really depends on how well the sample represents the population. The sample must contain the characteristics of the population in order to be a **representative sample**. We are interested in both the sample statistic and the population parameter in

inferential statistics. In a later chapter, we will use the sample statistic to test the validity of the established population parameter.

A **variable**, usually notated by capital letters such as X and Y , is a characteristic or measurement that can be determined for each member of a population. Variables may be **numerical** or **categorical**. **Numerical variables** take on values with equal units such as weight in pounds and time in hours. **Categorical variables** place the person or thing into a category. If we let X equal the number of points earned by one math student at the end of a term, then X is a numerical variable. If we let Y be a person's party affiliation, then some examples of Y include Republican, Democrat, and Independent. Y is a categorical variable. We could do some math with values of X (calculate the average number of points earned, for example), but it makes no sense to do math with values of Y (calculating an average party affiliation makes no sense).

Data are the actual values of the variable. They may be numbers or they may be words. **Datum** is a single value.

Two words that come up often in statistics are **mean** and **proportion**. If you were to take three exams in your math classes and obtain scores of 86, 75, and 92, you would calculate your mean score by adding the three exam scores and dividing by three (your mean score would be 84.3 to one decimal place). If, in your math class, there are 40 students and 22 are men and 18 are women, then the proportion of men students is $\frac{22}{40}$ and the proportion of women students is $\frac{18}{40}$. Mean and proportion are discussed in more detail in later chapters.

Note:

NOTE

The words "**mean**" and "**average**" are often used interchangeably. The substitution of one word for the other is common practice. The technical term is "arithmetic mean," and "average" is technically a center location. However, in practice among non-statisticians, "average" is commonly accepted for "arithmetic mean."

Example:**Exercise:****Problem:**

Determine what the key terms refer to in the following study. We want to know the average (mean) amount of money first year college students spend at ABC College on school supplies that do not include books. We randomly surveyed 100 first year students at the college. Three of those students spent \$150, \$200, and \$225, respectively.

Solution:

The **population** is all first year students attending ABC College this term.

The **sample** could be all students enrolled in one section of a beginning statistics course at ABC College (although this sample may not represent the entire population).

The **parameter** is the average (mean) amount of money spent (excluding books) by first year college students at ABC College this term.

The **statistic** is the average (mean) amount of money spent (excluding books) by first year college students in the sample.

The **variable** could be the amount of money spent (excluding books) by one first year student. Let X = the amount of money spent (excluding books) by one first year student attending ABC College.

The **data** are the dollar amounts spent by the first year students. Examples of the data are \$150, \$200, and \$225.

Note:

Try It

Exercise:

Problem:

Determine what the key terms refer to in the following study. We want to know the average (mean) amount of money spent on school uniforms each year by families with children at Knoll Academy. We randomly survey 100 families with children in the school. Three of the families spent \$65, \$75, and \$95, respectively.

Solution:

Try It Solutions

The **population** is all families with children attending Knoll Academy.

The **sample** is a random selection of 100 families with children attending Knoll Academy.

The **parameter** is the average (mean) amount of money spent on school uniforms by families with children at Knoll Academy.

The **statistic** is the average (mean) amount of money spent on school uniforms by families in the sample.

The **variable** is the amount of money spent by one family. Let X = the amount of money spent on school uniforms by one family with children attending Knoll Academy.

The **data** are the dollar amounts spent by the families. Examples of the data are \$65, \$75, and \$95.

Example:

Exercise:

Problem:

Determine what the key terms refer to in the following study.

A study was conducted at a local college to analyze the average cumulative GPA's of students who graduated last year. Fill in the letter of the phrase that best describes each of the items below.

1. Population _____ 2. Statistic _____ 3. Parameter _____ 4. Sample _____
5. Variable _____ 6. Data _____

- a) all students who attended the college last year
- b) the cumulative GPA of one student who graduated from the college last year
- c) 3.65, 2.80, 1.50, 3.90
- d) a group of students who graduated from the college last year, randomly selected
- e) the average cumulative GPA of students who graduated from the college last year
- f) all students who graduated from the college last year
- g) the average cumulative GPA of students in the study who graduated from the college last year

Solution:

1. f 2. g 3. e 4. d 5. b 6. c

Example:**Exercise:****Problem:**

Determine what the key terms refer to in the following study.

As part of a study designed to test the safety of automobiles, the National Transportation Safety Board collected and reviewed data about the effects of an automobile crash on test dummies. Here is the criterion they used:

Speed at which Cars Crashed	Location of “drive” (i.e. dummies)
35 miles/hour	Front Seat

Cars with dummies in the front seats were crashed into a wall at a speed of 35 miles per hour. We want to know the proportion of dummies in the driver’s seat that would have had head injuries, if they had been actual drivers. We start with a simple random sample of 75 cars.

Solution:

The **population** is all cars containing dummies in the front seat.

The **sample** is the 75 cars, selected by a simple random sample.

The **parameter** is the proportion of driver dummies (if they had been real people) who would have suffered head injuries in the population.

The **statistic** is proportion of driver dummies (if they had been real people) who would have suffered head injuries in the sample.

The **variable** X = the number of driver dummies (if they had been real people) who would have suffered head injuries.

The **data** are either: yes, had head injury, or no, did not.

Example:**Exercise:****Problem:**

Determine what the key terms refer to in the following study.

An insurance company would like to determine the proportion of all medical doctors who have been involved in one or more malpractice lawsuits. The company selects 500 doctors at random from a professional directory and determines the number in the sample who have been involved in a malpractice lawsuit.

Solution:

The **population** is all medical doctors listed in the professional directory.

The **parameter** is the proportion of medical doctors who have been involved in one or more malpractice suits in the population.

The **sample** is the 500 doctors selected at random from the professional directory.

The **statistic** is the proportion of medical doctors who have been involved in one or more malpractice suits in the sample.

The **variable** X = the number of medical doctors who have been involved in one or more malpractice suits.

The **data** are either: yes, was involved in one or more malpractice lawsuits, or no, was not.

Note:

Collaborative Exercise

Do the following exercise collaboratively with up to four people per group. Find a population, a sample, the parameter, the statistic, a variable, and data for the following study: You want to determine the average (mean) number of glasses of milk college students drink per day. Suppose yesterday, in your English class, you asked five students how many glasses of milk they drank the day before. The answers were 1, 0, 1, 3, and 4 glasses of milk.

References

The Data and Story Library,
<http://lib.stat.cmu.edu/DASL/Stories/CrashTestDummies.html> (accessed May 1, 2013).

Chapter Review

The mathematical theory of statistics is easier to learn when you know the language. This module presents important terms that will be used throughout the text.

Practice

Use the following information to answer the next five exercises. Studies are often done by pharmaceutical companies to determine the effectiveness of a treatment program. Suppose that a new AIDS antibody drug is currently under study. It is given to patients once the AIDS symptoms have revealed themselves. Of interest is the average (mean) length of time in months patients live once they start the treatment. Two researchers each follow a different set of 40 patients with AIDS from the start of treatment until their deaths. The following data (in months) are collected.

Researcher A:

3 4 11 15 16 17 22 44 37 16 14 24 25 15 26 27 33 29 35 44 13 21 22 10 12
8 40 32 26 27 31 34 29 17 8 24 18 47 33 34

Researcher B:

3 14 11 5 16 17 28 41 31 18 14 14 26 25 21 22 31 2 35 44 23 21 21 16 12
18 41 22 16 25 33 34 29 13 18 24 23 42 33 29

Determine what the key terms refer to in the example for Researcher A.

Exercise:

Problem: population

Solution:

AIDS patients.

Exercise:

Problem: sample

Exercise:

Problem: parameter

Solution:

The average length of time (in months) AIDS patients live after treatment.

Exercise:

Problem: statistic

Exercise:

Problem: variable

Solution:

X = the length of time (in months) AIDS patients live after treatment

HOMEWORK

For each of the following eight exercises, identify: a. the population, b. the sample, c. the parameter, d. the statistic, e. the variable, and f. the data. Give examples where appropriate.

Exercise:

Problem:

A fitness center is interested in the mean amount of time a client exercises in the center each week.

Exercise:

Problem:

Ski resorts are interested in the mean age that children take their first ski and snowboard lessons. They need this information to plan their ski classes optimally.

Solution:

- a. all children who take ski or snowboard lessons
- b. a group of these children
- c. the population mean age of children who take their first snowboard lesson
- d. the sample mean age of children who take their first snowboard lesson
- e. X = the age of one child who takes his or her first ski or snowboard lesson
- f. values for X , such as 3, 7, and so on

Exercise:

Problem:

A cardiologist is interested in the mean recovery period of her patients who have had heart attacks.

Exercise:

Problem:

Insurance companies are interested in the mean health costs each year of their clients, so that they can determine the costs of health insurance.

Solution:

- a. the clients of the insurance companies
- b. a group of the clients
- c. the mean health costs of the clients
- d. the mean health costs of the sample
- e. X = the health costs of one client
- f. values for X , such as 34, 9, 82, and so on

Exercise:**Problem:**

A politician is interested in the proportion of voters in his district who think he is doing a good job.

Exercise:**Problem:**

A marriage counselor is interested in the proportion of clients she counsels who stay married.

Solution:

- a. all the clients of this counselor
- b. a group of clients of this marriage counselor
- c. the proportion of all her clients who stay married
- d. the proportion of the sample of the counselor's clients who stay married
- e. X = the number of couples who stay married
- f. yes, no

Exercise:**Problem:**

Political pollsters may be interested in the proportion of people who will vote for a particular cause.

Exercise:**Problem:**

A marketing company is interested in the proportion of people who will buy a particular product.

Solution:

- a. all people (maybe in a certain geographic area, such as the United States)
- b. a group of the people
- c. the proportion of all people who will buy the product
- d. the proportion of the sample who will buy the product
- e. X = the number of people who will buy it
- f. buy, not buy

Use the following information to answer the next three exercises: A Lake Tahoe Community College instructor is interested in the mean number of days Lake Tahoe Community College math students are absent from class during a quarter.

Exercise:

Problem: What is the population she is interested in?

- a. all Lake Tahoe Community College students
- b. all Lake Tahoe Community College English students
- c. all Lake Tahoe Community College students in her classes
- d. all Lake Tahoe Community College math students

Exercise:

Problem: Consider the following:

X = number of days a Lake Tahoe Community College math student is absent

In this case, X is an example of a:

- a. variable.
- b. population.
- c. statistic.
- d. data.

Solution:

a

Exercise:

Problem:

The instructor's sample produces a mean number of days absent of 3.5 days. This value is an example of a:

- a. parameter.
- b. data.
- c. statistic.
- d. variable.

Glossary

Average

also called mean; a number that describes the central tendency of the data

Categorical Variable

variables that take on values that are names or labels

Data

a set of observations (a set of possible outcomes); most data can be put into two groups: **qualitative** (an attribute whose value is indicated by a label) or **quantitative** (an attribute whose value is indicated by a number). Quantitative data can be separated into two subgroups: **discrete** and **continuous**. Data is discrete if it is the result of counting (such as the number of students of a given ethnic group in a class or the number of books on a shelf). Data is continuous if it is the result of measuring (such as distance traveled or weight of luggage)

Numerical Variable

variables that take on values that are indicated by numbers

Parameter

a number that is used to represent a population characteristic and that generally cannot be determined easily

Population

all individuals, objects, or measurements whose properties are being studied

Probability

a number between zero and one, inclusive, that gives the likelihood that a specific event will occur

Proportion

the number of successes divided by the total number in the sample

Representative Sample

a subset of the population that has the same characteristics as the population

Sample

a subset of the population studied

Statistic

a numerical characteristic of the sample; a statistic estimates the corresponding population parameter.

Variable

a characteristic of interest for each person or object in a population

Frequency, Frequency Tables, and Levels of Measurement

Once you have a set of data, you will need to organize it so that you can analyze how frequently each datum occurs in the set. However, when calculating the frequency, you may need to round your answers so that they are as precise as possible.

Answers and Rounding Off

A simple way to round off answers is to carry your final answer one more decimal place than was present in the original data. Round off only the final answer. Do not round off any intermediate results, if possible. If it becomes necessary to round off intermediate results, carry them to at least twice as many decimal places as the final answer. For example, the average of the three quiz scores four, six, and nine is 6.3, rounded off to the nearest tenth, because the data are whole numbers. Most answers will be rounded off in this manner.

It is not necessary to reduce most fractions in this course. Especially in [Probability Topics](#), the chapter on probability, it is more helpful to leave an answer as an unreduced fraction.

Levels of Measurement

The way a set of data is measured is called its **level of measurement**. Correct statistical procedures depend on a researcher being familiar with levels of measurement. Not every statistical operation can be used with every set of data. Data can be classified into four levels of measurement. They are (from lowest to highest level):

- **Nominal scale level**
- **Ordinal scale level**
- **Interval scale level**
- **Ratio scale level**

Data that is measured using a **nominal scale** is **qualitative(categorical)**. Categories, colors, names, labels and favorite foods along with yes or no responses are examples of nominal level data. Nominal scale data are not ordered. For example, trying to classify people according to their favorite food does not make any sense. Putting pizza first and sushi second is not meaningful.

Smartphone companies are another example of nominal scale data. The data are the names of the companies that make smartphones, but there is no agreed upon order of these brands, even though people may have personal preferences. Nominal scale data cannot be used in calculations.

Data that is measured using an **ordinal scale** is similar to nominal scale data but there is a big difference. The ordinal scale data can be ordered. An example of ordinal scale data is a list of the top five national parks in the United States. The top five national parks in the United States can be ranked from one to five but we cannot measure differences between the data.

Another example of using the ordinal scale is a cruise survey where the responses to questions about the cruise are “excellent,” “good,” “satisfactory,” and “unsatisfactory.” These responses are ordered from the most desired response to the least desired. But the differences between two pieces

of data cannot be measured. Like the nominal scale data, ordinal scale data cannot be used in calculations.

Data that is measured using the **interval scale** is similar to ordinal level data because it has a definite ordering but there is a difference between data. The differences between interval scale data can be measured though the data does not have a starting point.

Temperature scales like Celsius (C) and Fahrenheit (F) are measured by using the interval scale. In both temperature measurements, 40° is equal to 100° minus 60°. Differences make sense. But 0 degrees does not because, in both scales, 0 is not the absolute lowest temperature. Temperatures like -10° F and -15° C exist and are colder than 0.

Interval level data can be used in calculations, but one type of comparison cannot be done. 80° C is not four times as hot as 20° C (nor is 80° F four times as hot as 20° F). There is no meaning to the ratio of 80 to 20 (or four to one).

Data that is measured using the **ratio scale** takes care of the ratio problem and gives you the most information. Ratio scale data is like interval scale data, but it has a 0 point and ratios can be calculated. For example, four multiple choice statistics final exam scores are 80, 68, 20 and 92 (out of a possible 100 points). The exams are machine-graded.

The data can be put in order from lowest to highest: 20, 68, 80, 92.

The differences between the data have meaning. The score 92 is more than the score 68 by 24 points. Ratios can be calculated. The smallest score is 0. So 80 is four times 20. The score of 80 is four times better than the score of 20.

Frequency

Twenty students were asked how many hours they worked per day. Their responses, in hours, are as follows: 56332475235654435253.

[\[link\]](#) lists the different data values in ascending order and their frequencies.

DATA VALUE	FREQUENCY
2	3
3	5
4	3
5	6

DATA VALUE	FREQUENCY
6	2
7	1

Frequency Table of Student Work Hours

A **frequency** is the number of times a value of the data occurs. According to [\[link\]](#), there are three students who work two hours, five students who work three hours, and so on. The sum of the values in the frequency column, 20, represents the total number of students included in the sample.

A **relative frequency** is the ratio (fraction or proportion) of the number of times a value of the data occurs in the set of all outcomes to the total number of outcomes. To find the relative frequencies, divide each frequency by the total number of students in the sample—in this case, 20. Relative frequencies can be written as fractions, percents, or decimals.

DATA VALUE	FREQUENCY	RELATIVE FREQUENCY
2	3	$\frac{3}{20}$ or 0.15
3	5	$\frac{5}{20}$ or 0.25
4	3	$\frac{3}{20}$ or 0.15
5	6	$\frac{6}{20}$ or 0.30
6	2	$\frac{2}{20}$ or 0.10
7	1	$\frac{1}{20}$ or 0.05

Frequency Table of Student Work Hours with Relative Frequencies

The sum of the values in the relative frequency column of [\[link\]](#) is $\frac{20}{20}$, or 1.

Cumulative relative frequency is the accumulation of the previous relative frequencies. To find the cumulative relative frequencies, add all the previous relative frequencies to the relative frequency for the current row, as shown in [\[link\]](#).

DATA VALUE	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
2	3	$\frac{3}{20}$ or 0.15	0.15
3	5	$\frac{5}{20}$ or 0.25	$0.15 + 0.25 = 0.40$
4	3	$\frac{3}{20}$ or 0.15	$0.40 + 0.15 = 0.55$
5	6	$\frac{6}{20}$ or 0.30	$0.55 + 0.30 = 0.85$
6	2	$\frac{2}{20}$ or 0.10	$0.85 + 0.10 = 0.95$
7	1	$\frac{1}{20}$ or 0.05	$0.95 + 0.05 = 1.00$

Frequency Table of Student Work Hours with Relative and Cumulative Relative Frequencies

The last entry of the cumulative relative frequency column is one, indicating that one hundred percent of the data has been accumulated.

Note:

NOTE

Because of rounding, the relative frequency column may not always sum to one, and the last entry in the cumulative relative frequency column may not be one. However, they each should be close to one.

[\[link\]](#) represents the heights, in inches, of a sample of 100 male semiprofessional soccer players.

HEIGHTS (INCHES)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
59.95–61.95	5	$\frac{5}{100} = 0.05$	0.05
61.95–63.95	3	$\frac{3}{100} = 0.03$	$0.05 + 0.03 = 0.08$

HEIGHTS (INCHES)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
63.95–65.95	15	$\frac{15}{100} = 0.15$	$0.08 + 0.15 = 0.23$
65.95–67.95	40	$\frac{40}{100} = 0.40$	$0.23 + 0.40 = 0.63$
67.95–69.95	17	$\frac{17}{100} = 0.17$	$0.63 + 0.17 = 0.80$
69.95–71.95	12	$\frac{12}{100} = 0.12$	$0.80 + 0.12 = 0.92$
71.95–73.95	7	$\frac{7}{100} = 0.07$	$0.92 + 0.07 = 0.99$
73.95–75.95	1	$\frac{1}{100} = 0.01$	$0.99 + 0.01 = 1.00$
	Total = 100	Total = 1.00	

Frequency Table of Soccer Player Height

The data in this table have been **grouped** into the following intervals:

- 59.95 to 61.95 inches
- 61.95 to 63.95 inches
- 63.95 to 65.95 inches
- 65.95 to 67.95 inches
- 67.95 to 69.95 inches
- 69.95 to 71.95 inches
- 71.95 to 73.95 inches
- 73.95 to 75.95 inches

Note:

Note

This example is used again in [Descriptive Statistics](#), where the method used to compute the intervals will be explained.

In this sample, there are **five** players whose heights fall within the interval 59.95–61.95 inches, **three** players whose heights fall within the interval 61.95–63.95 inches, **15** players whose heights fall within the interval 63.95–65.95 inches, **40** players whose heights fall within the interval 65.95–67.95 inches, **17** players whose heights fall within the interval 67.95–69.95 inches, **12** players whose heights fall within the interval 69.95–71.95, **seven** players whose heights fall within the interval 71.95–73.95, and **one** player whose heights fall within the interval 73.95–75.95. All heights fall between the endpoints of an interval and not at the endpoints.

Example:
Exercise:

Problem: From [\[link\]](#), find the percentage of heights that are less than 65.95 inches.

Solution:

If you look at the first, second, and third rows, the heights are all less than 65.95 inches. There are $5 + 3 + 15 = 23$ players whose heights are less than 65.95 inches. The percentage of heights less than 65.95 inches is then $\frac{23}{100}$ or 23%. This percentage is the cumulative relative frequency entry in the third row.

Note:
Try It
Exercise:

Problem: [\[link\]](#) shows the amount, in inches, of annual rainfall in a sample of towns.

Rainfall (Inches)	Frequency	Relative Frequency	Cumulative Relative Frequency
2.95–4.97	6	$\frac{6}{50} = 0.12$	0.12
4.97–6.99	7	$\frac{7}{50} = 0.14$	$0.12 + 0.14 = 0.26$
6.99–9.01	15	$\frac{15}{50} = 0.30$	$0.26 + 0.30 = 0.56$
9.01–11.03	8	$\frac{8}{50} = 0.16$	$0.56 + 0.16 = 0.72$
11.03–13.05	9	$\frac{9}{50} = 0.18$	$0.72 + 0.18 = 0.90$
13.05–15.07	5	$\frac{5}{50} = 0.10$	$0.90 + 0.10 = 1.00$
	Total = 50	Total = 1.00	

From [\[link\]](#), find the percentage of rainfall that is less than 9.01 inches.

Solution:
Try It Solutions

0.56 or 56%

Example:

Exercise:

Problem:

From [\[link\]](#), find the percentage of heights that fall between 61.95 and 65.95 inches.

Solution:

Add the relative frequencies in the second and third rows: $0.03 + 0.15 = 0.18$ or 18%.

Note:

Try It

Exercise:

Problem: From [\[link\]](#), find the percentage of rainfall that is between 6.99 and 13.05 inches.

Solution:

Try It Solutions

$0.30 + 0.16 + 0.18 = 0.64$ or 64%

Example:

Exercise:

Problem:

Use the heights of the 100 male semiprofessional soccer players in [\[link\]](#). Fill in the blanks and check your answers.

- The percentage of heights that are from 67.95 to 71.95 inches is: ____.
- The percentage of heights that are from 67.95 to 73.95 inches is: ____.
- The percentage of heights that are more than 65.95 inches is: ____.
- The number of players in the sample who are between 61.95 and 71.95 inches tall is: ____.
- What kind of data are the heights?
- Describe how you could gather this data (the heights) so that the data are characteristic of all male semiprofessional soccer players.

Remember, you **count frequencies**. To find the relative frequency, divide the frequency by the total number of data values. To find the cumulative relative frequency, add all of the

previous relative frequencies to the relative frequency for the current row.

Solution:

- a. 29%
- b. 36%
- c. 77%
- d. 87
- e. quantitative continuous
- f. get rosters from each team and choose a simple random sample from each

Note:

Try It

Exercise:

Problem:

From [\[link\]](#), find the number of towns that have rainfall between 2.95 and 9.01 inches.

Solution:

Try It Solutions

$$6 + 7 + 15 = 28 \text{ towns}$$

Note:

Collaborative Exercise

In your class, have someone conduct a survey of the number of siblings (brothers and sisters) each student has. Create a frequency table. Add to it a relative frequency column and a cumulative relative frequency column. Answer the following questions:

1. What percentage of the students in your class have no siblings?
2. What percentage of the students have from one to three siblings?
3. What percentage of the students have fewer than three siblings?

Example:

Nineteen people were asked how many miles, to the nearest mile, they commute to work each day. The data are as follows: 2 5 7 3 2 10 18 15 20 7 10 18 5 12 13 12 4 5 10. [\[link\]](#) was produced:

DATA	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
3	3	$\frac{3}{19}$	0.1579
4	1	$\frac{1}{19}$	0.2105
5	3	$\frac{3}{19}$	0.1579
7	2	$\frac{2}{19}$	0.2632
10	3	$\frac{4}{19}$	0.4737
12	2	$\frac{2}{19}$	0.7895
13	1	$\frac{1}{19}$	0.8421
15	1	$\frac{1}{19}$	0.8948
18	1	$\frac{1}{19}$	0.9474
20	1	$\frac{1}{19}$	1.0000

Frequency of Commuting Distances

Exercise:

Problem:

- Is the table correct? If it is not correct, what is wrong?
- True or False: Three percent of the people surveyed commute three miles. If the statement is not correct, what should it be? If the table is incorrect, make the corrections.
- What fraction of the people surveyed commute five or seven miles?
- What fraction of the people surveyed commute 12 miles or more? Less than 12 miles? Between five and 13 miles (not including five and 13 miles)?

Solution:

- No. The frequency column sums to 18, not 19. Not all cumulative relative frequencies are correct.
- False. The frequency for three miles should be one; for two miles (left out), two. The cumulative relative frequency column should read: 0.1052, 0.1579, 0.2105, 0.3684, 0.4737, 0.6316, 0.7368, 0.7895, 0.8421, 0.9474, 1.0000.
- $\frac{5}{19}$

d. $\frac{7}{19}, \frac{12}{19}, \frac{7}{19}$

Note:

Try It

Exercise:

Problem:

[\[link\]](#) represents the amount, in inches, of annual rainfall in a sample of towns. What fraction of towns surveyed get between 11.03 and 13.05 inches of rainfall each year?

Solution:

Try It Solutions

$$\frac{9}{50}$$

Example:

[\[link\]](#) contains the total number of deaths worldwide as a result of earthquakes for the period from 2000 to 2012.

Year	Total Number of Deaths
2000	231
2001	21,357
2002	11,685
2003	33,819
2004	228,802
2005	88,003
2006	6,605
2007	712

Year	Total Number of Deaths
2008	88,011
2009	1,790
2010	320,120
2011	21,953
2012	768
Total	823,856

Exercise:

Problem: Answer the following questions.

- What is the frequency of deaths measured from 2006 through 2009?
- What percentage of deaths occurred after 2009?
- What is the relative frequency of deaths that occurred in 2003 or earlier?
- What is the percentage of deaths that occurred in 2004?
- What kind of data are the numbers of deaths?
- The Richter scale is used to quantify the energy produced by an earthquake. Examples of Richter scale numbers are 2.3, 4.0, 6.1, and 7.0. What kind of data are these numbers?

Solution:

- 97,118 (11.8%)
- 41.6%
- $67,092/823,356$ or 0.081 or 8.1 %
- 27.8%
- Quantitative discrete
- Quantitative continuous

Note:

Try It

Exercise:

Problem:

[\[link\]](#) contains the total number of fatal motor vehicle traffic crashes in the United States for the period from 1994 to 2011.

Year	Total Number of Crashes	Year	Total Number of Crashes
1994	36,254	2004	38,444
1995	37,241	2005	39,252
1996	37,494	2006	38,648
1997	37,324	2007	37,435
1998	37,107	2008	34,172
1999	37,140	2009	30,862
2000	37,526	2010	30,296
2001	37,862	2011	29,757
2002	38,491	Total	653,782
2003	38,477		

Answer the following questions.

- What is the frequency of deaths measured from 2000 through 2004?
- What percentage of deaths occurred after 2006?
- What is the relative frequency of deaths that occurred in 2000 or before?
- What is the percentage of deaths that occurred in 2011?
- What is the cumulative relative frequency for 2006? Explain what this number tells you about the data.

Solution:

Try It Solutions

- 190,800 (29.2%)
- 24.9%
- $260,086/653,782$ or 39.8%
- 4.6%
- 75.1% of all fatal traffic crashes for the period from 1994 to 2011 happened from 1994 to 2006.

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Chapter Review

Some calculations generate numbers that are artificially precise. It is not necessary to report a value to eight decimal places when the measures that generated that value were only accurate to the nearest tenth. Round off your final answer to one more decimal place than was present in the original data. This means that if you have data measured to the nearest tenth of a unit, report the final statistic to the nearest hundredth.

In addition to rounding your answers, you can measure your data using the following four levels of measurement.

- **Nominal scale level:** data that cannot be ordered nor can it be used in calculations
- **Ordinal scale level:** data that can be ordered; the differences cannot be measured
- **Interval scale level:** data with a definite ordering but no starting point; the differences can be measured, but there is no such thing as a ratio.
- **Ratio scale level:** data with a starting point that can be ordered; the differences have meaning and ratios can be calculated.

When organizing data, it is important to know how many times a value appears. How many statistics students study five hours or more for an exam? What percent of families on our block own two pets? Frequency, relative frequency, and cumulative relative frequency are measures that answer questions like these.

Exercise:

Problem: What type of measure scale is being used? Nominal, ordinal, interval or ratio.

- a. High school soccer players classified by their athletic ability: Superior, Average, Above average
- b. Baking temperatures for various main dishes: 350, 400, 325, 250, 300
- c. The colors of crayons in a 24-crayon box

- d. Social security numbers
- e. Incomes measured in dollars
- f. A satisfaction survey of a social website by number: 1 = very satisfied, 2 = somewhat satisfied, 3 = not satisfied
- g. Political outlook: extreme left, left-of-center, right-of-center, extreme right
- h. Time of day on an analog watch
- i. The distance in miles to the closest grocery store
- j. The dates 1066, 1492, 1644, 1947, and 1944
- k. The heights of 21–65 year-old women
- l. Common letter grades: A, B, C, D, and F

Solution:

- a. ordinal
- b. interval
- c. nominal
- d. nominal
- e. ratio
- f. ordinal
- g. nominal
- h. interval
- i. ratio
- j. interval
- k. ratio
- l. ordinal

HOMEWORK

Exercise:

Problem:

Fifty part-time students were asked how many courses they were taking this term. The (incomplete) results are shown below:

# of Courses	Frequency	Relative Frequency	Cumulative Relative Frequency
1	30	0.6	
2	15		

# of Courses	Frequency	Relative Frequency	Cumulative Relative Frequency
3			

Part-time Student Course Loads

- Fill in the blanks in [\[link\]](#).
- What percent of students take exactly two courses?
- What percent of students take one or two courses?

Exercise:

Problem:

Sixty adults with gum disease were asked the number of times per week they used to floss before their diagnosis. The (incomplete) results are shown in [\[link\]](#).

# Flossing per Week	Frequency	Relative Frequency	Cumulative Relative Freq.
0	27	0.4500	
1	18		
3			0.9333
6	3	0.0500	
7	1	0.0167	

Flossing Frequency for Adults with Gum Disease

- Fill in the blanks in [\[link\]](#).
- What percent of adults flossed six times per week?
- What percent flossed at most three times per week?

Solution:

-

# Flossing per Week	Frequency	Relative Frequency	Cumulative Relative Frequency
0	27	0.4500	0.4500
1	18	0.3000	0.7500
3	11	0.1833	0.9333
6	3	0.0500	0.9833
7	1	0.0167	1

- b. 5.00%
- c. 93.33%

Exercise:

Problem:

Nineteen immigrants to the U.S were asked how many years, to the nearest year, they have lived in the U.S. The data are as follows: 2 5 7 2 2 10 20 15 0 7 0 20 5 12 15 12 4 5 10 .

[\[link\]](#) was produced.

Data	Frequency	Relative Frequency	Cumulative Relative Frequency
0	2	$\frac{2}{19}$	0.1053
2	3	$\frac{3}{19}$	0.2632
4	1	$\frac{1}{19}$	0.3158
5	3	$\frac{3}{19}$	0.4737
7	2	$\frac{2}{19}$	0.5789
10	2	$\frac{2}{19}$	0.6842
12	2	$\frac{2}{19}$	0.7895

Data	Frequency	Relative Frequency	Cumulative Relative Frequency
15	1	$\frac{1}{19}$	0.8421
20	1	$\frac{1}{19}$	1.0000

Frequency of Immigrant Survey Responses

- Fix the errors in [\[link\]](#). Also, explain how someone might have arrived at the incorrect number(s).
- Explain what is wrong with this statement: “47 percent of the people surveyed have lived in the U.S. for 5 years.”
- Fix the statement in **b** to make it correct.
- What fraction of the people surveyed have lived in the U.S. five or seven years?
- What fraction of the people surveyed have lived in the U.S. at most 12 years?
- What fraction of the people surveyed have lived in the U.S. fewer than 12 years?
- What fraction of the people surveyed have lived in the U.S. from five to 20 years, inclusive?

Exercise:

Problem:

How much time does it take to travel to work? [\[link\]](#) shows the mean commute time by state for workers at least 16 years old who are not working at home. Find the mean travel time, and round off the answer properly.

24.0	24.3	25.9	18.9	27.5	17.9	21.8	20.9	16.7	27.3
18.2	24.7	20.0	22.6	23.9	18.0	31.4	22.3	24.0	25.5
24.7	24.6	28.1	24.9	22.6	23.6	23.4	25.7	24.8	25.5
21.2	25.7	23.1	23.0	23.9	26.0	16.3	23.1	21.4	21.5
27.0	27.0	18.6	31.7	23.3	30.1	22.9	23.3	21.7	18.6

Solution:

The sum of the travel times is 1,173.1. Divide the sum by 50 to calculate the mean value: 23.462. Because each state’s travel time was measured to the nearest tenth, round this calculation to the nearest hundredth: 23.46.

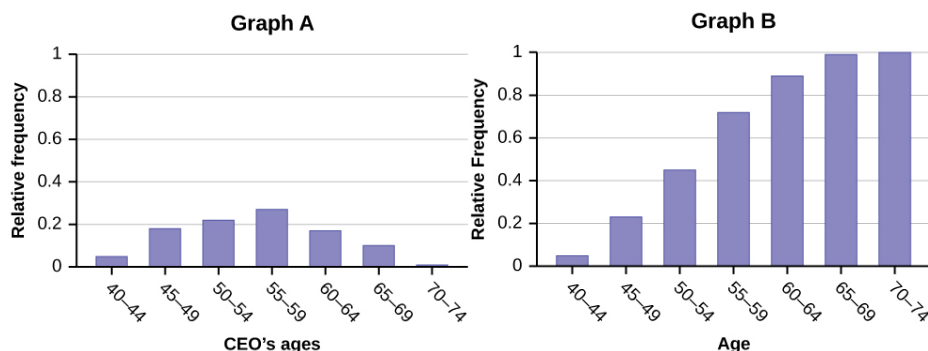
Exercise:

Problem:

Forbes magazine published data on the best small firms in 2012. These were firms which had been publicly traded for at least a year, have a stock price of at least \$5 per share, and have reported annual revenue between \$5 million and \$1 billion. [\[link\]](#) shows the ages of the chief executive officers for the first 60 ranked firms.

Age	Frequency	Relative Frequency	Cumulative Relative Frequency
40–44	3		
45–49	11		
50–54	13		
55–59	16		
60–64	10		
65–69	6		
70–74	1		

- What is the frequency for CEO ages between 54 and 65?
- What percentage of CEOs are 65 years or older?
- What is the relative frequency of ages under 50?
- What is the cumulative relative frequency for CEOs younger than 55?
- Which graph shows the relative frequency and which shows the cumulative relative frequency?



Use the following information to answer the next two exercises: [link](#) contains data on hurricanes that have made direct hits on the U.S. Between 1851 and 2004. A hurricane is given a strength category rating based on the minimum wind speed generated by the storm.

Category	Number of Direct Hits	Relative Frequency	Cumulative Frequency
1	109	0.3993	0.3993
2	72	0.2637	0.6630
3	71	0.2601	
4	18		0.9890
5	3	0.0110	1.0000
	Total = 273		

Frequency of Hurricane Direct Hits

Exercise:

Problem: What is the relative frequency of direct hits that were category 4 hurricanes?

- a. 0.0768
- b. 0.0659
- c. 0.2601
- d. Not enough information to calculate

Solution:

b

Exercise:

Problem:

What is the relative frequency of direct hits that were AT MOST a category 3 storm?

- a. 0.3480
- b. 0.9231
- c. 0.2601
- d. 0.3370

Glossary

Cumulative Relative Frequency

The term applies to an ordered set of observations from smallest to largest. The cumulative relative frequency is the sum of the relative frequencies for all values that are less than or equal to the given value.

Frequency

the number of times a value of the data occurs

Relative Frequency

the ratio of the number of times a value of the data occurs in the set of all outcomes to the number of all outcomes to the total number of outcomes

Stem-and-Leaf Graphs (Stemplots), Line Graphs, and Bar Graphs

One simple graph, the **stem-and-leaf graph** or **stemplot**, comes from the field of exploratory data analysis. It is a good choice when the data sets are small. To create the plot, divide each observation of data into a stem and a leaf. The leaf consists of a **final significant digit**. For example, 23 has stem two and leaf three. The number 432 has stem 43 and leaf two. Likewise, the number 5,432 has stem 543 and leaf two. The decimal 9.3 has stem nine and leaf three. Write the stems in a vertical line from smallest to largest. Draw a vertical line to the right of the stems. Then write the leaves in increasing order next to their corresponding stem.

Example:

For Susan Dean's spring pre-calculus class, scores for the first exam were as follows (smallest to largest):

33; 42; 49; 49; 53; 55; 55; 61; 63; 67; 68; 68; 69; 69; 72; 73; 74; 78; 80; 83; 88; 88; 88; 90; 92; 94; 94; 94; 94; 96; 100

Stem	Leaf
3	3
4	2 9 9
5	3 5 5
6	1 3 7 8 8 9 9
7	2 3 4 8
8	0 3 8 8 8
9	0 2 4 4 4 4 6
10	0

Stem-and-Leaf Graph

The stemplot shows that most scores fell in the 60s, 70s, 80s, and 90s. Eight out of the 31 scores or approximately 26% ($\frac{8}{31}$) were in the 90s or 100, a fairly high number of As.

Note:

Try It

Exercise:

Problem:

For the Park City basketball team, scores for the last 30 games were as follows (smallest to largest):

32; 32; 33; 34; 38; 40; 42; 42; 43; 44; 46; 47; 47; 48; 48; 48; 49; 50; 50; 51; 52; 52; 52; 53; 54; 56; 57; 57; 60; 61

Construct a stem plot for the data.

Solution:

Stem	Leaf
3	2 2 3 4 8
4	0 2 2 3 4 6 7 7 8 8 8 9
5	0 0 1 2 2 2 3 4 6 7 7
6	0 1

The stemplot is a quick way to graph data and gives an exact picture of the data. You want to look for an overall pattern and any outliers. An **outlier** is an observation of data that does not fit the rest of the data. It is sometimes called an **extreme value**. When you graph an outlier, it will appear not to fit the pattern of the graph. Some outliers are due to mistakes (for example, writing down 50 instead of 500) while others may indicate that something unusual is happening. It takes some background information to explain outliers, so we will cover them in more detail later.

Example:

The data are the distances (in kilometers) from a home to local supermarkets. Create a stemplot using the data:

1.1; 1.5; 2.3; 2.5; 2.7; 3.2; 3.3; 3.3; 3.5; 3.8; 4.0; 4.2; 4.5; 4.5; 4.7; 4.8; 5.5; 5.6; 6.5; 6.7; 12.3

Exercise:

Problem: Do the data seem to have any concentration of values?

Note:
NOTE

The leaves are to the right of the decimal.

Solution:

The value 12.3 may be an outlier. Values appear to concentrate at three and four kilometers.

Stem	Leaf
1	1 5
2	3 5 7
3	2 3 3 5 8
4	0 2 5 5 7 8
5	5 6
6	5 7
7	
8	
9	
10	
11	
12	3

Note:
Try It
Exercise:

Problem:

The following data show the distances (in miles) from the homes of off-campus statistics students to the college. Create a stem plot using the data and identify any outliers:

0.5; 0.7; 1.1; 1.2; 1.2; 1.3; 1.3; 1.5; 1.5; 1.7; 1.7; 1.8; 1.9; 2.0; 2.2; 2.5; 2.6; 2.8; 2.8; 2.8; 3.5; 3.8; 4.4; 4.8; 4.9; 5.2; 5.5; 5.7; 5.8; 8.0

Solution:

Stem	Leaf
0	5 7
1	1 2 2 3 3 5 5 7 7 8 9
2	0 2 5 6 8 8 8
3	5 8
4	4 8 9
5	2 5 7 8
6	
7	
8	0

The value 8.0 may be an outlier. Values appear to concentrate at one and two miles.

Example:**Exercise:****Problem:**

A **side-by-side stem-and-leaf plot** allows a comparison of the two data sets in two columns. In a side-by-side stem-and-leaf plot, two sets of leaves share the same stem. The leaves are to the left and the right of the stems. [\[link\]](#) and [\[link\]](#) show the ages of presidents at their inauguration and at their death. Construct a side-by-side stem-and-leaf plot using this data.

Solution:

Ages at Inauguration		Ages at Death
9 9 8 7 7 7 6 3 2	4	6 9
8 7 7 7 7 6 6 6 5 5 5 5 4 4 4 4 4 2 2 1 1 1 1 1 0	5	3 6 6 7 7 8
9 8 5 4 4 2 1 1 1 0	6	0 0 3 3 4 4 5 6 7 7 7 8
	7	0 0 1 1 1 4 7 8 8 9
	8	0 1 3 5 8
	9	0 0 3 3

President	Age	President	Age	President	Age
Washington	57	Lincoln	52	Hoover	54
J. Adams	61	A. Johnson	56	F. Roosevelt	51
Jefferson	57	Grant	46	Truman	60
Madison	57	Hayes	54	Eisenhower	62
Monroe	58	Garfield	49	Kennedy	43
J. Q. Adams	57	Arthur	51	L. Johnson	55
Jackson	61	Cleveland	47	Nixon	56
Van Buren	54	B. Harrison	55	Ford	61
W. H. Harrison	68	Cleveland	55	Carter	52

President	Age	President	Age	President	Age
Tyler	51	McKinley	54	Reagan	69
Polk	49	T. Roosevelt	42	G.H.W. Bush	64
Taylor	64	Taft	51	Clinton	47
Fillmore	50	Wilson	56	G. W. Bush	54
Pierce	48	Harding	55	Obama	47
Buchanan	65	Coolidge	51		

Presidential Ages at Inauguration

President	Age	President	Age	President	Age
Washington	67	Lincoln	56	Hoover	90
J. Adams	90	A. Johnson	66	F. Roosevelt	63
Jefferson	83	Grant	63	Truman	88
Madison	85	Hayes	70	Eisenhower	78
Monroe	73	Garfield	49	Kennedy	46
J. Q. Adams	80	Arthur	56	L. Johnson	64
Jackson	78	Cleveland	71	Nixon	81
Van Buren	79	B. Harrison	67	Ford	93
W. H. Harrison	68	Cleveland	71	Reagan	93
Tyler	71	McKinley	58		
Polk	53	T. Roosevelt	60		
Taylor	65	Taft	72		

President	Age	President	Age	President	Age
Fillmore	74	Wilson	67		
Pierce	64	Harding	57		
Buchanan	77	Coolidge	60		

Presidential Age at Death

Note:

Exercise:

Problem:

The table shows the number of wins and losses the Atlanta Hawks have had in 42 seasons. Create a side-by-side stem-and-leaf plot of these wins and losses.

Losses	Wins	Year	Losses	Wins	Year
34	48	1968–1969	41	41	1989–1990
34	48	1969–1970	39	43	1990–1991
46	36	1970–1971	44	38	1991–1992
46	36	1971–1972	39	43	1992–1993
36	46	1972–1973	25	57	1993–1994
47	35	1973–1974	40	42	1994–1995
51	31	1974–1975	36	46	1995–1996
53	29	1975–1976	26	56	1996–1997
51	31	1976–1977	32	50	1997–1998
41	41	1977–1978	19	31	1998–1999
36	46	1978–1979	54	28	1999–2000

Losses	Wins	Year	Losses	Wins	Year
32	50	1979–1980	57	25	2000–2001
51	31	1980–1981	49	33	2001–2002
40	42	1981–1982	47	35	2002–2003
39	43	1982–1983	54	28	2003–2004
42	40	1983–1984	69	13	2004–2005
48	34	1984–1985	56	26	2005–2006
32	50	1985–1986	52	30	2006–2007
25	57	1986–1987	45	37	2007–2008
32	50	1987–1988	35	47	2008–2009
30	52	1988–1989	29	53	2009–2010

Solution:

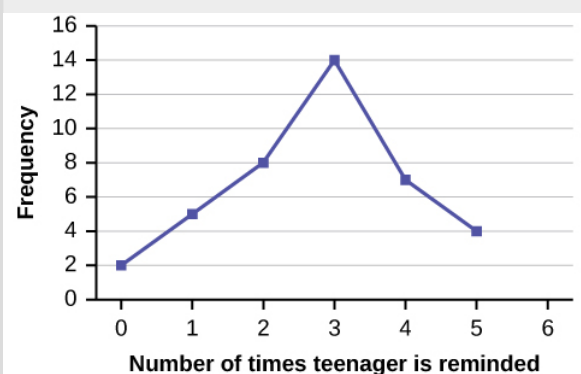
Atlanta Hawks Wins and Losses		
Number of Wins		Number of Losses
3	1	9
9 8 8 6 5	2	5 5 9
8 7 6 6 5 5 4 3 1 1 1 1 0	3	0 2 2 2 2 4 4 5 6 6 6 9 9 9
8 8 7 6 6 6 3 3 3 2 2 1 1 0	4	0 0 1 1 2 4 5 6 6 7 7 8 9
7 7 6 3 2 0 0 0 0	5	1 1 1 2 3 4 4 6 7
	6	9

Another type of graph that is useful for specific data values is a **line graph**. In the particular line graph shown in [\[link\]](#), the **x-axis** (horizontal axis) consists of **data values** and the **y-axis** (vertical axis) consists of **frequency points**. The frequency points are connected using line segments.

Example:

In a survey, 40 mothers were asked how many times per week a teenager must be reminded to do his or her chores. The results are shown in [\[link\]](#) and in [\[link\]](#).

Number of times teenager is reminded	Frequency
0	2
1	5
2	8
3	14
4	7
5	4



Note:

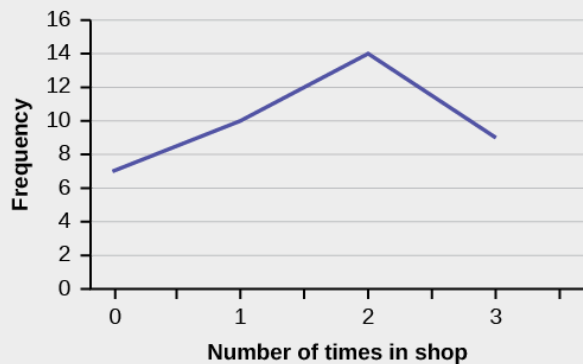
Try It

Exercise:

Problem:

In a survey, 40 people were asked how many times per year they had their car in the shop for repairs. The results are shown in [\[link\]](#). Construct a line graph.

Number of times in shop	Frequency
0	7
1	10
2	14
3	9

Solution:

Bar graphs consist of bars that are separated from each other. The bars can be rectangles or they can be rectangular boxes (used in three-dimensional plots), and they can be vertical or horizontal. The **bar graph** shown in [\[link\]](#) has age groups represented on the **x-axis** and proportions on the **y-axis**.

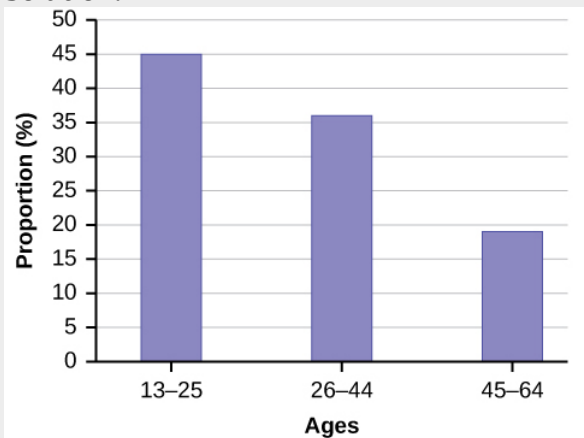
Example:**Exercise:**

Problem:

By the end of 2011, Facebook had over 146 million users in the United States. [\[link\]](#) shows three age groups, the number of users in each age group, and the proportion (%) of users in each age group. Construct a bar graph using this data.

Age groups	Number of Facebook users	Proportion (%) of Facebook users
13–25	65,082,280	45%
26–44	53,300,200	36%
45–64	27,885,100	19%

Solution:



Note:

Try It

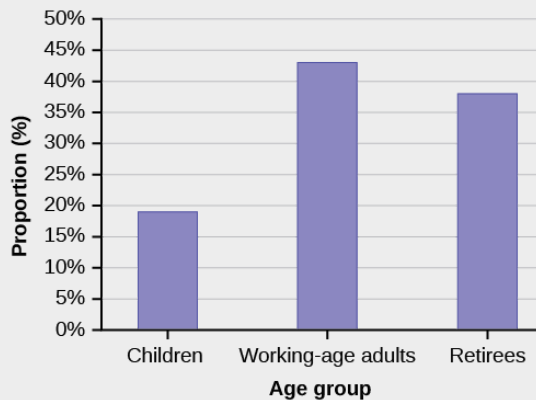
Exercise:

Problem:

The population in Park City is made up of children, working-age adults, and retirees. [\[link\]](#) shows the three age groups, the number of people in the town from each age group, and the proportion (%) of people in each age group. Construct a bar graph showing the proportions.

Age groups	Number of people	Proportion of population
Children	67,059	19%
Working-age adults	152,198	43%
Retirees	131,662	38%

Solution:



Example:

Exercise:

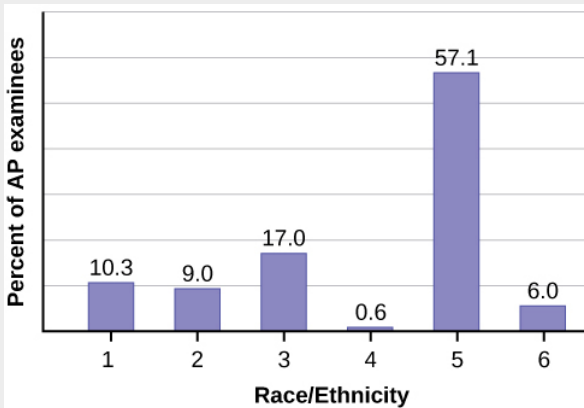
Problem:

The columns in [\[link\]](#) contain: the race or ethnicity of students in U.S. Public Schools for the class of 2011, percentages for the Advanced Placement examinee population for that class, and percentages for the overall student population. Create a bar graph with the student race or ethnicity (qualitative data) on the x -axis, and the Advanced Placement examinee population percentages on the y -axis.

Race/Ethnicity	AP Examinee Population	Overall Student Population
1 = Asian, Asian American or Pacific Islander	10.3%	5.7%
2 = Black or African American	9.0%	14.7%

Race/Ethnicity	AP Examinee Population	Overall Student Population
3 = Hispanic or Latino	17.0%	17.6%
4 = American Indian or Alaska Native	0.6%	1.1%
5 = White	57.1%	59.2%
6 = Not reported/other	6.0%	1.7%

Solution:



Note:

Try It

Exercise:

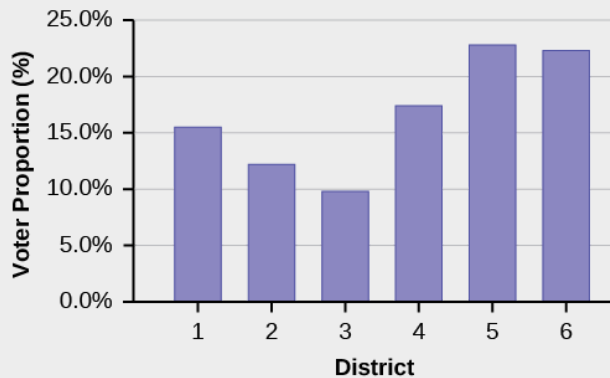
Problem:

Park city is broken down into six voting districts. The table shows the percent of the total registered voter population that lives in each district as well as the percent total of the entire population that lives in each district. Construct a bar graph that shows the registered voter population by district.

District	Registered voter population	Overall city population
1	15.5%	19.4%

District	Registered voter population	Overall city population
2	12.2%	15.6%
3	9.8%	9.0%
4	17.4%	18.5%
5	22.8%	20.7%
6	22.3%	16.8%

Solution:



References

Burbary, Ken. *Facebook Demographics Revisited – 2001 Statistics*, 2011. Available online at <http://www.kenburbary.com/2011/03/facebook-demographics-revisited-2011-statistics-2/> (accessed August 21, 2013).

“9th Annual AP Report to the Nation.” CollegeBoard, 2013. Available online at <http://apreport.collegeboard.org/goals-and-findings/promoting-equity> (accessed September 13, 2013).

“Overweight and Obesity: Adult Obesity Facts.” Centers for Disease Control and Prevention. Available online at <http://www.cdc.gov/obesity/data/adult.html> (accessed September 13, 2013).

Chapter Review

A **stem-and-leaf plot** is a way to plot data and look at the distribution. In a stem-and-leaf plot, all data values within a class are visible. The advantage in a stem-and-leaf plot is that all values are listed, unlike a histogram, which gives classes of data values. A **line graph** is often used to represent a set of data values in which a quantity varies with time. These graphs are useful for

finding trends. That is, finding a general pattern in data sets including temperature, sales, employment, company profit or cost over a period of time. A **bar graph** is a chart that uses either horizontal or vertical bars to show comparisons among categories. One axis of the chart shows the specific categories being compared, and the other axis represents a discrete value. Some bar graphs present bars clustered in groups of more than one (grouped bar graphs), and others show the bars divided into subparts to show cumulative effect (stacked bar graphs). Bar graphs are especially useful when categorical data is being used.

For each of the following data sets, create a stem plot and identify any outliers.

Exercise:

Problem:

The miles per gallon rating for 30 cars are shown below (lowest to highest).

19, 19, 19, 20, 21, 21, 25, 25, 25, 26, 26, 28, 29, 31, 31, 32, 32, 33, 34, 35, 36, 37, 37, 38, 38, 38, 38, 41, 43, 43

Solution:

Stem	Leaf
1	9 9 9
2	0 1 1 5 5 5 6 6 8 9
3	1 1 2 2 3 4 5 6 7 7 8 8 8 8
4	1 3 3

Exercise:

Problem:

The height in feet of 25 trees is shown below (lowest to highest).

25, 27, 33, 34, 34, 34, 35, 37, 37, 38, 39, 39, 39, 40, 41, 45, 46, 47, 49, 50, 50, 53, 53, 54, 54

Exercise:

Problem:

The data are the prices of different laptops at an electronics store. Round each value to the nearest ten.

249, 249, 260, 265, 265, 280, 299, 299, 309, 319, 325, 326, 350, 350, 350, 365, 369, 389, 409, 459, 489, 559, 569, 570, 610

Solution:

Stem	Leaf
2	5 5 6 7 7 8
3	0 0 1 2 3 3 5 5 5 7 7 9
4	1 6 9
5	6 7 7
6	1

Exercise:**Problem:**

The data are daily high temperatures in a town for one month.

61, 61, 62, 64, 66, 67, 67, 67, 68, 69, 70, 70, 70, 71, 71, 72, 74, 74, 74, 75, 75, 75, 76, 76, 77, 78, 78, 79, 79, 95

For the next three exercises, use the data to construct a line graph.

Exercise:**Problem:**

In a survey, 40 people were asked how many times they visited a store before making a major purchase. The results are shown in [\[link\]](#).

Number of times in store	Frequency
1	4
2	10
3	16

Number of times in store	Frequency
4	6
5	4

Solution:



Exercise:

Problem:

In a survey, several people were asked how many years it has been since they purchased a mattress. The results are shown in [\[link\]](#).

Years since last purchase	Frequency
0	2
1	8
2	13
3	22
4	16
5	9

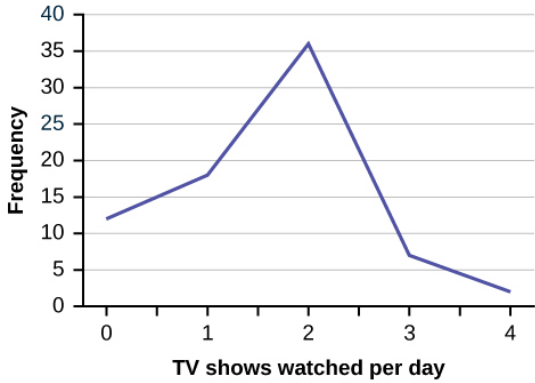
Exercise:

Problem:

Several children were asked how many TV shows they watch each day. The results of the survey are shown in [\[link\]](#).

Number of TV Shows	Frequency
0	12
1	18
2	36
3	7
4	2

Solution:



Exercise:

Problem:

The students in Ms. Ramirez’s math class have birthdays in each of the four seasons. [\[link\]](#) shows the four seasons, the number of students who have birthdays in each season, and the percentage (%) of students in each group. Construct a bar graph showing the number of students.

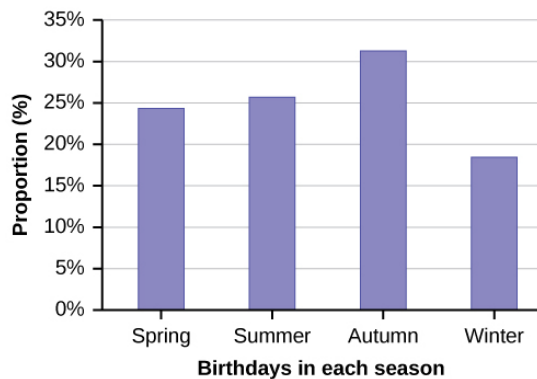
Seasons	Number of students	Proportion of population
Spring	8	24%
Summer	9	26%
Autumn	11	32%
Winter	6	18%

Exercise:

Problem:

Using the data from Mrs. Ramirez's math class supplied in [\[link\]](#), construct a bar graph showing the percentages.

Solution:



Exercise:

Problem:

David County has six high schools. Each school sent students to participate in a county-wide science competition. [\[link\]](#) shows the percentage breakdown of competitors from each school, and the percentage of the entire student population of the county that goes to each school. Construct a bar graph that shows the population percentage of competitors from each school.

High School	Science competition population	Overall student population
Alabaster	28.9%	8.6%
Concordia	7.6%	23.2%

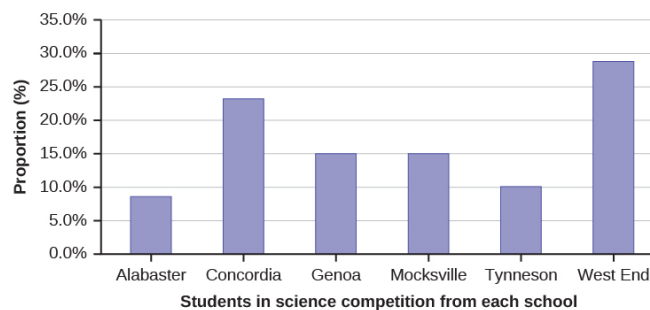
High School	Science competition population	Overall student population
Genoa	12.1%	15.0%
Mocksville	18.5%	14.3%
Tynneson	24.2%	10.1%
West End	8.7%	28.8%

Exercise:

Problem:

Use the data from the David County science competition supplied in [\[link\]](#). Construct a bar graph that shows the county-wide population percentage of students at each school.

Solution:



Homework

Exercise:

Problem: Student grades on a chemistry exam were: 77, 78, 76, 81, 86, 51, 79, 82, 84, 99

- Construct a stem-and-leaf plot of the data.
- Are there any potential outliers? If so, which scores are they? Why do you consider them outliers?

Exercise:

Problem: [\[link\]](#) contains the 2010 obesity rates in U.S. states and Washington, DC.

State	Percent (%)	State	Percent (%)	State	Percent (%)
Alabama	32.2	Kentucky	31.3	North Dakota	27.2
Alaska	24.5	Louisiana	31.0	Ohio	29.2
Arizona	24.3	Maine	26.8	Oklahoma	30.4
Arkansas	30.1	Maryland	27.1	Oregon	26.8
California	24.0	Massachusetts	23.0	Pennsylvania	28.6
Colorado	21.0	Michigan	30.9	Rhode Island	25.5
Connecticut	22.5	Minnesota	24.8	South Carolina	31.5
Delaware	28.0	Mississippi	34.0	South Dakota	27.3
Washington, DC	22.2	Missouri	30.5	Tennessee	30.8
Florida	26.6	Montana	23.0	Texas	31.0
Georgia	29.6	Nebraska	26.9	Utah	22.5
Hawaii	22.7	Nevada	22.4	Vermont	23.2
Idaho	26.5	New Hampshire	25.0	Virginia	26.0
Illinois	28.2	New Jersey	23.8	Washington	25.5
Indiana	29.6	New Mexico	25.1	West Virginia	32.5
Iowa	28.4	New York	23.9	Wisconsin	26.3
Kansas	29.4	North Carolina	27.8	Wyoming	25.1

- Use a random number generator to randomly pick eight states. Construct a bar graph of the obesity rates of those eight states.
- Construct a bar graph for all the states beginning with the letter "A."

c. Construct a bar graph for all the states beginning with the letter "M."

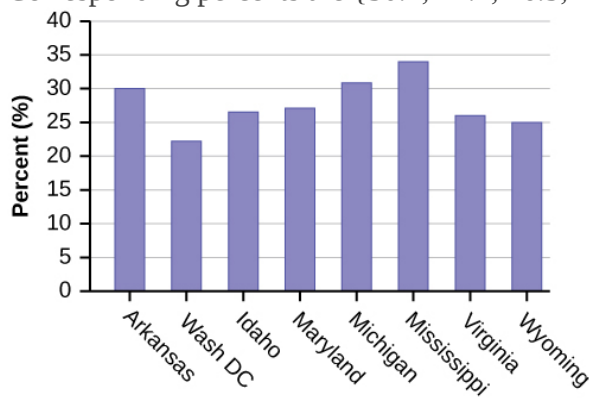
Solution:

a. Example solution for using the random number generator for the TI-84+ to generate a simple random sample of 8 states. Instructions are as follows.

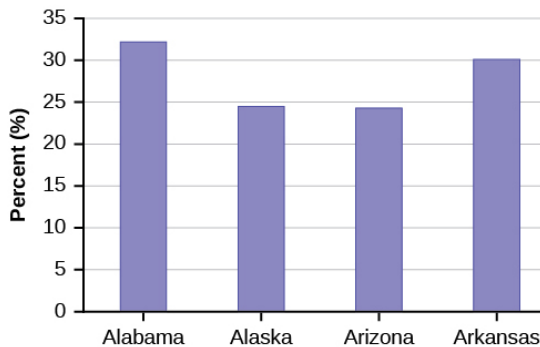
- Number the entries in the table 1–51 (Includes Washington, DC; Numbered vertically)
- Press MATH
- Arrow over to PRB
- Press 5:randInt(
- Enter 51,1,8)

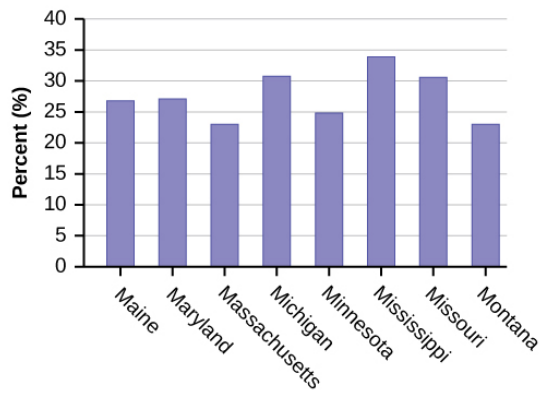
Eight numbers are generated (use the right arrow key to scroll through the numbers). The numbers correspond to the numbered states (for this example: {47 21 9 23 51 13 25 4}). If any numbers are repeated, generate a different number by using 5:randInt(51,1)). Here, the states (and Washington DC) are {Arkansas, Washington DC, Idaho, Maryland, Michigan, Mississippi, Virginia, Wyoming}.

Corresponding percents are {30.1, 22.2, 26.5, 27.1, 30.9, 34.0, 26.0, 25.1}.



b.





C.

Histograms, Frequency Polygons, and Time Series Graphs

For most of the work you do in this book, you will use a histogram to display the data. One advantage of a histogram is that it can readily display large data sets. A rule of thumb is to use a histogram when the data set consists of 100 values or more.

A **histogram** consists of contiguous (adjoining) boxes. It has both a horizontal axis and a vertical axis. The horizontal axis is labeled with what the data represents (for instance, distance from your home to school). The vertical axis is labeled either **frequency** or **relative frequency** (or percent frequency or probability). The graph will have the same shape with either label. The histogram (like the stemplot) can give you the shape of the data, the center, and the spread of the data.

The relative frequency is equal to the frequency for an observed value of the data divided by the total number of data values in the sample. (Remember, frequency is defined as the number of times an answer occurs.) If:

- f = frequency
- n = total number of data values (or the sum of the individual frequencies), and
- RF = relative frequency,

then:

Equation:

$$RF = \frac{f}{n}$$

For example, if three students in Mr. Ahab's English class of 40 students received from 90% to 100%, then, $f = 3$, $n = 40$, and $RF = \frac{f}{n} = \frac{3}{40} = 0.075$. 7.5% of the students received 90–100%. 90–100% are quantitative measures.

To construct a histogram, first decide how many **bars** or **intervals**, also called classes, represent the data. Many histograms consist of five to 15 bars or classes for clarity. The number of bars needs to be chosen. Choose a starting point for the first interval to be less than the smallest data value. A **convenient starting point** is a lower value carried out to one more decimal place than the value with the most decimal places. For example, if the value with the most decimal places is 6.1 and this is the smallest value, a convenient starting point is 6.05 ($6.1 - 0.05 = 6.05$). We say that 6.05 has more precision. If the value with the most decimal places is 2.23 and the lowest value is 1.5, a convenient starting point is 1.495 ($1.5 - 0.005 = 1.495$). If the value with the most decimal places is 3.234 and the lowest value is 1.0, a convenient starting point is 0.9995 ($1.0 - 0.0005 = 0.9995$). If all the data happen to be integers and the smallest value is two, then a convenient starting point is 1.5 ($2 - 0.5 = 1.5$). Also, when the starting point and other boundaries are carried to one additional decimal place, no data value will fall on a boundary. The next two examples go into detail about how to construct a histogram using continuous data and how to create a histogram using discrete data.

Example:

The following data are the heights (in inches to the nearest half inch) of 100 male semiprofessional soccer players. The heights are **continuous** data, since height is measured.

60; 60.5; 61; 61; 61.5

63.5; 63.5; 63.5

64; 64; 64; 64; 64; 64; 64; 64; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5

66; 66; 66; 66; 66; 66; 66; 66; 66; 66; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 67; 67; 67;

67; 67; 67; 67; 67; 67; 67; 67; 67; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5

68; 68; 69; 69; 69; 69; 69; 69; 69; 69; 69; 69; 69.5; 69.5; 69.5; 69.5; 69.5

70; 70; 70; 70; 70; 70; 70.5; 70.5; 71; 71; 71

72; 72; 72; 72.5; 72.5; 73; 73.5

74

The smallest data value is 60. Since the data with the most decimal places has one decimal (for instance, 61.5), we want our starting point to have two decimal places. Since the numbers 0.5, 0.05, 0.005, etc. are convenient numbers, use 0.05 and subtract it from 60, the smallest value, for the convenient starting point.

$60 - 0.05 = 59.95$ which is more precise than, say, 61.5 by one decimal place. The starting point is, then, 59.95.

The largest value is 74, so $74 + 0.05 = 74.05$ is the ending value.

Next, calculate the width of each bar or class interval. To calculate this width, subtract the starting point from the ending value and divide by the number of bars (you must choose the number of bars you desire). Suppose you choose eight bars.

Equation:

$$\frac{74.05 - 59.95}{8} = 1.76$$

Note:

NOTE

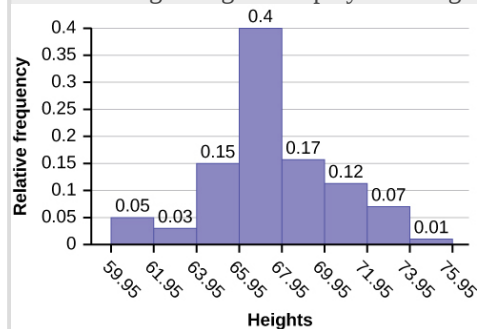
We will round up to two and make each bar or class interval two units wide. Rounding up to two is one way to prevent a value from falling on a boundary. Rounding to the next number is often necessary even if it goes against the standard rules of rounding. For this example, using 1.76 as the width would also work. A guideline that is followed by some for the number of bars or class intervals is to take the square root of the number of data values and then round to the nearest whole number, if necessary. For example, if there are 150 values of data, take the square root of 150 and round to 12 bars or intervals.

The boundaries are:

- 59.95
- $59.95 + 2 = 61.95$
- $61.95 + 2 = 63.95$
- $63.95 + 2 = 65.95$
- $65.95 + 2 = 67.95$
- $67.95 + 2 = 69.95$
- $69.95 + 2 = 71.95$
- $71.95 + 2 = 73.95$
- $73.95 + 2 = 75.95$

The heights 60 through 61.5 inches are in the interval 59.95–61.95. The heights that are 63.5 are in the interval 61.95–63.95. The heights that are 64 through 64.5 are in the interval 63.95–65.95. The heights 66 through 67.5 are in the interval 65.95–67.95. The heights 68 through 69.5 are in the interval 67.95–69.95. The heights 70 through 71 are in the interval 69.95–71.95. The heights 72 through 73.5 are in the interval 71.95–73.95. The height 74 is in the interval 73.95–75.95.

The following histogram displays the heights on the x-axis and relative frequency on the y-axis.



Note:

Try It

Exercise:**Problem:**

The following data are the shoe sizes of 50 male students. The sizes are continuous data since shoe size is measured. Construct a histogram and calculate the width of each bar or class interval. Suppose you choose six bars.

9; 9; 9.5; 9.5; 10; 10; 10; 10; 10; 10; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5
11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5
12; 12; 12; 12; 12; 12; 12.5; 12.5; 12.5; 12.5; 14

Solution:

Smallest value: 9

Largest value: 14

Convenient starting value: $9 - 0.05 = 8.95$

Convenient ending value: $14 + 0.05 = 14.05$

$$\frac{14.05 - 8.95}{6} = 0.85$$

The calculations suggests using 0.85 as the width of each bar or class interval. You can also use an interval with a width equal to one.

Example:

Create a histogram for the following data: the number of books bought by 50 part-time college students at ABC College. the number of books bought by 50 part-time college students at ABC College. The number of books is **discrete data**, since books are counted.

1; 1; 1; 1; 1; 1; 1; 1; 1; 1
2; 2; 2; 2; 2; 2; 2; 2; 2; 2
3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3
4; 4; 4; 4; 4; 4
5; 5; 5; 5; 5
6; 6

Eleven students buy one book. Ten students buy two books. Sixteen students buy three books. Six students buy four books. Five students buy five books. Two students buy six books.

Because the data are integers, subtract 0.5 from 1, the smallest data value and add 0.5 to 6, the largest data value. Then the starting point is 0.5 and the ending value is 6.5.

Exercise:**Problem:**

Next, calculate the width of each bar or class interval. If the data are discrete and there are not too many different values, a width that places the data values in the middle of the bar or class interval is the most convenient. Since the data consist of the numbers 1, 2, 3, 4, 5, 6, and the starting point is 0.5, a width of one places the 1 in the middle of the interval from 0.5 to 1.5, the 2 in the middle of the interval from 1.5 to 2.5, the 3 in the middle of the interval from 2.5 to 3.5, the 4 in the middle of the interval from _____ to _____, the 5 in the middle of the interval from _____ to _____, and the _____ in the middle of the interval from _____ to _____.

Solution:

1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1
2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2

3; 3; 3; 3; 3; 3; 3; 3

20 student athletes play one sport. 22 student athletes play two sports. Eight student athletes play three sports.

Fill in the blanks for the following sentence. Since the data consist of the numbers 1, 2, 3, and the starting point is 0.5, a width of one places the 1 in the middle of the interval 0.5 to _____, the 2 in the middle of the interval from _____ to _____, and the 3 in the middle of the interval from _____ to _____.

Solution:

1.5

1.5 to 2.5

2.5 to 3.5

Example:

Exercise:

Problem: Using this data set, construct a histogram.

Number of Hours My Classmates Spent Playing Video Games on Weekends

9.95	10	2.25	16.75	0
19.5	22.5	7.5	15	12.75
5.5	11	10	20.75	17.5
23	21.9	24	23.75	18
20	15	22.9	18.8	20.5

Solution:



Some values in this data set fall on boundaries for the class intervals. A value is counted in a class interval if it falls on the left boundary, but not if it falls on the right boundary. Different researchers may set up histograms for the same data in different ways. There is more than one correct way to set up a histogram.

Note:

Try It

Exercise:

Problem:

The following data represent the number of employees at various restaurants in New York City. Using this data, create a histogram.

22351526 40281820 25343942 24221927 22344020 38and 28

Use 10–19 as the first interval.

Note:

Count the money (bills and change) in your pocket or purse. Your instructor will record the amounts. As a class, construct a histogram displaying the data. Discuss how many intervals you think is appropriate. You may want to experiment with the number of intervals.

Frequency Polygons

Frequency polygons are analogous to line graphs, and just as line graphs make continuous data visually easy to interpret, so too do frequency polygons.

To construct a frequency polygon, first examine the data and decide on the number of intervals, or class intervals, to use on the x -axis and y -axis. After choosing the appropriate ranges, begin plotting the data points. After all the points are plotted, draw line segments to connect them.

Example:

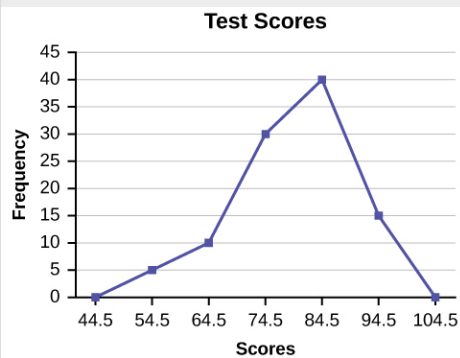
A frequency polygon was constructed from the frequency table below.

Frequency Distribution for Calculus Final Test Scores

Lower Bound	Upper Bound	Frequency	Cumulative Frequency
49.5	59.5	5	5
59.5	69.5	10	15
69.5	79.5	30	45

Frequency Distribution for Calculus Final Test Scores

Lower Bound	Upper Bound	Frequency	Cumulative Frequency
79.5	89.5	40	85
89.5	99.5	15	100



The first label on the x -axis is 44.5. This represents an interval extending from 39.5 to 49.5. Since the lowest test score is 54.5, this interval is used only to allow the graph to touch the x -axis. The point labeled 54.5 represents the next interval, or the first “real” interval from the table, and contains five scores. This reasoning is followed for each of the remaining intervals with the point 104.5 representing the interval from 99.5 to 109.5. Again, this interval contains no data and is only used so that the graph will touch the x -axis. Looking at the graph, we say that this distribution is skewed because one side of the graph does not mirror the other side.

Note:

Try It

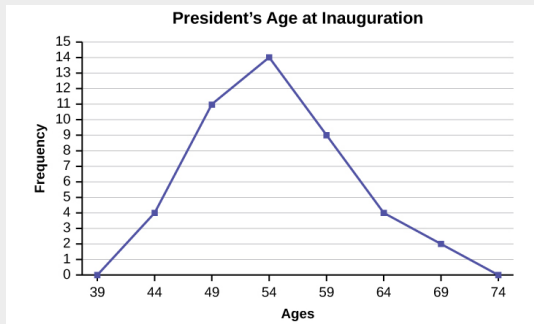
Exercise:

Problem: Construct a frequency polygon of U.S. Presidents’ ages at inauguration shown in [\[link\]](#).

Age at Inauguration	Frequency
41.5–46.5	4
46.5–51.5	11
51.5–56.5	14
56.5–61.5	9
61.5–66.5	4
66.5–71.5	2

Solution:

The first label on the x -axis is 39. This represents an interval extending from 36.5 to 41.5. Since there are no ages less than 41.5, this interval is used only to allow the graph to touch the x -axis. The point labeled 44 represents the next interval, or the first “real” interval from the table, and contains four scores. This reasoning is followed for each of the remaining intervals with the point 74 representing the interval from 71.5 to 76.5. Again, this interval contains no data and is only used so that the graph will touch the x -axis. Looking at the graph, we say that this distribution is skewed because one side of the graph does not mirror the other side.



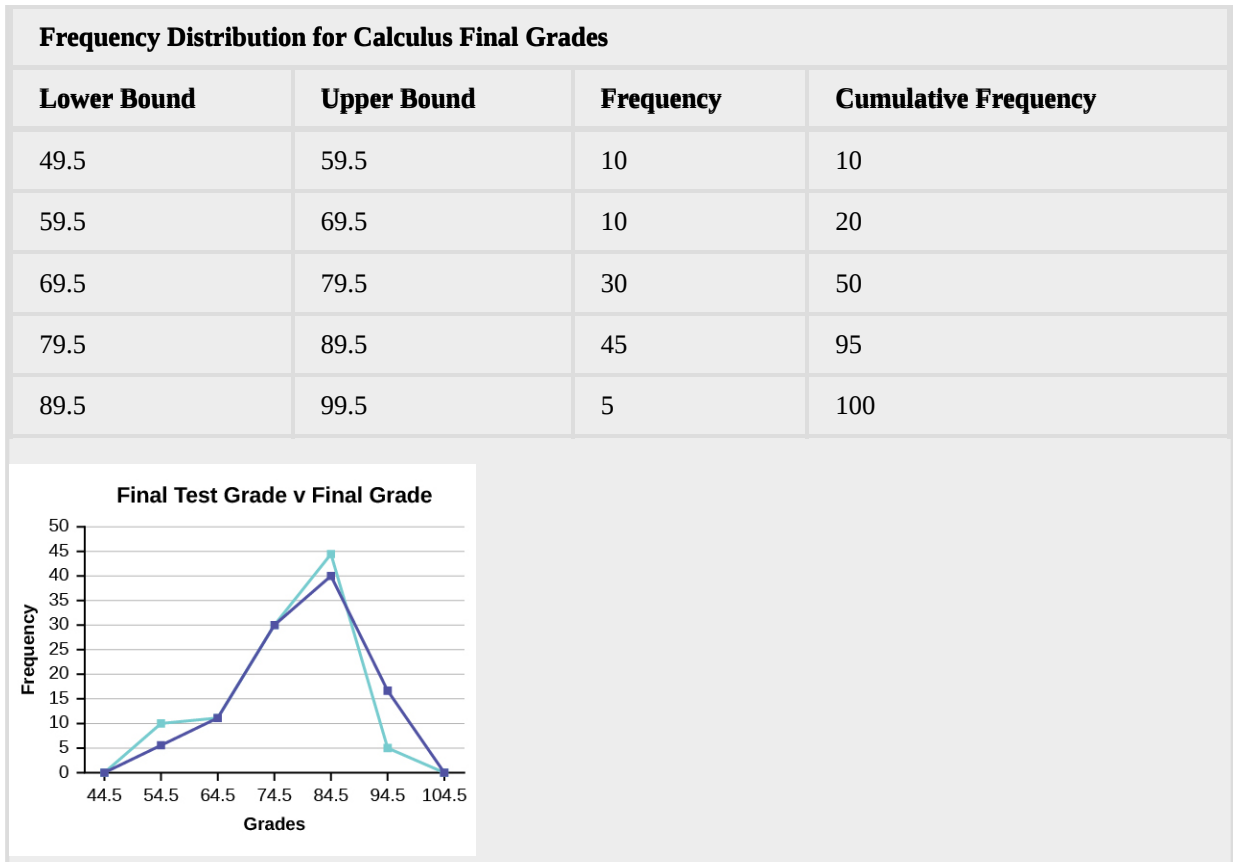
Frequency polygons are useful for comparing distributions. This is achieved by overlaying the frequency polygons drawn for different data sets.

Example:

We will construct an overlay frequency polygon comparing the scores from [\[link\]](#) with the students' final numeric grade.

Frequency Distribution for Calculus Final Test Scores

Lower Bound	Upper Bound	Frequency	Cumulative Frequency
49.5	59.5	5	5
59.5	69.5	10	15
69.5	79.5	30	45
79.5	89.5	40	85
89.5	99.5	15	100



Suppose that we want to study the temperature range of a region for an entire month. Every day at noon we note the temperature and write this down in a log. A variety of statistical studies could be done with this data. We could find the mean or the median temperature for the month. We could construct a histogram displaying the number of days that temperatures reach a certain range of values. However, all of these methods ignore a portion of the data that we have collected.

One feature of the data that we may want to consider is that of time. Since each date is paired with the temperature reading for the day, we don't have to think of the data as being random. We can instead use the times given to impose a chronological order on the data. A graph that recognizes this ordering and displays the changing temperature as the month progresses is called a time series graph.

Constructing a Time Series Graph

To construct a time series graph, we must look at both pieces of our **paired data set**. We start with a standard Cartesian coordinate system. The horizontal axis is used to plot the date or time increments, and the vertical axis is used to plot the values of the variable that we are measuring. By doing this, we make each point on the graph correspond to a date and a measured quantity. The points on the graph are typically connected by straight lines in the order in which they occur.

Example:

Exercise:

Problem:

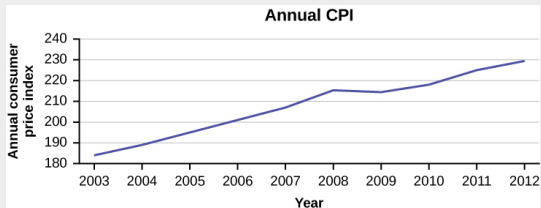
The following data shows the Annual Consumer Price Index, each month, for ten years. Construct a time series graph for the Annual Consumer Price Index data only.

Year	Jan	Feb	Mar	Apr	May	Jun	Jul
2003	181.7	183.1	184.2	183.8	183.5	183.7	183.9
2004	185.2	186.2	187.4	188.0	189.1	189.7	189.4
2005	190.7	191.8	193.3	194.6	194.4	194.5	195.4
2006	198.3	198.7	199.8	201.5	202.5	202.9	203.5
2007	202.416	203.499	205.352	206.686	207.949	208.352	208.299
2008	211.080	211.693	213.528	214.823	216.632	218.815	219.964
2009	211.143	212.193	212.709	213.240	213.856	215.693	215.351
2010	216.687	216.741	217.631	218.009	218.178	217.965	218.011
2011	220.223	221.309	223.467	224.906	225.964	225.722	225.922
2012	226.665	227.663	229.392	230.085	229.815	229.478	229.104

Year	Aug	Sep	Oct	Nov	Dec	Annual
2003	184.6	185.2	185.0	184.5	184.3	184.0
2004	189.5	189.9	190.9	191.0	190.3	188.9
2005	196.4	198.8	199.2	197.6	196.8	195.3
2006	203.9	202.9	201.8	201.5	201.8	201.6
2007	207.917	208.490	208.936	210.177	210.036	207.342
2008	219.086	218.783	216.573	212.425	210.228	215.303
2009	215.834	215.969	216.177	216.330	215.949	214.537
2010	218.312	218.439	218.711	218.803	219.179	218.056

Year	Aug	Sep	Oct	Nov	Dec	Annual
2011	226.545	226.889	226.421	226.230	225.672	224.939
2012	230.379	231.407	231.317	230.221	229.601	229.594

Solution:



Note:

Try It

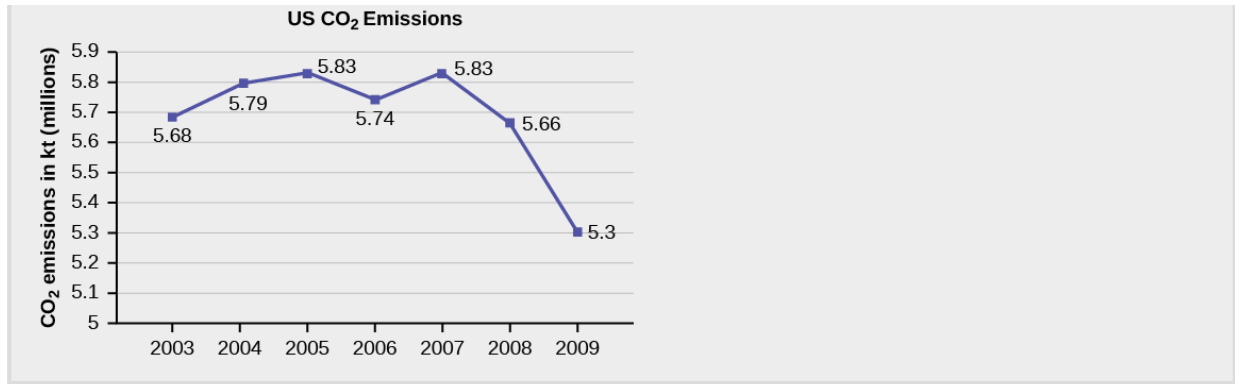
Exercise:

Problem:

The following table is a portion of a data set from www.worldbank.org. Use the table to construct a time series graph for CO₂ emissions for the United States.

CO2 Emissions			
	Ukraine	United Kingdom	United States
2003	352,259	540,640	5,681,664
2004	343,121	540,409	5,790,761
2005	339,029	541,990	5,826,394
2006	327,797	542,045	5,737,615
2007	328,357	528,631	5,828,697
2008	323,657	522,247	5,656,839
2009	272,176	474,579	5,299,563

Solution:



Uses of a Time Series Graph

Time series graphs are important tools in various applications of statistics. When recording values of the same variable over an extended period of time, sometimes it is difficult to discern any trend or pattern. However, once the same data points are displayed graphically, some features jump out. Time series graphs make trends easy to spot.

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Chapter Review

A **histogram** is a graphic version of a frequency distribution. The graph consists of bars of equal width drawn adjacent to each other. The horizontal scale represents classes of quantitative data values and the vertical scale represents frequencies. The heights of the bars correspond to frequency values. Histograms are typically used for large, continuous, quantitative data sets. A frequency polygon can also be used when graphing large data sets with data points that repeat. The data usually goes on *y*-axis with the frequency being graphed on the *x*-axis. Time series graphs can be helpful when looking at large amounts of data for one variable over a period of time.

Exercise:

Problem:

Sixty-five randomly selected car salespersons were asked the number of cars they generally sell in one week. Fourteen people answered that they generally sell three cars; nineteen generally sell four cars; twelve generally sell five cars; nine generally sell six cars; eleven generally sell seven cars. Complete the table.

Data Value (# cars)	Frequency	Relative Frequency	Cumulative Relative Frequency

Exercise:

Problem: What does the frequency column in [\[link\]](#) sum to? Why?

Solution:

65

Exercise:

Problem: What does the relative frequency column in [\[link\]](#) sum to? Why?

Exercise:

Problem: What is the difference between relative frequency and frequency for each data value in [\[link\]](#)?

Solution:

The relative frequency shows the *proportion* of data points that have each value. The frequency tells the *number* of data points that have each value.

Exercise:

Problem:

What is the difference between cumulative relative frequency and relative frequency for each data value?

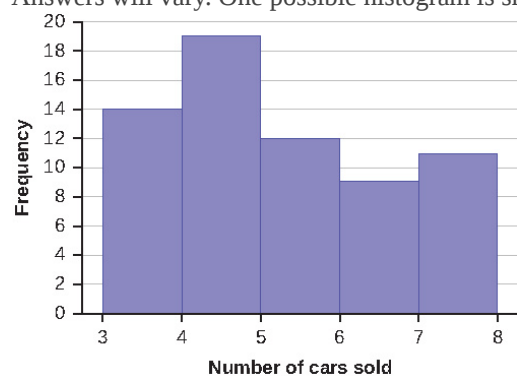
Exercise:

Problem:

To construct the histogram for the data in [\[link\]](#), determine appropriate minimum and maximum x and y values and the scaling. Sketch the histogram. Label the horizontal and vertical axes with words. Include numerical scaling.

**Solution:**

Answers will vary. One possible histogram is shown:

**Exercise:**

Problem: Construct a frequency polygon for the following:

a.	Pulse Rates for Women	Frequency
	60–69	12
	70–79	14
	80–89	11
	90–99	1
	100–109	1
	110–119	0
	120–129	1

b.

Actual Speed in a 30 MPH Zone	Frequency
42–45	25
46–49	14
50–53	7
54–57	3
58–61	1

c.

Tar (mg) in Nonfiltered Cigarettes	Frequency
10–13	1
14–17	0
18–21	15
22–25	7
26–29	2

Exercise:

Problem:

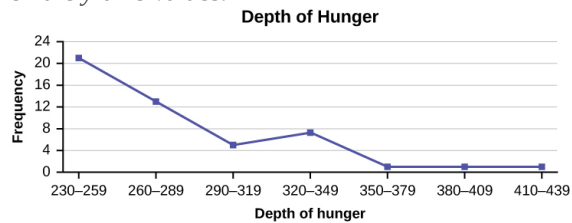
Construct a frequency polygon from the frequency distribution for the 50 highest ranked countries for depth of hunger.

Depth of Hunger	Frequency
230–259	21
260–289	13
290–319	5
320–349	7
350–379	1
380–409	1

Depth of Hunger	Frequency
410–439	1

Solution:

Find the midpoint for each class. These will be graphed on the x -axis. The frequency values will be graphed on the y -axis values.



Exercise:

Problem:

Use the two frequency tables to compare the life expectancy of men and women from 20 randomly selected countries. Include an overlaid frequency polygon and discuss the shapes of the distributions, the center, the spread, and any outliers. What can we conclude about the life expectancy of women compared to men?

Life Expectancy at Birth – Women	Frequency
49–55	3
56–62	3
63–69	1
70–76	3
77–83	8
84–90	2

Life Expectancy at Birth – Men	Frequency
49–55	3
56–62	3

Life Expectancy at Birth – Men	Frequency
63–69	1
70–76	1
77–83	7
84–90	5

Exercise:

Problem:

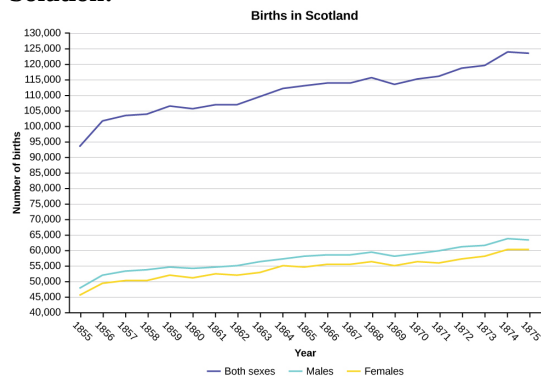
Construct a times series graph for (a) the number of male births, (b) the number of female births, and (c) the total number of births.

Sex/Year	1855	1856	1857	1858	1859	1860	1861
Female	45,545	49,582	50,257	50,324	51,915	51,220	52,403
Male	47,804	52,239	53,158	53,694	54,628	54,409	54,606
Total	93,349	101,821	103,415	104,018	106,543	105,629	107,009

Sex/Year	1862	1863	1864	1865	1866	1867	1868	1869
Female	51,812	53,115	54,959	54,850	55,307	55,527	56,292	55,514
Male	55,257	56,226	57,374	58,220	58,360	58,517	59,222	58,514
Total	107,069	109,341	112,333	113,070	113,667	114,044	115,514	114,028

Sex/Year	1870	1871	1872	1873	1874	1875
Female	56,431	56,099	57,472	58,233	60,109	60,146
Male	58,959	60,029	61,293	61,467	63,602	63,432
Total	115,390	116,128	118,765	119,700	123,711	123,578

Solution:



Exercise:

Problem:

The following data sets list full time police per 100,000 citizens along with homicides per 100,000 citizens for the city of Detroit, Michigan during the period from 1961 to 1973.

Year	1961	1962	1963	1964	1965	1966	1967
Police	260.35	269.8	272.04	272.96	272.51	261.34	268.89
Homicides	8.6	8.9	8.52	8.89	13.07	14.57	21.36

Year	1968	1969	1970	1971	1972	1973
Police	295.99	319.87	341.43	356.59	376.69	390.19
Homicides	28.03	31.49	37.39	46.26	47.24	52.33

- Construct a double time series graph using a common x -axis for both sets of data.
- Which variable increased the fastest? Explain.
- Did Detroit's increase in police officers have an impact on the murder rate? Explain.

Homework

Exercise:

Problem:

Suppose that three book publishers were interested in the number of fiction paperbacks adult consumers purchase per month. Each publisher conducted a survey. In the survey, adult consumers were asked the number of fiction paperbacks they had purchased the previous month. The results are as follows:

# of books	Freq.	Rel. Freq.
0	10	
1	12	
2	16	
3	12	
4	8	
5	6	
6	2	
8	2	

Publisher A

# of books	Freq.	Rel. Freq.
0	18	
1	24	
2	24	
3	22	
4	15	
5	10	
7	5	
9	1	

Publisher B

# of books	Freq.	Rel. Freq.
0–1	20	
2–3	35	
4–5	12	
6–7	2	
8–9	1	

Publisher C

- Find the relative frequencies for each survey. Write them in the charts.
- Using either a graphing calculator, computer, or by hand, use the frequency column to construct a histogram for each publisher's survey. For Publishers A and B, make bar widths of one. For Publisher C, make bar widths of two.
- In complete sentences, give two reasons why the graphs for Publishers A and B are not identical.
- Would you have expected the graph for Publisher C to look like the other two graphs? Why or why not?
- Make new histograms for Publisher A and Publisher B. This time, make bar widths of two.
- Now, compare the graph for Publisher C to the new graphs for Publishers A and B. Are the graphs more similar or more different? Explain your answer.

Exercise:

Problem:

Often, cruise ships conduct all on-board transactions, with the exception of gambling, on a cashless basis. At the end of the cruise, guests pay one bill that covers all onboard transactions. Suppose that 60 single travelers and 70 couples were surveyed as to their on-board bills for a seven-day cruise from Los Angeles to the Mexican Riviera. Following is a summary of the bills for each group.

Amount(\$)	Frequency	Rel. Frequency
51–100	5	
101–150	10	
151–200	15	
201–250	15	
251–300	10	
301–350	5	

Singles

Amount(\$)	Frequency	Rel. Frequency
100–150	5	
201–250	5	
251–300	5	
301–350	5	
351–400	10	
401–450	10	
451–500	10	
501–550	10	
551–600	5	
601–650	5	

Couples

- Fill in the relative frequency for each group.
- Construct a histogram for the singles group. Scale the x -axis by \$50 widths. Use relative frequency on the y -axis.
- Construct a histogram for the couples group. Scale the x -axis by \$50 widths. Use relative frequency on the y -axis.
- Compare the two graphs:
 - List two similarities between the graphs.
 - List two differences between the graphs.
 - Overall, are the graphs more similar or different?
- Construct a new graph for the couples by hand. Since each couple is paying for two individuals, instead of scaling the x -axis by \$50, scale it by \$100. Use relative frequency on the y -axis.
- Compare the graph for the singles with the new graph for the couples:
 - List two similarities between the graphs.
 - Overall, are the graphs more similar or different?
- How did scaling the couples graph differently change the way you compared it to the singles graph?
- Based on the graphs, do you think that individuals spend the same amount, more or less, as singles as they do person by person as a couple? Explain why in one or two complete sentences.

Solution:

Amount(\$)	Frequency	Relative Frequency
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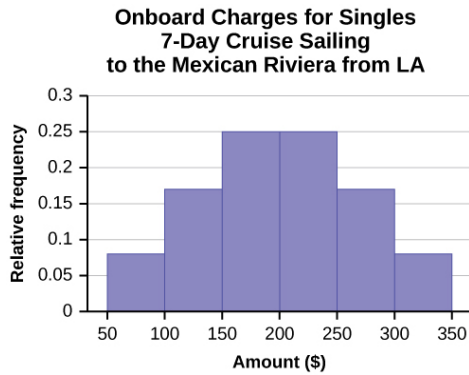
Amount(\$)	Frequency	Relative Frequency
51–100	5	0.08
101–150	10	0.17
151–200	15	0.25
201–250	15	0.25
251–300	10	0.17
301–350	5	0.08

Singles

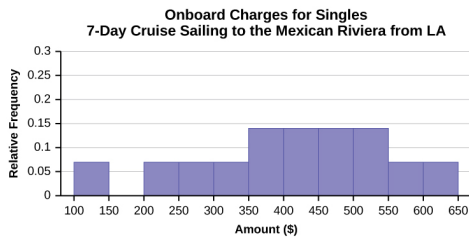
Amount(\$)	Frequency	Relative Frequency
100–150	5	0.07
201–250	5	0.07
251–300	5	0.07
301–350	5	0.07
351–400	10	0.14
401–450	10	0.14
451–500	10	0.14
501–550	10	0.14
551–600	5	0.07
601–650	5	0.07

Couples

- See [\[link\]](#) and [\[link\]](#).
- In the following histogram data values that fall on the right boundary are counted in the class interval, while values that fall on the left boundary are not counted (with the exception of the first interval where both boundary values are included).



- c. In the following histogram, the data values that fall on the right boundary are counted in the class interval, while values that fall on the left boundary are not counted (with the exception of the first interval where values on both boundaries are included).



- d. Compare the two graphs:

- i. Answers may vary. Possible answers include:

- Both graphs have a single peak.
- Both graphs use class intervals with width equal to \$50.

- ii. Answers may vary. Possible answers include:

- The couples graph has a class interval with no values.
- It takes almost twice as many class intervals to display the data for couples.

- iii. Answers may vary. Possible answers include: The graphs are more similar than different because the overall patterns for the graphs are the same.

- e. Check student's solution.

- f. Compare the graph for the Singles with the new graph for the Couples:

- i.
 - Both graphs have a single peak.
 - Both graphs display 6 class intervals.
 - Both graphs show the same general pattern.

- ii. Answers may vary. Possible answers include: Although the width of the class intervals for couples is double that of the class intervals for singles, the graphs are more similar than they are different.

- g. Answers may vary. Possible answers include: You are able to compare the graphs interval by interval. It is easier to compare the overall patterns with the new scale on the Couples graph. Because a couple represents two individuals, the new scale leads to a more accurate comparison.

- h. Answers may vary. Possible answers include: Based on the histograms, it seems that spending does not vary much from singles to individuals who are part of a couple. The overall patterns are the same. The range of spending for couples is approximately double the range for individuals.

Exercise:

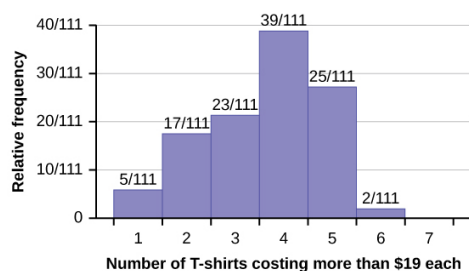
Problem:

Twenty-five randomly selected students were asked the number of movies they watched the previous week. The results are as follows.

# of movies	Frequency	Relative Frequency	Cumulative Relative Frequency
0	5		
1	9		
2	6		
3	4		
4	1		

- Construct a histogram of the data.
- Complete the columns of the chart.

Use the following information to answer the next two exercises: Suppose one hundred eleven people who shopped in a special t-shirt store were asked the number of t-shirts they own costing more than \$19 each.

**Exercise:****Problem:**

The percentage of people who own at most three t-shirts costing more than \$19 each is approximately:

- 21
- 59
- 41
- Cannot be determined

Solution:

c

Exercise:

Problem:

If the data were collected by asking the first 111 people who entered the store, then the type of sampling is:

- a. cluster
- b. simple random
- c. stratified
- d. convenience

Exercise:

Problem: Following are the 2010 obesity rates by U.S. states and Washington, DC.

State	Percent (%)	State	Percent (%)	State	Percent (%)
Alabama	32.2	Kentucky	31.3	North Dakota	27.2
Alaska	24.5	Louisiana	31.0	Ohio	29.2
Arizona	24.3	Maine	26.8	Oklahoma	30.4
Arkansas	30.1	Maryland	27.1	Oregon	26.8
California	24.0	Massachusetts	23.0	Pennsylvania	28.6
Colorado	21.0	Michigan	30.9	Rhode Island	25.5
Connecticut	22.5	Minnesota	24.8	South Carolina	31.5
Delaware	28.0	Mississippi	34.0	South Dakota	27.3
Washington, DC	22.2	Missouri	30.5	Tennessee	30.8
Florida	26.6	Montana	23.0	Texas	31.0
Georgia	29.6	Nebraska	26.9	Utah	22.5
Hawaii	22.7	Nevada	22.4	Vermont	23.2
Idaho	26.5	New Hampshire	25.0	Virginia	26.0
Illinois	28.2	New Jersey	23.8	Washington	25.5

State	Percent (%)	State	Percent (%)	State	Percent (%)
Indiana	29.6	New Mexico	25.1	West Virginia	32.5
Iowa	28.4	New York	23.9	Wisconsin	26.3
Kansas	29.4	North Carolina	27.8	Wyoming	25.1

Construct a bar graph of obesity rates of your state and the four states closest to your state. Hint: Label the x -axis with the states.

Solution:

Answers will vary.

Glossary

Frequency

the number of times a value of the data occurs

Histogram

a graphical representation in x - y form of the distribution of data in a data set; x represents the data and y represents the frequency, or relative frequency. The graph consists of contiguous rectangles.

Relative Frequency

the ratio of the number of times a value of the data occurs in the set of all outcomes to the number of all outcomes

Measures of the Location of the Data

The common measures of location are **quartiles** and **percentiles**

Quartiles are special percentiles. The first quartile, Q_1 , is the same as the 25th percentile, and the third quartile, Q_3 , is the same as the 75th percentile. The median, M , is called both the second quartile and the 50th percentile.

To calculate quartiles and percentiles, the data must be ordered from smallest to largest. Quartiles divide ordered data into quarters. Percentiles divide ordered data into hundredths. To score in the 90th percentile of an exam does not mean, necessarily, that you received 90% on a test. It means that 90% of test scores are the same or less than your score and 10% of the test scores are the same or greater than your test score.

Percentiles are useful for comparing values. For this reason, universities and colleges use percentiles extensively. One instance in which colleges and universities use percentiles is when SAT results are used to determine a minimum testing score that will be used as an acceptance factor. For example, suppose Duke accepts SAT scores at or above the 75th percentile. That translates into a score of at least 1220.

Percentiles are mostly used with very large populations. Therefore, if you were to say that 90% of the test scores are less (and not the same or less) than your score, it would be acceptable because removing one particular data value is not significant.

The **median** is a number that measures the "center" of the data. You can think of the median as the "middle value," but it does not actually have to be one of the observed values. It is a number that separates ordered data into halves. Half the values are the same number or smaller than the median, and half the values are the same number or larger. For example, consider the following data.

1; 11.5; 6; 7.2; 4; 8; 9; 10; 6.8; 8.3; 2; 2; 10; 1

Ordered from smallest to largest:

1; 1; 2; 2; 4; 6; 6.8; 7.2; 8; 8.3; 9; 10; 10; 11.5

Since there are 14 observations, the median is between the seventh value, 6.8, and the eighth value, 7.2. To find the median, add the two values together and divide by two.

Equation:

$$\frac{6.8 + 7.2}{2} = 7$$

The median is seven. Half of the values are smaller than seven and half of the values are larger than seven.

Quartiles are numbers that separate the data into quarters. Quartiles may or may not be part of the data. To find the quartiles, first find the median or second quartile. The first quartile, Q_1 , is the middle value of the lower half of the data, and the third quartile, Q_3 , is the middle value, or median, of the upper half of the data. To get the idea, consider the same data set:

1; 1; 2; 2; 4; 6; 6.8; 7.2; 8; 8.3; 9; 10; 10; 11.5

The median or **second quartile** is seven. The lower half of the data are 1, 1, 2, 2, 4, 6, 6.8. The middle value of the lower half is two.

1; 1; 2; 2; 4; 6; 6.8

The number two, which is part of the data, is the **first quartile**. One-fourth of the entire sets of values are the same as or less than two and three-fourths of the values are more than two.

The upper half of the data is 7.2, 8, 8.3, 9, 10, 10, 11.5. The middle value of the upper half is nine.

The **third quartile**, Q_3 , is nine. Three-fourths (75%) of the ordered data set are less than nine. One-fourth (25%) of the ordered data set are greater than nine. The third quartile is part of the data set in this example.

The **interquartile range** is a number that indicates the spread of the middle half or the middle 50% of the data. It is the difference between the third quartile (Q_3) and the first quartile (Q_1).

$$IQR = Q_3 - Q_1$$

The *IQR* can help to determine potential **outliers**. A value is suspected to be a **potential outlier if it is less than $(1.5)(IQR)$ below the first quartile or more than $(1.5)(IQR)$ above the third quartile**. Potential outliers always require further investigation.

Note:**NOTE**

A potential outlier is a data point that is significantly different from the other data points. These special data points may be errors or some kind of abnormality or they may be a key to understanding the data.

Example:**Exercise:****Problem:**

For the following 13 real estate prices, calculate the *IQR* and determine if any prices are potential outliers. Prices are in dollars.

389,950; 230,500; 158,000; 479,000; 639,000; 114,950; 5,500,000;
387,000; 659,000; 529,000; 575,000; 488,800; 1,095,000

Solution:

Order the data from smallest to largest.

114,950; 158,000; 230,500; 387,000; 389,950; 479,000; 488,800; 529,000;
575,000; 639,000; 659,000; 1,095,000; 5,500,000

$$M = 488,800$$

$$Q_1 = \frac{230,500 + 387,000}{2} = 308,750$$

$$Q_3 = \frac{639,000 + 659,000}{2} = 649,000$$

$$IQR = 649,000 - 308,750 = 340,250$$

$$(1.5)(IQR) = (1.5)(340,250) = 510,375$$

$$Q_1 - (1.5)(IQR) = 308,750 - 510,375 = -201,625$$

$$Q_3 + (1.5)(IQR) = 649,000 + 510,375 = 1,159,375$$

No house price is less than $-201,625$. However, 5,500,000 is more than 1,159,375. Therefore, 5,500,000 is a potential **outlier**.

Note:

Try It

Exercise:**Problem:**

For the following 11 salaries, calculate the *IQR* and determine if any salaries are outliers. The salaries are in dollars.

\$33,000 \$64,500 \$28,000 \$54,000 \$72,000 \$68,500 \$69,000 \$42,000
\$54,000 \$120,000 \$40,500

Solution:

Order the data from smallest to largest.

\$28,000 \$33,000 \$40,500 \$42,000 \$54,000 \$54,000 \$64,500 \$68,500
\$69,000 \$72,000 \$120,000

Median = \$54,000

$Q_1 = \$40,500$

$Q_3 = \$69,000$

$IQR = \$69,000 - \$40,500 = \$28,500$

$(1.5)(IQR) = (1.5)(\$28,500) = \$42,750$

$Q_1 - (1.5)(IQR) = \$40,500 - \$42,750 = -\$2,250$

$Q_3 + (1.5)(IQR) = \$69,000 + \$42,750 = \$111,750$

No salary is less than $-\$2,250$. However, \$120,000 is more than \$11,750, so \$120,000 is a potential outlier.

Example:**Exercise:**

Problem:

For the two data sets in the [test scores example](#), find the following:

- The interquartile range. Compare the two interquartile ranges.
- Any outliers in either set.

Solution:

The five number summary for the day and night classes is

	Minimum	Q_1	Median	Q_3	Maximum
Day	32	56	74.5	82.5	99
Night	25.5	78	81	89	98

- a. The IQR for the day group is $Q_3 - Q_1 = 82.5 - 56 = 26.5$

The IQR for the night group is $Q_3 - Q_1 = 89 - 78 = 11$

The interquartile range (the spread or variability) for the day class is larger than the night class *IQR*. This suggests more variation will be found in the day class's class test scores.

- b. Day class outliers are found using the IQR times 1.5 rule. So,

- $Q_1 - IQR(1.5) = 56 - 26.5(1.5) = 16.25$
- $Q_3 + IQR(1.5) = 82.5 + 26.5(1.5) = 122.25$

Since the minimum and maximum values for the day class are greater than 16.25 and less than 122.25, there are no outliers.

Night class outliers are calculated as:

- $Q_1 - IQR(1.5) = 78 - 11(1.5) = 61.5$
- $Q_3 + IQR(1.5) = 89 + 11(1.5) = 105.5$

For this class, any test score less than 61.5 is an outlier. Therefore, the scores of 45 and 25.5 are outliers. Since no test score is greater than 105.5, there is no upper end outlier.

Note:

Try It

Exercise:

Problem:

Find the interquartile range for the following two data sets and compare them.

Test Scores for Class A

69; 96; 81; 79; 65; 76; 83; 99; 89; 67; 90; 77; 85; 98; 66; 91; 77; 69; 80; 94

Test Scores for Class B

90; 72; 80; 92; 90; 97; 92; 75; 79; 68; 70; 80; 99; 95; 78; 73; 71; 68; 95; 100

Solution:

Class A

Order the data from smallest to largest.

65 66 67 69 69 76 77 77 79 80 81 83 85 89 90 91 94 96 98 99

$$\text{Median} = \frac{80+81}{2} = 80.5$$

$$Q_1 = \frac{69+76}{2} = 72.5$$

$$Q_3 = \frac{90+91}{2} = 90.5$$

$$IQR = 90.5 - 72.5 = 18$$

Class *B*

Order the data from smallest to largest.

68 68 70 71 72 73 75 78 79 80 80 90 90 92 92 95 95 97 99 100

$$\text{Median} = \frac{80+80}{2} = 80$$

$$Q_1 = \frac{72+73}{2} = 72.5$$

$$Q_3 = \frac{92+95}{2} = 93.5$$

$$IQR = 93.5 - 72.5 = 21$$

The data for Class *B* has a larger *IQR*, so the scores between Q_3 and Q_1 (middle 50%) for the data for Class *B* are more spread out and not clustered about the median.

Example:

Fifty statistics students were asked how much sleep they get per school night (rounded to the nearest hour). The results were:

AMOUNT OF SLEEP PER SCHOOL NIGHT (HOURS)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
4	2	0.04	0.04
5	5	0.10	0.14

AMOUNT OF SLEEP PER SCHOOL NIGHT (HOURS)	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE RELATIVE FREQUENCY
6	7	0.14	0.28
7	12	0.24	0.52
8	14	0.28	0.80
9	7	0.14	0.94
10	3	0.06	1.00

Find the 28th percentile. Notice the 0.28 in the "cumulative relative frequency" column. Twenty-eight percent of 50 data values is 14 values. There are 14 values less than the 28th percentile. They include the two 4s, the five 5s, and the seven 6s. The 28th percentile is between the last six and the first seven. **The 28th percentile is 6.5.**

Find the median. Look again at the "cumulative relative frequency" column and find 0.52. The median is the 50th percentile or the second quartile. 50% of 50 is 25. There are 25 values less than the median. They include the two 4s, the five 5s, the seven 6s, and eleven of the 7s. The median or 50th percentile is between the 25th, or seven, and 26th, or seven, values. **The median is seven.**

Find the third quartile. The third quartile is the same as the 75th percentile. You can "eyeball" this answer. If you look at the "cumulative relative frequency" column, you find 0.52 and 0.80. When you have all the fours, fives, sixes and sevens, you have 52% of the data. When you include all the 8s, you have 80% of the data. **The 75th percentile, then, must be an eight.** Another way to look at the problem is to find 75% of 50, which is 37.5, and round up to 38. The third quartile, Q_3 , is the 38th value, which is an eight. You can check this answer by counting the values. (There are 37 values below the third quartile and 12 values above.)

Note:

Try it

Exercise:**Problem:**

Forty bus drivers were asked how many hours they spend each day running their routes (rounded to the nearest hour). Find the 65th percentile.

Amount of time spent on route (hours)	Frequency	Relative Frequency	Cumulative Relative Frequency
2	12	0.30	0.30
3	14	0.35	0.65
4	10	0.25	0.90
5	4	0.10	1.00

Solution:

The 65th percentile is between the last three and the first four.

The 65th percentile is 3.5.

Example:**Exercise:**

Problem: Using [\[link\]](#):

- a. Find the 80th percentile.
- b. Find the 90th percentile.
- c. Find the first quartile. What is another name for the first quartile?

Solution:

Using the data from the frequency table, we have:

- a. The 80th percentile is between the last eight and the first nine in the table (between the 40th and 41st values). Therefore, we need to take the mean of the 40th and 41st values. The 80th percentile $= \frac{8+9}{2} = 8.5$
- b. The 90th percentile will be the 45th data value (location is $0.90(50) = 45$) and the 45th data value is nine.
- c. Q_1 is also the 25th percentile. The 25th percentile location calculation: $P_{25} = 0.25(50) = 12.5 \approx 13$ the 13th data value. Thus, the 25th percentile is six.

Note:

Try It

Exercise:

Problem:

Refer to the [link](#). Find the third quartile. What is another name for the third quartile?

Solution:

The third quartile is the 75th percentile, which is four. The 65th percentile is between three and four, and the 90th percentile is between four and 5.75. The third quartile is between 65 and 90, so it must be four.

Note:

Collaborative Statistics

Your instructor or a member of the class will ask everyone in class how many sweaters they own. Answer the following questions:

1. How many students were surveyed?
2. What kind of sampling did you do?
3. Construct two different histograms. For each, starting value = _____ ending value = _____.
4. Find the median, first quartile, and third quartile.
5. Construct a table of the data to find the following:
 - a. the 10th percentile
 - b. the 70th percentile
 - c. the percent of students who own less than four sweaters

A Formula for Finding the k th Percentile

If you were to do a little research, you would find several formulas for calculating the k^{th} percentile. Here is one of them.

k = the k^{th} percentile. It may or may not be part of the data.

i = the index (ranking or position of a data value)

n = the total number of data

- Order the data from smallest to largest.
- Calculate $i = \frac{k}{100}(n + 1)$
- If i is an integer, then the k^{th} percentile is the data value in the i^{th} position in the ordered set of data.
- If i is not an integer, then round i up and round i down to the nearest integers. Average the two data values in these two positions in the ordered data set. This is easier to understand in an example.

Example:

Exercise:

Problem:

Listed are 29 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 21; 22; 25; 26; 27; 29; 30; 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

- a. Find the 70th percentile.
- b. Find the 83rd percentile.

Solution:

- a.
 - $k = 70$
 - $i = \text{the index}$
 - $n = 29$

$i = \frac{k}{100} (n + 1) = (\frac{70}{100})(29 + 1) = 21$. Twenty-one is an integer, and the data value in the 21st position in the ordered data set is 64. The 70th percentile is 64 years.

- b.
 - $k = 83^{\text{rd}}$ percentile
 - $i = \text{the index}$
 - $n = 29$

$i = \frac{k}{100} (n + 1) = (\frac{83}{100})(29 + 1) = 24.9$, which is NOT an integer. Round it down to 24 and up to 25. The age in the 24th position is 71 and the age in the 25th position is 72. Average 71 and 72. The 83rd percentile is 71.5 years.

Note:

Try It

Exercise:

Problem:

Listed are 29 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 21; 22; 25; 26; 27; 29; 30; 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

Calculate the 20th percentile and the 55th percentile.

Solution:

$k = 20$. Index $= i = \frac{k}{100}(n + 1) = \frac{20}{100}(29 + 1) = 6$. The age in the sixth position is 27. The 20th percentile is 27 years.

$k = 55$. Index $= i = \frac{k}{100}(n + 1) = \frac{55}{100}(29 + 1) = 16.5$. Round down to 16 and up to 17. The age in the 16th position is 52 and the age in the 17th position is 55. The average of 52 and 55 is 53.5. The 55th percentile is 53.5 years.

Note:**NOTE**

You can calculate percentiles using calculators and computers. There are a variety of online calculators.

A Formula for Finding the Percentile of a Value in a Data Set

- Order the data from smallest to largest.
- x = the number of data values counting from the bottom of the data list up to but not including the data value for which you want to find the percentile.
- y = the number of data values equal to the data value for which you want to find the percentile.
- n = the total number of data.
- Calculate $\frac{x+0.5y}{n}(100)$. Then round to the nearest integer.

Example:**Exercise:****Problem:**

Listed are 29 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 21; 22; 25; 26; 27; 29; 30; 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

- a. Find the percentile for 58.
- b. Find the percentile for 25.

Solution:

- a. Counting from the bottom of the list, there are 18 data values less than 58. There is one value of 58.

$x = 18$ and $y = 1. \frac{x+0.5y}{n} (100) = \frac{18+0.5(1)}{29} (100) = 63.80$. 58 is the 64th percentile.

- b. Counting from the bottom of the list, there are three data values less than 25. There is one value of 25.

$x = 3$ and $y = 1. \frac{x+0.5y}{n} (100) = \frac{3+0.5(1)}{29} (100) = 12.07$. Twenty-five is the 12th percentile.

Note:

Try It

Exercise:**Problem:**

Listed are 30 ages for Academy Award winning best actors in order from smallest to largest.

18; 21; 22; 25; 26; 27; 29; 30; 31, 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

Find the percentiles for 47 and 31.

Solution:

Percentile for 47: Counting from the bottom of the list, there are 15 data values less than 47. There is one value of 47.

$x = 15$ and $y = 1$. $\frac{x+0.5y}{n}(100) = \frac{15+0.5(1)}{29}(100) = 53.45$. 47 is the 53rd percentile.

Percentile for 31: Counting from the bottom of the list, there are eight data values less than 31. There are two values of 31.

$x = 15$ and $y = 2$. $\frac{x+0.5y}{n}(100) = \frac{15+0.5(2)}{29}(100) = 31.03$. 31 is the 31st percentile.

Interpreting Percentiles, Quartiles, and Median

A percentile indicates the relative standing of a data value when data are sorted into numerical order from smallest to largest. Percentages of data values are less than or equal to the p th percentile. For example, 15% of data values are less than or equal to the 15th percentile.

- Low percentiles always correspond to lower data values.
- High percentiles always correspond to higher data values.

A percentile may or may not correspond to a value judgment about whether it is "good" or "bad." The interpretation of whether a certain percentile is "good" or "bad" depends on the context of the situation to which the data applies. In some situations, a low percentile would be considered "good;" in other contexts a high percentile might be considered "good". In many situations, there is no value judgment that applies.

Understanding how to interpret percentiles properly is important not only when describing data, but also when calculating probabilities in later chapters of this text.

Note:**NOTE**

When writing the interpretation of a percentile in the context of the given data, the sentence should contain the following information.

- information about the context of the situation being considered
- the data value (value of the variable) that represents the percentile
- the percent of individuals or items with data values below the percentile
- the percent of individuals or items with data values above the percentile.

Example:**Exercise:****Problem:**

On a timed math test, the first quartile for time it took to finish the exam was 35 minutes. Interpret the first quartile in the context of this situation.

Solution:

- Twenty-five percent of students finished the exam in 35 minutes or less.
- Seventy-five percent of students finished the exam in 35 minutes or more.
- A low percentile could be considered good, as finishing more quickly on a timed exam is desirable. (If you take too long, you might not be able to finish.)

Note:

Try It

Exercise:

Problem:

For the 100-meter dash, the third quartile for times for finishing the race was 11.5 seconds. Interpret the third quartile in the context of the situation.

Solution:

Twenty-five percent of runners finished the race in 11.5 seconds or more. Seventy-five percent of runners finished the race in 11.5 seconds or less. A lower percentile is good because finishing a race more quickly is desirable.

Example:**Exercise:****Problem:**

On a 20 question math test, the 70th percentile for number of correct answers was 16. Interpret the 70th percentile in the context of this situation.

Solution:

- Seventy percent of students answered 16 or fewer questions correctly.
- Thirty percent of students answered 16 or more questions correctly.
- A higher percentile could be considered good, as answering more questions correctly is desirable.

Note:

Try It

Exercise:**Problem:**

On a 60 point written assignment, the 80th percentile for the number of points earned was 49. Interpret the 80th percentile in the context of this situation.

Solution:

Eighty percent of students earned 49 points or fewer. Twenty percent of students earned 49 or more points. A higher percentile is good because getting more points on an assignment is desirable.

Example:**Exercise:****Problem:**

At a community college, it was found that the 30th percentile of credit units that students are enrolled for is seven units. Interpret the 30th percentile in the context of this situation.

Solution:

- Thirty percent of students are enrolled in seven or fewer credit units.
- Seventy percent of students are enrolled in seven or more credit units.
- In this example, there is no "good" or "bad" value judgment associated with a higher or lower percentile. Students attend community college for varied reasons and needs, and their course load varies according to their needs.

Note:

Try It

Exercise:**Problem:**

During a season, the 40th percentile for points scored per player in a game is eight. Interpret the 40th percentile in the context of this situation.

Solution:

Forty percent of players scored eight points or fewer. Sixty percent of players scored eight points or more. A higher percentile is good because getting more points in a basketball game is desirable.

Example:

Sharpe Middle School is applying for a grant that will be used to add fitness equipment to the gym. The principal surveyed 15 anonymous students to determine how many minutes a day the students spend exercising. The results from the 15 anonymous students are shown.

0 minutes; 40 minutes; 60 minutes; 30 minutes; 60 minutes

10 minutes; 45 minutes; 30 minutes; 300 minutes; 90 minutes;

30 minutes; 120 minutes; 60 minutes; 0 minutes; 20 minutes

Determine the following five values.

- $\text{Min} = 0$
- $Q_1 = 20$
- $\text{Med} = 40$
- $Q_3 = 60$
- $\text{Max} = 300$

If you were the principal, would you be justified in purchasing new fitness equipment? Since 75% of the students exercise for 60 minutes or less daily, and since the *IQR* is 40 minutes ($60 - 20 = 40$), we know that half of the students surveyed exercise between 20 minutes and 60 minutes daily. This seems a reasonable amount of time spent exercising, so the principal would be justified in purchasing the new equipment.

However, the principal needs to be careful. The value 300 appears to be a potential outlier.

$$Q_3 + 1.5(IQR) = 60 + (1.5)(40) = 120.$$

The value 300 is greater than 120 so it is a potential outlier. If we delete it and calculate the five values, we get the following values:

- $\text{Min} = 0$
- $Q_1 = 20$
- $Q_3 = 60$
- $\text{Max} = 120$

We still have 75% of the students exercising for 60 minutes or less daily and half of the students exercising between 20 and 60 minutes a day. However, 15 students is a small sample and the principal should survey more students to be sure of his survey results.

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Chapter Review

The values that divide a rank-ordered set of data into 100 equal parts are called percentiles. Percentiles are used to compare and interpret data. For example, an observation at the 50th percentile would be greater than 50 percent of the other observations in the set. Quartiles divide data into quarters. The first quartile (Q_1) is the 25th percentile, the second quartile (Q_2 or median) is 50th percentile, and the third quartile (Q_3) is the 75th percentile. The interquartile range, or *IQR*, is the range of the middle 50 percent of the data values. The *IQR* is found by subtracting Q_1 from Q_3 , and can help determine outliers by using the following two expressions.

- $Q_3 + IQR(1.5)$
- $Q_1 - IQR(1.5)$

Formula Review

$$i = \left(\frac{k}{100} \right) (n + 1)$$

where i = the ranking or position of a data value,

k = the k th percentile,

n = total number of data.

Expression for finding the percentile of a data value: $\left(\frac{x + 0.5y}{n} \right) (100)$

where x = the number of values counting from the bottom of the data list up to but not including the data value for which you want to find the percentile,

y = the number of data values equal to the data value for which you want to find the percentile,

n = total number of data

Exercise:

Problem:

Listed are 29 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 21; 22; 25; 26; 27; 29; 30; 31; 33; 36; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

- Find the 40th percentile.
- Find the 78th percentile.

Solution:

- The 40th percentile is 37 years.
- The 78th percentile is 70 years.

Exercise:

Problem:

Listed are 32 ages for Academy Award winning best actors *in order from smallest to largest*.

18; 18; 21; 22; 25; 26; 27; 29; 30; 31; 31; 33; 36; 37; 37; 41; 42; 47; 52; 55; 57; 58; 62; 64; 67; 69; 71; 72; 73; 74; 76; 77

- a. Find the percentile of 37.
- b. Find the percentile of 72.

Exercise:**Problem:**

Jesse was ranked 37th in his graduating class of 180 students. At what percentile is Jesse's ranking?

Solution:

Jesse graduated 37th out of a class of 180 students. There are $180 - 37 = 143$ students ranked below Jesse. There is one rank of 37.

$x = 143$ and $y = 1$. $\frac{x+0.5y}{n}(100) = \frac{143+0.5(1)}{180}(100) = 79.72$. Jesse's rank of 37 puts him at the 80th percentile.

Exercise:**Problem:**

- a. For runners in a race, a low time means a faster run. The winners in a race have the shortest running times. Is it more desirable to have a finish time with a high or a low percentile when running a race?
- b. The 20th percentile of run times in a particular race is 5.2 minutes. Write a sentence interpreting the 20th percentile in the context of the situation.
- c. A bicyclist in the 90th percentile of a bicycle race completed the race in 1 hour and 12 minutes. Is he among the fastest or slowest cyclists in the race? Write a sentence interpreting the 90th percentile in the context of the situation.

Exercise:**Problem:**

- a. For runners in a race, a higher speed means a faster run. Is it more desirable to have a speed with a high or a low percentile when running a race?
- b. The 40th percentile of speeds in a particular race is 7.5 miles per hour. Write a sentence interpreting the 40th percentile in the context of the situation.

Solution:

- a. For runners in a race it is more desirable to have a high percentile for speed. A high percentile means a higher speed which is faster.
- b. 40% of runners ran at speeds of 7.5 miles per hour or less (slower). 60% of runners ran at speeds of 7.5 miles per hour or more (faster).

Exercise:**Problem:**

On an exam, would it be more desirable to earn a grade with a high or low percentile? Explain.

Exercise:**Problem:**

Mina is waiting in line at the Department of Motor Vehicles (DMV). Her wait time of 32 minutes is the 85th percentile of wait times. Is that good or bad? Write a sentence interpreting the 85th percentile in the context of this situation.

Solution:

When waiting in line at the DMV, the 85th percentile would be a long wait time compared to the other people waiting. 85% of people had shorter wait times than Mina. In this context, Mina would prefer a wait time corresponding to a lower percentile. 85% of people at the DMV waited 32 minutes or less. 15% of people at the DMV waited 32 minutes or longer.

Exercise:**Problem:**

In a survey collecting data about the salaries earned by recent college graduates, Li found that her salary was in the 78th percentile. Should Li be pleased or upset by this result? Explain.

Exercise:**Problem:**

In a study collecting data about the repair costs of damage to automobiles in a certain type of crash tests, a certain model of car had \$1,700 in damage and was in the 90th percentile. Should the manufacturer and the consumer be pleased or upset by this result? Explain and write a sentence that interprets the 90th percentile in the context of this problem.

Solution:

The manufacturer and the consumer would be upset. This is a large repair cost for the damages, compared to the other cars in the sample.

INTERPRETATION: 90% of the crash tested cars had damage repair costs of \$1700 or less; only 10% had damage repair costs of \$1700 or more.

Exercise:**Problem:**

The University of California has two criteria used to set admission standards for freshman to be admitted to a college in the UC system:

- a. Students' GPAs and scores on standardized tests (SATs and ACTs) are entered into a formula that calculates an "admissions index" score. The admissions index score is used to set eligibility standards intended to meet the goal of admitting the top 12% of high school students in the state. In this context, what percentile does the top 12% represent?
- b. Students whose GPAs are at or above the 96th percentile of all students at their high school are eligible (called eligible in the local context), even if they are not in the top 12% of all students in the state. What percentage of students from each high school are "eligible in the local context"?

Exercise:**Problem:**

Suppose that you are buying a house. You and your realtor have determined that the most expensive house you can afford is the 34th percentile. The 34th percentile of housing prices is \$240,000 in the town you want to move to. In this town, can you afford 34% of the houses or 66% of the houses?

Solution:

You can afford 34% of houses. 66% of the houses are too expensive for your budget. INTERPRETATION: 34% of houses cost \$240,000 or less. 66% of houses cost \$240,000 or more.

Use the following information to answer the next six exercises. Sixty-five randomly selected car salespersons were asked the number of cars they generally sell in one week. Fourteen people answered that they generally sell three cars; nineteen generally sell four cars; twelve generally sell five cars; nine generally sell six cars; eleven generally sell seven cars.

Exercise:

Problem: First quartile = _____

Exercise:

Problem: Second quartile = median = 50th percentile = _____

Solution:

4

Exercise:

Problem: Third quartile = _____

Exercise:

Problem: Interquartile range (*IQR*) = _____ - _____ = _____

Solution:

$$6 - 4 = 2$$

Exercise:

Problem: 10th percentile = _____

Exercise:

Problem: 70th percentile = _____

Solution:

6

Homework

Exercise:

Problem:

The median age for U.S. blacks currently is 30.9 years; for U.S. whites it is 42.3 years.

- Based upon this information, give two reasons why the black median age could be lower than the white median age.
- Does the lower median age for blacks necessarily mean that blacks die younger than whites? Why or why not?
- How might it be possible for blacks and whites to die at approximately the same age, but for the median age for whites to be higher?

Exercise:

Problem:

Six hundred adult Americans were asked by telephone poll, "What do you think constitutes a middle-class income?" The results are in [\[link\]](#). Also, include left endpoint, but not the right endpoint.

Salary (\$)	Relative Frequency
< 20,000	0.02
20,000–25,000	0.09
25,000–30,000	0.19
30,000–40,000	0.26
40,000–50,000	0.18
50,000–75,000	0.17
75,000–99,999	0.02
100,000+	0.01

- What percentage of the survey answered "not sure"?
- What percentage think that middle-class is from \$25,000 to \$50,000?
- Construct a histogram of the data.
 - Should all bars have the same width, based on the data? Why or why not?
 - How should the <20,000 and the 100,000+ intervals be handled? Why?
- Find the 40th and 80th percentiles
- Construct a bar graph of the data

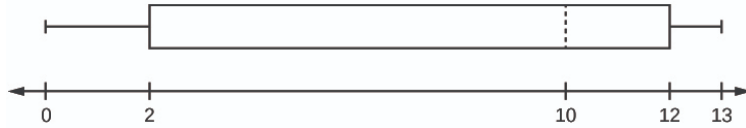
Solution:

- $1 - (0.02 + 0.09 + 0.19 + 0.26 + 0.18 + 0.17 + 0.02 + 0.01) = 0.06$
- $0.19 + 0.26 + 0.18 = 0.63$
- Check student's solution.
- 40th percentile will fall between 30,000 and 40,000
 80th percentile will fall between 50,000 and 75,000

e. Check student's solution.

Exercise:

Problem: Given the following box plot:

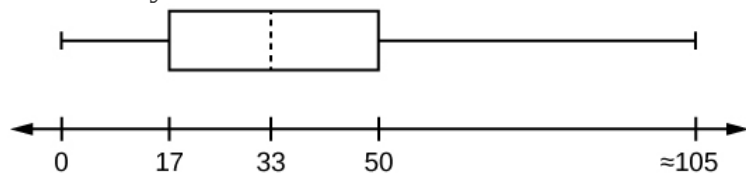


- which quarter has the smallest spread of data? What is that spread?
- which quarter has the largest spread of data? What is that spread?
- find the interquartile range (*IQR*).
- are there more data in the interval 5–10 or in the interval 10–13? How do you know this?
- which interval has the fewest data in it? How do you know this?
 - 0–2
 - 2–4
 - 10–12
 - 12–13
 - need more information

Exercise:

Problem:

The following box plot shows the U.S. population for 1990, the latest available year.



- Are there fewer or more children (age 17 and under) than senior citizens (age 65 and over)? How do you know?
- 12.6% are age 65 and over. Approximately what percentage of the population are working age adults (above age 17 to age 65)?

Solution:

- a. more children; the left whisker shows that 25% of the population are children 17 and younger. The right whisker shows that 25% of the population are adults 50 and older, so adults 65 and over represent less than 25%.
- b. 62.4%

Glossary

Interquartile Range

or *IQR*, is the range of the middle 50 percent of the data values; the *IQR* is found by subtracting the first quartile from the third quartile.

Outlier

an observation that does not fit the rest of the data

Percentile

a number that divides ordered data into hundredths; percentiles may or may not be part of the data. The median of the data is the second quartile and the 50th percentile. The first and third quartiles are the 25th and the 75th percentiles, respectively.

Quartiles

the numbers that separate the data into quarters; quartiles may or may not be part of the data. The second quartile is the median of the data.

Box Plots

Box plots (also called **box-and-whisker plots** or **box-whisker plots**) give a good graphical image of the concentration of the data. They also show how far the extreme values are from most of the data. A box plot is constructed from five values: the minimum value, the first quartile, the median, the third quartile, and the maximum value. We use these values to compare how close other data values are to them.

To construct a box plot, use a horizontal or vertical number line and a rectangular box. The smallest and largest data values label the endpoints of the axis. The first quartile marks one end of the box and the third quartile marks the other end of the box. Approximately **the middle 50 percent of the data fall inside the box**. The "whiskers" extend from the ends of the box to the smallest and largest data values. The median or second quartile can be between the first and third quartiles, or it can be one, or the other, or both. The box plot gives a good, quick picture of the data.

Note:

NOTE

You may encounter box-and-whisker plots that have dots marking outlier values. In those cases, the whiskers are not extending to the minimum and maximum values.

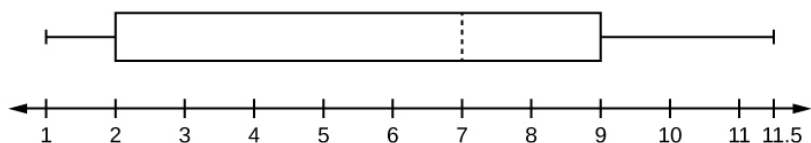
Consider, again, this dataset.

1 1 2 2 4 6 6.8 7.2 8 8.3 9 10 10 11.5

The first quartile is two, the median is seven, and the third quartile is nine. The smallest value is one, and the largest value is 11.5. The following image shows the constructed box plot.

Note:**NOTE**

See the calculator instructions on the [TI web site](#) or in the appendix.



The two whiskers extend from the first quartile to the smallest value and from the third quartile to the largest value. The median is shown with a dashed line.

Note:**NOTE**

It is important to start a box plot with a **scaled number line**. Otherwise the box plot may not be useful.

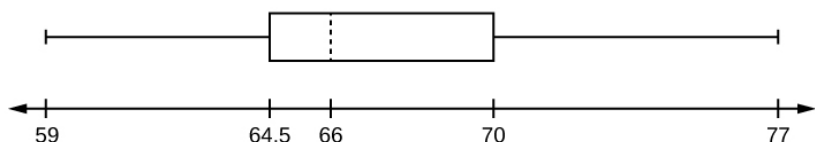
Example:

The following data are the heights of 40 students in a statistics class.

59 60 61 62 62 63 63 64 64 64 65 65 65 65 65 65 65 65 65 66 66 67 67 68
68 69 70 70 70 70 70 71 71 72 72 73 74 74 75 77

Construct a box plot with the following properties; the calculator instructions for the minimum and maximum values as well as the quartiles follow the example.

- Minimum value = 59
- Maximum value = 77
- Q1: First quartile = 64.5
- Q2: Second quartile or median = 66
- Q3: Third quartile = 70



- Each quarter has approximately 25% of the data.
- The spreads of the four quarters are $64.5 - 59 = 5.5$ (first quarter), $66 - 64.5 = 1.5$ (second quarter), $70 - 66 = 4$ (third quarter), and $77 - 70 = 7$ (fourth quarter). So, the second quarter has the smallest spread and the fourth quarter has the largest spread.
- Range = maximum value – the minimum value = $77 - 59 = 18$
- Interquartile Range: $IQR = Q3 - Q1 = 70 - 64.5 = 5.5$.
- The interval 59–65 has more than 25% of the data so it has more data in it than the interval 66 through 70 which has 25% of the data.
- The middle 50% (middle half) of the data has a range of 5.5 inches.

Note:

To find the minimum, maximum, and quartiles:

Enter data into the list editor (Pres STAT 1:EDIT). If you need to clear the list, arrow up to the name L1, press CLEAR, and then arrow down.

Put the data values into the list L1.

Press STAT and arrow to CALC. Press 1:1-VarStats. Enter L1.

Press ENTER.

Use the down and up arrow keys to scroll.

Smallest value = 59.

Largest value = 77.

Q_1 : First quartile = 64.5.

Q_2 : Second quartile or median = 66.

Q_3 : Third quartile = 70.

To construct the box plot:

Press 4:Plotsoff. Press ENTER.

Arrow down and then use the right arrow key to go to the fifth picture, which is the box plot. Press ENTER.

Arrow down to Xlist: Press 2nd 1 for L1

Arrow down to Freq: Press ALPHA. Press 1.

Press Zoom. Press 9: ZoomStat.

Press TRACE, and use the arrow keys to examine the box plot.

Note:

Try It

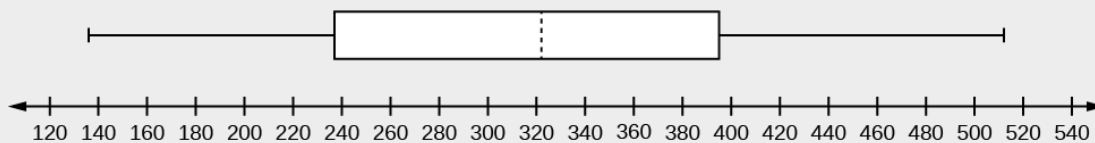
Exercise:

Problem:

The following data are the number of pages in 40 books on a shelf. Construct a box plot using a graphing calculator, and state the interquartile range.

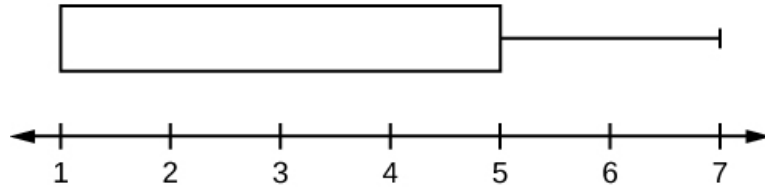
136 140 178 190 205 215 217 218 232 234 240 255 270 275 290 301
303 315 317 318 326 333 343 349 360 369 377 388 391 392 398 400
402 405 408 422 429 450 475 512

Solution:



$$IQR = 158$$

For some sets of data, some of the largest value, smallest value, first quartile, median, and third quartile may be the same. For instance, you might have a data set in which the median and the third quartile are the same. In this case, the diagram would not have a dotted line inside the box displaying the median. The right side of the box would display both the third quartile and the median. For example, if the smallest value and the first quartile were both one, the median and the third quartile were both five, and the largest value was seven, the box plot would look like:



In this case, at least 25% of the values are equal to one. Twenty-five percent of the values are between one and five, inclusive. At least 25% of the values are equal to five. The top 25% of the values fall between five and seven, inclusive.

Example:

Test scores for a college statistics class held during the day are:

99 56 78 55.5 32 90 80 81 56 59 45 77 84.5 84 70 72 68 32 79 90

Test scores for a college statistics class held during the evening are:

98 78 68 83 81 89 88 76 65 45 98 90 80 84.5 85 79 78 98 90 79 81 25.5

Exercise:

Problem:

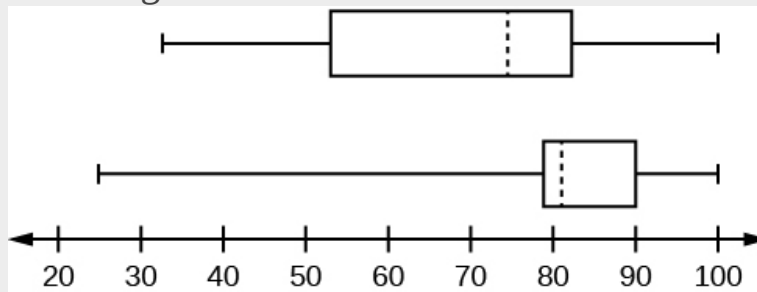
- Find the smallest and largest values, the median, and the first and third quartile for the day class.
- Find the smallest and largest values, the median, and the first and third quartile for the night class.
- For each data set, what percentage of the data is between the smallest value and the first quartile? the first quartile and the median? the median and the third quartile? the third quartile and the largest value? What percentage of the data is between the first quartile and the largest value?
- Create a box plot for each set of data. Use one number line for both box plots.
- Which box plot has the widest spread for the middle 50% of the data (the data between the first and third quartiles)? What does this mean for that set of data in comparison to the other set of data?

Solution:

- a.
- $\text{Min} = 32$
 - $Q_1 = 56$
 - $M = 74.5$
 - $Q_3 = 82.5$
 - $\text{Max} = 99$

- b.
- $\text{Min} = 25.5$
 - $Q_1 = 78$
 - $M = 81$
 - $Q_3 = 89$
 - $\text{Max} = 98$

- c. Day class: There are six data values ranging from 32 to 56: 30%. There are six data values ranging from 56 to 74.5: 30%. There are five data values ranging from 74.5 to 82.5: 25%. There are five data values ranging from 82.5 to 99: 25%. There are 16 data values between the first quartile, 56, and the largest value, 99: 75%. Night class:



- d.
- e. The first data set has the wider spread for the middle 50% of the data. The *IQR* for the first data set is greater than the *IQR* for the second set. This means that there is more variability in the middle 50% of the first data set.

Note:

Try It

Exercise:

Problem:

The following data set shows the heights in inches for the boys in a class of 40 students.

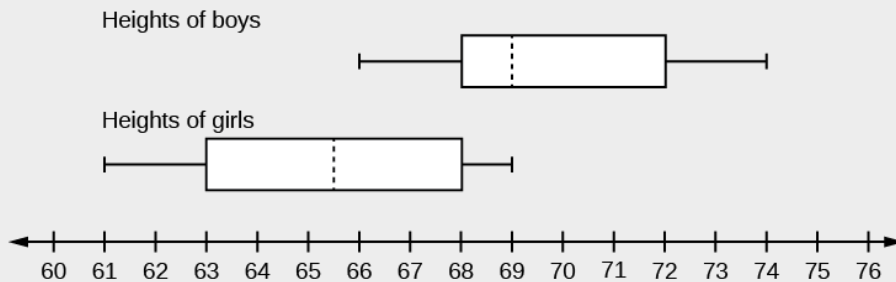
66; 66; 67; 67; 68; 68; 68; 68; 68; 69; 69; 69; 70; 71; 72; 72; 72; 73; 73; 74

The following data set shows the heights in inches for the girls in a class of 40 students.

61; 61; 62; 62; 63; 63; 63; 65; 65; 65; 66; 66; 66; 67; 68; 68; 68; 69; 69; 69

Construct a box plot using a graphing calculator for each data set, and state which box plot has the wider spread for the middle 50% of the data.

Solution:



IQR for the boys = 4

IQR for the girls = 5

The box plot for the heights of the girls has the wider spread for the middle 50% of the data.

Example:

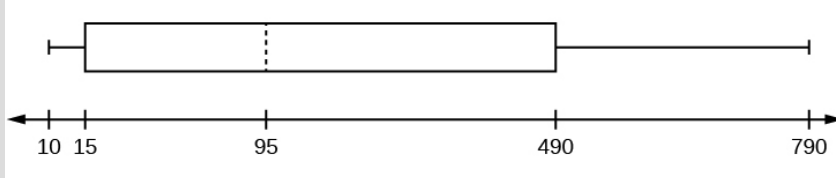
Graph a box-and-whisker plot for the data values shown.

1010101535759095100175420490515515790

The five numbers used to create a box-and-whisker plot are:

- Min: 10
- Q_1 : 15
- Med: 95
- Q_3 : 490
- Max: 790

The following graph shows the box-and-whisker plot.



Note:

Try It

Exercise:

Problem:

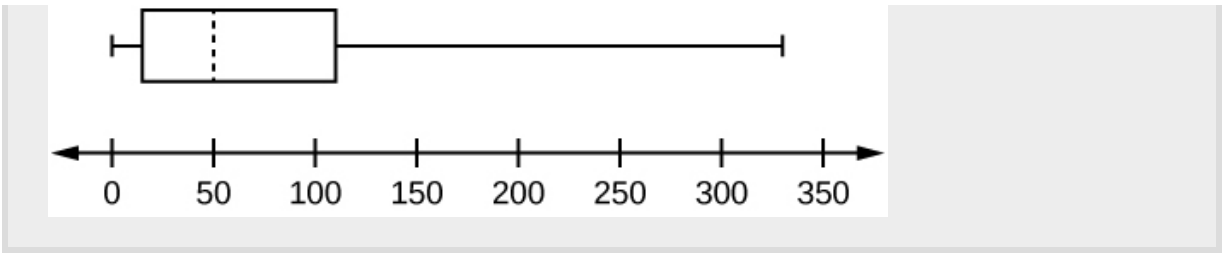
Follow the steps you used to graph a box-and-whisker plot for the data values shown.

0551530304550506075110140240330

Solution:

The data are in order from least to greatest. There are 15 values, so the eighth number in order is the median: 50. There are seven data values written to the left of the median and 7 values to the right. The five values that are used to create the boxplot are:

- Min: 0
- Q_1 : 15
- Med: 50
- Q_3 : 110
- Max: 330



References

Data from *West Magazine*.

Chapter Review

Box plots are a type of graph that can help visually organize data. To graph a box plot the following data points must be calculated: the minimum value, the first quartile, the median, the third quartile, and the maximum value. Once the box plot is graphed, you can display and compare distributions of data.

Use the following information to answer the next two exercises. Sixty-five randomly selected car salespersons were asked the number of cars they generally sell in one week. Fourteen people answered that they generally sell three cars; nineteen generally sell four cars; twelve generally sell five cars; nine generally sell six cars; eleven generally sell seven cars.

Exercise:

Problem:

Construct a box plot below. Use a ruler to measure and scale accurately.

Exercise:

Problem:

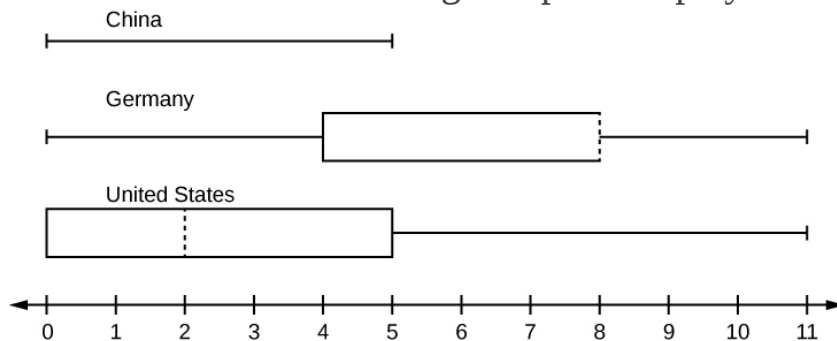
Looking at your box plot, does it appear that the data are concentrated together, spread out evenly, or concentrated in some areas, but not in others? How can you tell?

Solution:

More than 25% of salespersons sell four cars in a typical week. You can see this concentration in the box plot because the first quartile is equal to the median. The top 25% and the bottom 25% are spread out evenly; the whiskers have the same length.

Homework**Exercise:****Problem:**

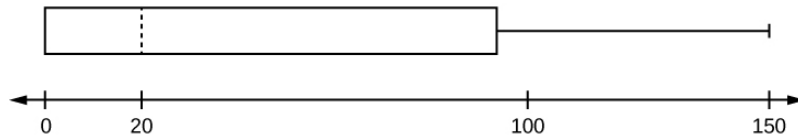
In a survey of 20-year-olds in China, Germany, and the United States, people were asked the number of foreign countries they had visited in their lifetime. The following box plots display the results.



- In complete sentences, describe what the shape of each box plot implies about the distribution of the data collected.
- Have more Americans or more Germans surveyed been to over eight foreign countries?
- Compare the three box plots. What do they imply about the foreign travel of 20-year-old residents of the three countries when compared to each other?

Exercise:

Problem: Given the following box plot, answer the questions.



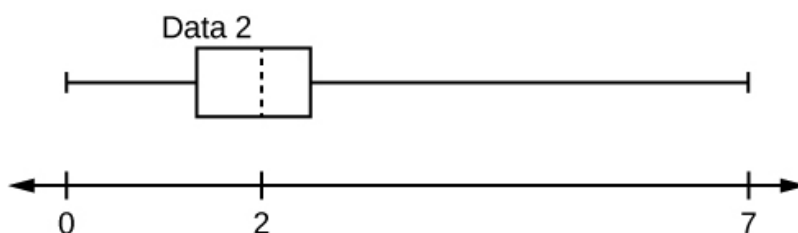
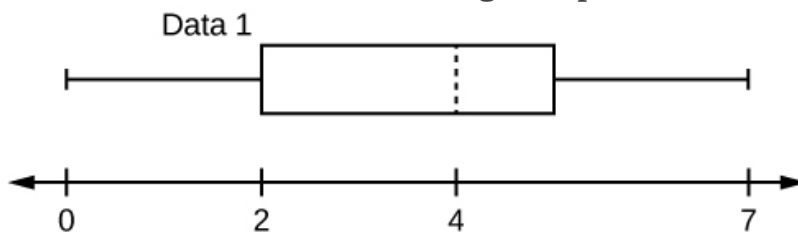
- Think of an example (in words) where the data might fit into the above box plot. In 2–5 sentences, write down the example.
- What does it mean to have the first and second quartiles so close together, while the second to third quartiles are far apart?

Solution:

- Answers will vary. Possible answer: State University conducted a survey to see how involved its students are in community service. The box plot shows the number of community service hours logged by participants over the past year.
- Because the first and second quartiles are close, the data in this quarter is very similar. There is not much variation in the values. The data in the third quarter is much more variable, or spread out. This is clear because the second quartile is so far away from the third quartile.

Exercise:

Problem: Given the following box plots, answer the questions.

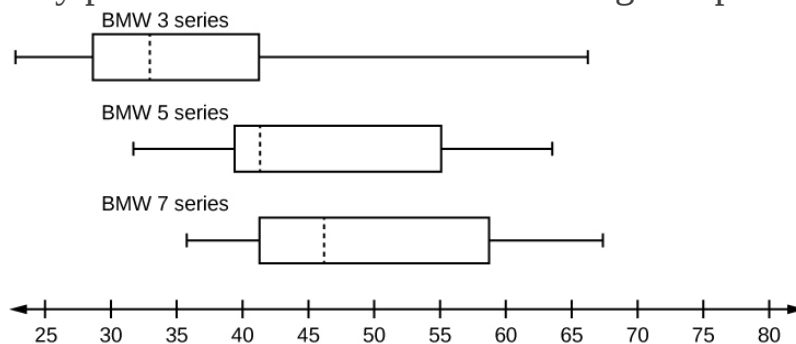


- a. In complete sentences, explain why each statement is false.
- Data 1** has more data values above two than **Data 2** has above two.
 - The data sets cannot have the same mode.
 - For **Data 1**, there are more data values below four than there are above four.
- b. For which group, Data 1 or Data 2, is the value of “7” more likely to be an outlier? Explain why in complete sentences.

Exercise:

Problem:

A survey was conducted of 130 purchasers of new BMW 3 series cars, 130 purchasers of new BMW 5 series cars, and 130 purchasers of new BMW 7 series cars. In it, people were asked the age they were when they purchased their car. The following box plots display the results.



- In complete sentences, describe what the shape of each box plot implies about the distribution of the data collected for that car series.
- Which group is most likely to have an outlier? Explain how you determined that.
- Compare the three box plots. What do they imply about the age of purchasing a BMW from the series when compared to each other?
- Look at the BMW 5 series. Which quarter has the smallest spread of data? What is the spread?

- e. Look at the BMW 5 series. Which quarter has the largest spread of data? What is the spread?
 - f. Look at the BMW 5 series. Estimate the interquartile range (IQR).
 - g. Look at the BMW 5 series. Are there more data in the interval 31 to 38 or in the interval 45 to 55? How do you know this?
 - h. Look at the BMW 5 series. Which interval has the fewest data in it? How do you know this?
 - i. 31–35
 - ii. 38–41
 - iii. 41–64
-

Solution:

- a. Each box plot is spread out more in the greater values. Each plot is skewed to the right, so the ages of the top 50% of buyers are more variable than the ages of the lower 50%.
- b. The BMW 3 series is most likely to have an outlier. It has the longest whisker.
- c. Comparing the median ages, younger people tend to buy the BMW 3 series, while older people tend to buy the BMW 7 series. However, this is not a rule, because there is so much variability in each data set.
- d. The second quarter has the smallest spread. There seems to be only a three-year difference between the first quartile and the median.
- e. The third quarter has the largest spread. There seems to be approximately a 14-year difference between the median and the third quartile.
- f. $IQR \sim 17$ years
- g. There is not enough information to tell. Each interval lies within a quarter, so we cannot tell exactly where the data in that quarter is concentrated.
- h. The interval from 31 to 35 years has the fewest data values. Twenty-five percent of the values fall in the interval 38 to 41, and

25% fall between 41 and 64. Since 25% of values fall between 31 and 38, we know that fewer than 25% fall between 31 and 35.

Exercise:

Problem:

Twenty-five randomly selected students were asked the number of movies they watched the previous week. The results are as follows:

# of movies	Frequency
0	5
1	9
2	6
3	4
4	1

Construct a box plot of the data.

Bringing It Together

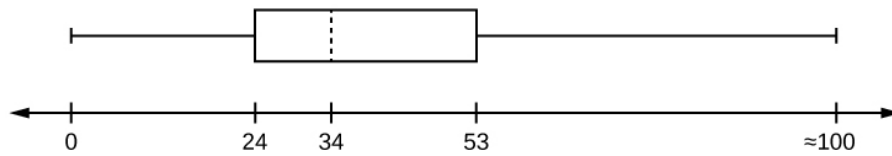
Exercise:

Problem:

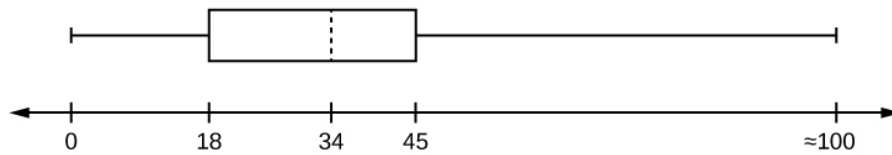
Santa Clara County, CA, has approximately 27,873 Japanese-Americans. Their ages are as follows:

Age Group	Percent of Community
0–17	18.9
18–24	8.0
25–34	22.8
35–44	15.0
45–54	13.1
55–64	11.9
65+	10.3

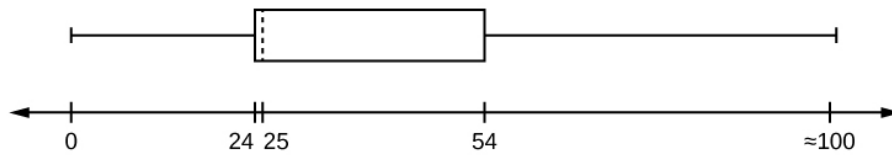
- Construct a histogram of the Japanese-American community in Santa Clara County, CA. The bars will **not** be the same width for this example. Why not? What impact does this have on the reliability of the graph?
- What percentage of the community is under age 35?
- Which box plot most resembles the information above?



(a)



(b)



(c)

Solution:

- For graph, check student's solution.
- 49.7% of the community is under the age of 35.
- Based on the information in the table, graph (a) most closely represents the data.

Glossary

Box plot

a graph that gives a quick picture of the middle 50% of the data

First Quartile

the value that is the median of the of the lower half of the ordered data set

Frequency Polygon

looks like a line graph but uses intervals to display ranges of large amounts of data

Interval

also called a class interval; an interval represents a range of data and is used when displaying large data sets

Paired Data Set

two data sets that have a one to one relationship so that:

- both data sets are the same size, and
- each data point in one data set is matched with exactly one point from the other set.

Skewed

used to describe data that is not symmetrical; when the right side of a graph looks “chopped off” compared the left side, we say it is “skewed to the left.” When the left side of the graph looks “chopped off” compared to the right side, we say the data is “skewed to the right.” Alternatively: when the lower values of the data are more spread out, we say the data are skewed to the left. When the greater values are more spread out, the data are skewed to the right.

Measures of the Center of the Data

The "center" of a data set is also a way of describing location. The two most widely used measures of the "center" of the data are the **mean** (average) and the **median**. To calculate the **mean weight** of 50 people, add the 50 weights together and divide by 50. To find the **median weight** of the 50 people, order the data and find the number that splits the data into two equal parts. The median is generally a better measure of the center when there are extreme values or outliers because it is not affected by the precise numerical values of the outliers. The mean is the most common measure of the center.

Note:

NOTE

The words "mean" and "average" are often used interchangeably. The substitution of one word for the other is common practice. The technical term is "arithmetic mean" and "average" is technically a center location. However, in practice among non-statisticians, "average" is commonly accepted for "arithmetic mean."

When each value in the data set is not unique, the mean can be calculated by multiplying each distinct value by its frequency and then dividing the sum by the total number of data values. The letter used to represent the **sample mean** is an x with a bar over it (pronounced " x bar"): $\bar{x}$.

The Greek letter μ (pronounced "mew") represents the **population mean**. One of the requirements for the **sample mean** to be a good estimate of the **population mean** is for the sample taken to be truly random.

To see that both ways of calculating the mean are the same, consider the sample:

1; 1; 1; 2; 2; 3; 4; 4; 4; 4; 4

Equation:

$$\bar{x} = \frac{1 + 1 + 1 + 2 + 2 + 3 + 4 + 4 + 4 + 4 + 4}{11} = 2.7$$

Equation:

$$\bar{x} = \frac{3(1) + 2(2) + 1(3) + 5(4)}{11} = 2.7$$

In the second calculation, the frequencies are 3, 2, 1, and 5.

You can quickly find the location of the median by using the expression $\frac{n+1}{2}$.

The letter n is the total number of data values in the sample. If n is an odd number, the median is the middle value of the ordered data (ordered smallest to largest). If n is an even number, the median is equal to the two middle values added together and divided by two after the data has been ordered. For example, if the total number of data values is 97, then $\frac{n+1}{2} = \frac{97+1}{2} = 49$. The median is the 49th value in the ordered data. If the total number of data values is 100, then $\frac{n+1}{2} = \frac{100+1}{2} = 50.5$. The median occurs midway between the 50th and 51st values. The location of the median and the value of the median are **not** the same. The upper case letter M is often used to represent the median. The next example illustrates the location of the median and the value of the median.

Example:**Exercise:****Problem:**

AIDS data indicating the number of months a patient with AIDS lives after taking a new antibody drug are as follows (smallest to largest):

3; 4; 8; 8; 10; 11; 12; 13; 14; 15; 15; 16; 16; 17; 17; 18; 21; 22; 22; 24; 24; 25; 26; 26; 27; 27; 29; 29; 31; 32; 33; 33; 34; 34; 35; 37; 40; 44; 44; 47;

Calculate the mean and the median.

Solution:

The calculation for the mean is:

$$x = \frac{[3+4+(8)(2)+10+11+12+13+14+(15)(2)+(16)(2)+\dots+35+37+40+(44)(2)+47]}{40} = 23.6$$

To find the median, M , first use the formula for the location. The location is:

$$\frac{n+1}{2} = \frac{40+1}{2} = 20.5$$

Starting at the smallest value, the median is located between the 20th and 21st values (the two 24s):

3; 4; 8; 8; 10; 11; 12; 13; 14; 15; 15; 16; 16; 17; 17; 18; 21; 22; 22; 24; 24; 25; 26; 26; 27; 27; 29; 29; 31; 32; 33; 33; 34; 34; 35; 37; 40; 44; 44; 47;

$$M = \frac{24+24}{2} = 24$$

Note:

To find the mean and the median:

Clear list L1. Press STAT 4:ClrList. Enter 2nd 1 for list L1. Press ENTER.

Enter data into the list editor. Press STAT 1:EDIT.

Put the data values into list L1.

Press STAT and arrow to CALC. Press 1:1-VarStats. Press 2nd 1 for L1 and then ENTER.

Press the down and up arrow keys to scroll.

$x = 23.6$, $M = 24$

Note:

Try It

Exercise:**Problem:**

The following data show the number of months patients typically wait on a transplant list before getting surgery. The data are ordered from smallest to largest. Calculate the mean and median.

3 4 5 7 7 7 7 8 8 9 9 10 10 10 10 10 11 12 12 13 14 14 15 15 17 17 18 19 19 19 21 21 22 22 23 24 24 24 24

Solution:

Mean: $3 + 4 + 5 + 7 + 7 + 7 + 7 + 8 + 8 + 9 + 9 + 10 + 10 + 10 + 10 + 10 + 11 + 12 + 12 + 13 + 14 + 14 + 15 + 15 + 17 + 17 + 18 + 19 + 19 + 19 + 21 + 21 + 22 + 22 + 23 + 24 + 24 + 24 = 544$
 $\frac{544}{39} = 13.95$

Median: Starting at the smallest value, the median is the 20th term, which is 13.

Example:

Exercise:

Problem:

Suppose that in a small town of 50 people, one person earns \$5,000,000 per year and the other 49 each earn \$30,000. Which is the better measure of the "center": the mean or the median?

Solution:

$$x = \frac{5,000,000 + 49(30,000)}{50} = 129,400$$

$$M = 30,000$$

(There are 49 people who earn \$30,000 and one person who earns \$5,000,000.)

The median is a better measure of the "center" than the mean because 49 of the values are 30,000 and one is 5,000,000. The 5,000,000 is an outlier. The 30,000 gives us a better sense of the middle of the data.

Note:

Try It

Exercise:

Problem:

In a sample of 60 households, one house is worth \$2,500,000. Half of the rest are worth \$280,000, and all the others are worth \$315,000. Which is the better measure of the "center": the mean or the median?

Solution:

The median is the better measure of the "center" than the mean because 59 of the values are \$280,000 and one is \$2,500,000. The \$2,500,000 is an outlier. Either \$280,000 or \$315,000 gives us a better sense of the middle of the data.

Another measure of the center is the mode. The **mode** is the most frequent value. There can be more than one mode in a data set as long as those values have the same frequency and that frequency is the highest. A data set with two modes is called bimodal.

Example:

Statistics exam scores for 20 students are as follows:

5053595963637272727272767881838484849093

Exercise:

Problem: Find the mode.

Solution:

The most frequent score is 72, which occurs five times. Mode = 72.

Note:

Try It

Exercise:

Problem: The number of books checked out from the library from 25 students are as follows:

0001233445577778889101011111212

Find the mode.

Solution:

The most frequent number of books is 7, which occurs four times. Mode = 7.

Example:

Five real estate exam scores are 430, 430, 480, 480, 495. The data set is bimodal because the scores 430 and 480 each occur twice.

When is the mode the best measure of the "center"? Consider a weight loss program that advertises a mean weight loss of six pounds the first week of the program. The mode might indicate that most people lose two pounds the first week, making the program less appealing.

Note:**NOTE**

The mode can be calculated for qualitative data as well as for quantitative data. For example, if the data set is: red, red, red, green, green, yellow, purple, black, blue, the mode is red.

Statistical software will easily calculate the mean, the median, and the mode. Some graphing calculators can also make these calculations. In the real world, people make these calculations using software.

Note:

Try It

Exercise:**Problem:**

Five credit scores are 680, 680, 700, 720, 720. The data set is bimodal because the scores 680 and 720 each occur twice. Consider the annual earnings of workers at a factory. The mode is \$25,000 and occurs 150 times out of 301. The median is \$50,000 and the mean is \$47,500. What would be the best measure of the “center”?

Solution:

Because \$25,000 occurs nearly half the time, the mode would be the best measure of the center because the median and mean don’t represent what most people make at the factory.

The Law of Large Numbers and the Mean

The Law of Large Numbers says that if you take samples of larger and larger size from any population, then the mean $\bar{x}$ of the sample is very likely to get closer and closer to μ . This is discussed in more detail later in the text.

Sampling Distributions and Statistic of a Sampling Distribution

You can think of a **sampling distribution** as a **relative frequency distribution** with a great many samples. (See **Sampling and Data** for a review of relative frequency). Suppose thirty randomly selected students were asked the number of movies they watched the previous week. The results are in the **relative frequency table** shown below.

# of movies	Relative Frequency
0	$\frac{5}{30}$
1	$\frac{15}{30}$
2	$\frac{6}{30}$

# of movies	Relative Frequency
3	$\frac{3}{30}$
4	$\frac{1}{30}$

If you let the number of samples get very large (say, 300 million or more), the relative frequency table becomes a relative frequency distribution.

A **statistic** is a number calculated from a sample. Statistic examples include the mean, the median and the mode as well as others. The sample mean $\bar{x}$ is an example of a statistic which estimates the population mean μ .

Calculating the Mean of Grouped Frequency Tables

When only grouped data is available, you do not know the individual data values (we only know intervals and interval frequencies); therefore, you cannot compute an exact mean for the data set. What we must do is estimate the actual mean by calculating the mean of a frequency table. A frequency table is a data representation in which grouped data is displayed along with the corresponding frequencies. To calculate the mean from a grouped frequency table we can apply the basic definition of mean: $mean = \frac{\text{data sum}}{\text{number of data values}}$ We simply need to modify the definition to fit within the restrictions of a frequency table.

Since we do not know the individual data values we can instead find the midpoint of each interval. The midpoint is $\frac{\text{lower boundary} + \text{upper boundary}}{2}$. We can now modify the mean definition to be

$Mean\ of\ Frequency\ Table = \frac{\sum fm}{\sum f}$ where f = the frequency of the interval and m = the midpoint of the interval.

Example:

Exercise:

Problem:

A frequency table displaying professor Blount's last statistic test is shown. Find the best estimate of the class mean.

Grade Interval	Number of Students
----------------	--------------------

Grade Interval	Number of Students
50–56.5	1
56.5–62.5	0
62.5–68.5	4
68.5–74.5	4
74.5–80.5	2
80.5–86.5	3
86.5–92.5	4
92.5–98.5	1

Solution:

- Find the midpoints for all intervals

Grade Interval	Midpoint
50–56.5	53.25
56.5–62.5	59.5
62.5–68.5	65.5
68.5–74.5	71.5
74.5–80.5	77.5
80.5–86.5	83.5
86.5–92.5	89.5
92.5–98.5	95.5

- Calculate the sum of the product of each interval frequency and midpoint. $\sum fm$

$$53.25(1) + 59.5(0) + 65.5(4) + 71.5(4) + 77.5(2) + 83.5(3) + 89.5(4) + 95.5(1) = 1460.25$$

$$\bullet \mu = \frac{\sum fm}{\sum f} = \frac{1460.25}{19} = 76.86$$

Note:

Try It

Exercise:

Problem:

Maris conducted a study on the effect that playing video games has on memory recall. As part of her study, she compiled the following data:

Hours Teenagers Spend on Video Games	Number of Teenagers
0–3.5	3
3.5–7.5	7
7.5–11.5	12
11.5–15.5	7
15.5–19.5	9

What is the best estimate for the mean number of hours spent playing video games?

Solution:

Find the midpoint of each interval, multiply by the corresponding number of teenagers, add the results and then divide by the total number of teenagers

The midpoints are 1.75, 5.5, 9.5, 13.5, 17.5.

$$\text{Mean} = \frac{(1.75)(3) + (5.5)(7) + (9.5)(12) + (13.5)(7) + (17.5)(9)}{(3 + 7 + 12 + 7 + 9)} = \frac{409.75}{38} = 10.78$$

References

Data from The World Bank, available online at <http://www.worldbank.org> (accessed April 3, 2013).

“Demographics: Obesity – adult prevalence rate.” Indexmundi. Available online at <http://www.indexmundi.com/g/r.aspx?t=50&v=2228&l=en> (accessed April 3, 2013).

Chapter Review

The mean and the median can be calculated to help you find the "center" of a data set. The mean is the best estimate for the actual data set, but the median is the best measurement when a data set contains several outliers or extreme values. The mode will tell you the most frequently occurring datum (or data) in your data set. The mean, median, and mode are extremely helpful when you need to analyze your data, but if your data set consists of ranges which lack specific values, the mean may seem impossible to calculate. However, the mean can be approximated if you add the lower boundary with the upper boundary and divide by two to find the midpoint of each interval. Multiply each midpoint by the number of values found in the corresponding range. Divide the sum of these values by the total number of data values in the set.

Formula Review

$$\mu = \frac{\sum fm}{\sum f}$$

Where f = interval frequencies and m = interval midpoints.

Exercise:

Problem: Find the mean for the following frequency tables.

a.

Grade	Frequency
49.5–59.5	2
59.5–69.5	3
69.5–79.5	8
79.5–89.5	12
89.5–99.5	5

b.

Daily Low Temperature	Frequency
49.5–59.5	53
59.5–69.5	32
69.5–79.5	15
79.5–89.5	1

Daily Low Temperature	Frequency
89.5–99.5	0

c.

Points per Game	Frequency
49.5–59.5	14
59.5–69.5	32
69.5–79.5	15
79.5–89.5	23
89.5–99.5	2

Use the following information to answer the next three exercises: The following data show the lengths of boats moored in a marina. The data are ordered from smallest to largest:

161719202021232425252526262727272829303233333435373940

Exercise:

Problem: Calculate the mean.

Solution:

Mean: $16 + 17 + 19 + 20 + 20 + 21 + 23 + 24 + 25 + 25 + 25 + 26 + 26 + 27 + 27 + 27 + 28 + 29 + 30 + 32 + 33 + 33 + 34 + 35 + 37 + 39 + 40 = 738$;

$$\frac{738}{27} = 27.33$$

Exercise:

Problem: Identify the median.

Exercise:

Problem: Identify the mode.

Solution:

The most frequent lengths are 25 and 27, which occur three times. Mode = 25, 27

Use the following information to answer the next three exercises: Sixty-five randomly selected car

salespersons were asked the number of cars they generally sell in one week. Fourteen people answered that they generally sell three cars; nineteen generally sell four cars; twelve generally sell five cars; nine generally sell six cars; eleven generally sell seven cars. Calculate the following:

Exercise:

Problem: sample mean = $\bar{x}$ = _____

Exercise:

Problem: median = _____

Solution:

4

Exercise:

Problem: mode = _____

Homework

Exercise:

Problem:

The most obese countries in the world have obesity rates that range from 11.4% to 74.6%. This data is summarized in the following table.

Percent of Population Obese	Number of Countries
11.4–20.45	29
20.45–29.45	13
29.45–38.45	4
38.45–47.45	0
47.45–56.45	2
56.45–65.45	1
65.45–74.45	0
74.45–83.45	1

- What is the best estimate of the average obesity percentage for these countries?
- The United States has an average obesity rate of 33.9%. Is this rate above average or below?
- How does the United States compare to other countries?

Exercise:

Problem:

[link](#) gives the percent of children under five considered to be underweight. What is the best estimate for the mean percentage of underweight children?

Percent of Underweight Children	Number of Countries
16–21.45	23
21.45–26.9	4
26.9–32.35	9
32.35–37.8	7
37.8–43.25	6
43.25–48.7	1

Solution:

The mean percentage, $x = \frac{1328.65}{50} = 26.75$

Bringing It Together

Exercise:

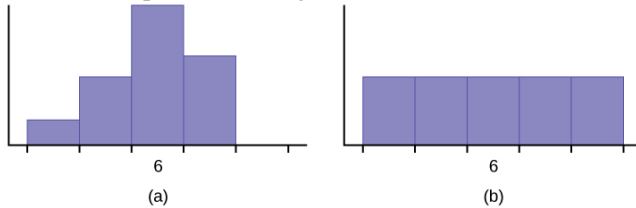
Problem:

Javier and Ercilia are supervisors at a shopping mall. Each was given the task of estimating the mean distance that shoppers live from the mall. They each randomly surveyed 100 shoppers. The samples yielded the following information.

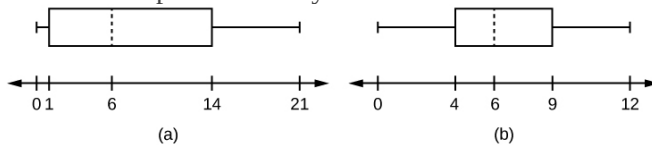
	Javier	Ercilia
--	---------------	----------------

	Javier	Ercilia
$\bar{x}$	6.0 miles	6.0 miles
s	4.0 miles	7.0 miles

- How can you determine which survey was correct ?
- Explain what the difference in the results of the surveys implies about the data.
- If the two histograms depict the distribution of values for each supervisor, which one depicts Ercilia's sample? How do you know?



- If the two box plots depict the distribution of values for each supervisor, which one depicts Ercilia's sample? How do you know?



Use the following information to answer the next three exercises: We are interested in the number of years students in a particular elementary statistics class have lived in California. The information in the following table is from the entire section.

Number of years	Frequency	Number of years	Frequency
7	1	22	1
14	3	23	1
15	1	26	1
18	1	40	2
19	4	42	2
20	3		
			Total = 20

Exercise:**Problem:** What is the *IQR*?

- a. 8
- b. 11
- c. 15
- d. 35

Solution:

a

Exercise:**Problem:** What is the mode?

- a. 19
- b. 19.5
- c. 14 and 20
- d. 22.65

Exercise:**Problem:** Is this a sample or the entire population?

- a. sample
- b. entire population
- c. neither

Solution:

b

Glossary**Frequency Table**

a data representation in which grouped data is displayed along with the corresponding frequencies

Mean

a number that measures the central tendency of the data; a common name for mean is 'average.' The term 'mean' is a shortened form of 'arithmetic mean.' By definition, the mean for a sample (denoted by x) is $x = \frac{\text{Sum of all values in the sample}}{\text{Number of values in the sample}}$, and the mean for a population (denoted by μ) is $\mu = \frac{\text{Sum of all values in the population}}{\text{Number of values in the population}}$.

Median

a number that separates ordered data into halves; half the values are the same number or smaller than the median and half the values are the same number or larger than the median. The median may

or may not be part of the data.

Midpoint

the mean of an interval in a frequency table

Mode

the value that appears most frequently in a set of data

Independent and Mutually Exclusive Events

Independent and mutually exclusive do **not** mean the same thing.

Independent Events

Two events are independent if the following are true:

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \text{ AND } B) = P(A)P(B)$

Two events A and B are **independent** if the knowledge that one occurred does not affect the chance the other occurs. For example, the outcomes of two rolls of a fair die are independent events. The outcome of the first roll does not change the probability for the outcome of the second roll. To show two events are independent, you must show **only one** of the above conditions. If two events are NOT independent, then we say that they are **dependent**.

Sampling may be done **with replacement** or **without replacement**.

- **With replacement:** If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once. When sampling is done with replacement, then events are considered to be independent, meaning the result of the first pick will not change the probabilities for the second pick.
- **Without replacement:** When sampling is done without replacement, each member of a population may be chosen only once. In this case, the probabilities for the second pick are affected by the result of the first pick. The events are considered to be dependent or not independent.

If it is not known whether A and B are independent or dependent, **assume they are dependent until you can show otherwise**.

Example:

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), *K* (king) of that suit.

a. Sampling with replacement:

Suppose you pick three cards with replacement. The first card you pick out of the 52 cards is the *Q* of spades. You put this card back, reshuffle the cards and pick a second card from the 52-card deck. It is the ten of clubs. You put this card back, reshuffle the cards and pick a third card from the 52-card deck. This time, the card is the *Q* of spades again. Your picks are {*Q* of spades, ten of clubs, *Q* of spades}. You have picked the *Q* of spades twice. You pick each card from the 52-card deck.

b. Sampling without replacement:

Suppose you pick three cards without replacement. The first card you pick out of the 52 cards is the *K* of hearts. You put this card aside and pick the second card from the 51 cards remaining in the deck. It is the three of diamonds. You put this card aside and pick the third card from the remaining 50 cards in the deck. The third card is the *J* of spades. Your picks are {*K* of hearts, three of diamonds, *J* of spades}. Because you have picked the cards without replacement, you cannot pick the same card twice.

Note:

Try It

Exercise:**Problem:**

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), *K* (king) of that suit. Three cards are picked at random.

- a. Suppose you know that the picked cards are *Q* of spades, *K* of hearts and *Q* of spades. Can you decide if the sampling was with

or without replacement?

- b. Suppose you know that the picked cards are Q of spades, K of hearts, and J of spades. Can you decide if the sampling was with or without replacement?

Solution:

- a. With replacement
b. No

Example:

Exercise:

Problem:

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts, and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J (jack), Q (queen), and K (king) of that suit. S = spades, H = Hearts, D = Diamonds, C = Clubs.

- a. Suppose you pick four cards, but do not put any cards back into the deck. Your cards are QS , $1D$, $1C$, QD .
b. Suppose you pick four cards and put each card back before you pick the next card. Your cards are KH , $7D$, $6D$, KH .

Which of a. or b. did you sample with replacement and which did you sample without replacement?

Solution:

- a. Without replacement; b. With replacement

Note:

Try It

Exercise:**Problem:**

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts, and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), and *K* (king) of that suit. *S* = spades, *H* = Hearts, *D* = Diamonds, *C* = Clubs. Suppose that you sample four cards without replacement. Which of the following outcomes are possible? Answer the same question for sampling with replacement.

- a. *QS*, *1D*, *1C*, *QD*
- b. *KH*, *7D*, *6D*, *KH*
- c. *QS*, *7D*, *6D*, *KS*

Solution:

without replacement: 1. Possible; 2. Impossible, 3. Possible

with replacement: 1. Possible; 2. Possible, 3. Possible

Mutually Exclusive Events

A and *B* are **mutually exclusive** events if they cannot occur at the same time. This means that *A* and *B* do not share any outcomes and $P(A \text{ AND } B) = 0$.

For example, suppose the sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7, 8\}$, and $C = \{7, 9\}$. $A \text{ AND } B = \{4, 5\}$. $P(A \text{ AND } B) = \frac{2}{10}$ and is not equal to zero. Therefore, *A* and *B* are not mutually exclusive. *A* and *C* do not have any numbers in common so $P(A \text{ AND } C) = 0$. Therefore, *A* and *C* are mutually exclusive.

If it is not known whether A and B are mutually exclusive, **assume they are not until you can show otherwise**. The following examples illustrate these definitions and terms.

Example:

Flip two fair coins. (This is an experiment.)

The sample space is $\{HH, HT, TH, TT\}$ where T = tails and H = heads. The outcomes are HH, HT, TH , and TT . The outcomes HT and TH are different. The HT means that the first coin showed heads and the second coin showed tails. The TH means that the first coin showed tails and the second coin showed heads.

- Let A = the event of getting **at most one tail**. (At most one tail means zero or one tail.) Then A can be written as $\{HH, HT, TH\}$. The outcome HH shows zero tails. HT and TH each show one tail.
- Let B = the event of getting all tails. B can be written as $\{TT\}$. B is the **complement** of A , so $B = A'$. Also, $P(A) + P(B) = P(A) + P(A') = 1$.
- The probabilities for A and for B are $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{4}$.
- Let C = the event of getting all heads. $C = \{HH\}$. Since $B = \{TT\}$, $P(B \text{ AND } C) = 0$. B and C are mutually exclusive. (B and C have no members in common because you cannot have all tails and all heads at the same time.)
- Let D = event of getting **more than one tail**. $D = \{TT\}$. $P(D) = \frac{1}{4}$
- Let E = event of getting a head on the first roll. (This implies you can get either a head or tail on the second roll.) $E = \{HT, HH\}$. $P(E) = \frac{2}{4}$
- Find the probability of getting **at least one** (one or two) tail in two flips. Let F = event of getting at least one tail in two flips. $F = \{HT, TH, TT\}$. $P(F) = \frac{3}{4}$

Note:

Try It

Exercise:

Problem:

Draw two cards from a standard 52-card deck with replacement. Find the probability of getting at least one black card.

Solution:**Try It Solutions**

The sample space of drawing two cards with replacement from a standard 52-card deck with respect to color is $\{BB, BR, RB, RR\}$.

Event A = Getting at least one black card = $\{BB, BR, RB\}$

$$P(A) = \frac{3}{4} = 0.75$$

Example:**Exercise:**

Problem: Flip two fair coins. Find the probabilities of the events.

- Let F = the event of getting at most one tail (zero or one tail).
- Let G = the event of getting two faces that are the same.
- Let H = the event of getting a head on the first flip followed by a head or tail on the second flip.
- Are F and G mutually exclusive?
- Let J = the event of getting all tails. Are J and H mutually exclusive?

Solution:

Look at the sample space in [\[link\]](#).

- Zero (0) or one (1) tails occur when the outcomes HH, TH, HT show up. $P(F) = \frac{3}{4}$

- b. Two faces are the same if HH or TT show up. $P(G) = \frac{2}{4}$
- c. A head on the first flip followed by a head or tail on the second flip occurs when HH or HT show up. $P(H) = \frac{2}{4}$
- d. F and G share HH so $P(F \text{ AND } G)$ is not equal to zero (0). F and G are not mutually exclusive.
- e. Getting all tails occurs when tails shows up on both coins (TT). H 's outcomes are HH and HT .

J and H have nothing in common so $P(J \text{ AND } H) = 0$. J and H are mutually exclusive.

Note:

Try It

Exercise:

Problem:

A box has two balls, one white and one red. We select one ball, put it back in the box, and select a second ball (sampling with replacement). Find the probability of the following events:

- a. Let F = the event of getting the white ball twice.
- b. Let G = the event of getting two balls of different colors.
- c. Let H = the event of getting white on the first pick.
- d. Are F and G mutually exclusive?
- e. Are G and H mutually exclusive?

Solution:

- a. $P(F) = \frac{1}{4}$
- b. $P(G) = \frac{1}{2}$
- c. $P(H) = \frac{1}{2}$
- d. Yes
- e. No

Example:

Roll one fair, six-sided die. The sample space is $\{1, 2, 3, 4, 5, 6\}$. Let event A = a face is odd. Then $A = \{1, 3, 5\}$. Let event B = a face is even. Then $B = \{2, 4, 6\}$.

- Find the complement of A , A' . The complement of A , A' , is B because A and B together make up the sample space. $P(A) + P(B) = P(A) + P(A') = 1$. Also, $P(A) = \frac{3}{6}$ and $P(B) = \frac{3}{6}$.
- Let event C = odd faces larger than two. Then $C = \{3, 5\}$. Let event D = all even faces smaller than five. Then $D = \{2, 4\}$. $P(C \text{ AND } D) = 0$ because you cannot have an odd and even face at the same time. Therefore, C and D are mutually exclusive events.
- Let event E = all faces less than five. $E = \{1, 2, 3, 4\}$.

Exercise:**Problem:**

Are C and E mutually exclusive events? (Answer yes or no.) Why or why not?

Solution:

No. $C = \{3, 5\}$ and $E = \{1, 2, 3, 4\}$. $P(C \text{ AND } E) = \frac{1}{6}$. To be mutually exclusive, $P(C \text{ AND } E)$ must be zero.

- Find $P(C|A)$. This is a conditional probability. Recall that the event C is $\{3, 5\}$ and event A is $\{1, 3, 5\}$. To find $P(C|A)$, find the probability of C using the sample space A . You have reduced the sample space from the original sample space $\{1, 2, 3, 4, 5, 6\}$ to $\{1, 3, 5\}$. So, $P(C|A) = \frac{2}{3}$.

Note:

Try It

Exercise:**Problem:**

Let event A = learning Spanish. Let event B = learning German. Then $A \text{ AND } B$ = learning Spanish and German. Suppose $P(A) = 0.4$ and $P(B) = 0.2$. $P(A \text{ AND } B) = 0.08$. Are events A and B independent?

Hint: You must show ONE of the following:

- $P(A|B) = P(A)$
- $P(B|A) = P(B)$
- $P(A \text{ AND } B) = P(A)P(B)$

Solution:

$$P(A|B) = \frac{P(A \text{ AND } B)}{P(B)} = \frac{0.08}{0.2} = 0.4 = P(A)$$

The events are independent because $P(A|B) = P(A)$.

Example:

Let event G = taking a math class. Let event H = taking a science class. Then, $G \text{ AND } H$ = taking a math class and a science class. Suppose $P(G) = 0.6$, $P(H) = 0.5$, and $P(G \text{ AND } H) = 0.3$. Are G and H independent?

If G and H are independent, then you must show **ONE** of the following:

- $P(G|H) = P(G)$
- $P(H|G) = P(H)$
- $P(G \text{ AND } H) = P(G)P(H)$

Note:**NOTE**

The choice you make depends on the information you have. You could choose any of the methods here because you have the necessary information.

Exercise:

Problem: a. Show that $P(G|H) = P(G)$.

Solution:

$$P(G|H) = \frac{P(G \text{ AND } H)}{P(H)} = \frac{0.3}{0.5} = 0.6 = P(G)$$

Exercise:

Problem: b. Show $P(G \text{ AND } H) = P(G)P(H)$.

Solution:

$$P(G)P(H) = (0.6)(0.5) = 0.3 = P(G \text{ AND } H)$$

Since G and H are independent, knowing that a person is taking a science class does not change the chance that he or she is taking a math class. If the two events had not been independent (that is, they are dependent) then knowing that a person is taking a science class would change the chance he or she is taking math. For practice, show that $P(H|G) = P(H)$ to show that G and H are independent events.

Note:

Try It

Exercise:

Problem:

In a bag, there are six red marbles and four green marbles. The red marbles are marked with the numbers 1, 2, 3, 4, 5, and 6. The green marbles are marked with the numbers 1, 2, 3, and 4.

- R = a red marble
- G = a green marble
- O = an odd-numbered marble
- The sample space is $S = \{R1, R2, R3, R4, R5, R6, G1, G2, G3, G4\}$.

S has ten outcomes. What is $P(G \text{ AND } O)$?

Solution:

Event G and $O = \{G1, G3\}$

$$P(G \text{ and } O) = \frac{2}{10} = 0.2$$

Example:**Exercise:****Problem:**

Let event C = taking an English class. Let event D = taking a speech class.

Suppose $P(C) = 0.75$, $P(D) = 0.3$, $P(C|D) = 0.75$ and $P(C \text{ AND } D) = 0.225$.

Justify your answers to the following questions numerically.

- a. Are C and D independent?
- b. Are C and D mutually exclusive?
- c. What is $P(D|C)$?

Solution:

- a. Yes, because $P(C|D) = P(C)$.
- b. No, because $P(C \text{ AND } D)$ is not equal to zero.
- c. $P(D|C) = \frac{P(C \text{ AND } D)}{P(C)} = \frac{0.225}{0.75} = 0.3$

Note:

Try It

Exercise:**Problem:**

A student goes to the library. Let events B = the student checks out a book and D = the student checks out a DVD. Suppose that $P(B) = 0.40$, $P(D) = 0.30$ and $P(B \text{ AND } D) = 0.20$.

- a. Find $P(B|D)$.
- b. Find $P(D|B)$.
- c. Are B and D independent?
- d. Are B and D mutually exclusive?

Solution:

- a. $P(B|D) = 0.6667$
- b. $P(D|B) = 0.5$
- c. No
- d. No

Example:

In a box there are three red cards and five blue cards. The red cards are marked with the numbers 1, 2, and 3, and the blue cards are marked with the numbers 1, 2, 3, 4, and 5. The cards are well-shuffled. You reach into the box (you cannot see into it) and draw one card.

Let R = red card is drawn, B = blue card is drawn, E = even-numbered card is drawn.

The sample space $S = R1, R2, R3, B1, B2, B3, B4, B5$. S has eight outcomes.

- $P(R) = \frac{3}{8}$. $P(B) = \frac{5}{8}$. $P(R \text{ AND } B) = 0$. (You cannot draw one card that is both red and blue.)
- $P(E) = \frac{3}{8}$. (There are three even-numbered cards, $R2$, $B2$, and $B4$.)
- $P(E|B) = \frac{2}{5}$. (There are five blue cards: $B1$, $B2$, $B3$, $B4$, and $B5$. Out of the blue cards, there are two even cards; $B2$ and $B4$.)
- $P(B|E) = \frac{2}{3}$. (There are three even-numbered cards: $R2$, $B2$, and $B4$. Out of the even-numbered cards, two are blue; $B2$ and $B4$.)
- The events R and B are mutually exclusive because $P(R \text{ AND } B) = 0$.
- Let G = card with a number greater than 3. $G = \{B4, B5\}$. $P(G) = \frac{2}{8}$. Let H = blue card numbered between one and four, inclusive. $H = \{B1, B2, B3, B4\}$. $P(G|H) = \frac{1}{4}$. (The only card in H that has a number greater than three is $B4$.) Since $\frac{2}{8} = \frac{1}{4}$, $P(G) = P(G|H)$, which means that G and H are independent.

Note:

Try It

Exercise:

Problem: In a basketball arena,

- 70% of the fans are rooting for the home team.
- 25% of the fans are wearing blue.
- 20% of the fans are wearing blue and are rooting for the away team.

- Of the fans rooting for the away team, 67% are wearing blue.

Let A be the event that a fan is rooting for the away team.

Let B be the event that a fan is wearing blue.

Are the events of rooting for the away team and wearing blue independent? Are they mutually exclusive?

Solution:

$$P(B|A) = 0.67$$

$$P(B) = 0.25$$

So $P(B)$ does not equal $P(B|A)$ which means that B and A are not independent (wearing blue and rooting for the away team are not independent). They are also not mutually exclusive, because $P(B \text{ AND } A) = 0.20$, not 0.

Example:

In a particular college class, 60% of the students are female. Fifty percent of all students in the class have long hair. Forty-five percent of the students are female and have long hair. Of the female students, 75% have long hair. Let F be the event that a student is female. Let L be the event that a student has long hair. One student is picked randomly. Are the events of being female and having long hair independent?

- The following probabilities are given in this example:
- $P(F) = 0.60$; $P(L) = 0.50$
- $P(F \text{ AND } L) = 0.45$
- $P(L|F) = 0.75$

Note:

NOTE

The choice you make depends on the information you have. You could use the first or last condition on the list for this example. You do not know $P(F|L)$ yet, so you cannot use the second condition.

Solution 1

Check whether $P(F \text{ AND } L) = P(F)P(L)$. We are given that $P(F \text{ AND } L) = 0.45$, but $P(F)P(L) = (0.60)(0.50) = 0.30$. The events of being female and having long hair are not independent because $P(F \text{ AND } L)$ does not equal $P(F)P(L)$.

Solution 2

Check whether $P(L|F)$ equals $P(L)$. We are given that $P(L|F) = 0.75$, but $P(L) = 0.50$; they are not equal. The events of being female and having long hair are not independent.

Interpretation of Results

The events of being female and having long hair are not independent; knowing that a student is female changes the probability that a student has long hair.

Note:

Try It

Exercise:

Problem:

Mark is deciding which route to take to work. His choices are I = the Interstate and F = Fifth Street.

- $P(I) = 0.44$ and $P(F) = 0.56$
- $P(I \text{ AND } F) = 0$ because Mark will take only one route to work.

What is the probability of $P(I \text{ OR } F)$?

Solution:

Because $P(I \text{ AND } F) = 0$,

$$P(I \text{ OR } F) = P(I) + P(F) - P(I \text{ AND } F) = 0.44 + 0.56 - 0 = 1$$

Example:**Exercise:****Problem:**

- a. Toss one fair coin (the coin has two sides, H and T). The outcomes are _____. Count the outcomes. There are _____ outcomes.
- b. Toss one fair, six-sided die (the die has 1, 2, 3, 4, 5 or 6 dots on a side). The outcomes are _____. Count the outcomes. There are _____ outcomes.
- c. Multiply the two numbers of outcomes. The answer is _____.
- d. If you flip one fair coin and follow it with the toss of one fair, six-sided die, the answer in part c. is the number of outcomes (size of the sample space). What are the outcomes? (Hint: Two of the outcomes are $H1$ and $T6$.)
- e. Event A = heads (H) on the coin followed by an even number (2, 4, 6) on the die.
 $A = \{ \text{_____} \}$. Find $P(A)$.
- f. Event B = heads on the coin followed by a three on the die. $B = \{ \text{_____} \}$. Find $P(B)$.
- g. Are A and B mutually exclusive? (Hint: What is $P(A \text{ AND } B)$? If $P(A \text{ AND } B) = 0$, then A and B are mutually exclusive.)
- h. Are A and B independent? (Hint: Is $P(A \text{ AND } B) = P(A)P(B)$? If $P(A \text{ AND } B) = P(A)P(B)$, then A and B are independent. If not, then they are dependent).

Solution:

- a. H and T ; 2
- b. 1, 2, 3, 4, 5, 6; 6
- c. $2(6) = 12$
- d. $T1, T2, T3, T4, T5, T6, H1, H2, H3, H4, H5, H6$

- e. $A = \{H2, H4, H6\}; P(A) = \frac{3}{12}$
- f. $B = \{H3\}; P(B) = \frac{1}{12}$
- g. Yes, because $P(A \text{ AND } B) = 0$
- h. $P(A \text{ AND } B) = 0. P(A)P(B) = \left(\frac{3}{12}\right)\left(\frac{1}{12}\right)$. $P(A \text{ AND } B)$ does not equal $P(A)P(B)$, so A and B are dependent.

Note:

Try It

Exercise:

Problem:

A box has two balls, one white and one red. We select one ball, put it back in the box, and select a second ball (sampling with replacement). Let T be the event of getting the white ball twice, F the event of picking the white ball first, S the event of picking the white ball in the second drawing.

- a. Compute $P(T)$.
- b. Compute $P(T|F)$.
- c. Are T and F independent?
- d. Are F and S mutually exclusive?
- e. Are F and S independent?

Solution:

- a. $P(T) = \frac{1}{4}$
- b. $P(T|F) = \frac{1}{2}$
- c. No
- d. No
- e. Yes

References

Lopez, Shane, Preeti Sidhu. "U.S. Teachers Love Their Lives, but Struggle in the Workplace." Gallup Wellbeing, 2013.

<http://www.gallup.com/poll/161516/teachers-love-lives-struggle-workplace.aspx> (accessed May 2, 2013).

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Chapter Review

Two events A and B are independent if the knowledge that one occurred does not affect the chance the other occurs. If two events are not independent, then we say that they are dependent.

In sampling with replacement, each member of a population is replaced after it is picked, so that member has the possibility of being chosen more than once, and the events are considered to be independent. In sampling without replacement, each member of a population may be chosen only once, and the events are considered not to be independent. When events do not share outcomes, they are mutually exclusive of each other.

Formula Review

If A and B are independent, $P(A \text{ AND } B) = P(A)P(B)$, $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

If A and B are mutually exclusive, $P(A \text{ OR } B) = P(A) + P(B)$ and $P(A \text{ AND } B) = 0$.

Exercise:

Problem:

E and F are mutually exclusive events. $P(E) = 0.4$; $P(F) = 0.5$. Find $P(E|F)$.

Exercise:

Problem: J and K are independent events. $P(J|K) = 0.3$. Find $P(J)$.

Solution:

$$P(J) = 0.3$$

Exercise:

Problem:

U and V are mutually exclusive events. $P(U) = 0.26$; $P(V) = 0.37$.
Find:

- a. $P(U \text{ AND } V) =$
- b. $P(U|V) =$
- c. $P(U \text{ OR } V) =$

Exercise:

Problem:

Q and R are independent events. $P(Q) = 0.4$ and $P(Q \text{ AND } R) = 0.1$.
Find $P(R)$.

Solution:

$$P(Q \text{ AND } R) = P(Q)P(R)$$

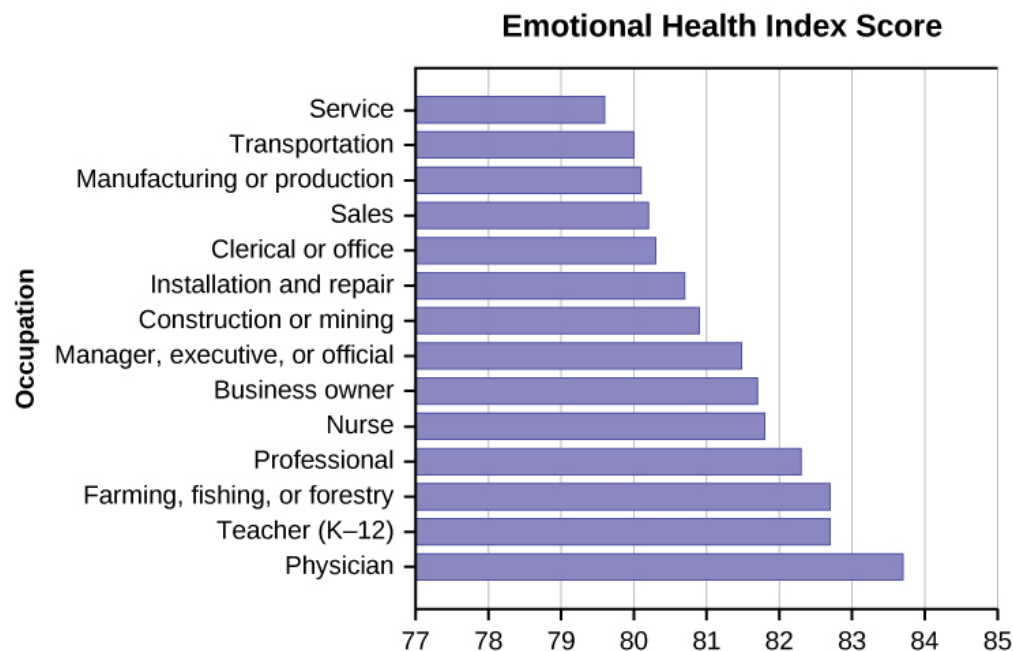
$$0.1 = (0.4)P(R)$$

$$P(R) = 0.25$$

Homework

Use the following information to answer the next 12 exercises. The graph shown is based on more than 170,000 interviews done by Gallup that took place from January through December 2012. The sample consists of

employed Americans 18 years of age or older. The Emotional Health Index Scores are the sample space. We randomly sample one Emotional Health Index Score.



Exercise:

Problem:

Find the probability that an Emotional Health Index Score is 82.7.

Exercise:

Problem:

Find the probability that an Emotional Health Index Score is 81.0.

Solution:

0

Exercise:

Problem:

Find the probability that an Emotional Health Index Score is more than 81?

Exercise:**Problem:**

Find the probability that an Emotional Health Index Score is between 80.5 and 82?

Solution:

0.3571

Exercise:**Problem:**

If we know an Emotional Health Index Score is 81.5 or more, what is the probability that it is 82.7?

Exercise:**Problem:**

What is the probability that an Emotional Health Index Score is 80.7 or 82.7?

Solution:

0.2142

Exercise:**Problem:**

What is the probability that an Emotional Health Index Score is less than 80.2 given that it is already less than 81.

Exercise:

Problem: What occupation has the highest emotional index score?

Solution:

Physician (83.7)

Exercise:

Problem: What occupation has the lowest emotional index score?

Exercise:

Problem: What is the range of the data?

Solution:

$$83.7 - 79.6 = 4.1$$

Exercise:

Problem: Compute the average EHIS.

Exercise:

Problem:

If all occupations are equally likely for a certain individual, what is the probability that he or she will have an occupation with lower than average EHIS?

Solution:

$$P(\text{Occupation} < 81.3) = 0.5$$

Bringing It Together

Exercise:

Problem:

A previous year, the weights of the members of the **San Francisco 49ers** and the **Dallas Cowboys** were published in the *San Jose Mercury News*. The factual data are compiled into [\[link\]](#).

Shirt#	≤ 210	211–250	251–290	$290 \leq$
1–33	21	5	0	0
34–66	6	18	7	4
66–99	6	12	22	5

For the following, suppose that you randomly select one player from the 49ers or Cowboys.

If having a shirt number from one to 33 and weighing at most 210 pounds were independent events, then what should be true about $P(\text{Shirt# } 1\text{--}33 | \leq 210 \text{ pounds})$?

Exercise:**Problem:**

The probability that a male develops some form of cancer in his lifetime is 0.4567. The probability that a male has at least one false positive test result (meaning the test comes back for cancer when the man does not have it) is 0.51. Some of the following questions do not have enough information for you to answer them. Write “not enough information” for those answers. Let C = a man develops cancer in his lifetime and P = man has at least one false positive.

- a. $P(C) = \underline{\hspace{2cm}}$
 - b. $P(P|C) = \underline{\hspace{2cm}}$
 - c. $P(P|C') = \underline{\hspace{2cm}}$
 - d. If a test comes up positive, based upon numerical values, can you assume that man has cancer? Justify numerically and explain why or why not.
-

Solution:

- a. $P(C) = 0.4567$
- b. not enough information
- c. not enough information
- d. No, because over half (0.51) of men have at least one false positive text

Exercise:

Problem:

Given events G and H : $P(G) = 0.43$; $P(H) = 0.26$; $P(H \text{ AND } G) = 0.14$

- a. Find $P(H \text{ OR } G)$.
- b. Find the probability of the complement of event $(H \text{ AND } G)$.
- c. Find the probability of the complement of event $(H \text{ OR } G)$.

Exercise:

Problem:

Given events J and K : $P(J) = 0.18$; $P(K) = 0.37$; $P(J \text{ OR } K) = 0.45$

- a. Find $P(J \text{ AND } K)$.
 - b. Find the probability of the complement of event $(J \text{ AND } K)$.
 - c. Find the probability of the complement of event $(J \text{ OR } K)$.
-

Solution:

- a. $P(J \text{ OR } K) = P(J) + P(K) - P(J \text{ AND } K)$; $0.45 = 0.18 + 0.37 - P(J \text{ AND } K)$; solve to find $P(J \text{ AND } K) = 0.10$
- b. $P(\text{NOT } (J \text{ AND } K)) = 1 - P(J \text{ AND } K) = 1 - 0.10 = 0.90$
- c. $P(\text{NOT } (J \text{ OR } K)) = 1 - P(J \text{ OR } K) = 1 - 0.45 = 0.55$

Glossary

Dependent Events

If two events are NOT independent, then we say that they are dependent.

Sampling with Replacement

If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once.

Sampling without Replacement

When sampling is done without replacement, each member of a population may be chosen only once.

The Conditional Probability of One Event Given Another Event

$P(A|B)$ is the probability that event A will occur given that the event B has already occurred.

The OR of Two Events

An outcome is in the event $A \text{ OR } B$ if the outcome is in A , is in B , or is in both A and B .

Two Basic Rules of Probability

When calculating probability, there are two rules to consider when determining if two events are independent or dependent and if they are mutually exclusive or not.

The Multiplication Rule

If A and B are two events defined on a **sample space**, then: $P(A \text{ AND } B) = P(B)P(A|B)$.

This rule may also be written as: $P(A|B) = \frac{P(A \text{ AND } B)}{P(B)}$

(The probability of A given B equals the probability of A and B divided by the probability of B .)

If A and B are **independent**, then $P(A|B) = P(A)$. Then $P(A \text{ AND } B) = P(A|B)P(B)$ becomes $P(A \text{ AND } B) = P(A)P(B)$.

The Addition Rule

If A and B are defined on a sample space, then: $P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$.

If A and B are **mutually exclusive**, then $P(A \text{ AND } B) = 0$. Then $P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$ becomes $P(A \text{ OR } B) = P(A) + P(B)$.

Example:

Klaus is trying to choose where to go on vacation. His two choices are: A = New Zealand and B = Alaska

- Klaus can only afford one vacation. The probability that he chooses A is $P(A) = 0.6$ and the probability that he chooses B is $P(B) = 0.35$.
- $P(A \text{ AND } B) = 0$ because Klaus can only afford to take one vacation
- Therefore, the probability that he chooses either New Zealand or Alaska is $P(A \text{ OR } B) = P(A) + P(B) = 0.6 + 0.35 = 0.95$. Note that the

probability that he does not choose to go anywhere on vacation must be 0.05.

Example:

Carlos plays college soccer. He makes a goal 65% of the time he shoots. Carlos is going to attempt two goals in a row in the next game. A = the event Carlos is successful on his first attempt. $P(A) = 0.65$. B = the event Carlos is successful on his second attempt. $P(B) = 0.65$. Carlos tends to shoot in streaks. The probability that he makes the second goal **GIVEN** that he made the first goal is 0.90.

Exercise:

Problem: a. What is the probability that he makes both goals?

Solution:

a. The problem is asking you to find $P(A \text{ AND } B) = P(B \text{ AND } A)$. Since $P(B|A) = 0.90$: $P(B \text{ AND } A) = P(B|A) P(A) = (0.90)(0.65) = 0.585$

Carlos makes the first and second goals with probability 0.585.

Exercise:

Problem:

b. What is the probability that Carlos makes either the first goal or the second goal?

Solution:

b. The problem is asking you to find $P(A \text{ OR } B)$.

$$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B) = 0.65 + 0.65 - 0.585 = 0.715$$

Carlos makes either the first goal or the second goal with probability 0.715.

Exercise:

Problem: c. Are A and B independent?

Solution:

c. No, they are not, because $P(B \text{ AND } A) = 0.585$.

$$P(B)P(A) = (0.65)(0.65) = 0.423$$

$$0.423 \neq 0.585 = P(B \text{ AND } A)$$

So, $P(B \text{ AND } A)$ is **not** equal to $P(B)P(A)$.

Exercise:

Problem: d. Are A and B mutually exclusive?

Solution:

d. No, they are not because $P(A \text{ and } B) = 0.585$.

To be mutually exclusive, $P(A \text{ AND } B)$ must equal zero.

Note:

Try It

Exercise:

Problem:

Helen plays basketball. For free throws, she makes the shot 75% of the time. Helen must now attempt two free throws. C = the event that Helen makes the first shot. $P(C) = 0.75$. D = the event Helen makes the second shot. $P(D) = 0.75$. The probability that Helen makes the second free throw given that she made the first is 0.85. What is the probability that Helen makes both free throws?

Solution:

$$P(D|C) = 0.85$$

$$P(C \text{ AND } D) = P(D \text{ AND } C)$$

$$P(D \text{ AND } C) = P(D|C)P(C) = (0.85)(0.75) = 0.6375$$

Helen makes the first and second free throws with probability 0.6375.

Example:

A community swim team has **150** members. **Seventy-five** of the members are advanced swimmers. **Forty-seven** of the members are intermediate swimmers. The remainder are novice swimmers. **Forty** of the advanced swimmers practice four times a week. **Thirty** of the intermediate swimmers practice four times a week. **Ten** of the novice swimmers practice four times a week. Suppose one member of the swim team is chosen randomly.

Exercise:**Problem:**

a. What is the probability that the member is a novice swimmer?

Solution:

a. $\frac{28}{150}$

Exercise:

Problem:

b. What is the probability that the member practices four times a week?

Solution:

b. $\frac{80}{150}$

Exercise:**Problem:**

c. What is the probability that the member is an advanced swimmer and practices four times a week?

Solution:

c. $\frac{40}{150}$

Exercise:**Problem:**

d. What is the probability that a member is an advanced swimmer and an intermediate swimmer? Are being an advanced swimmer and an intermediate swimmer mutually exclusive? Why or why not?

Solution:

d. $P(\text{advanced AND intermediate}) = 0$, so these are mutually exclusive events. A swimmer cannot be an advanced swimmer and an intermediate swimmer at the same time.

Exercise:

Problem:

e. Are being a novice swimmer and practicing four times a week independent events? Why or why not?

Solution:

e. No, these are not independent events.

$$P(\text{novice AND practices four times per week}) = 0.0667$$

$$P(\text{novice})P(\text{practices four times per week}) = 0.0996$$

$$0.0667 \neq 0.0996$$

Note:

Try It

Exercise:**Problem:**

A school has 200 seniors of whom 140 will be going to college next year. Forty will be going directly to work. The remainder are taking a gap year. Fifty of the seniors going to college play sports. Thirty of the seniors going directly to work play sports. Five of the seniors taking a gap year play sports. What is the probability that a senior is taking a gap year?

Solution:

$$P = \frac{200 - 140 - 40}{200} = \frac{20}{200} = 0.1$$

Example:

Felicity attends Modesto JC in Modesto, CA. The probability that Felicity enrolls in a math class is 0.2 and the probability that she enrolls in a speech class is 0.65. The probability that she enrolls in a math class GIVEN that she enrolls in speech class is 0.25.

Let: M = math class, S = speech class, $M|S$ = math given speech

Exercise:

Problem:

- a. What is the probability that Felicity enrolls in math and speech?
Find $P(M \text{ AND } S) = P(M|S)P(S)$.
- b. What is the probability that Felicity enrolls in math or speech classes?
Find $P(M \text{ OR } S) = P(M) + P(S) - P(M \text{ AND } S)$.
- c. Are M and S independent? Is $P(M|S) = P(M)$?
- d. Are M and S mutually exclusive? Is $P(M \text{ AND } S) = 0$?

Solution:

a. 0.1625, b. 0.6875, c. No, d. No

Note:

Try It

Exercise:

Problem:

A student goes to the library. Let events B = the student checks out a book and D = the student check out a DVD. Suppose that $P(B) = 0.40$, $P(D) = 0.30$ and $P(D|B) = 0.5$.

- a. Find $P(B \text{ AND } D)$.
- b. Find $P(B \text{ OR } D)$.

Solution:

- a. $P(B \text{ AND } D) = P(D|B)P(B) = (0.5)(0.4) = 0.20$.
- b. $P(B \text{ OR } D) = P(B) + P(D) - P(B \text{ AND } D) = 0.40 + 0.30 - 0.20 = 0.50$

Example:

Studies show that about one woman in seven (approximately 14.3%) who live to be 90 will develop breast cancer. Suppose that of those women who develop breast cancer, a test is negative 2% of the time. Also suppose that in the general population of women, the test for breast cancer is negative about 85% of the time. Let B = woman develops breast cancer and let N = tests negative. Suppose one woman is selected at random.

Exercise:**Problem:**

- a. What is the probability that the woman develops breast cancer? What is the probability that woman tests negative?

Solution:

- a. $P(B) = 0.143$; $P(N) = 0.85$

Exercise:**Problem:**

- b. Given that the woman has breast cancer, what is the probability that she tests negative?

Solution:

- b. $P(N|B) = 0.02$

Exercise:**Problem:**

- c. What is the probability that the woman has breast cancer AND tests negative?

Solution:

c. $P(B \text{ AND } N) = P(B)P(N|B) = (0.143)(0.02) = 0.0029$

Exercise:**Problem:**

d. What is the probability that the woman has breast cancer or tests negative?

Solution:

d. $P(B \text{ OR } N) = P(B) + P(N) - P(B \text{ AND } N) = 0.143 + 0.85 - 0.0029 = 0.9901$

Exercise:**Problem:**

e. Are having breast cancer and testing negative independent events?

Solution:

e. No. $P(N) = 0.85$; $P(N|B) = 0.02$. So, $P(N|B)$ does not equal $P(N)$.

Exercise:**Problem:**

f. Are having breast cancer and testing negative mutually exclusive?

Solution:

f. No. $P(B \text{ AND } N) = 0.0029$. For B and N to be mutually exclusive, $P(B \text{ AND } N)$ must be zero.

Note:

Try It

Exercise:**Problem:**

A school has 200 seniors of whom 140 will be going to college next year. Forty will be going directly to work. The remainder are taking a gap year. Fifty of the seniors going to college play sports. Thirty of the seniors going directly to work play sports. Five of the seniors taking a gap year play sports. What is the probability that a senior is going to college and plays sports?

Solution:

Let A = student is a senior going to college.

Let B = student plays sports.

$$P(B) = \frac{140}{200}$$

$$P(B|A) = \frac{50}{140}$$

$$P(A \text{ AND } B) = P(B|A)P(A)$$

$$P(A \text{ AND } B) = \left(\frac{140}{200} \right) \left(\frac{50}{140} \right) = \frac{1}{4}$$

Example:**Exercise:**

Problem: Refer to the information in [\[link\]](#). P = tests positive.

- Given that a woman develops breast cancer, what is the probability that she tests positive. Find $P(P|B) = 1 - P(N|B)$.
- What is the probability that a woman develops breast cancer and tests positive. Find $P(B \text{ AND } P) = P(P|B)P(B)$.

- c. What is the probability that a woman does not develop breast cancer. Find $P(B') = 1 - P(B)$.
- d. What is the probability that a woman tests positive for breast cancer. Find $P(P) = 1 - P(N)$.

Solution:

a. 0.98; b. 0.1401; c. 0.857; d. 0.15

Note:

Try It

Exercise:

Problem:

A student goes to the library. Let events B = the student checks out a book and D = the student checks out a DVD. Suppose that $P(B) = 0.40$, $P(D) = 0.30$ and $P(D|B) = 0.5$.

- a. Find $P(B')$.
- b. Find $P(D \text{ AND } B)$.
- c. Find $P(B|D)$.
- d. Find $P(D \text{ AND } B')$.
- e. Find $P(D|B')$.

Solution:

- a. $P(B') = 0.60$
- b. $P(D \text{ AND } B) = P(D|B)P(B) = 0.20$
- c. $P(B|D) = \frac{P(B \text{ AND } D)}{P(D)} = \frac{(0.20)}{(0.30)} = 0.66$
- d. $P(D \text{ AND } B') = P(D) - P(D \text{ AND } B) = 0.30 - 0.20 = 0.10$
- e. $P(D|B') = P(D \text{ AND } B')P(B') = (P(D) - P(D \text{ AND } B))(0.60) = (0.10)(0.60) = 0.06$

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Chapter Review

The multiplication rule and the addition rule are used for computing the probability of A and B , as well as the probability of A or B for two given events A, B defined on the sample space. In sampling with replacement each member of a population is replaced after it is picked, so that member has the possibility of being chosen more than once, and the events are considered to be independent. In sampling without replacement, each member of a population may be chosen only once, and the events are considered to be not independent. The events A and B are mutually exclusive events when they do not have any outcomes in common.

Formula Review

The multiplication rule: $P(A \text{ AND } B) = P(A|B)P(B)$

The addition rule: $P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$

Use the following information to answer the next ten exercises. Forty-eight percent of all Californians registered voters prefer life in prison without parole over the death penalty for a person convicted of first degree murder. Among Latino California registered voters, 55% prefer life in prison without parole over the death penalty for a person convicted of first degree murder. 37.6% of all Californians are Latino.

In this problem, let:

- C = Californians (registered voters) preferring life in prison without parole over the death penalty for a person convicted of first degree murder.
- L = Latino Californians

Suppose that one Californian is randomly selected.

Exercise:

Problem: Find $P(C)$.

Exercise:

Problem: Find $P(L)$.

Solution:

0.376

Exercise:

Problem: Find $P(C|L)$.

Exercise:

Problem: In words, what is $C|L$?

Solution:

$C|L$ means, given the person chosen is a Latino Californian, the person is a registered voter who prefers life in prison without parole for a person convicted of first degree murder.

Exercise:

Problem: Find $P(L \text{ AND } C)$.

Exercise:

Problem: In words, what is $L \text{ AND } C$?

Solution:

$L \text{ AND } C$ is the event that the person chosen is a Latino California registered voter who prefers life without parole over the death penalty for a person convicted of first degree murder.

Exercise:

Problem: Are L and C independent events? Show why or why not.

Exercise:

Problem: Find $P(L \text{ OR } C)$.

Solution:

0.6492

Exercise:

Problem: In words, what is $L \text{ OR } C$?

Exercise:

Problem:

Are L and C mutually exclusive events? Show why or why not.

Solution:

No, because $P(L \text{ AND } C)$ does not equal 0.

Homework

Exercise:

Problem:

On February 28, 2013, a Field Poll Survey reported that 61% of California registered voters approved of allowing two people of the same gender to marry and have regular marriage laws apply to them. Among 18 to 39 year olds (California registered voters), the approval rating was 78%. Six in ten California registered voters said that the upcoming Supreme Court's ruling about the constitutionality of California's Proposition 8 was either very or somewhat important to them. Out of those CA registered voters who support same-sex marriage, 75% say the ruling is important to them.

In this problem, let:

- C = California registered voters who support same-sex marriage.

- B = California registered voters who say the Supreme Court's ruling about the constitutionality of California's Proposition 8 is very or somewhat important to them
 - A = California registered voters who are 18 to 39 years old.
- a. Find $P(C)$.
 - b. Find $P(B)$.
 - c. Find $P(C|A)$.
 - d. Find $P(B|C)$.
 - e. In words, what is $C|A$?
 - f. In words, what is $B|C$?
 - g. Find $P(C \text{ AND } B)$.
 - h. In words, what is $C \text{ AND } B$?
 - i. Find $P(C \text{ OR } B)$.
 - j. Are C and B mutually exclusive events? Show why or why not.

Exercise:

Problem:

After Rob Ford, the mayor of Toronto, announced his plans to cut budget costs in late 2011, the Forum Research polled 1,046 people to measure the mayor's popularity. Everyone polled expressed either approval or disapproval. These are the results their poll produced:

- In early 2011, 60 percent of the population approved of Mayor Ford's actions in office.
 - In mid-2011, 57 percent of the population approved of his actions.
 - In late 2011, the percentage of popular approval was measured at 42 percent.
- a. What is the sample size for this study?
 - b. What proportion in the poll disapproved of Mayor Ford, according to the results from late 2011?
 - c. How many people polled responded that they approved of Mayor Ford in late 2011?
 - d. What is the probability that a person supported Mayor Ford, based on the data collected in mid-2011?

- e. What is the probability that a person supported Mayor Ford, based on the data collected in early 2011?

Solution:

- a. The Forum Research surveyed 1,046 Torontonians.
- b. 58%
- c. 42% of 1,046 = 439 (rounding to the nearest integer)
- d. 0.57
- e. 0.60.

Use the following information to answer the next three exercises. The casino game, roulette, allows the gambler to bet on the probability of a ball, which spins in the roulette wheel, landing on a particular color, number, or range of numbers. The table used to place bets contains of 38 numbers, and each number is assigned to a color and a range.

00		3	6	9	12	15	18	21	24	27	30	33	36	2 to 12 to 1	
0		2	5	8	11	14	17	20	23	26	29	32	35		
		1	4	7	10	13	16	19	22	25	28	31	34		
1st Dozen					2nd Dozen					3rd Dozen					
1 to 18		EVEN						ODD		19 to 36					

(credit: film8ker/wikibooks)

Exercise:

Problem:

- List the sample space of the 38 possible outcomes in roulette.
- You bet on red. Find $P(\text{red})$.
- You bet on -1st 12- (1st Dozen). Find $P(-1\text{st } 12-)$.
- You bet on an even number. Find $P(\text{even number})$.
- Is getting an odd number the complement of getting an even number? Why?
- Find two mutually exclusive events.
- Are the events Even and 1st Dozen independent?

Exercise:

Problem:

Compute the probability of winning the following types of bets:

- Betting on two lines that touch each other on the table as in 1-2-3-4-5-6
- Betting on three numbers in a line, as in 1-2-3
- Betting on one number
- Betting on four numbers that touch each other to form a square, as in 10-11-13-14
- Betting on two numbers that touch each other on the table, as in 10-11 or 10-13
- Betting on 0-00-1-2-3
- Betting on 0-1-2; or 0-00-2; or 00-2-3

Solution:

- $P(\text{Betting on two line that touch each other on the table}) = \frac{6}{38}$
- $P(\text{Betting on three numbers in a line}) = \frac{3}{38}$
- $P(\text{Betttng on one number}) = \frac{1}{38}$
- $P(\text{Betting on four number that touch each other to form a square}) = \frac{4}{38}$
- $P(\text{Betting on two number that touch each other on the table}) = \frac{2}{38}$
- $P(\text{Betting on 0-00-1-2-3}) = \frac{5}{38}$
- $P(\text{Betting on 0-1-2; or 0-00-2; or 00-2-3}) = \frac{3}{38}$

Exercise:**Problem:**

Compute the probability of winning the following types of bets:

- a. Betting on a color
- b. Betting on one of the dozen groups
- c. Betting on the range of numbers from 1 to 18
- d. Betting on the range of numbers 19–36
- e. Betting on one of the columns
- f. Betting on an even or odd number (excluding zero)

Exercise:**Problem:**

Suppose that you have eight cards. Five are green and three are yellow. The five green cards are numbered 1, 2, 3, 4, and 5. The three yellow cards are numbered 1, 2, and 3. The cards are well shuffled. You randomly draw one card.

- G = card drawn is green
 - E = card drawn is even-numbered
- a. List the sample space.
 - b. $P(G) = \underline{\hspace{2cm}}$
 - c. $P(G|E) = \underline{\hspace{2cm}}$
 - d. $P(G \text{ AND } E) = \underline{\hspace{2cm}}$
 - e. $P(G \text{ OR } E) = \underline{\hspace{2cm}}$
 - f. Are G and E mutually exclusive? Justify your answer numerically.

Solution:

- a. $\{G1, G2, G3, G4, G5, Y1, Y2, Y3\}$
- b. $\frac{5}{8}$
- c. $\frac{2}{3}$

- d. $\frac{2}{8}$
- e. $\frac{6}{8}$
- f. No, because $P(G \text{ AND } E)$ does not equal 0.

Exercise:

Problem: Roll two fair dice separately. Each die has six faces.

- a. List the sample space.
- b. Let A be the event that either a three or four is rolled first, followed by an even number. Find $P(A)$.
- c. Let B be the event that the sum of the two rolls is at most seven. Find $P(B)$.
- d. In words, explain what " $P(A|B)$ " represents. Find $P(A|B)$.
- e. Are A and B mutually exclusive events? Explain your answer in one to three complete sentences, including numerical justification.
- f. Are A and B independent events? Explain your answer in one to three complete sentences, including numerical justification.

Exercise:

Problem:

A special deck of cards has ten cards. Four are green, three are blue, and three are red. When a card is picked, its color of it is recorded. An experiment consists of first picking a card and then tossing a coin.

- a. List the sample space.
- b. Let A be the event that a blue card is picked first, followed by landing a head on the coin toss. Find $P(A)$.
- c. Let B be the event that a red or green is picked, followed by landing a head on the coin toss. Are the events A and B mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.
- d. Let C be the event that a red or blue is picked, followed by landing a head on the coin toss. Are the events A and C mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.

Solution:**Note:****NOTE**

The coin toss is independent of the card picked first.

- a. $\{(G,H) (G,T) (B,H) (B,T) (R,H) (R,T)\}$
- b. $P(A) = P(\text{blue})P(\text{head}) = \left(\frac{3}{10}\right)\left(\frac{1}{2}\right) = \frac{3}{20}$
- c. Yes, A and B are mutually exclusive because they cannot happen at the same time; you cannot pick a card that is both blue and also (red or green). $P(A \text{ AND } B) = 0$
- d. No, A and C are not mutually exclusive because they can occur at the same time. In fact, C includes all of the outcomes of A ; if the card chosen is blue it is also (red or blue). $P(A \text{ AND } C) = P(A) = \frac{3}{20}$

Exercise:**Problem:**

An experiment consists of first rolling a die and then tossing a coin.

- a. List the sample space.
- b. Let A be the event that either a three or a four is rolled first, followed by landing a head on the coin toss. Find $P(A)$.
- c. Let B be the event that the first and second tosses land on heads. Are the events A and B mutually exclusive? Explain your answer in one to three complete sentences, including numerical justification.

Exercise:

Problem:

An experiment consists of tossing a nickel, a dime, and a quarter. Of interest is the side the coin lands on.

- List the sample space.
 - Let A be the event that there are at least two tails. Find $P(A)$.
 - Let B be the event that the first and second tosses land on heads.
Are the events A and B mutually exclusive? Explain your answer in one to three complete sentences, including justification.
-

Solution:

- $S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\}$
- $\frac{4}{8}$
- Yes, because if A has occurred, it is impossible to obtain two tails.
In other words, $P(A \text{ AND } B) = 0$.

Exercise:

Consider the following scenario:

Let $P(C) = 0.4$.

Let $P(D) = 0.5$.

Problem: Let $P(C|D) = 0.6$.

- Find $P(C \text{ AND } D)$.
- Are C and D mutually exclusive? Why or why not?
- Are C and D independent events? Why or why not?
- Find $P(C \text{ OR } D)$.
- Find $P(D|C)$.

Exercise:

Problem: Y and Z are independent events.

- a. Rewrite the basic Addition Rule $P(Y \text{ OR } Z) = P(Y) + P(Z) - P(Y \text{ AND } Z)$ using the information that Y and Z are independent events.
 - b. Use the rewritten rule to find $P(Z)$ if $P(Y \text{ OR } Z) = 0.71$ and $P(Y) = 0.42$.
-

Solution:

- a. If Y and Z are independent, then $P(Y \text{ AND } Z) = P(Y)P(Z)$, so $P(Y \text{ OR } Z) = P(Y) + P(Z) - P(Y)P(Z)$.
- b. 0.5

Exercise:

Problem: G and H are mutually exclusive events. $P(G) = 0.5$ $P(H) = 0.3$

- a. Explain why the following statement MUST be false: $P(H|G) = 0.4$.
- b. Find $P(H \text{ OR } G)$.
- c. Are G and H independent or dependent events? Explain in a complete sentence.

Exercise:

Problem:

Approximately 281,000,000 people over age five live in the United States. Of these people, 55,000,000 speak a language other than English at home. Of those who speak another language at home, 62.3% speak Spanish.

Let: E = speaks English at home; E' = speaks another language at home;
 S = speaks Spanish;

Finish each probability statement by matching the correct answer.

Probability Statements	Answers
a. $P(E')$ =	i. 0.8043
b. $P(E)$ =	ii. 0.623
c. $P(S \text{ and } E')$ =	iii. 0.1957
d. $P(S E')$ =	iv. 0.1219

Solution:

iii i iv ii

Exercise:

Problem:

1994, the U.S. government held a lottery to issue 55,000 Green Cards (permits for non-citizens to work legally in the U.S.). Renate Deutsch, from Germany, was one of approximately 6.5 million people who entered this lottery. Let G = won green card.

- What was Renate's chance of winning a Green Card? Write your answer as a probability statement.
- In the summer of 1994, Renate received a letter stating she was one of 110,000 finalists chosen. Once the finalists were chosen, assuming that each finalist had an equal chance to win, what was Renate's chance of winning a Green Card? Write your answer as a conditional probability statement. Let F = was a finalist.
- Are G and F independent or dependent events? Justify your answer numerically and also explain why.
- Are G and F mutually exclusive events? Justify your answer numerically and explain why.

Exercise:

Problem:

Three professors at George Washington University did an experiment to determine if economists are more selfish than other people. They dropped 64 stamped, addressed envelopes with \$10 cash in different classrooms on the George Washington campus. 44% were returned overall. From the economics classes 56% of the envelopes were returned. From the business, psychology, and history classes 31% were returned.

Let: R = money returned; E = economics classes; O = other classes

- a. Write a probability statement for the overall percent of money returned.
- b. Write a probability statement for the percent of money returned out of the economics classes.
- c. Write a probability statement for the percent of money returned out of the other classes.
- d. Is money being returned independent of the class? Justify your answer numerically and explain it.
- e. Based upon this study, do you think that economists are more selfish than other people? Explain why or why not. Include numbers to justify your answer.

Solution:

- a. $P(R) = 0.44$
- b. $P(R|E) = 0.56$
- c. $P(R|O) = 0.31$
- d. No, whether the money is returned is not independent of which class the money was placed in. There are several ways to justify this mathematically, but one is that the money placed in economics classes is not returned at the same overall rate; $P(R|E) \neq P(R)$.
- e. No, this study definitely does not support that notion; in fact, it suggests the opposite. The money placed in the economics classrooms was returned at a higher rate than the money placed in all classes collectively; $P(R|E) > P(R)$.

Exercise:**Problem:**

The following table of data obtained from www.baseball-almanac.com shows hit information for four players. Suppose that one hit from the table is randomly selected.

Name	Single	Double	Triple	Home Run	Total Hits
Babe Ruth	1,517	506	136	714	2,873
Jackie Robinson	1,054	273	54	137	1,518
Ty Cobb	3,603	174	295	114	4,189
Hank Aaron	2,294	624	98	755	3,771
Total	8,471	1,577	583	1,720	12,351

Are "the hit being made by Hank Aaron" and "the hit being a double" independent events?

- Yes, because $P(\text{hit by Hank Aaron}|\text{hit is a double}) = P(\text{hit by Hank Aaron})$
- No, because $P(\text{hit by Hank Aaron}|\text{hit is a double}) \neq P(\text{hit is a double})$
- No, because $P(\text{hit is by Hank Aaron}|\text{hit is a double}) \neq P(\text{hit by Hank Aaron})$

- d. Yes, because $P(\text{hit is by Hank Aaron}|\text{hit is a double}) = P(\text{hit is a double})$

Exercise:

Problem:

United Blood Services is a blood bank that serves more than 500 hospitals in 18 states. According to their website, a person with type O blood and a negative Rh factor (Rh-) can donate blood to any person with any bloodtype. Their data show that 43% of people have type O blood and 15% of people have Rh- factor; 52% of people have type O or Rh- factor.

- Find the probability that a person has both type O blood and the Rh- factor.
- Find the probability that a person does NOT have both type O blood and the Rh- factor.

Solution:

a. $P(\text{type O OR Rh-}) = P(\text{type O}) + P(\text{Rh-}) - P(\text{type O AND Rh-})$

$$0.52 = 0.43 + 0.15 - P(\text{type O AND Rh-}); \text{ solve to find } P(\text{type O AND Rh-}) = 0.06$$

6% of people have type O, Rh- blood

b. $P(\text{NOT}(\text{type O AND Rh-})) = 1 - P(\text{type O AND Rh-}) = 1 - 0.06 = 0.94$

94% of people do not have type O, Rh- blood

Exercise:

Problem:

At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam. Let F be the event that a course has a final exam. Let R be the event that a course requires a research paper.

- Find the probability that a course has a final exam or a research project.
- Find the probability that a course has NEITHER of these two requirements.

Exercise:**Problem:**

In a box of assorted cookies, 36% contain chocolate and 12% contain nuts. Of those, 8% contain both chocolate and nuts. Sean is allergic to both chocolate and nuts.

- Find the probability that a cookie contains chocolate or nuts (he can't eat it).
- Find the probability that a cookie does not contain chocolate or nuts (he can eat it).

Solution:

- Let C = be the event that the cookie contains chocolate. Let N = the event that the cookie contains nuts.
- $P(C \text{ OR } N) = P(C) + P(N) - P(C \text{ AND } N) = 0.36 + 0.12 - 0.08 = 0.40$
- $P(\text{NEITHER chocolate NOR nuts}) = 1 - P(C \text{ OR } N) = 1 - 0.40 = 0.60$

Exercise:

Problem:

A college finds that 10% of students have taken a distance learning class and that 40% of students are part time students. Of the part time students, 20% have taken a distance learning class. Let D = event that a student takes a distance learning class and E = event that a student is a part time student

- a. Find $P(D \text{ AND } E)$.
- b. Find $P(E|D)$.
- c. Find $P(D \text{ OR } E)$.
- d. Using an appropriate test, show whether D and E are independent.
- e. Using an appropriate test, show whether D and E are mutually exclusive.

Glossary**Independent Events**

The occurrence of one event has no effect on the probability of the occurrence of another event. Events A and B are independent if one of the following is true:

1. $P(A|B) = P(A)$
2. $P(B|A) = P(B)$
3. $P(A \text{ AND } B) = P(A)P(B)$

Mutually Exclusive

Two events are mutually exclusive if the probability that they both happen at the same time is zero. If events A and B are mutually exclusive, then $P(A \text{ AND } B) = 0$.

Data, Sampling, and Variation in Data and Sampling

Data may come from a population or from a sample. Lowercase letters like x or y generally are used to represent data values. Most data can be put into the following categories:

- Qualitative
- Quantitative

Qualitative data are the result of categorizing or describing attributes of a population. Qualitative data are also often called **categorical data**. Hair color, blood type, ethnic group, the car a person drives, and the street a person lives on are examples of qualitative data. Qualitative data are generally described by words or letters. For instance, hair color might be black, dark brown, light brown, blonde, gray, or red. Blood type might be AB+, O-, or B+. Researchers often prefer to use quantitative data over qualitative data because it lends itself more easily to mathematical analysis. For example, it does not make sense to find an average hair color or blood type.

Quantitative data are always numbers. Quantitative data are the result of **counting** or **measuring** attributes of a population. Amount of money, pulse rate, weight, number of people living in your town, and number of students who take statistics are examples of quantitative data. Quantitative data may be either **discrete** or **continuous**.

All data that are the result of counting are called **quantitative discrete data**. These data take on only certain numerical values. If you count the number of phone calls you receive for each day of the week, you might get values such as zero, one, two, or three.

Data that are not only made up of counting numbers, but that may include fractions, decimals, or irrational numbers, are called **quantitative continuous data**. Continuous data are often the results of measurements like lengths, weights, or times. A list of the lengths in minutes for all the phone calls that you make in a week, with numbers like 2.4, 7.5, or 11.0, would be quantitative continuous data.

Example:**Data Sample of Quantitative Discrete Data**

The data are the number of books students carry in their backpacks. You sample five students. Two students carry three books, one student carries four books, one student carries two books, and one student carries one book. The numbers of books (three, four, two, and one) are the quantitative discrete data.

Note:

Try It

Exercise:**Problem:**

The data are the number of machines in a gym. You sample five gyms. One gym has 12 machines, one gym has 15 machines, one gym has ten machines, one gym has 22 machines, and the other gym has 20 machines. What type of data is this?

Solution:**Try It Solutions**

quantitative discrete data

Example:**Data Sample of Quantitative Continuous Data**

The data are the weights of backpacks with books in them. You sample the same five students. The weights (in pounds) of their backpacks are 6.2, 7, 6.8, 9.1, 4.3. Notice that backpacks carrying three books can have different weights. Weights are quantitative continuous data.

Note:

Try It

Exercise:

Problem:

The data are the areas of lawns in square feet. You sample five houses. The areas of the lawns are 144 sq. feet, 160 sq. feet, 190 sq. feet, 180 sq. feet, and 210 sq. feet. What type of data is this?

Solution:**Try It Solutions**

quantitative continuous data

Example:

You go to the supermarket and purchase three cans of soup (19 ounces tomato bisque, 14.1 ounces lentil, and 19 ounces Italian wedding), two packages of nuts (walnuts and peanuts), four different kinds of vegetable (broccoli, cauliflower, spinach, and carrots), and two desserts (16 ounces pistachio ice cream and 32 ounces chocolate chip cookies).

Exercise:**Problem:**

Name data sets that are quantitative discrete, quantitative continuous, and qualitative.

Solution:

One Possible Solution:

- The three cans of soup, two packages of nuts, four kinds of vegetables and two desserts are quantitative discrete data because you count them.
- The weights of the soups (19 ounces, 14.1 ounces, 19 ounces) are quantitative continuous data because you measure weights as precisely as possible.
- Types of soups, nuts, vegetables and desserts are qualitative data because they are categorical.

Try to identify additional data sets in this example.

Example:

The data are the colors of backpacks. Again, you sample the same five students. One student has a red backpack, two students have black backpacks, one student has a green backpack, and one student has a gray backpack. The colors red, black, black, green, and gray are qualitative data.

Note:

Try It

Exercise:

Problem:

The data are the colors of houses. You sample five houses. The colors of the houses are white, yellow, white, red, and white. What type of data is this?

Solution:

Try It Solutions

qualitative data

Note:

Note

You may collect data as numbers and report it categorically. For example, the quiz scores for each student are recorded throughout the term. At the end of the term, the quiz scores are reported as A, B, C, D, or F.

Example:

Exercise:

Problem:

Work collaboratively to determine the correct data type (quantitative or qualitative). Indicate whether quantitative data are continuous or discrete. Hint: Data that are discrete often start with the words "the number of."

- a. the number of pairs of shoes you own
- b. the type of car you drive
- c. the distance it is from your home to the nearest grocery store
- d. the number of classes you take per school year.
- e. the type of calculator you use
- f. weights of sumo wrestlers
- g. number of correct answers on a quiz
- h. IQ scores (This may cause some discussion.)

Solution:

Items a, d, and g are quantitative discrete; items c, f, and h are quantitative continuous; items b and e are qualitative, or categorical.

Note:

Try It

Exercise:**Problem:**

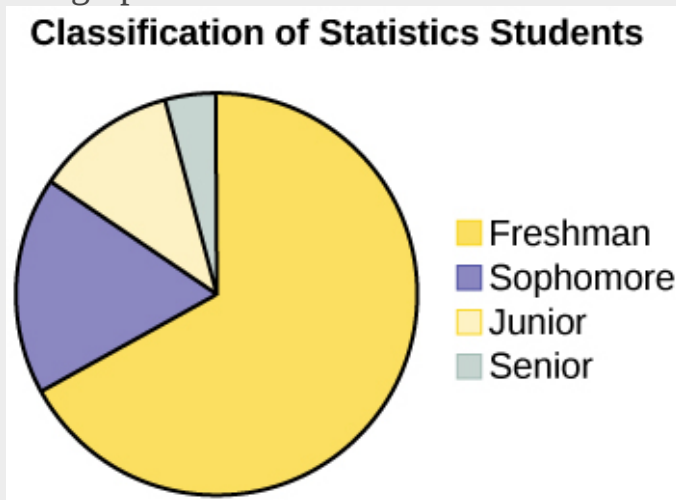
Determine the correct data type (quantitative or qualitative) for the number of cars in a parking lot. Indicate whether quantitative data are continuous or discrete.

Solution:**Try It Solutions**

quantitative discrete

Example:**Exercise:****Problem:**

A statistics professor collects information about the classification of her students as freshmen, sophomores, juniors, or seniors. The data she collects are summarized in the pie chart [\[link\]](#). What type of data does this graph show?

**Solution:**

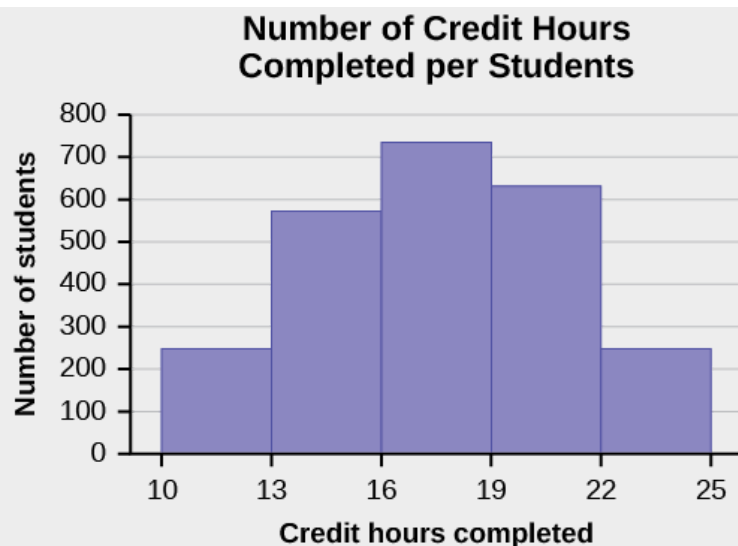
This pie chart shows the students in each year, which is **qualitative (or categorical) data**.

Note:

Try It

Exercise:**Problem:**

The registrar at State University keeps records of the number of credit hours students complete each semester. The data he collects are summarized in the histogram. The class boundaries are 10 to less than 13, 13 to less than 16, 16 to less than 19, 19 to less than 22, and 22 to less than 25.



What type of data does this graph show?

Solution:

Try It Solutions

A histogram is used to display quantitative data: the numbers of credit hours completed. Because students can complete only a whole number of hours (no fractions of hours allowed), this data is quantitative discrete.

Qualitative Data Discussion

Below are tables comparing the number of part-time and full-time students at De Anza College and Foothill College enrolled for the spring 2010 quarter. The tables display counts (frequencies) and percentages or proportions (relative frequencies). The percent columns make comparing the same categories in the colleges easier. Displaying percentages along with the numbers is often helpful, but it is particularly important when comparing sets of data that do not have the same totals, such as the total enrollments for both colleges in this example. Notice how much larger the percentage for part-time students at Foothill College is compared to De Anza College.

De Anza College				Foothill College		
	Number	Percent			Number	Percent
Full-time	9,200	40.9%		Full-time	4,059	28.6%
Part-time	13,296	59.1%		Part-time	10,124	71.4%
Total	22,496	100%		Total	14,183	100%

Fall Term 2007 (Census day)

Tables are a good way of organizing and displaying data. But graphs can be even more helpful in understanding the data. There are no strict rules concerning which graphs to use. Two graphs that are used to display qualitative data are pie charts and bar graphs.

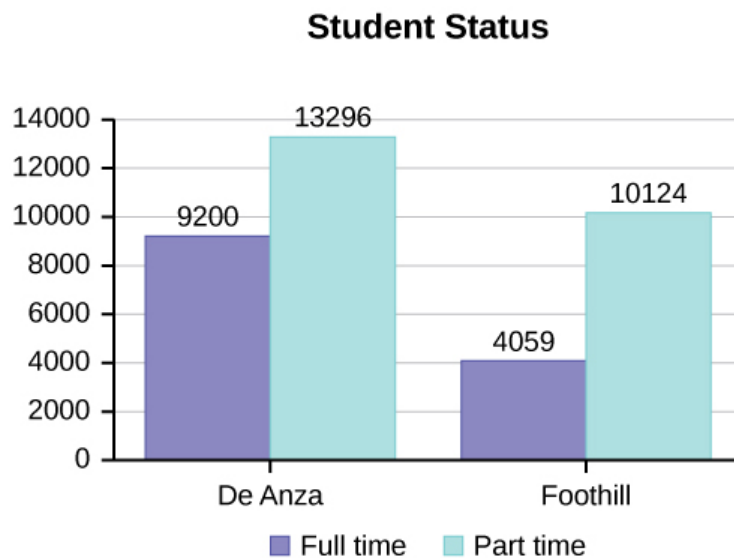
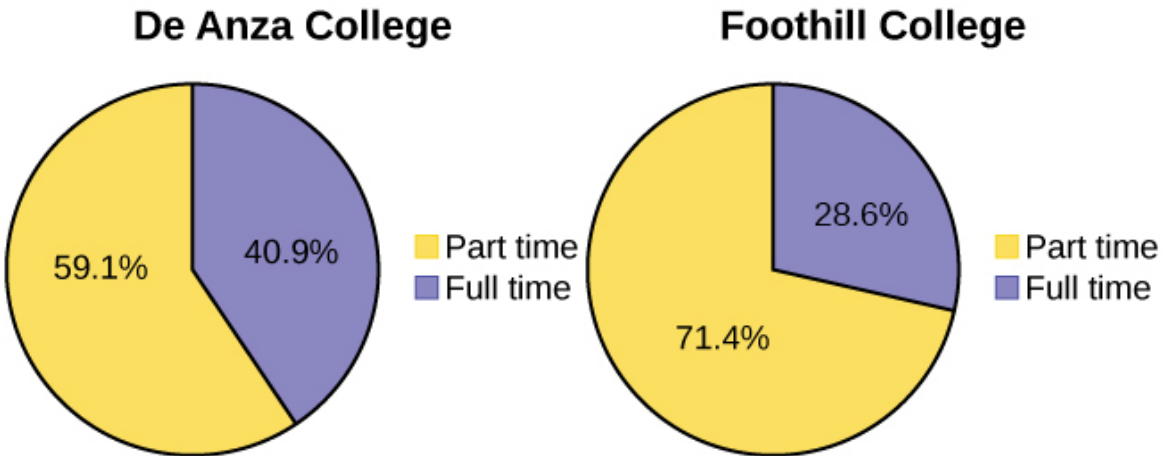
In a **pie chart**, categories of data are represented by wedges in a circle and are proportional in size to the percent of individuals in each category.

In a **bar graph**, the length of the bar for each category is proportional to the number or percent of individuals in each category. Bars may be vertical or horizontal.

A **Pareto chart** consists of bars that are sorted into order by category size (largest to smallest).

Look at [\[link\]](#) and [\[link\]](#) and determine which graph (pie or bar) you think displays the comparisons better.

It is a good idea to look at a variety of graphs to see which is the most helpful in displaying the data. We might make different choices of what we think is the “best” graph depending on the data and the context. Our choice also depends on what we are using the data for.

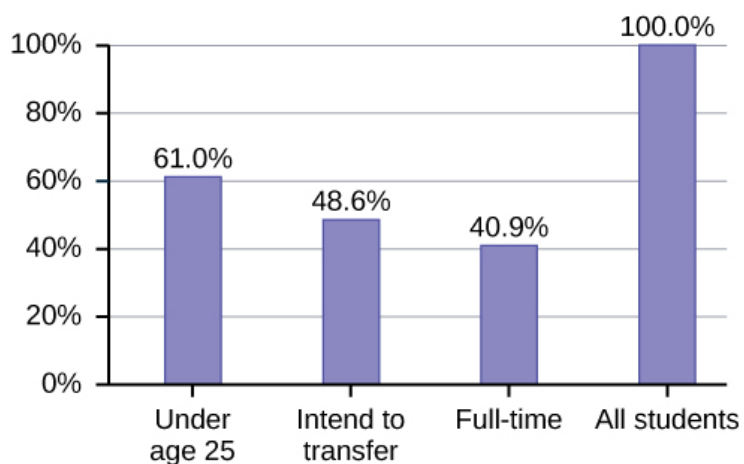


Percentages That Add to More (or Less) Than 100%

Sometimes percentages add up to be more than 100% (or less than 100%). In the graph, the percentages add to more than 100% because students can be in more than one category. A bar graph is appropriate to compare the relative size of the categories. A pie chart cannot be used. It also could not be used if the percentages added to less than 100%.

Characteristic/Category	Percent
Full-Time Students	40.9%
Students who intend to transfer to a 4-year educational institution	48.6%
Students under age 25	61.0%
TOTAL	150.5%

De Anza College Spring 2010

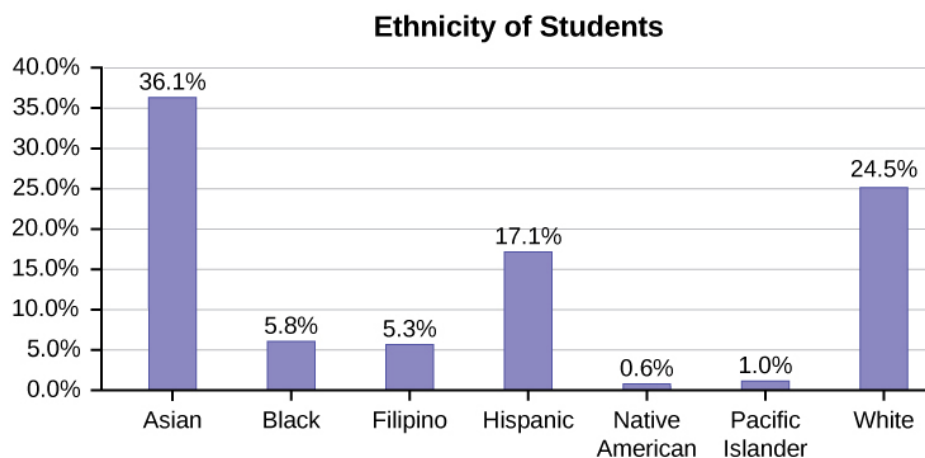


Omitting Categories/Missing Data

The table displays Ethnicity of Students but is missing the "Other/Unknown" category. This category contains people who did not feel they fit into any of the ethnicity categories or declined to respond. Notice that the frequencies do not add up to the total number of students. In this situation, create a bar graph and not a pie chart.

	Frequency	Percent
Asian	8,794	36.1%
Black	1,412	5.8%
Filipino	1,298	5.3%
Hispanic	4,180	17.1%
Native American	146	0.6%
Pacific Islander	236	1.0%
White	5,978	24.5%
TOTAL	22,044 out of 24,382	90.4% out of 100%

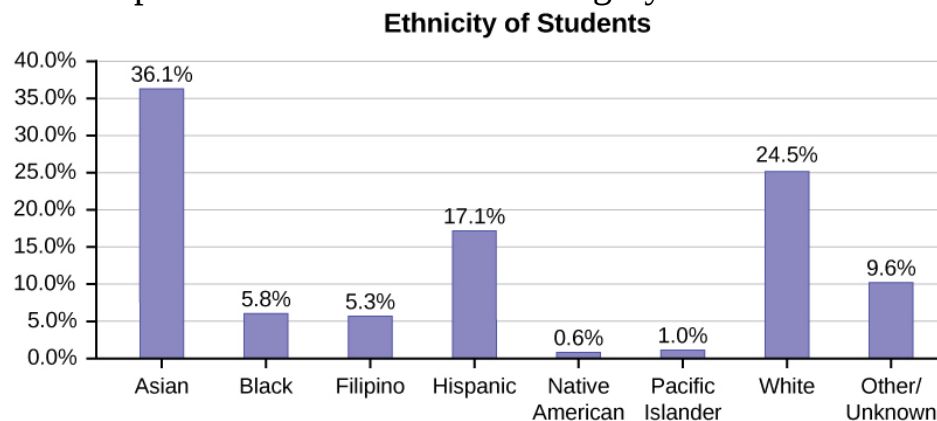
Ethnicity of Students at De Anza College Fall Term 2007 (Census Day)



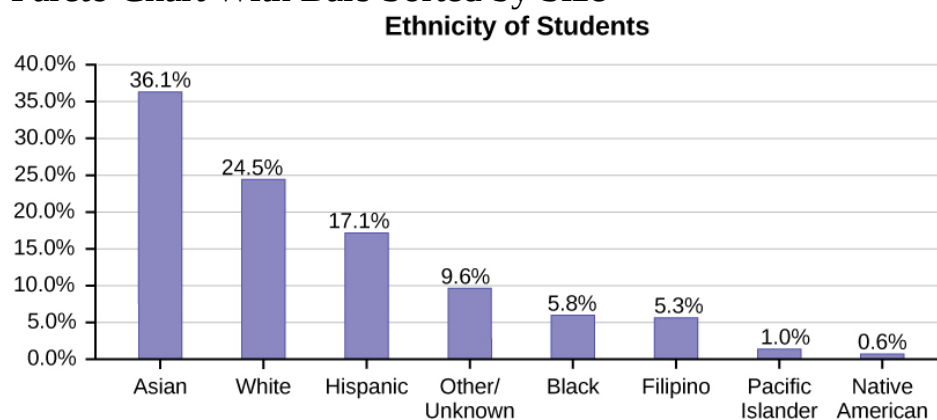
The following graph is the same as the previous graph but the “Other/Unknown” percent (9.6%) has been included. The “Other/Unknown” category is large compared to some of the other categories (Native American, 0.6%, Pacific Islander 1.0%). This is important to know when we think about what the data are telling us.

This particular bar graph in [\[link\]](#) can be difficult to understand visually. The graph in [\[link\]](#) is a Pareto chart. The Pareto chart has the bars sorted from largest to smallest and is easier to read and interpret.

Bar Graph with Other/Unknown Category

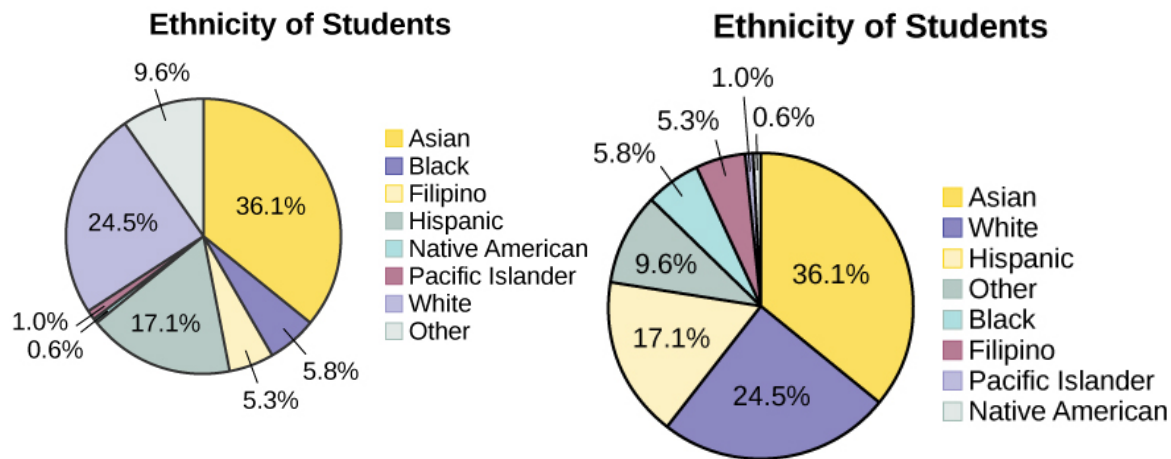


Pareto Chart With Bars Sorted by Size



Pie Charts: No Missing Data

The following pie charts have the “Other/Unknown” category included (since the percentages must add to 100%). The chart in [\[link\]](#) is organized by the size of each wedge, which makes it a more visually informative graph than the unsorted, alphabetical graph in [\[link\]](#).



Sampling

Gathering information about an entire population often costs too much or is virtually impossible. Instead, we use a sample of the population. **A sample should have the same characteristics as the population it is representing.** Most statisticians use various methods of random sampling in an attempt to achieve this goal. This section will describe a few of the most common methods. There are several different methods of **random sampling**. In each form of random sampling, each member of a population initially has an equal chance of being selected for the sample. Each method has pros and cons. The easiest method to describe is called a **simple random sample**. Any group of n individuals is equally likely to be chosen as any other group of n individuals if the simple random sampling technique is used. In other words, each sample of the same size has an equal chance of being selected. For example, suppose Lisa wants to form a four-person study group (herself and three other people) from her pre-calculus class, which has 31 members not including Lisa. To choose a simple random sample of size three from the other members of her class, Lisa could put all 31 names in a hat, shake the hat, close her eyes, and pick out three names. A more technological way is for Lisa to first list the last names of the members of her class together with a two-digit number, as in [\[link\]](#):

ID	Name	ID	Name	ID	Name
00	Anselmo	11	King	21	Roquero
01	Bautista	12	Legeny	22	Roth
02	Bayani	13	Lundquist	23	Rowell
03	Cheng	14	Macierz	24	Salangsang
04	Cuarismo	15	Motogawa	25	Slade
05	Cunningham	16	Okimoto	26	Stratcher
06	Fontecha	17	Patel	27	Tallai
07	Hong	18	Price	28	Tran
08	Hoobler	19	Quizon	29	Wai
09	Jiao	20	Reyes	30	Wood
10	Khan				

Class Roster

Lisa can use a table of random numbers (found in many statistics books and mathematical handbooks), a calculator, or a computer to generate random numbers. For this example, suppose Lisa chooses to generate random numbers from a calculator. The numbers generated are as follows:

0.94360 0.99832 0.14669 0.51470 0.40581 0.73381 0.04399

Lisa reads two-digit groups until she has chosen three class members (that is, she reads 0.94360 as the groups 94, 43, 36, 60). Each random number may only contribute one class member. If she needed to, Lisa could have generated more random numbers.

The random numbers 0.94360 and 0.99832 do not contain appropriate two digit numbers. However the third random number, 0.14669, contains 14 (the fourth random number also contains 14), the fifth random number contains 05, and the seventh random number contains 04. The two-digit number 14 corresponds to Macierz, 05 corresponds to Cuningham, and 04 corresponds to Cuarismo. Besides herself, Lisa's group will consist of Marcierz, Cuningham, and Cuarismo.

Note:

To generate random numbers:

- Press MATH.
- Arrow over to PRB.
- Press 5:randInt(. Enter 0, 30).
- Press ENTER for the first random number.
- Press ENTER two more times for the other 2 random numbers. If there is a repeat press ENTER again.

Note: randInt(0, 30, 3) will generate 3 random numbers.

```
randInt(0,30) 29
randInt(0,30) 28
randInt(0,30) 4
```

Besides simple random sampling, there are other forms of sampling that involve a chance process for getting the sample. **Other well-known random sampling methods are the stratified sample, the cluster sample, and the systematic sample.**

To choose a **stratified sample**, divide the population into groups called strata and then take a **proportionate** number from each stratum. For example, you could stratify (group) your college population by department and then choose a proportionate simple random sample from each stratum (each department) to

get a stratified random sample. To choose a simple random sample from each department, number each member of the first department, number each member of the second department, and do the same for the remaining departments. Then use simple random sampling to choose proportionate numbers from the first department and do the same for each of the remaining departments. Those numbers picked from the first department, picked from the second department, and so on represent the members who make up the stratified sample.

To choose a **cluster sample**, divide the population into clusters (groups) and then randomly select some of the clusters. All the members from these clusters are in the cluster sample. For example, if you randomly sample four departments from your college population, the four departments make up the cluster sample. Divide your college faculty by department. The departments are the clusters. Number each department, and then choose four different numbers using simple random sampling. All members of the four departments with those numbers are the cluster sample.

To choose a **systematic sample**, randomly select a starting point and take every n^{th} piece of data from a listing of the population. For example, suppose you have to do a phone survey. Your phone book contains 20,000 residence listings. You must choose 400 names for the sample. Number the population 1–20,000 and then use a simple random sample to pick a number that represents the first name in the sample. Then choose every fiftieth name thereafter until you have a total of 400 names (you might have to go back to the beginning of your phone list). Systematic sampling is frequently chosen because it is a simple method.

A type of sampling that is non-random is convenience sampling. **Convenience sampling** involves using results that are readily available. For example, a computer software store conducts a marketing study by interviewing potential customers who happen to be in the store browsing through the available software. The results of convenience sampling may be very good in some cases and highly biased (favor certain outcomes) in others.

Sampling data should be done very carefully. Collecting data carelessly can have devastating results. Surveys mailed to households and then returned may be very biased (they may favor a certain group). It is better for the person conducting the survey to select the sample respondents.

True random sampling is done **with replacement**. That is, once a member is picked, that member goes back into the population and thus may be chosen more than once. However for practical reasons, in most populations, simple random sampling is done **without replacement**. Surveys are typically done without replacement. That is, a member of the population may be chosen only once. Most samples are taken from large populations and the sample tends to be small in comparison to the population. Since this is the case, sampling without replacement is approximately the same as sampling with replacement because the chance of picking the same individual more than once with replacement is very low.

In a college population of 10,000 people, suppose you want to pick a sample of 1,000 randomly for a survey. **For any particular sample of 1,000**, if you are sampling **with replacement**,

- the chance of picking the first person is 1,000 out of 10,000 (0.1000);
- the chance of picking a different second person for this sample is 999 out of 10,000 (0.0999);
- the chance of picking the same person again is 1 out of 10,000 (very low).

If you are sampling **without replacement**,

- the chance of picking the first person for any particular sample is 1000 out of 10,000 (0.1000);
- the chance of picking a different second person is 999 out of 9,999 (0.0999);
- you do not replace the first person before picking the next person.

Compare the fractions $999/10,000$ and $999/9,999$. For accuracy, carry the decimal answers to four decimal places. To four decimal places, these numbers are equivalent (0.0999).

Sampling without replacement instead of sampling with replacement becomes a mathematical issue only when the population is small. For example, if the population is 25 people, the sample is ten, and you are sampling **with replacement for any particular sample**, then the chance of picking the first person is ten out of 25, and the chance of picking a different second person is nine out of 25 (you replace the first person).

If you sample **without replacement**, then the chance of picking the first person is ten out of 25, and then the chance of picking the second person (who is different) is nine out of 24 (you do not replace the first person).

Compare the fractions $9/25$ and $9/24$. To four decimal places, $9/25 = 0.3600$ and $9/24 = 0.3750$. To four decimal places, these numbers are not equivalent.

When you analyze data, it is important to be aware of **sampling errors** and nonsampling errors. The actual process of sampling causes sampling errors. For example, the sample may not be large enough. Factors not related to the sampling process cause **nonsampling errors**. A defective counting device can cause a nonsampling error.

In reality, a sample will never be exactly representative of the population so there will always be some sampling error. As a rule, the larger the sample, the smaller the sampling error.

In statistics, a **sampling bias** is created when a sample is collected from a population and some members of the population are not as likely to be chosen as others (remember, each member of the population should have an equally likely chance of being chosen). When a sampling bias happens, there can be incorrect conclusions drawn about the population that is being studied.

Critical Evaluation

We need to evaluate the statistical studies we read about critically and analyze them before accepting the results of the studies. Common problems to be aware of include

- Problems with samples: A sample must be representative of the population. A sample that is not representative of the population is biased. Biased samples that are not representative of the population give results that are inaccurate and not valid.
- Self-selected samples: Responses only by people who choose to respond, such as call-in surveys, are often unreliable.
- Sample size issues: Samples that are too small may be unreliable. Larger samples are better, if possible. In some situations, having small samples

is unavoidable and can still be used to draw conclusions. Examples: crash testing cars or medical testing for rare conditions

- Undue influence: collecting data or asking questions in a way that influences the response
- Non-response or refusal of subject to participate: The collected responses may no longer be representative of the population. Often, people with strong positive or negative opinions may answer surveys, which can affect the results.
- Causality: A relationship between two variables does not mean that one causes the other to occur. They may be related (correlated) because of their relationship through a different variable.
- Self-funded or self-interest studies: A study performed by a person or organization in order to support their claim. Is the study impartial? Read the study carefully to evaluate the work. Do not automatically assume that the study is good, but do not automatically assume the study is bad either. Evaluate it on its merits and the work done.
- Misleading use of data: improperly displayed graphs, incomplete data, or lack of context
- Confounding: When the effects of multiple factors on a response cannot be separated. Confounding makes it difficult or impossible to draw valid conclusions about the effect of each factor.

Note:

Collaborative Exercise

As a class, determine whether or not the following samples are representative. If they are not, discuss the reasons.

1. To find the average GPA of all students in a university, use all honor students at the university as the sample.
2. To find out the most popular cereal among young people under the age of ten, stand outside a large supermarket for three hours and speak to every twentieth child under age ten who enters the supermarket.
3. To find the average annual income of all adults in the United States, sample U.S. congressmen. Create a cluster sample by considering each state as a stratum (group). By using simple random sampling, select states to be part of the cluster. Then survey every U.S. congressman in the cluster.

4. To determine the proportion of people taking public transportation to work, survey 20 people in New York City. Conduct the survey by sitting in Central Park on a bench and interviewing every person who sits next to you.
5. To determine the average cost of a two-day stay in a hospital in Massachusetts, survey 100 hospitals across the state using simple random sampling.

Example:

Exercise:

Problem:

A study is done to determine the average tuition that San Jose State undergraduate students pay per semester. Each student in the following samples is asked how much tuition he or she paid for the Fall semester. What is the type of sampling in each case?

- a. A sample of 100 undergraduate San Jose State students is taken by organizing the students' names by classification (freshman, sophomore, junior, or senior), and then selecting 25 students from each.
- b. A random number generator is used to select a student from the alphabetical listing of all undergraduate students in the Fall semester. Starting with that student, every 50th student is chosen until 75 students are included in the sample.
- c. A completely random method is used to select 75 students. Each undergraduate student in the fall semester has the same probability of being chosen at any stage of the sampling process.
- d. The freshman, sophomore, junior, and senior years are numbered one, two, three, and four, respectively. A random number generator is used to pick two of those years. All students in those two years are in the sample.
- e. An administrative assistant is asked to stand in front of the library one Wednesday and to ask the first 100 undergraduate students he

encounters what they paid for tuition the Fall semester. Those 100 students are the sample.

Solution:

a. stratified; b. systematic; c. simple random; d. cluster; e. convenience

Note:

Try It

You are going to use the random number generator to generate different types of samples from the data.

This table displays six sets of quiz scores (each quiz counts 10 points) for an elementary statistics class.

#1	#2	#3	#4	#5	#6
5	7	10	9	8	3
10	5	9	8	7	6
9	10	8	6	7	9
9	10	10	9	8	9
7	8	9	5	7	4
9	9	9	10	8	7
7	7	10	9	8	8

#1	#2	#3	#4	#5	#6
8	8	9	10	8	8
9	7	8	7	7	8
8	8	10	9	8	7

Instructions: Use the Random Number Generator to pick samples.

Exercise:

Problem:

1. Create a stratified sample by column. Pick three quiz scores randomly from each column.
 - Number each row one through ten.
 - On your calculator, press Math and arrow over to PRB.
 - For column 1, Press 5:randInt(and enter 1,10). Press ENTER. Record the number. Press ENTER 2 more times (even the repeats). Record these numbers. Record the three quiz scores in column one that correspond to these three numbers.
 - Repeat for columns two through six.
 - These 18 quiz scores are a stratified sample.
2. Create a cluster sample by picking two of the columns. Use the column numbers: one through six.
 - Press MATH and arrow over to PRB.
 - Press 5:randInt(and enter 1,6). Press ENTER. Record the number. Press ENTER and record that number.
 - The two numbers are for two of the columns.
 - The quiz scores (20 of them) in these 2 columns are the cluster sample.
3. Create a simple random sample of 15 quiz scores.
 - Use the numbering one through 60.

- Press MATH. Arrow over to PRB. Press 5:randInt(and enter 1, 60).
- Press ENTER 15 times and record the numbers.
- Record the quiz scores that correspond to these numbers.
- These 15 quiz scores are the systematic sample.

4. Create a systematic sample of 12 quiz scores.

- Use the numbering one through 60.
- Press MATH. Arrow over to PRB. Press 5:randInt(and enter 1, 60).
- Press ENTER. Record the number and the first quiz score. From that number, count ten quiz scores and record that quiz score. Keep counting ten quiz scores and recording the quiz score until you have a sample of 12 quiz scores. You may wrap around (go back to the beginning).

Example:

Exercise:

Problem:

Determine the type of sampling used (simple random, stratified, systematic, cluster, or convenience).

- a. A soccer coach selects six players from a group of boys aged eight to ten, seven players from a group of boys aged 11 to 12, and three players from a group of boys aged 13 to 14 to form a recreational soccer team.
- b. A pollster interviews all human resource personnel in five different high tech companies.
- c. A high school educational researcher interviews 50 high school female teachers and 50 high school male teachers.
- d. A medical researcher interviews every third cancer patient from a list of cancer patients at a local hospital.

- e. A high school counselor uses a computer to generate 50 random numbers and then picks students whose names correspond to the numbers.
- f. A student interviews classmates in his algebra class to determine how many pairs of jeans a student owns, on the average.

Solution:

a. stratified; b. cluster; c. stratified; d. systematic; e. simple random; f. convenience

Note:

Try It

Exercise:

Problem:

Determine the type of sampling used (simple random, stratified, systematic, cluster, or convenience).

A high school principal polls 50 freshmen, 50 sophomores, 50 juniors, and 50 seniors regarding policy changes for after school activities.

Solution:

stratified

If we were to examine two samples representing the same population, even if we used random sampling methods for the samples, they would not be exactly the same. Just as there is variation in data, there is variation in samples. As you become accustomed to sampling, the variability will begin to seem natural.

Example:

Suppose ABC College has 10,000 part-time students (the population). We are interested in the average amount of money a part-time student spends on books in the fall term. Asking all 10,000 students is an almost impossible task.

Suppose we take two different samples.

First, we use convenience sampling and survey ten students from a first term organic chemistry class. Many of these students are taking first term calculus in addition to the organic chemistry class. The amount of money they spend on books is as follows:

\$128 \$87 \$173 \$116 \$130 \$204 \$147 \$189 \$93 \$153

The second sample is taken using a list of senior citizens who take P.E. classes and taking every fifth senior citizen on the list, for a total of ten senior citizens. They spend:

\$50 \$40 \$36 \$15 \$50 \$100 \$40 \$53 \$22 \$22

It is unlikely that any student is in both samples.

Exercise:**Problem:**

- a. Do you think that either of these samples is representative of (or is characteristic of) the entire 10,000 part-time student population?

Solution:

- a. No. The first sample probably consists of science-oriented students. Besides the chemistry course, some of them are also taking first-term calculus. Books for these classes tend to be expensive. Most of these students are, more than likely, paying more than the average part-time student for their books. The second sample is a group of senior citizens who are, more than likely, taking courses for health and interest. The amount of money they spend on books is probably much less than the average parttime student. Both samples are biased. Also, in both cases, not all students have a chance to be in either sample.

Exercise:

Problem:

b. Since these samples are not representative of the entire population, is it wise to use the results to describe the entire population?

Solution:

b. No. For these samples, each member of the population did not have an equally likely chance of being chosen.

Now, suppose we take a third sample. We choose ten different part-time students from the disciplines of chemistry, math, English, psychology, sociology, history, nursing, physical education, art, and early childhood development. (We assume that these are the only disciplines in which part-time students at ABC College are enrolled and that an equal number of part-time students are enrolled in each of the disciplines.) Each student is chosen using simple random sampling. Using a calculator, random numbers are generated and a student from a particular discipline is selected if he or she has a corresponding number. The students spend the following amounts:

\$180 \$50 \$150 \$85 \$260 \$75 \$180 \$200 \$200 \$150

Exercise:

Problem: c. Is the sample biased?

Solution:

c. The sample is unbiased, but a larger sample would be recommended to increase the likelihood that the sample will be close to representative of the population. However, for a biased sampling technique, even a large sample runs the risk of not being representative of the population.

Students often ask if it is "good enough" to take a sample, instead of surveying the entire population. If the survey is done well, the answer is yes.

Note:

Try It

Exercise:

Problem:

A local radio station has a fan base of 20,000 listeners. The station wants to know if its audience would prefer more music or more talk shows. Asking all 20,000 listeners is an almost impossible task.

The station uses convenience sampling and surveys the first 200 people they meet at one of the station's music concert events. 24 people said they'd prefer more talk shows, and 176 people said they'd prefer more music.

Do you think that this sample is representative of (or is characteristic of) the entire 20,000 listener population?

Solution:**Try It Solutions**

The sample probably consists more of people who prefer music because it is a concert event. Also, the sample represents only those who showed up to the event earlier than the majority. The sample probably doesn't represent the entire fan base and is probably biased towards people who would prefer music.

Variation in Data

Variation is present in any set of data. For example, 16-ounce cans of beverage may contain more or less than 16 ounces of liquid. In one study, eight 16 ounce cans were measured and produced the following amount (in ounces) of beverage:

15.8 16.1 15.2 14.8 15.8 15.9 16.0 15.5

Measurements of the amount of beverage in a 16-ounce can may vary because different people make the measurements or because the exact amount, 16 ounces of liquid, was not put into the cans. Manufacturers regularly run tests

to determine if the amount of beverage in a 16-ounce can falls within the desired range.

Be aware that as you take data, your data may vary somewhat from the data someone else is taking for the same purpose. This is completely natural. However, if two or more of you are taking the same data and get very different results, it is time for you and the others to reevaluate your data-taking methods and your accuracy.

Variation in Samples

It was mentioned previously that two or more **samples** from the same **population**, taken randomly, and having close to the same characteristics of the population will likely be different from each other. Suppose Doreen and Jung both decide to study the average amount of time students at their college sleep each night. Doreen and Jung each take samples of 500 students. Doreen uses systematic sampling and Jung uses cluster sampling. Doreen's sample will be different from Jung's sample. Even if Doreen and Jung used the same sampling method, in all likelihood their samples would be different. Neither would be wrong, however.

Think about what contributes to making Doreen's and Jung's samples different.

If Doreen and Jung took larger samples (i.e. the number of data values is increased), their sample results (the average amount of time a student sleeps) might be closer to the actual population average. But still, their samples would be, in all likelihood, different from each other. This **variability in samples** cannot be stressed enough.

Size of a Sample

The size of a sample (often called the number of observations) is important. The examples you have seen in this book so far have been small. Samples of only a few hundred observations, or even smaller, are sufficient for many purposes. In polling, samples that are from 1,200 to 1,500 observations are

considered large enough and good enough if the survey is random and is well done. You will learn why when you study confidence intervals.

Be aware that many large samples are biased. For example, call-in surveys are invariably biased, because people choose to respond or not.

Note:
Collaborative Exercise
Divide into groups of two, three, or four. Your instructor will give each group one six-sided die. Try this experiment twice. Roll one fair die (six-sided) 20 times. Record the number of ones, twos, threes, fours, fives, and sixes you get in [\[link\]](#) and [\[link\]](#) (“frequency” is the number of times a particular face of the die occurs):

Face on Die	Frequency
1	
2	
3	
4	
5	
6	

First Experiment (20 rolls)

Face on Die	Frequency
1	
2	
3	
4	
5	
6	

Second Experiment (20 rolls)

Did the two experiments have the same results? Probably not. If you did the experiment a third time, do you expect the results to be identical to the first or second experiment? Why or why not?

Which experiment had the correct results? They both did. The job of the statistician is to see through the variability and draw appropriate conclusions.

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LBCC Distance Learning (DL) program data in 2010-2011,
<http://de.lbcc.edu/reports/2010-11/future/highlights.html#focus> (accessed May 1, 2013).

Data from San Jose Mercury News

Chapter Review

Data are individual items of information that come from a population or sample. Data may be classified as qualitative(categorical), quantitative continuous, or quantitative discrete.

Because it is not practical to measure the entire population in a study, researchers use samples to represent the population. A random sample is a representative group from the population chosen by using a method that gives each individual in the population an equal chance of being included in the sample. Random sampling methods include simple random sampling, stratified sampling, cluster sampling, and systematic sampling. Convenience sampling is a nonrandom method of choosing a sample that often produces biased data.

Samples that contain different individuals result in different data. This is true even when the samples are well-chosen and representative of the population. When properly selected, larger samples model the population more closely than smaller samples. There are many different potential problems that can affect the reliability of a sample. Statistical data needs to be critically analyzed, not simply accepted.

Practice

Exercise:

Problem: “Number of times per week” is what type of data?

- a. qualitative(categorical) b. quantitative discrete c. quantitative continuous

Use the following information to answer the next four exercises: A study was done to determine the age, number of times per week, and the duration (amount of time) of residents using a local park in San Antonio, Texas. The first house in the neighborhood around the park was selected randomly, and then the resident of every eighth house in the neighborhood around the park was interviewed.

Exercise:

Problem: The sampling method was

- a. simple random b. systematic c. stratified d. cluster

Solution:

b

Exercise:

Problem: “Duration (amount of time)” is what type of data?

- a. qualitative(categorical) b. quantitative discrete c. quantitative continuous

Exercise:**Problem:**

The colors of the houses around the park are what kind of data?

a. qualitative(categorical) b. quantitative discrete c. quantitative continuous

Solution:

a

Exercise:

Problem: The population is _____

Exercise:**Problem:**

[\[link\]](#) contains the total number of deaths worldwide as a result of earthquakes from 2000 to 2012.

Year	Total Number of Deaths
2000	231
2001	21,357
2002	11,685
2003	33,819
2004	228,802

Year	Total Number of Deaths
2005	88,003
2006	6,605
2007	712
2008	88,011
2009	1,790
2010	320,120
2011	21,953
2012	768
Total	823,856

Use [\[link\]](#) to answer the following questions.

- What is the proportion of deaths between 2007 and 2012?
- What percent of deaths occurred before 2001?
- What is the percent of deaths that occurred in 2003 or after 2010?
- What is the fraction of deaths that happened before 2012?
- What kind of data is the number of deaths?
- Earthquakes are quantified according to the amount of energy they produce (examples are 2.1, 5.0, 6.7). What type of data is that?
- What contributed to the large number of deaths in 2010? In 2004? Explain.

Solution:

- 0.5242
- 0.03%
- 6.86%

- d. $\frac{823,088}{823,856}$
- e. quantitative discrete
- f. quantitative continuous
- g. In both years, underwater earthquakes produced massive tsunamis.

For the following four exercises, determine the type of sampling used (simple random, stratified, systematic, cluster, or convenience).

Exercise:

Problem:

A group of test subjects is divided into twelve groups; then four of the groups are chosen at random.

Exercise:

Problem:

A market researcher polls every tenth person who walks into a store.

Solution:

systematic

Exercise:

Problem:

The first 50 people who walk into a sporting event are polled on their television preferences.

Exercise:

Problem:

A computer generates 100 random numbers, and 100 people whose names correspond with the numbers on the list are chosen.

Solution:

simple random

Use the following information to answer the next seven exercises: Studies are often done by pharmaceutical companies to determine the effectiveness of a treatment program. Suppose that a new AIDS antibody drug is currently under study. It is given to patients once the AIDS symptoms have revealed themselves. Of interest is the average (mean) length of time in months patients live once starting the treatment. Two researchers each follow a different set of 40 AIDS patients from the start of treatment until their deaths. The following data (in months) are collected.

Researcher A: 3; 4; 11; 15; 16; 17; 22; 44; 37; 16; 14; 24; 25; 15; 26; 27; 33; 29; 35; 44; 13; 21; 22; 10; 12; 8; 40; 32; 26; 27; 31; 34; 29; 17; 8; 24; 18; 47; 33; 34

Researcher B: 3; 14; 11; 5; 16; 17; 28; 41; 31; 18; 14; 14; 26; 25; 21; 22; 31; 2; 35; 44; 23; 21; 21; 16; 12; 18; 41; 22; 16; 25; 33; 34; 29; 13; 18; 24; 23; 42; 33; 29

Exercise:

Problem: Complete the tables using the data provided:

Survival Length (in months)	Frequency	Relative Frequency	Cumulative Relative Frequency
0.5–6.5			
6.5–12.5			
12.5–18.5			
18.5–24.5			

Survival Length (in months)	Frequency	Relative Frequency	Cumulative Relative Frequency
24.5–30.5			
30.5–36.5			
36.5–42.5			
42.5–48.5			

Researcher A

Survival Length (in months)	Frequency	Relative Frequency	Cumulative Relative Frequency
0.5–6.5			
6.5–12.5			
12.5–18.5			
18.5–24.5			
24.5–30.5			
30.5–36.5			
36.5–45.5			

Researcher B

Exercise:

Problem:

Determine what the key term data refers to in the above example for Researcher A.

Solution:

values for X , such as 3, 4, 11, and so on

Exercise:

Problem: List two reasons why the data may differ.

Exercise:

Problem:

Can you tell if one researcher is correct and the other one is incorrect? Why?

Solution:

No, we do not have enough information to make such a claim.

Exercise:

Problem: Would you expect the data to be identical? Why or why not?

Exercise:

Problem:

Suggest at least two methods the researchers might use to gather random data.

Solution:

Take a simple random sample from each group. One way is by assigning a number to each patient and using a random number generator to randomly select patients.

Exercise:**Problem:**

Suppose that the first researcher conducted his survey by randomly choosing one state in the nation and then randomly picking 40 patients from that state. What sampling method would that researcher have used?

Exercise:**Problem:**

Suppose that the second researcher conducted his survey by choosing 40 patients he knew. What sampling method would that researcher have used? What concerns would you have about this data set, based upon the data collection method?

Solution:

This would be convenience sampling and is not random.

Use the following data to answer the next five exercises: Two researchers are gathering data on hours of video games played by school-aged children and young adults. They each randomly sample different groups of 150 students from the same school. They collect the following data.

Hours Played per Week	Frequency	Relative Frequency	Cumulative Relative Frequency
0–2	26	0.17	0.17
2–4	30	0.20	0.37
4–6	49	0.33	0.70

Hours Played per Week	Frequency	Relative Frequency	Cumulative Relative Frequency
6–8	25	0.17	0.87
8–10	12	0.08	0.95
10–12	8	0.05	1

Researcher A

Hours Played per Week	Frequency	Relative Frequency	Cumulative Relative Frequency
0–2	48	0.32	0.32
2–4	51	0.34	0.66
4–6	24	0.16	0.82
6–8	12	0.08	0.90
8–10	11	0.07	0.97
10–12	4	0.03	1

Researcher B

Exercise:

Problem: Give a reason why the data may differ.

Exercise:**Problem:**

Would the sample size be large enough if the population is the students in the school?

Solution:

Yes, the sample size of 150 would be large enough to reflect a population of one school.

Exercise:**Problem:**

Would the sample size be large enough if the population is school-aged children and young adults in the United States?

Exercise:**Problem:**

Researcher A concludes that most students play video games between four and six hours each week. Researcher B concludes that most students play video games between two and four hours each week. Who is correct?

Solution:

Even though the specific data support each researcher's conclusions, the different results suggest that more data need to be collected before the researchers can reach a conclusion.

Exercise:**Problem:**

As part of a way to reward students for participating in the survey, the researchers gave each student a gift card to a video game store. Would this affect the data if students knew about the award before the study?

Use the following data to answer the next five exercises: A pair of studies was performed to measure the effectiveness of a new software program designed to help stroke patients regain their problem-solving skills. Patients were asked to use the software program twice a day, once in the morning and once in the evening. The studies observed 200 stroke patients recovering over a period of several weeks. The first study collected the data in [\[link\]](#). The second study collected the data in [\[link\]](#).

Group	Showed improvement	No improvement	Deterioration
Used program	142	43	15
Did not use program	72	110	18

Group	Showed improvement	No improvement	Deterioration
Used program	105	74	19
Did not use program	89	99	12

Exercise:

Problem: Given what you know, which study is correct?

Solution:

There is not enough information given to judge if either one is correct or incorrect.

Exercise:

Problem:

The first study was performed by the company that designed the software program. The second study was performed by the American Medical Association. Which study is more reliable?

Exercise:

Problem:

Both groups that performed the study concluded that the software works. Is this accurate?

Solution:

The software program seems to work because the second study shows that more patients improve while using the software than not. Even though the difference is not as large as that in the first study, the results from the second study are likely more reliable and still show improvement.

Exercise:

Problem:

The company takes the two studies as proof that their software causes mental improvement in stroke patients. Is this a fair statement?

Exercise:

Problem:

Patients who used the software were also a part of an exercise program whereas patients who did not use the software were not. Does this change the validity of the conclusions from [\[link\]](#)?

Solution:

Yes, because we cannot tell if the improvement was due to the software or the exercise; the data is confounded, and a reliable conclusion cannot be drawn. New studies should be performed.

Exercise:**Problem:**

Is a sample size of 1,000 a reliable measure for a population of 5,000?

Exercise:**Problem:**

Is a sample of 500 volunteers a reliable measure for a population of 2,500?

Solution:

No, even though the sample is large enough, the fact that the sample consists of volunteers makes it a self-selected sample, which is not reliable.

Exercise:**Problem:**

A question on a survey reads: "Do you prefer the delicious taste of Brand X or the taste of Brand Y?" Is this a fair question?

Exercise:

Problem: Is a sample size of two representative of a population of five?

Solution:

No, even though the sample is a large portion of the population, two responses are not enough to justify any conclusions. Because the population is so small, it would be better to include everyone in the population to get the most accurate data.

Exercise:**Problem:**

Is it possible for two experiments to be well run with similar sample sizes to get different data?

HOMEWORK

For the following exercises, identify the type of data that would be used to describe a response (quantitative discrete, quantitative continuous, or qualitative), and give an example of the data.

Exercise:

Problem: number of tickets sold to a concert

Solution:

quantitative discrete, 150

Exercise:

Problem: percent of body fat

Exercise:

Problem: favorite baseball team

Solution:

qualitative, Oakland A's

Exercise:

Problem: time in line to buy groceries

Exercise:

Problem: number of students enrolled at Evergreen Valley College

Solution:

quantitative discrete, 11,234 students

Exercise:

Problem: most-watched television show

Exercise:

Problem: brand of toothpaste

Solution:

qualitative, Crest

Exercise:

Problem: distance to the closest movie theatre

Exercise:

Problem: age of executives in Fortune 500 companies

Solution:

quantitative continuous, 47.3 years

Exercise:

Problem: number of competing computer spreadsheet software packages

Use the following information to answer the next two exercises: A study was done to determine the age, number of times per week, and the duration (amount of time) of resident use of a local park in San Jose. The first house in the neighborhood around the park was selected randomly and then every 8th house in the neighborhood around the park was interviewed.

Exercise:

Problem: “Number of times per week” is what type of data?

- a. qualitative
 - b. quantitative discrete
 - c. quantitative continuous
-

Solution:

b

Exercise:

Problem: “Duration (amount of time)” is what type of data?

- a. qualitative
- b. quantitative discrete
- c. quantitative continuous

Exercise:

Problem:

Airline companies are interested in the consistency of the number of babies on each flight, so that they have adequate safety equipment. Suppose an airline conducts a survey. Over Thanksgiving weekend, it surveys six flights from Boston to Salt Lake City to determine the number of babies on the flights. It determines the amount of safety equipment needed by the result of that study.

- a. Using complete sentences, list three things wrong with the way the survey was conducted.

- b. Using complete sentences, list three ways that you would improve the survey if it were to be repeated.

Solution:

- a. The survey was conducted using six similar flights.
The survey would not be a true representation of the entire population of air travelers.
Conducting the survey on a holiday weekend will not produce representative results.
- b. Conduct the survey during different times of the year.
Conduct the survey using flights to and from various locations.
Conduct the survey on different days of the week.

Exercise:

Problem:

Suppose you want to determine the mean number of students per statistics class in your state. Describe a possible sampling method in three to five complete sentences. Make the description detailed.

Exercise:

Problem:

Suppose you want to determine the mean number of cans of soda drunk each month by students in their twenties at your school. Describe a possible sampling method in three to five complete sentences. Make the description detailed.

Solution:

Answers will vary. Sample Answer: You could use a systematic sampling method. Stop the tenth person as they leave one of the buildings on campus at 9:50 in the morning. Then stop the tenth person as they leave a different building on campus at 1:50 in the afternoon.

Exercise:

Problem:

List some practical difficulties involved in getting accurate results from a telephone survey.

Exercise:**Problem:**

List some practical difficulties involved in getting accurate results from a mailed survey.

Solution:

Answers will vary. Sample Answer: Many people will not respond to mail surveys. If they do respond to the surveys, you can't be sure who is responding. In addition, mailing lists can be incomplete.

Exercise:**Problem:**

With your classmates, brainstorm some ways you could overcome these problems if you needed to conduct a phone or mail survey.

Exercise:**Problem:**

The instructor takes her sample by gathering data on five randomly selected students from each Lake Tahoe Community College math class. The type of sampling she used is

- a. cluster sampling
- b. stratified sampling
- c. simple random sampling
- d. convenience sampling

Solution:

b

Exercise:**Problem:**

A study was done to determine the age, number of times per week, and the duration (amount of time) of residents using a local park in San Jose. The first house in the neighborhood around the park was selected randomly and then every eighth house in the neighborhood around the park was interviewed. The sampling method was:

- a. simple random
- b. systematic
- c. stratified
- d. cluster

Exercise:**Problem:**

Name the sampling method used in each of the following situations:

- a. A woman in the airport is handing out questionnaires to travelers asking them to evaluate the airport's service. She does not ask travelers who are hurrying through the airport with their hands full of luggage, but instead asks all travelers who are sitting near gates and not taking naps while they wait.
- b. A teacher wants to know if her students are doing homework, so she randomly selects rows two and five and then calls on all students in row two and all students in row five to present the solutions to homework problems to the class.
- c. The marketing manager for an electronics chain store wants information about the ages of its customers. Over the next two weeks, at each store location, 100 randomly selected customers are given questionnaires to fill out asking for information about age, as well as about other variables of interest.
- d. The librarian at a public library wants to determine what proportion of the library users are children. The librarian has a tally sheet on which she marks whether books are checked out by an adult or a child. She records this data for every fourth patron who checks out books.

- e. A political party wants to know the reaction of voters to a debate between the candidates. The day after the debate, the party's polling staff calls 1,200 randomly selected phone numbers. If a registered voter answers the phone or is available to come to the phone, that registered voter is asked whom he or she intends to vote for and whether the debate changed his or her opinion of the candidates.

Solution:

convenience cluster stratified systematic simple random

Exercise:

Problem:

A “random survey” was conducted of 3,274 people of the “microprocessor generation” (people born since 1971, the year the microprocessor was invented). It was reported that 48% of those individuals surveyed stated that if they had \$2,000 to spend, they would use it for computer equipment. Also, 66% of those surveyed considered themselves relatively savvy computer users.

- a. Do you consider the sample size large enough for a study of this type? Why or why not?
- b. Based on your “gut feeling,” do you believe the percents accurately reflect the U.S. population for those individuals born since 1971? If not, do you think the percents of the population are actually higher or lower than the sample statistics? Why?

Additional information: The survey, reported by Intel Corporation, was filled out by individuals who visited the Los Angeles Convention Center to see the Smithsonian Institute's road show called “America's Smithsonian.”

- c. With this additional information, do you feel that all demographic and ethnic groups were equally represented at the event? Why or why not?
- d. With the additional information, comment on how accurately you think the sample statistics reflect the population parameters.

Exercise:

Problem:

The Well-Being Index is a survey that follows trends of U.S. residents on a regular basis. There are six areas of health and wellness covered in the survey: Life Evaluation, Emotional Health, Physical Health, Healthy Behavior, Work Environment, and Basic Access. Some of the questions used to measure the Index are listed below.

Identify the type of data obtained from each question used in this survey: qualitative, quantitative discrete, or quantitative continuous.

- a. Do you have any health problems that prevent you from doing any of the things people your age can normally do?
 - b. During the past 30 days, for about how many days did poor health keep you from doing your usual activities?
 - c. In the last seven days, on how many days did you exercise for 30 minutes or more?
 - d. Do you have health insurance coverage?
-

Solution:

- a. qualitative
- b. quantitative discrete
- c. quantitative discrete
- d. qualitative

Exercise:**Problem:**

In advance of the 1936 Presidential Election, a magazine titled Literary Digest released the results of an opinion poll predicting that the republican candidate Alf Landon would win by a large margin. The magazine sent post cards to approximately 10,000,000 prospective voters. These prospective voters were selected from the subscription list of the magazine, from automobile registration lists, from phone lists, and from club membership lists. Approximately 2,300,000 people returned the postcards.

- a. Think about the state of the United States in 1936. Explain why a sample chosen from magazine subscription lists, automobile registration lists, phone books, and club membership lists was not representative of the population of the United States at that time.
- b. What effect does the low response rate have on the reliability of the sample?
- c. Are these problems examples of sampling error or nonsampling error?
- d. During the same year, George Gallup conducted his own poll of 30,000 prospective voters. These researchers used a method they called "quota sampling" to obtain survey answers from specific subsets of the population. Quota sampling is an example of which sampling method described in this module?

Exercise:

Problem:

Crime-related and demographic statistics for 47 US states in 1960 were collected from government agencies, including the FBI's *Uniform Crime Report*. One analysis of this data found a strong connection between education and crime indicating that higher levels of education in a community correspond to higher crime rates.

Which of the potential problems with samples discussed in [\[link\]](#) could explain this connection?

Solution:

Causality: The fact that two variables are related does not guarantee that one variable is influencing the other. We cannot assume that crime rate impacts education level or that education level impacts crime rate.

Confounding: There are many factors that define a community other than education level and crime rate. Communities with high crime rates and high education levels may have other lurking variables that distinguish them from communities with lower crime rates and lower education levels. Because we cannot isolate these variables of interest, we cannot draw valid conclusions about the connection between education and

crime. Possible lurking variables include police expenditures, unemployment levels, region, average age, and size.

Exercise:

Problem:

YouPolls is a website that allows anyone to create and respond to polls. One question posted April 15 asks:

“Do you feel happy paying your taxes when members of the Obama administration are allowed to ignore their tax liabilities?”[\[footnote\]](#)
lastbaldeagle. 2013. On Tax Day, House to Call for Firing Federal Workers Who Owe Back Taxes. Opinion poll posted online at: <http://www.youpolls.com/details.aspx?id=12328> (accessed May 1, 2013).

As of April 25, 11 people responded to this question. Each participant answered “NO!”

Which of the potential problems with samples discussed in this module could explain this connection?

Exercise:

Problem:

A scholarly article about response rates begins with the following quote:

“Declining contact and cooperation rates in random digit dial (RDD) national telephone surveys raise serious concerns about the validity of estimates drawn from such research.”[\[footnote\]](#)

Scott Keeter et al., “Gauging the Impact of Growing Nonresponse on Estimates from a National RDD Telephone Survey,” *Public Opinion Quarterly* 70 no. 5 (2006), <http://poq.oxfordjournals.org/content/70/5/759.full> (accessed May 1, 2013).

The Pew Research Center for People and the Press admits:

“The percentage of people we interview – out of all we try to interview – has been declining over the past decade or more.”[\[footnote\]](#)

Frequently Asked Questions, Pew Research Center for the People & the Press, <http://www.people-press.org/methodology/frequently-asked-questions/#dont-you-have-trouble-getting-people-to-answer-your-polls> (accessed May 1, 2013).

- a. What are some reasons for the decline in response rate over the past decade?
- b. Explain why researchers are concerned with the impact of the declining response rate on public opinion polls.

Solution:

- a. Possible reasons: increased use of caller id, decreased use of landlines, increased use of private numbers, voice mail, privacy managers, hectic nature of personal schedules, decreased willingness to be interviewed
- b. When a large number of people refuse to participate, then the sample may not have the same characteristics of the population. Perhaps the majority of people willing to participate are doing so because they feel strongly about the subject of the survey.

Bringing It Together

Exercise:

Problem:

Seven hundred and seventy-one distance learning students at Long Beach City College responded to surveys in the 2010-11 academic year. Highlights of the summary report are listed in [\[link\]](#).

Have computer at home	96%
-----------------------	-----

Unable to come to campus for classes	65%
Age 41 or over	24%
Would like LBCC to offer more DL courses	95%
Took DL classes due to a disability	17%
Live at least 16 miles from campus	13%
Took DL courses to fulfill transfer requirements	71%

LBCC Distance Learning Survey Results

- What percent of the students surveyed do not have a computer at home?
- About how many students in the survey live at least 16 miles from campus?
- If the same survey were done at Great Basin College in Elko, Nevada, do you think the percentages would be the same? Why?

Exercise:

Problem:

Several online textbook retailers advertise that they have lower prices than on-campus bookstores. However, an important factor is whether the Internet retailers actually have the textbooks that students need in stock. Students need to be able to get textbooks promptly at the beginning of the college term. If the book is not available, then a student would not be able to get the textbook at all, or might get a delayed delivery if the book is back ordered.

A college newspaper reporter is investigating textbook availability at online retailers. He decides to investigate one textbook for each of the following seven subjects: calculus, biology, chemistry, physics, statistics, geology, and general engineering. He consults textbook industry sales

data and selects the most popular nationally used textbook in each of these subjects. He visits websites for a random sample of major online textbook sellers and looks up each of these seven textbooks to see if they are available in stock for quick delivery through these retailers. Based on his investigation, he writes an article in which he draws conclusions about the overall availability of all college textbooks through online textbook retailers.

Write an analysis of his study that addresses the following issues: Is his sample representative of the population of all college textbooks? Explain why or why not. Describe some possible sources of bias in this study, and how it might affect the results of the study. Give some suggestions about what could be done to improve the study.

Solution:

Answers will vary. Sample answer: The sample is not representative of the population of all college textbooks. Two reasons why it is not representative are that he only sampled seven subjects and he only investigated one textbook in each subject. There are several possible sources of bias in the study. The seven subjects that he investigated are all in mathematics and the sciences; there are many subjects in the humanities, social sciences, and other subject areas, (for example: literature, art, history, psychology, sociology, business) that he did not investigate at all. It may be that different subject areas exhibit different patterns of textbook availability, but his sample would not detect such results.

He also looked only at the most popular textbook in each of the subjects he investigated. The availability of the most popular textbooks may differ from the availability of other textbooks in one of two ways:

- the most popular textbooks may be more readily available online, because more new copies are printed, and more students nationwide are selling back their used copies OR
- the most popular textbooks may be harder to find available online, because more student demand exhausts the supply more quickly.

In reality, many college students do not use the most popular textbook in their subject, and this study gives no useful information about the situation for those less popular textbooks.

He could improve this study by:

- expanding the selection of subjects he investigates so that it is more representative of all subjects studied by college students, and
- expanding the selection of textbooks he investigates within each subject to include a mixed representation of both the most popular and less popular textbooks.

Glossary

Cluster Sampling

a method for selecting a random sample and dividing the population into groups (clusters); use simple random sampling to select a set of clusters. Every individual in the chosen clusters is included in the sample.

Continuous Random Variable

a random variable (RV) whose outcomes are measured; the height of trees in the forest is a continuous RV.

Convenience Sampling

a nonrandom method of selecting a sample; this method selects individuals that are easily accessible and may result in biased data.

Discrete Random Variable

a random variable (RV) whose outcomes are counted

Nonsampling Error

an issue that affects the reliability of sampling data other than natural variation; it includes a variety of human errors including poor study design, biased sampling methods, inaccurate information provided by study participants, data entry errors, and poor analysis.

Qualitative Data

See [Data](#).

Quantitative Data

See [Data](#).

Random Sampling

a method of selecting a sample that gives every member of the population an equal chance of being selected.

Sampling Bias

not all members of the population are equally likely to be selected

Sampling Error

the natural variation that results from selecting a sample to represent a larger population; this variation decreases as the sample size increases, so selecting larger samples reduces sampling error.

Sampling with Replacement

Once a member of the population is selected for inclusion in a sample, that member is returned to the population for the selection of the next individual.

Sampling without Replacement

A member of the population may be chosen for inclusion in a sample only once. If chosen, the member is not returned to the population before the next selection.

Simple Random Sampling

a straightforward method for selecting a random sample; give each member of the population a number. Use a random number generator to select a set of labels. These randomly selected labels identify the members of your sample.

Stratified Sampling

a method for selecting a random sample used to ensure that subgroups of the population are represented adequately; divide the population into groups (strata). Use simple random sampling to identify a proportionate number of individuals from each stratum.

Systematic Sampling

a method for selecting a random sample; list the members of the population. Use simple random sampling to select a starting point in the population. Let $k = (\text{number of individuals in the population})/(\text{number of individuals needed in the sample})$. Choose every k th individual in the list starting with the one that was randomly selected. If necessary, return to the beginning of the population list to complete your sample.

Introduction

class="introduction"

Note the
many
individual
shapes in
this
building.
(credit:
Bert
Kaufmann
, Flickr)



We are surrounded by all sorts of geometry. Architects use geometry to design buildings. Artists create vivid images out of colorful geometric shapes. Street signs, automobiles, and product packaging all take advantage of geometric properties. In this chapter, we will begin by considering a formal approach to solving problems and use it to solve a variety of common problems, including making decisions about money. Then we will explore geometry and relate it to everyday situations, using the problem-solving strategy we develop.

Use a Problem Solving Strategy

By the end of this section, you will be able to:

- Approach word problems with a positive attitude
- Use a problem solving strategy for word problems
- Solve number problems

Note:

Before you get started, take this readiness quiz.

1. Translate “6 less than twice x ” into an algebraic expression.

If you missed this problem, review [\[link\]](#).

2. Solve: $\frac{2}{3}x = 24$.

If you missed this problem, review [\[link\]](#).

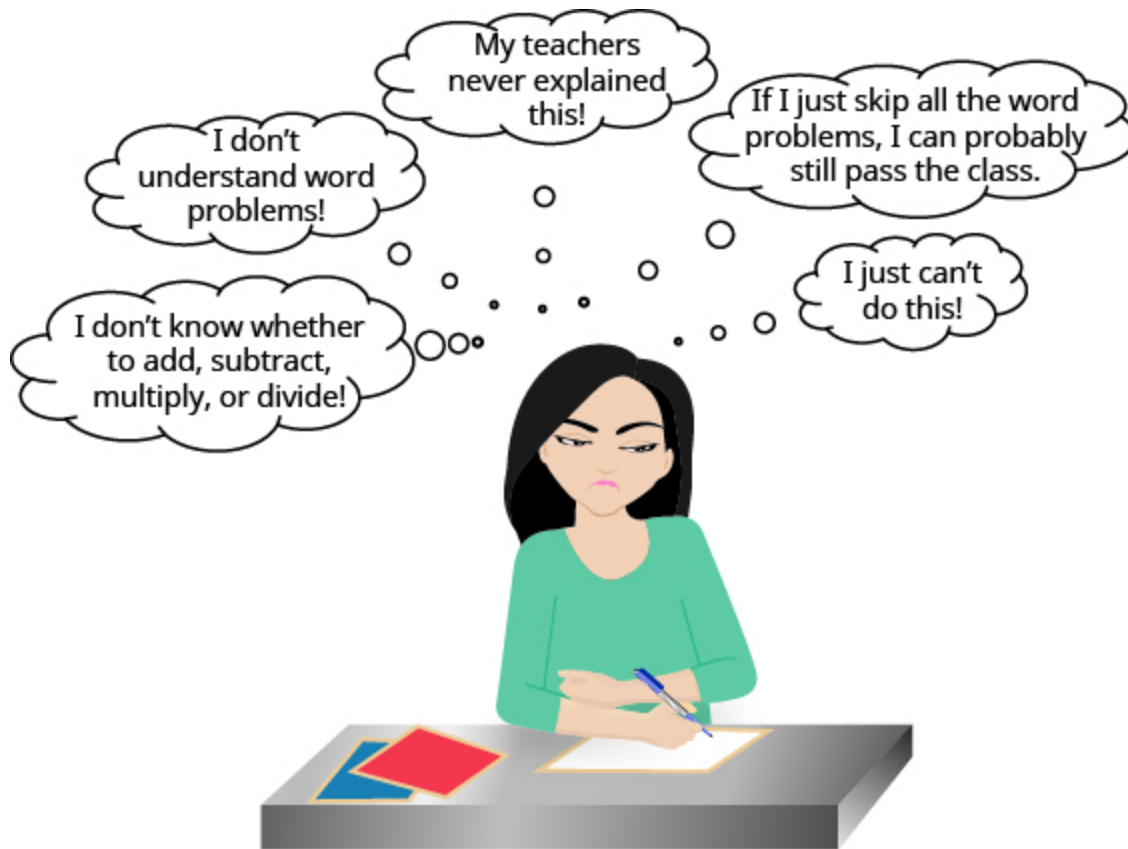
3. Solve: $3x + 8 = 14$.

If you missed this problem, review [\[link\]](#).

Approach Word Problems with a Positive Attitude

The world is full of word problems. How much money do I need to fill the car with gas? How much should I tip the server at a restaurant? How many socks should I pack for vacation? How big a turkey do I need to buy for Thanksgiving dinner, and what time do I need to put it in the oven? If my sister and I buy our mother a present, how much will each of us pay?

Now that we can solve equations, we are ready to apply our new skills to word problems. Do you know anyone who has had negative experiences in the past with word problems? Have you ever had thoughts like the student in [\[link\]](#)?



Negative thoughts about word problems can be barriers to success.

When we feel we have no control, and continue repeating negative thoughts, we set up barriers to success. We need to calm our fears and change our negative feelings.

Start with a fresh slate and begin to think positive thoughts like the student in [\[link\]](#). Read the positive thoughts and say them out loud.



When it comes to word problems, a positive attitude is a big step toward success.

If we take control and believe we can be successful, we will be able to master word problems.

Think of something that you can do now but couldn't do three years ago. Whether it's driving a car, snowboarding, cooking a gourmet meal, or speaking a new language, you have been able to learn and master a new skill. Word problems are no different. Even if you have struggled with word problems in the past, you have acquired many new math skills that will help you succeed now!

Use a Problem-solving Strategy for Word Problems

In earlier chapters, you translated word phrases into algebraic expressions, using some basic mathematical vocabulary and symbols. Since then you've increased your math vocabulary as you learned about more algebraic procedures, and you've had more practice translating from words into algebra.

You have also translated word sentences into algebraic equations and solved some word problems. The word problems applied math to everyday situations. You had to restate the situation in one sentence, assign a variable, and then write an equation to solve. This method works as long as the situation is familiar to you and the math is not too complicated.

Now we'll develop a strategy you can use to solve any word problem. This strategy will help you become successful with word problems. We'll demonstrate the strategy as we solve the following problem.

Example:

Exercise:

Problem:

Pete bought a shirt on sale for \$18, which is one-half the original price. What was the original price of the shirt?

Solution:

Solution

Step 1. **Read** the problem. Make sure you understand all the words and ideas. You may need to read the problem two or more times. If there are words you don't understand, look them up in a dictionary or on the Internet.

- *In this problem, do you understand what is being discussed? Do you understand every word?*

Step 2. **Identify** what you are looking for. It's hard to find something if you are not sure what it is! Read the problem again and look for words that tell you what you are looking for!

- *In this problem, the words “what was the original price of the shirt” tell you that what you are looking for: the original price of the shirt.*

Step 3. **Name** what you are looking for. Choose a variable to represent that quantity. You can use any letter for the variable, but it may help to choose one that helps you remember what it represents.

- *Let p = the original price of the shirt*

Step 4. **Translate** into an equation. It may help to first restate the problem in one sentence, with all the important information. Then translate the sentence into an equation.

18	is	one-half	of	the original price .
18	=	$\frac{1}{2}$	·	p

Step 5. **Solve** the equation using good algebra techniques. Even if you know the answer right away, using algebra will better prepare you to solve problems that do not have obvious answers.

Write the equation.

$$18 = \frac{1}{2}p$$

Multiply both sides by 2.

$$2 \cdot 18 = 2 \cdot \frac{1}{2}p$$

Simplify.

$$36 = p$$

Step 6. **Check** the answer in the problem and make sure it makes sense.

- *We found that $p = 36$, which means the original price was \$36. Does \$36 make sense in the problem? Yes, because 18 is one-half of 36, and the shirt was on sale at half the original price.*

Step 7. **Answer** the question with a complete sentence.

- *The problem asked “What was the original price of the shirt?” The answer to the question is: “The original price of the shirt was \$36 .”*

If this were a homework exercise, our work might look like this:

Let p = the original price.

18 is one-half the original price.

$$18 = \frac{1}{2} p$$

$$2 \cdot 18 = 2 \cdot \frac{1}{2} p$$

$$36 = p$$

Check:

Is \$36 a reasonable price for a shirt? Yes.

Is 18 one-half of 36? Yes.

The original price of the shirt was \$36.

Note:

Exercise:

Problem:

Joaquin bought a bookcase on sale for \$120, which was two-thirds the original price. What was the original price of the bookcase?

Solution:

\$180

Note:

Exercise:

Problem:

Two-fifths of the people in the senior center dining room are men. If there are 16 men, what is the total number of people in the dining room?

Solution:

40

We list the steps we took to solve the previous example.

Note:

Problem-Solving Strategy

Read the word problem. Make sure you understand all the words and ideas.

You may need to read the problem two or more times. If there are words you don't understand, look them up in a dictionary or on the internet.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation. It may be helpful to first restate the problem in one sentence before translating.

Solve the equation using good algebra techniques.

Check the answer in the problem. Make sure it makes sense.

Answer the question with a complete sentence.

Let's use this approach with another example.

Example:

Exercise:

Problem:

Yash brought apples and bananas to a picnic. The number of apples was three more than twice the number of bananas. Yash brought 11 apples to the picnic. How many bananas did he bring?

Solution:

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	How many bananas did he bring?
Step 3. Name what you are looking for.	Let b = number of bananas

Choose a variable to represent the number of bananas.	
Step 4. Translate. Restate the problem in one sentence with all the important information. Translate into an equation.	
Step 5. Solve the equation.	$11 = 2b + 3$
Subtract 3 from each side.	$11 - 3 = 2b + 3 - 3$
Simplify.	$8 = 2b$
Divide each side by 2.	$\frac{8}{2} = \frac{2b}{2}$
Simplify.	$4 = b$
Step 6. Check: First, is our answer reasonable? Yes, bringing four bananas to a picnic seems reasonable. The problem says the number of apples was three more than twice the number of bananas. If there are four bananas, does that make eleven apples? Twice 4 bananas is 8. Three more than 8 is 11.	
Step 7. Answer the question.	Yash brought 4 bananas to the picnic.

Note:

Exercise:

Problem:

Guillermo bought textbooks and notebooks at the bookstore. The number of textbooks was 3 more than the number of notebooks. He bought 5 textbooks. How many notebooks did he buy?

Solution:

2

Note:

Exercise:

Problem:

Gerry worked Sudoku puzzles and crossword puzzles this week. The number of Sudoku puzzles he completed is seven more than the number of crossword puzzles. He completed 14 Sudoku puzzles. How many crossword puzzles did he complete?

Solution:

7

In [Solve Sales Tax, Commission, and Discount Applications](#), we learned how to translate and solve basic percent equations and used them to solve

sales tax and commission applications. In the next example, we will apply our Problem Solving Strategy to more applications of percent.

Example:

Exercise:

Problem:

Nga's car insurance premium increased by \$60, which was 8% of the original cost. What was the original cost of the premium?

Solution:

Solution

Step 1. Read the problem. Remember, if there are words you don't understand, look them up.	
Step 2. Identify what you are looking for.	the original cost of the premium
Step 3. Name. Choose a variable to represent the original cost of premium.	Let c = the original cost
Step 4. Translate. Restate as one sentence. Translate into an equation.	$\underbrace{\\$60}_{60} \underbrace{\text{was}}{=} \underbrace{8\%}_{0.08} \underbrace{\text{of}}{\cdot} \underbrace{\text{the original cost}}_c.$
Step 5. Solve the equation.	

	$60 = 0.08c$
Divide both sides by 0.08.	$\frac{60}{0.08} = \frac{0.08c}{0.08}$
Simplify.	$c = 750$
Step 6. Check: Is our answer reasonable? Yes, a \$750 premium on auto insurance is reasonable. Now let's check our algebra. Is 8% of 750 equal to 60? $750 = c$ $0.08(750) = 60$ $60 = 60 \checkmark$	
Step 7. Answer the question.	The original cost of Nga's premium was \$750.

Note:

Exercise:

Problem:

Pilar's rent increased by 4%. The increase was \$38. What was the original amount of Pilar's rent?

Solution:

\$950

Note:**Exercise:****Problem:**

Steve saves 12% of his paycheck each month. If he saved \$504 last month, how much was his paycheck?

Solution:

\$4,200

Solve Number Problems

Now we will translate and solve **number problems**. In number problems, you are given some clues about one or more numbers, and you use these clues to build an equation. Number problems don't usually arise on an everyday basis, but they provide a good introduction to practicing the Problem Solving Strategy. Remember to look for clue words such as *difference*, *of*, and *and*.

Example:

Exercise:

Problem: The difference of a number and six is 13. Find the number.

Solution:

Solution

Step 1. **Read** the problem. Do you understand all the words?

Step 2. **Identify** what you are looking for.

the number

Step 3. **Name.** Choose a variable to represent the number.

Let n = the number

Step 4. **Translate.** Restate as one sentence.
Translate into an equation.

The difference of a number and 6 is 13.
 $n - 6 = 13$

Step 5. **Solve** the equation.
Add 6 to both sides.
Simplify.

$$n - 6 = 13$$

$$n - 6 + 6 = 13 + 6$$

$$n = 19$$

Step 6. **Check:**
The difference of 19 and 6 is 13. It

checks.

Step 7. **Answer** the question.

The number is 19.

Note:

Exercise:

Problem:

The difference of a number and eight is 17. Find the number.

Solution:

25

Note:

Exercise:

Problem:

The difference of a number and eleven is -7 . Find the number.

Solution:

4

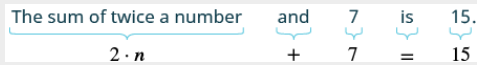
Example:

Exercise:

Problem:

The sum of twice a number and seven is 15. Find the number.

Solution:**Solution**

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the number
Step 3. Name. Choose a variable to represent the number.	Let n = the number
Step 4. Translate. Restate the problem as one sentence. Translate into an equation.	
Step 5. Solve the equation.	$2n + 7 = 15$
Subtract 7 from each side and simplify.	$2n = 8$
Divide each side by 2 and simplify.	$n = 4$
Step 6. Check: is the sum of twice 4 and 7 equal to 15?	

$$2 \cdot 4 + 7 = 15$$

$$8 + 7 = 15$$

$$15 = 15 \checkmark$$

Step 7. **Answer** the question.

The number is 4.

Note:

Exercise:

Problem:

The sum of four times a number and two is 14. Find the number.

Solution:

3

Note:

Exercise:

Problem:

The sum of three times a number and seven is 25. Find the number.

Solution:

6

Some number word problems ask you to find two or more numbers. It may be tempting to name them all with different variables, but so far we have only solved equations with one variable. We will define the numbers in terms of the same variable. Be sure to read the problem carefully to discover how all the numbers relate to each other.

Example:

Exercise:

Problem:

One number is five more than another. The sum of the numbers is twenty-one. Find the numbers.

Solution:

Solution

Step 1. Read the problem.		
Step 2. Identify what you are looking for.		You are looking for two numbers.
Step 3. Name. Choose a variable to represent the first number. What do you know about the second number? Translate.		Let $n = 1\text{st number}$ One number is five more than another. $x + 5 = 2^{\text{nd}}\text{number}$
Step 4. Translate. Restate the problem as one		The sum of the

sentence with all the important information. Translate into an equation. Substitute the variable expressions.		numbers is 21. The sum of the 1st number and the 2nd number is 21. $\begin{array}{rcl} \text{1st number} & + & \text{2nd number} & = & 21 \\ n & + & n + 5 & = & 21 \end{array}$
Step 5. Solve the equation.		$n + n + 5 = 21$
Combine like terms.		$2n + 5 = 21$
Subtract five from both sides and simplify.		$2n = 16$
Divide by two and simplify.		$n = 8$ 1 st number
Find the second number too.		$n + 5$ 2 nd number
Substitute $n = 8$		$8 + 5$
		13
Step 6. Check:		
Do these numbers check in the problem? Is one number 5 more than the other? Is thirteen, 5 more than 8? Yes.	$13 \stackrel{?}{=} 8 + 5$ $13 = 13 \checkmark$	

Is the sum of the two numbers 21?	$8 + 13 \neq 21$ $21 = 21 \checkmark$	
Step 7. Answer the question.		The numbers are 8 and 13.

Note:

Exercise:

Problem:

One number is six more than another. The sum of the numbers is twenty-four. Find the numbers.

Solution:

9, 15

Note:

Exercise:

Problem:

The sum of two numbers is fifty-eight. One number is four more than the other. Find the numbers.

Solution:

Example:**Exercise:****Problem:**

The sum of two numbers is negative fourteen. One number is four less than the other. Find the numbers.

Solution:**Solution**

Step 1. **Read** the problem.

Step 2. **Identify** what you are looking for.

two numbers

Step 3. **Name.**
Choose a variable.
What do you know about the second number?
Translate.

Let $n = 1^{\text{st}}$ number
One number is 4 less than the other.
 $n - 4 = 2^{\text{nd}}$ number

Step 4. **Translate.**
Write as one sentence.
Translate into an equation.

The sum of two numbers is negative fourteen.

$$\begin{array}{rcl} \underbrace{1^{\text{st}} \text{ number}} & + & \underbrace{2^{\text{nd}} \text{ number}} & = & -14 \\ n & + & n - 4 & = & -14 \end{array}$$

Substitute the variable expressions.		
Step 5. Solve the equation.		$n + n - 4 = -14$
Combine like terms.		$2n - 4 = -14$
Add 4 to each side and simplify.		$2n = -10$
Divide by 2.		$n = -5$ 1 st number
Substitute $n = -5$ to find the 2 nd number.		$n - 4$ 2 nd number
		$-5 - 4$
		-9
Step 6. Check:		
Is -9 four less than -5 ?	$-5 - 4 \stackrel{?}{=} -9$	
Is their sum -14 ?	$-9 = -9 \checkmark$	
	$-5 + (-9) \stackrel{?}{=} -14$	

$$-14 = -14 \checkmark$$

Step 7. **Answer** the question.

The numbers are -5 and -9 .

Note:

Exercise:

Problem:

The sum of two numbers is negative twenty-three. One number is 7 less than the other. Find the numbers.

Solution:

$-8, -15$

Note:

Exercise:

Problem:

The sum of two numbers is negative eighteen. One number is 40 more than the other. Find the numbers.

Solution:

$-29, 11$

Example:

Exercise:

Problem:

One number is ten more than twice another. Their sum is one. Find the numbers.

Solution:

Solution

Step 1. Read the problem.		
Step 2. Identify what you are looking for.		two numbers
Step 3. Name. Choose a variable. One number is ten more than twice another.		Let $x = 1^{\text{st}}$ number $2x + 10 = 2^{\text{nd}}$ number
Step 4. Translate. Restate as one sentence.		Their sum is one.
Translate into an equation		The sum of the two numbers is 1. $x + (2x + 10) = 1$
Step 5. Solve the equation.		$x + 2x + 10 = 1$

Combine like terms.		$3x + 10 = 1$
Subtract 10 from each side.		$3x = -9$
Divide each side by 3 to get the first number.		$x = -3$
Substitute to get the second number.		$2x + 10$
		$2(-3) + 10$
		4
Step 6. Check.		
Is 4 ten more than twice -3 ?	$2(-3) + 10 \stackrel{?}{=} 4$	
Is their sum 1?	$-6 + 10 = 4$	
	$4 = 4 \checkmark$	
	$-3 + 4 \stackrel{?}{=} 1$	
	$1 = 1 \checkmark$	

Step 7. **Answer** the question.

The numbers are -3 and 4 .

Note:

Exercise:

Problem:

One number is eight more than twice another. Their sum is negative four. Find the numbers.

Solution:

$-4, 0$

Note:

Exercise:

Problem:

One number is three more than three times another. Their sum is negative five. Find the numbers.

Solution:

$-2, -3$

Consecutive integers are integers that immediately follow each other. Some examples of consecutive integers are:

Equation:

...1, 2, 3, 4,...

Equation:

...-10, -9, -8, -7,...

Equation:

...150, 151, 152, 153,...

Notice that each number is one more than the number preceding it. So if we define the first integer as n , the next consecutive integer is $n + 1$. The one after that is one more than $n + 1$, so it is $n + 1 + 1$, or $n + 2$.

Equation:

n	1st integer
$n + 1$	2nd consecutive integer
$n + 2$	3rd consecutive integer

Example:

Exercise:

Problem:

The sum of two consecutive integers is 47. Find the numbers.

Solution:

Solution

Step 1. Read the problem.		
Step 2. Identify what you are looking for.		two consecutive integers
Step 3. Name.		Let $n = 1^{\text{st}}$ integer $n + 1 = \text{next consecutive integer}$
Step 4. Translate. Restate as one sentence. Translate into an equation.		The sum of the integers $n + n + 1$ is $=$ 47. 47
Step 5. Solve the equation.		$n + n + 1 = 47$
Combine like terms.		$2n + 1 = 47$
Subtract 1 from each side.		$2n = 46$
Divide each side by 2.		$n = 23$ 1^{st} integer
Substitute to get the second number.		$n + 1$ 2^{nd} integer
		$23 + 1$

		24
Step 6. Check:	$23 + 24 \stackrel{?}{=} 47$ $47 = 47 \checkmark$	
Step 7. Answer the question.		The two consecutive integers are 23 and 24.

Note:

Exercise:

Problem:

The sum of two consecutive integers is 95. Find the numbers.

Solution:

47, 48

Note:

Exercise:

Problem:

The sum of two consecutive integers is -31 . Find the numbers.

Solution:

$-15, -16$

Example:

Exercise:

Problem: Find three consecutive integers whose sum is 42.

Solution:

Solution

Step 1. **Read** the problem.

Step 2. **Identify** what you are looking for.

three consecutive integers

Step 3. **Name.**

Let $n = 1^{\text{st}}$ integer
 $n + 1 = 2^{\text{nd}}$ consecutive integer
 $n + 2 = 3^{\text{rd}}$ consecutive integer

Step 4. **Translate.**
Restate as one sentence.
Translate into an equation.

The sum of the three integers is 42.

$$n + n + 1 + n + 2 = 42$$

Step 5. **Solve** the equation.

$$n + n + 1 + n + 2 = 42$$

Combine like terms.		$3n + 3 = 42$
Subtract 3 from each side.		$3n = 39$
Divide each side by 3.		$n = 13$ 1 st integer
Substitute to get the second number.		$n + 1$ 2 nd integer
		$13 + 1$
		24
Substitute to get the third number.		$n + 2$ 3 rd integer
		$13 + 2$
		15
Step 6. Check:	$13 + 14 + 15 \stackrel{?}{=} 42$ $42 = 42 \checkmark$	
Step 7. Answer the		The three consecutive

question.

integers are 13, 14, and 15.

Note:

Exercise:

Problem: Find three consecutive integers whose sum is 96.

Solution:

31, 32, 33

Note:

Exercise:

Problem: Find three consecutive integers whose sum is -36 .

Solution:

$-11, -12, -13$

Note: The Links to Literacy activities *Math Curse*, *Missing Mittens* and *Among the Odds and Evens* will provide you with another view of the topics covered in this section.

Key Concepts

- **Problem Solving Strategy**

Read the word problem. Make sure you understand all the words and ideas. You may need to read the problem two or more times. If there are words you don't understand, look them up in a dictionary or on the internet.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation. It may be helpful to first restate the problem in one sentence before translating.

Solve the equation using good algebra techniques.

Check the answer in the problem. Make sure it makes sense.

Answer the question with a complete sentence.

Practice Makes Perfect

Use a Problem-solving Strategy for Word Problems

In the following exercises, use the problem-solving strategy for word problems to solve. Answer in complete sentences.

Exercise:

Problem:

Two-thirds of the children in the fourth-grade class are girls. If there are 20 girls, what is the total number of children in the class?

Solution:

There are 30 children in the class.

Exercise:

Problem:

Three-fifths of the members of the school choir are women. If there are 24 women, what is the total number of choir members?

Exercise:

Problem:

Zachary has 25 country music CDs, which is one-fifth of his CD collection. How many CDs does Zachary have?

Solution:

Zachary has 125 CDs.

Exercise:

Problem:

One-fourth of the candies in a bag are red. If there are 23 red candies, how many candies are in the bag?

Exercise:

Problem:

There are 16 girls in a school club. The number of girls is 4 more than twice the number of boys. Find the number of boys in the club.

Solution:

There are 6 boys in the club.

Exercise:

Problem:

There are 18 Cub Scouts in Troop 645. The number of scouts is 3 more than five times the number of adult leaders. Find the number of adult leaders.

Exercise:

Problem:

Lee is emptying dishes and glasses from the dishwasher. The number of dishes is 8 less than the number of glasses. If there are 9 dishes, what is the number of glasses?

Solution:

There are 17 glasses.

Exercise:**Problem:**

The number of puppies in the pet store window is twelve less than the number of dogs in the store. If there are 6 puppies in the window, what is the number of dogs in the store?

Exercise:**Problem:**

After 3 months on a diet, Lisa had lost 12% of her original weight. She lost 21 pounds. What was Lisa's original weight?

Solution:

Lisa's original weight was 175 pounds.

Exercise:**Problem:**

Tricia got a 6% raise on her weekly salary. The raise was \$30 per week. What was her original weekly salary?

Exercise:**Problem:**

Tim left a \$9 tip for a \$50 restaurant bill. What percent tip did he leave?

Solution:

18%

Exercise:

Problem:

Rashid left a \$15 tip for a \$75 restaurant bill. What percent tip did he leave?

Exercise:

Problem:

Yuki bought a dress on sale for \$72. The sale price was 60% of the original price. What was the original price of the dress?

Solution:

The original price was \$120.

Exercise:

Problem:

Kim bought a pair of shoes on sale for \$40.50. The sale price was 45% of the original price. What was the original price of the shoes?

Solve Number Problems

In the following exercises, solve each number word problem.

Exercise:

Problem: The sum of a number and eight is 12. Find the number.

Solution:

4

Exercise:

Problem: The sum of a number and nine is 17. Find the number.

Exercise:

Problem:

The difference of a number and twelve is 3. Find the number.

Solution:

15

Exercise:

Problem: The difference of a number and eight is 4. Find the number.

Exercise:

Problem:

The sum of three times a number and eight is 23. Find the number.

Solution:

5

Exercise:

Problem: The sum of twice a number and six is 14. Find the number.

Exercise:

Problem:

The difference of twice a number and seven is 17. Find the number.

Solution:

12

Exercise:

Problem:

The difference of four times a number and seven is 21. Find the number.

Exercise:**Problem:**

Three times the sum of a number and nine is 12. Find the number.

Solution:

-5

Exercise:**Problem:**

Six times the sum of a number and eight is 30. Find the number.

Exercise:**Problem:**

One number is six more than the other. Their sum is forty-two. Find the numbers.

Solution:

18, 24

Exercise:**Problem:**

One number is five more than the other. Their sum is thirty-three. Find the numbers.

Exercise:

Problem:

The sum of two numbers is twenty. One number is four less than the other. Find the numbers.

Solution:

8, 12

Exercise:**Problem:**

The sum of two numbers is twenty-seven. One number is seven less than the other. Find the numbers.

Exercise:**Problem:**

A number is one more than twice another number. Their sum is negative five. Find the numbers.

Solution:

-2, -3

Exercise:**Problem:**

One number is six more than five times another. Their sum is six. Find the numbers.

Exercise:**Problem:**

The sum of two numbers is fourteen. One number is two less than three times the other. Find the numbers.

Solution:

4, 10

Exercise:

Problem:

The sum of two numbers is zero. One number is nine less than twice the other. Find the numbers.

Exercise:

Problem:

One number is fourteen less than another. If their sum is increased by seven, the result is 85. Find the numbers.

Solution:

32, 46

Exercise:

Problem:

One number is eleven less than another. If their sum is increased by eight, the result is 71. Find the numbers.

Exercise:

Problem:

The sum of two consecutive integers is 77. Find the integers.

Solution:

38, 39

Exercise:

Problem:

The sum of two consecutive integers is 89. Find the integers.

Exercise:

Problem:

The sum of two consecutive integers is -23 . Find the integers.

Solution:

$-11, -12$

Exercise:**Problem:**

The sum of two consecutive integers is -37 . Find the integers.

Exercise:**Problem:**

The sum of three consecutive integers is 78 . Find the integers.

Solution:

$25, 26, 27$

Exercise:**Problem:**

The sum of three consecutive integers is 60 . Find the integers.

Exercise:

Problem: Find three consecutive integers whose sum is -36 .

Solution:

$-11, -12, -13$

Exercise:

Problem: Find three consecutive integers whose sum is -3 .

Everyday Math

Exercise:

Problem:

Shopping Patty paid \$35 for a purse on sale for \$10 off the original price. What was the original price of the purse?

Solution:

The original price was \$45.

Exercise:

Problem:

Shopping Travis bought a pair of boots on sale for \$25 off the original price. He paid \$60 for the boots. What was the original price of the boots?

Exercise:

Problem:

Shopping Minh spent \$6.25 on 5 sticker books to give his nephews. Find the cost of each sticker book.

Solution:

Each sticker book cost \$1.25.

Exercise:

Problem:

Shopping Alicia bought a package of 8 peaches for \$3.20. Find the cost of each peach.

Exercise:

Problem:

Shopping Tom paid \$1,166.40 for a new refrigerator, including \$86.40 tax. What was the price of the refrigerator before tax?

Solution:

The price of the refrigerator before tax was \$1,080.

Exercise:**Problem:**

Shopping Kenji paid \$2,279 for a new living room set, including \$129 tax. What was the price of the living room set before tax?

Writing Exercises**Exercise:****Problem:**

Write a few sentences about your thoughts and opinions of word problems. Are these thoughts positive, negative, or neutral? If they are negative, how might you change your way of thinking in order to do better?

Solution:

Answers will vary.

Exercise:**Problem:**

When you start to solve a word problem, how do you decide what to let the variable represent?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
approach word problems with a positive attitude.			
use a problem solving strategy for word problems.			
solve number problems.			

Ⓑ If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

...no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Simplify and Use Square Roots

By the end of this section, you will be able to:

- Simplify expressions with square roots
- Estimate square roots
- Approximate square roots
- Simplify variable expressions with square roots
- Use square roots in applications

Note:

Before you get started, take this readiness quiz.

1. Simplify: $(-9)^2$.
If you missed this problem, review [\[link\]](#).
2. Round 3.846 to the nearest hundredth.
If you missed this problem, review [\[link\]](#).
3. Evaluate $12d$ for $d = 80$.
If you missed this problem, review [\[link\]](#).

Simplify Expressions with Square Roots

To start this section, we need to review some important vocabulary and notation.

Remember that when a number n is multiplied by itself, we can write this as n^2 , which we read aloud as “ n squared.” For example, 8^2 is read as “8 squared.”

We call 64 the *square* of 8 because $8^2 = 64$. Similarly, 121 is the square of 11, because $11^2 = 121$.

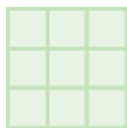
Note:

Square of a Number

If $n^2 = m$, then m is the square of n .

Modeling Squares

Do you know why we use the word *square*? If we construct a square with three tiles on each side, the total number of tiles would be nine.



This is why we say that the square of three is nine.

Equation:

$$3^2 = 9$$

The number 9 is called a perfect square because it is the square of a whole number.

Note: Doing the Manipulative Mathematics activity Square Numbers will help you develop a better understanding of perfect square numbers

The chart shows the squares of the counting numbers 1 through 15. You can refer to it to help you identify the perfect squares.

Number	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Note:

Perfect Squares

A **perfect square** is the square of a whole number.

What happens when you square a negative number?

Equation:

$$\begin{aligned}(-8)^2 &= (-8)(-8) \\ &= 64\end{aligned}$$

When we multiply two negative numbers, the product is always positive. So, the square of a negative number is always positive.

The chart shows the squares of the negative integers from -1 to -15 .

Number	n	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
Square	n^2	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

Did you notice that these squares are the same as the squares of the positive numbers?

Square Roots

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We can also say that 10 is a square root of 100.

Note:

Square Root of a Number

A number whose square is m is called a square root of m .

If $n^2 = m$, then n is a **square root** of m .

Notice $(-10)^2 = 100$ also, so -10 is also a square root of 100. Therefore, both 10 and -10 are square roots of 100.

So, every positive number has two square roots: one positive and one negative.

What if we only want the positive square root of a positive number? The *radical sign*, $\sqrt{}$, stands for the positive square root. The positive square root is also called the **principal square root**.

Note:

Square Root Notation

$\sqrt{m}$ is read as “the square root of m .”

If $m = n^2$, then $\sqrt{m} = n$ for $n \geq 0$.

radical sign $\longrightarrow \sqrt{m} \longleftarrow$ radicand

We can also use the radical sign for the square root of zero. Because $0^2 = 0$, $\sqrt{0} = 0$. Notice that zero has only one square root.

The chart shows the square roots of the first 15 perfect square numbers.

$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$	$\sqrt{169}$	$\sqrt{196}$	$\sqrt{225}$
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Example:

Exercise:

Problem: Simplify: ① $\sqrt{25}$ ② $\sqrt{121}$.

Solution:

Solution

①	
	$\sqrt{25}$
Since $5^2 = 25$	5

ⓑ	
	$\sqrt{121}$
Since $11^2 = 121$	-11

Note:

Exercise:

Problem: Simplify: ⓐ $\sqrt{36}$ ⓑ $\sqrt{169}$.

Solution:

ⓐ 6

ⓑ 13

Note:

Exercise:

Problem: Simplify: ⓐ $\sqrt{16}$ ⓑ $\sqrt{196}$.

Solution:

ⓐ 4

ⓑ 14

Every positive number has two square roots and the radical sign indicates the positive one. We write $\sqrt{100} = 10$. If we want to find the negative square root of a number, we place a negative in front of the radical sign. For example, $-\sqrt{100} = -10$.

Example:

Exercise:

Problem: Simplify. Ⓐ $-\sqrt{9}$ Ⓑ $-\sqrt{144}$.

Solution:

Solution

Ⓐ	
	$-\sqrt{9}$
The negative is in front of the radical sign.	-3

Ⓑ	
	$-\sqrt{144}$
The negative is in front of the radical sign.	-12

Note:

Exercise:

Problem: Simplify: Ⓐ $-\sqrt{4}$ Ⓑ $-\sqrt{225}$.

Solution:

- Ⓐ -2
- Ⓑ -15

Note:

Exercise:

Problem: Simplify: Ⓐ $-\sqrt{81}$ Ⓑ $-\sqrt{64}$.

Solution:

- Ⓐ -9
- Ⓑ -8

Square Root of a Negative Number

Can we simplify $\sqrt{-25}$? Is there a number whose square is -25 ?

Equation:

$$(\quad)^2 = -25?$$

None of the numbers that we have dealt with so far have a square that is -25 . Why? Any positive number squared is positive, and any negative number squared is also positive. In the next chapter we will see that all the numbers we work with are called the real numbers. So we say there is no real number equal to $\sqrt{-25}$. If we are asked to find the square root of any negative number, we say that the solution is not a real number.

Example:

Exercise:

Problem: Simplify: Ⓐ $\sqrt{-169}$ Ⓑ $-\sqrt{121}$.

Solution:
Solution

- Ⓐ There is no real number whose square is -169 . Therefore, $\sqrt{-169}$ is not a real number.
- Ⓑ The negative is in front of the radical sign, so we find the opposite of the square root of 121.

	$-\sqrt{121}$
The negative is in front of the radical.	-11

Note:
Exercise:

Problem: Simplify: Ⓐ $\sqrt{-196}$ Ⓑ $-\sqrt{81}$.

Solution:

- Ⓐ not a real number
Ⓑ -9

Note:
Exercise:

Problem: Simplify: Ⓐ $\sqrt{-49}$ Ⓑ $-\sqrt{121}$.

Solution:

- Ⓐ -7
- Ⓑ not a real number

Square Roots and the Order of Operations

When using the order of operations to simplify an expression that has square roots, we treat the radical sign as a grouping symbol. We simplify any expressions under the radical sign before performing other operations.

Example:

Exercise:

Problem: Simplify: Ⓐ $\sqrt{25} + \sqrt{144}$ Ⓑ $\sqrt{25 + 144}$.

Solution:

Solution

Ⓐ Use the order of operations.	
	$\sqrt{25} + \sqrt{144}$
Simplify each radical.	$5 + 12$
Add.	17

ⓑ Use the order of operations.	
	$\sqrt{25 + 144}$
Add under the radical sign.	$\sqrt{169}$
Simplify.	13

Note:

Exercise:

Problem: Simplify: ⓐ $\sqrt{9} + \sqrt{16}$ ⓑ $\sqrt{9 + 16}$.

Solution:

- ⓐ 7
- ⓑ 5

Note:

Exercise:

Problem: Simplify: ⓐ $\sqrt{64 + 225}$ ⓑ $\sqrt{64} + \sqrt{225}$.

Solution:

- ⓐ 17
- ⓑ 23

Notice the different answers in parts ⓐ and ⓑ of [\[link\]](#). It is important to follow the order of operations correctly. In ⓐ, we took each square root first and then added

them. In ⑥, we added under the radical sign first and then found the square root.

Estimate Square Roots

So far we have only worked with square roots of perfect squares. The square roots of other numbers are not whole numbers.

Number	Square root
4	$\sqrt{4} = 2$
5	$\sqrt{5}$
6	$\sqrt{6}$
7	$\sqrt{7}$
8	$\sqrt{8}$
9	$\sqrt{9} = 3$

We might conclude that the square roots of numbers between 4 and 9 will be between 2 and 3, and they will not be whole numbers. Based on the pattern in the table above, we could say that $\sqrt{5}$ is between 2 and 3. Using inequality symbols, we write

Equation:

$$2 < \sqrt{5} < 3$$

Example:

Exercise:

Problem: Estimate $\sqrt{60}$ between two consecutive whole numbers.

Solution:

Solution

Think of the perfect squares closest to 60. Make a small table of these perfect squares and their squares roots.

Number	Square root
36	6
49	7
64	8
81	9

60

$\sqrt{60}$

Locate 60 between two consecutive perfect squares.

$$49 < 60 < 64$$

$\sqrt{60}$ is between their square roots.

$$7 < \sqrt{60} < 8$$

Note:

Exercise:

Problem: Estimate $\sqrt{38}$ between two consecutive whole numbers.

Solution:

$$6 < \sqrt{38} < 7$$

Note:

Exercise:

Problem: Estimate $\sqrt{84}$ between two consecutive whole numbers.

Solution:

$$9 < \sqrt{84} < 10$$

Approximate Square Roots with a Calculator

There are mathematical methods to approximate square roots, but it is much more convenient to use a calculator to find square roots. Find the $\sqrt{}$ or $\sqrt{x}$ key on your calculator. You will use this key to approximate square roots. When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact number. It is an approximation, to the number of digits shown on your calculator's display. The symbol for an approximation is $\approx$ and it is read *approximately*.

Suppose your calculator has a 10-digit display. Using it to find the square root of 5 will give 2.236067977. This is the approximate square root of 5. When we report the answer, we should use the “approximately equal to” sign instead of an equal sign.

Equation:

$$\sqrt{5} \approx 2.236067978$$

You will seldom use this many digits for applications in algebra. So, if you wanted to round $\sqrt{5}$ to two decimal places, you would write

Equation:

$$\sqrt{5} \approx 2.24$$

How do we know these values are approximations and not the exact values? Look at what happens when we square them.

Equation:

$$\begin{aligned} 2.236067978^2 &= 5.000000002 \\ 2.24^2 &= 5.0176 \end{aligned}$$

The squares are close, but not exactly equal, to 5.

Example:

Exercise:

Problem: Round $\sqrt{17}$ to two decimal places using a calculator.

Solution:
Solution

	$\sqrt{17}$
Use the calculator square root key.	4.123105626
Round to two decimal places.	4.12
	$\sqrt{17} \approx 4.12$

Note:
Exercise:

Problem: Round $\sqrt{11}$ to two decimal places.

Solution:

≈ 3.32

Note:
Exercise:

Problem: Round $\sqrt{13}$ to two decimal places.

Solution:

≈ 3.61

Simplify Variable Expressions with Square Roots

Expressions with square root that we have looked at so far have not had any variables. What happens when we have to find a square root of a variable expression?

Consider $\sqrt{9x^2}$, where $x \geq 0$. Can you think of an expression whose square is $9x^2$?

Equation:

$$\begin{aligned} (?)^2 &= 9x^2 \\ (3x)^2 &= 9x^2 \quad \text{so } \sqrt{9x^2} = 3x \end{aligned}$$

When we use a variable in a square root expression, for our work, we will assume that the variable represents a non-negative number. In every example and exercise that follows, each variable in a square root expression is greater than or equal to zero.

Example:

Exercise:

Problem: Simplify: $\sqrt{x^2}$.

Solution:

Solution

Think about what we would have to square to get x^2 . Algebraically, $(?)^2 = x^2$

	$\sqrt{x^2}$
Since $(x)^2 = x^2$	x

Note:

Exercise:

Problem: Simplify: $\sqrt{y^2}$.

Solution:

y

Note:

Exercise:

Problem: Simplify: $\sqrt{m^2}$.

Solution:

m

Example:

Exercise:

Problem: Simplify: $\sqrt{16x^2}$.

Solution:

Solution

	$\sqrt{16x^2}$
Since $(4x)^2 = 16x^2$	$4x$

Note:

Exercise:

Problem: Simplify: $\sqrt{64x^2}$.

Solution:

$8x$

Note:

Exercise:

Problem: Simplify: $\sqrt{169y^2}$.

Solution:

$13y$

Example:

Exercise:

Problem: Simplify: $-\sqrt{81y^2}$.

Solution:

Solution

$-\sqrt{81y^2}$

Since $(9y)^2 = 81y^2$

$-9y$

Note:

Exercise:

Problem: Simplify: $-\sqrt{121y^2}$.

Solution:

$-11y$

Note:

Exercise:

Problem: Simplify: $-\sqrt{100p^2}$.

Solution:

$-10p$

Example:

Exercise:

Problem: Simplify: $\sqrt{36x^2y^2}$.

Solution:

Solution

	$\sqrt{36x^2y^2}$
Since $(6xy)^2 = 36x^2y^2$	$6xy$

Note:

Exercise:

Problem: Simplify: $\sqrt{100a^2b^2}$.

Solution:

$10ab$

Note:

Exercise:

Problem: Simplify: $\sqrt{225m^2n^2}$.

Solution:

$15mn$

Use Square Roots in Applications

As you progress through your college courses, you'll encounter several applications of square roots. Once again, if we use our strategy for applications, it will give us a plan for finding the answer!

Note:

Use a strategy for applications with square roots.

Identify what you are asked to find.
Write a phrase that gives the information to find it.
Translate the phrase to an expression.
Simplify the expression.
Write a complete sentence that answers the question.

Square Roots and Area

We have solved applications with area before. If we were given the length of the sides of a square, we could find its area by squaring the length of its sides. Now we can find the length of the sides of a square if we are given the area, by finding the square root of the area.

If the area of the square is A square units, the length of a side is $\sqrt{A}$ units. See [\[link\]](#).

Area (square units)	Length of side (units)
9	$\sqrt{9} = 3$
144	$\sqrt{144} = 12$
A	$\sqrt{A}$

Example:

Exercise:

Problem:

Mike and Lychelle want to make a square patio. They have enough concrete for an area of 200 square feet. To the nearest tenth of a foot, how long can a side of their square patio be?

Solution:
Solution

We know the area of the square is 200 square feet and want to find the length of the side. If the area of the square is A square units, the length of a side is $\sqrt{A}$ units.

What are you asked to find?	The length of each side of a square patio
Write a phrase.	The length of a side
Translate to an expression.	$\sqrt{A}$
Evaluate $\sqrt{A}$ when $A = 200$.	$\sqrt{200}$
Use your calculator.	14.142135...
Round to one decimal place.	14.1 feet
Write a sentence.	Each side of the patio should be 14.1 feet.

Note:
Exercise:

Problem:

Katie wants to plant a square lawn in her front yard. She has enough sod to cover an area of 370 square feet. To the nearest tenth of a foot, how long can a side of her square lawn be?

Solution:

19.2 feet

Note:**Exercise:****Problem:**

Sergio wants to make a square mosaic as an inlay for a table he is building. He has enough tile to cover an area of 2704 square centimeters. How long can a side of his mosaic be?

Solution:

52 centimeters

Square Roots and Gravity

Another application of square roots involves gravity. On Earth, if an object is dropped from a height of h feet, the time in seconds it will take to reach the ground is found by evaluating the expression $\frac{\sqrt{h}}{4}$. For example, if an object is dropped from a height of 64 feet, we can find the time it takes to reach the ground by evaluating $\frac{\sqrt{64}}{4}$.

	$\frac{\sqrt{64}}{4}$
Take the square root of 64.	$\frac{8}{4}$
Simplify the fraction.	2

It would take 2 seconds for an object dropped from a height of 64 feet to reach the ground.

Example:

Exercise:

Problem:

Christy dropped her sunglasses from a bridge 400 feet above a river. How many seconds does it take for the sunglasses to reach the river?

Solution:

Solution

What are you asked to find?	The number of seconds it takes for the sunglasses to reach the river
Write a phrase.	The time it will take to reach the river
Translate to an expression.	$\frac{\sqrt{h}}{4}$
Evaluate $\frac{\sqrt{h}}{4}$ when $h = 400$.	$\frac{\sqrt{400}}{4}$
Find the square root of 400.	$\frac{20}{4}$
Simplify.	5
Write a sentence.	It will take 5 seconds for the sunglasses to reach the river.

Note:

Exercise:

Problem:

A helicopter drops a rescue package from a height of 1296 feet. How many seconds does it take for the package to reach the ground?

Solution:

9 seconds

Note:

Exercise:

Problem:

A window washer drops a squeegee from a platform 196 feet above the sidewalk. How many seconds does it take for the squeegee to reach the sidewalk?

Solution:

3.5 seconds

Square Roots and Accident Investigations

Police officers investigating car accidents measure the length of the skid marks on the pavement. Then they use square roots to determine the speed, in miles per hour, a car was going before applying the brakes. According to some formulas, if the length of the skid marks is d feet, then the speed of the car can be found by evaluating $\sqrt{24d}$.

Example:

Exercise:

Problem:

After a car accident, the skid marks for one car measured 190 feet. To the nearest tenth, what was the speed of the car (in mph) before the brakes were applied?

Solution:
Solution

What are you asked to find?	The speed of the car before the brakes were applied
Write a phrase.	The speed of the car
Translate to an expression.	$\sqrt{24d}$
Evaluate $\sqrt{24d}$ when $d = 190$.	$\sqrt{24 \cdot 190}$
Multiply.	$\sqrt{4,560}$
Use your calculator.	67.527772...
Round to tenths.	67.5
Write a sentence.	The speed of the car was approximately 67.5 miles per hour.

Note:**Exercise:**

Problem:

An accident investigator measured the skid marks of a car and found their length was 76 feet. To the nearest tenth, what was the speed of the car before the brakes were applied?

Solution:

42.7 mph

Note:**Exercise:****Problem:**

The skid marks of a vehicle involved in an accident were 122 feet long. To the nearest tenth, how fast had the vehicle been going before the brakes were applied?

Solution:

54.1 mph

Note: The *Links to Literacy* activity "Sea Squares" will provide you with another view of the topics covered in this section.

Note:**ACCESS ADDITIONAL ONLINE RESOURCES**

- [Introduction to Square Roots](#)
- [Estimating Square Roots with a Calculator](#)

Key Concepts

- **Square Root Notation** $\sqrt{m}$ is read ‘the square root of m ’
If $m = n^2$, then $\sqrt{m} = n$, for $n \geq 0$.

radical sign $\longrightarrow \sqrt{m} \longleftarrow$ radicand

- **Use a strategy for applications with square roots.**
 - Identify what you are asked to find.
 - Write a phrase that gives the information to find it.
 - Translate the phrase to an expression.
 - Simplify the expression.
 - Write a complete sentence that answers the question.

Section Exercises

Practice Makes Perfect

Simplify Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{36}$

Solution:

6

Exercise:

Problem: $\sqrt{4}$

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{144}$

Exercise:

Problem: $-\sqrt{4}$

Solution:

-2

Exercise:

Problem: $-\sqrt{100}$

Exercise:

Problem: $-\sqrt{1}$

Solution:

-1

Exercise:

Problem: $-\sqrt{121}$

Exercise:

Problem: $\sqrt{-121}$

Solution:

not a real number

Exercise:

Problem: $\sqrt{-36}$

Exercise:

Problem: $\sqrt{-9}$

Solution:

not a real number

Exercise:

Problem: $\sqrt{-49}$

Exercise:

Problem: $\sqrt{9 + 16}$

Solution:

5

Exercise:

Problem: $\sqrt{25 + 144}$

Exercise:

Problem: $\sqrt{9} + \sqrt{16}$

Solution:

7

Exercise:

Problem: $\sqrt{25} + \sqrt{144}$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

Exercise:

Problem: $\sqrt{70}$

Solution:

$$8 < \sqrt{70} < 9$$

Exercise:

Problem: $\sqrt{55}$

Exercise:

Problem: $\sqrt{200}$

Solution:

$$14 < \sqrt{200} < 15$$

Exercise:

Problem: $\sqrt{172}$

Approximate Square Roots with a Calculator

In the following exercises, use a calculator to approximate each square root and round to two decimal places.

Exercise:

Problem: $\sqrt{19}$

Solution:

$$4.36$$

Exercise:

Problem: $\sqrt{21}$

Exercise:

Problem: $\sqrt{53}$

Solution:

$$7.28$$

Exercise:

Problem: $\sqrt{47}$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

Exercise:

Problem: $\sqrt{y^2}$

Solution:

$$y$$

Exercise:

Problem: $\sqrt{b^2}$

Exercise:

Problem: $\sqrt{49x^2}$

Solution:

$$7x$$

Exercise:

Problem: $\sqrt{100y^2}$

Exercise:

Problem: $-\sqrt{64a^2}$

Solution:

$$-8a$$

Exercise:

Problem: $-\sqrt{25x^2}$

Exercise:

Problem: $\sqrt{144x^2y^2}$

Solution:

$12xy$

Exercise:

Problem: $\sqrt{196a^2b^2}$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

Exercise:

Problem:

Landscaping Reed wants to have a square garden plot in his backyard. He has enough compost to cover an area of 75 square feet. How long can a side of his garden be?

Solution:

8.7 feet

Exercise:

Problem:

Landscaping Vince wants to make a square patio in his yard. He has enough concrete to pave an area of 130 square feet. How long can a side of his patio be?

Exercise:

Problem:

Gravity An airplane dropped a flare from a height of 1,024 feet above a lake. How many seconds did it take for the flare to reach the water?

Solution:

8 seconds

Exercise:

Problem:

Gravity A hang glider dropped his cell phone from a height of 350 feet. How many seconds did it take for the cell phone to reach the ground?

Exercise:

Problem:

Gravity A construction worker dropped a hammer while building the Grand Canyon skywalk, 4,000 feet above the Colorado River. How many seconds did it take for the hammer to reach the river?

Solution:

15.8 seconds

Exercise:

Problem:

Accident investigation The skid marks from a car involved in an accident measured 54 feet. What was the speed of the car before the brakes were applied?

Exercise:

Problem:

Accident investigation The skid marks from a car involved in an accident measured 216 feet. What was the speed of the car before the brakes were applied?

Solution:

72 mph

Exercise:

Problem:

Accident investigation An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 175 feet. What was the speed of the vehicle before the brakes were applied?

Exercise:**Problem:**

Accident investigation An accident investigator measured the skid marks of one of the vehicles involved in an accident. The length of the skid marks was 117 feet. What was the speed of the vehicle before the brakes were applied?

Solution:

53.0 mph

Everyday Math**Exercise:****Problem:**

Decorating Denise wants to install a square accent of designer tiles in her new shower. She can afford to buy 625 square centimeters of the designer tiles. How long can a side of the accent be?

Exercise:**Problem:**

Decorating Morris wants to have a square mosaic inlaid in his new patio. His budget allows for 2,025 tiles. Each tile is square with an area of one square inch. How long can a side of the mosaic be?

Solution:

45 inches

Writing Exercises

Exercise:

Problem: Why is there no real number equal to $\sqrt{-64}$?

Exercise:

Problem: What is the difference between 9^2 and $\sqrt{9}$?

Solution:

Answers will vary. 9^2 reads: “nine squared” and means nine times itself. The expression $\sqrt{9}$ reads: “the square root of nine” which gives us the number such that if it were multiplied by itself would give you the number inside of the square root.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			
simplify variable expressions with square roots.			
use square roots in applications.			

Ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Decimals

Name Decimals

In the following exercises, name each decimal.

Exercise:

Problem: 0.8

Exercise:

Problem: 0.375

Solution:

three hundred seventy-five thousandths

Exercise:

Problem: 0.007

Exercise:

Problem: 5.24

Solution:

five and twenty-four hundredths

Exercise:

Problem: -12.5632

Exercise:

Problem: -4.09

Solution:

negative four and nine hundredths

Write Decimals

In the following exercises, write as a decimal.

Exercise:

Problem: three tenths

Exercise:

Problem: nine hundredths

Solution:

0.09

Exercise:

Problem: twenty-seven hundredths

Exercise:

Problem: ten and thirty-five thousandths

Solution:

10.035

Exercise:

Problem: negative twenty and three tenths

Exercise:

Problem: negative five hundredths

Solution:

-0.05

Convert Decimals to Fractions or Mixed Numbers

In the following exercises, convert each decimal to a fraction. Simplify the answer if possible.

Exercise:

Problem: 0.43

Exercise:

Problem: 0.825

Solution:

$$\frac{33}{40}$$

Exercise:

Problem: 9.7

Exercise:

Problem: 3.64

Solution:

$$3\frac{16}{25}$$

Locate Decimals on the Number Line

Exercise:

Problem: Ⓐ 0.6

Ⓑ -0.9

Ⓒ 2.2

Ⓓ -1.3

Order Decimals

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

Exercise:

Problem: 0.6 ____ 0.8

Solution:

<

Exercise:

Problem: $0.2 \underline{\hspace{1cm}} 0.15$

Exercise:

Problem: $0.803 \underline{\hspace{1cm}} 0.83$

Solution:

<

Exercise:

Problem: $-0.56 \underline{\hspace{1cm}} -0.562$

Round Decimals

In the following exercises, round each number to the nearest: (a) hundredth (b) tenth
(c) whole number.

Exercise:

Problem: 12.529

Solution:

- (a) 12.53
- (b) 12.5
- (c) 13

Exercise:

Problem: 4.8447

Exercise:

Problem: 5.897

Solution:

- Ⓐ 5.90
- Ⓑ 5.9
- Ⓒ 6

Decimal Operations

Add and Subtract Decimals

In the following exercises, add or subtract.

Exercise:

Problem: $5.75 + 8.46$

Exercise:

Problem: $32.89 - 8.22$

Solution:

24.67

Exercise:

Problem: $24 - 19.31$

Exercise:

Problem: $10.2 + 14.631$

Solution:

24.831

Exercise:

Problem: $-6.4 + (-2.9)$

Exercise:

Problem: $1.83 - 4.2$

Solution:

-2.37

Multiply Decimals

In the following exercises, multiply.

Exercise:

Problem: $(0.3)(0.7)$

Exercise:

Problem: $(-6.4)(0.25)$

Solution:

-1.6

Exercise:

Problem: $(-3.35)(-12.7)$

Exercise:

Problem: $(15.4)(1000)$

Solution:

$15,400$

Divide Decimals

In the following exercises, divide.

Exercise:

Problem: $0.48 \div 6$

Exercise:

Problem: $4.32 \div 24$

Solution:

0.18

Exercise:

Problem: $\$6.29 \div 12$

Exercise:

Problem: $(-0.8) \div (-0.2)$

Solution:

4

Exercise:

Problem: $1.65 \div 0.15$

Exercise:

Problem: $9 \div 0.045$

Solution:

200

Use Decimals in Money Applications

In the following exercises, use the strategy for applications to solve.

Exercise:

Problem:

Miranda got \$40 from her ATM. She spent \$9.32 on lunch and \$16.99 on a book. How much money did she have left? Round to the nearest cent if necessary.

Exercise:

Problem:

Jessie put 8 gallons of gas in her car. One gallon of gas costs \$3.528. How much did Jessie owe for all the gas?

Solution:

\$28.22

Exercise:**Problem:**

A pack of 16 water bottles cost \$6.72. How much did each bottle cost?

Exercise:**Problem:**

Alice bought a roll of paper towels that cost \$2.49. She had a coupon for \$0.35 off, and the store doubled the coupon. How much did Alice pay for the paper towels?

Solution:

\$1.79

Decimals and Fractions

Convert Fractions to Decimals

In the following exercises, convert each fraction to a decimal.

Exercise:

Problem: $\frac{3}{5}$

Exercise:

Problem: $\frac{7}{8}$

Solution:

0.875

Exercise:

Problem: $-\frac{19}{20}$

Exercise:

Problem: $-\frac{21}{4}$

Solution:

-5.25

Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{6}{11}$

Solution:

$0.\overline{54}$

Order Decimals and Fractions

In the following exercises, order each pair of numbers, using $<$ or $>$.

Exercise:

Problem: $\frac{1}{2}$ ____ 0.2

Exercise:

Problem: $\frac{3}{5}$ ____ 0.

Solution:

$>$

Exercise:

Problem: $-\frac{7}{8}$ ____ -0.84

Exercise:

Problem: $-\frac{5}{12}$ ____ -0.42

Solution:

>

Exercise:

Problem: 0.625 ____ $\frac{13}{20}$

Exercise:

Problem: 0.33 ____ $\frac{5}{16}$

Solution:

>

In the following exercises, write each set of numbers in order from least to greatest.

Exercise:

Problem: $\frac{2}{3}$, $\frac{17}{20}$, 0.65

Exercise:

Problem: $\frac{7}{9}$, 0.75 , $\frac{11}{15}$

Solution:

$\frac{11}{15}$, 0.75 , $\frac{7}{9}$

Simplify Expressions Using the Order of Operations

In the following exercises, simplify

Exercise:

Problem: $4(10.3 - 5.8)$

Exercise:

Problem: $\frac{3}{4}(15.44 - 7.4)$

Solution:

6.03

Exercise:

Problem: $30 \div (0.45 + 0.15)$

Exercise:

Problem: $1.6 + \frac{3}{8}$

Solution:

1.975

Exercise:

Problem: $52(0.5) + (0.4)^2$

Exercise:

Problem: $-\frac{2}{5} \cdot \frac{9}{10} + 0.14$

Solution:

-0.22

Find the Circumference and Area of Circles

In the following exercises, approximate the Ⓐ circumference and Ⓑ area of each circle.

Exercise:

Problem: radius = 6 in.

Exercise:

Problem: radius = 3.5 ft.

Solution:

- Ⓐ 21.98 ft.
- Ⓑ 38.465 sq.ft.

Exercise:

Problem: radius = $\frac{7}{33}$ m

Exercise:

Problem: diameter = 11 cm

Solution:

- Ⓐ 34.54 cm
- Ⓑ 379.94 sq.cm

Solve Equations with Decimals

Determine Whether a Decimal is a Solution of an Equation

In the following exercises, determine whether the each number is a solution of the given equation.

Exercise:

$$x - 0.4 = 2.1$$

Problem: Ⓐ $x = 1.7$ Ⓑ $x = 2.5$

Exercise:

$$y + 3.2 = -1.5$$

Problem: Ⓐ $y = 1.7$ Ⓑ $y = -4.7$

Solution:

- Ⓐ no
Ⓑ yes

Exercise:

$$\frac{u}{2.5} = -12.5$$

Problem: Ⓐ $u = -5$ Ⓑ $u = -31.25$

Exercise:

$$0.45v = -40.5$$

Problem: Ⓐ $v = -18.225$ Ⓑ $v = -90$

Solution:

- Ⓐ no
Ⓑ yes

Solve Equations with Decimals

In the following exercises, solve.

Exercise:

Problem: $m + 3.8 = 7.5$

Exercise:

Problem: $h + 5.91 = 2.4$

Solution:

$$h = -3.51$$

Exercise:

Problem: $a + 2.26 = -1.1$

Exercise:

Problem: $p - 4.3 = -1.65$

Solution:

$$p = 2.65$$

Exercise:

Problem: $x - 0.24 = -8.6$

Exercise:

Problem: $j - 7.42 = -3.7$

Solution:

$$j = 3.72$$

Exercise:

Problem: $0.6p = 13.2$

Exercise:

Problem: $-8.6x = 34.4$

Solution:

$$x = -4$$

Exercise:

Problem: $-22.32 = -2.4z$

Exercise:

Problem: $\frac{a}{0.3} = -24$

Solution:

$$a = -7.2$$

Exercise:

Problem: $\frac{p}{-7} = -4.2$

Exercise:

Problem: $\frac{s}{-2.5} = -10$

Solution:

$$s = 25$$

Translate to an Equation and Solve

In the following exercises, translate and solve.

Exercise:

Problem: The difference of n and 15.2 is 4.4.

Exercise:

Problem: The product of -5.9 and x is -3.54 .

Solution:

$$-5.9x = -3.54; x = 0.6$$

Exercise:

Problem: The quotient of y and -1.8 is -9 .

Exercise:

Problem: The sum of m and -4.03 is 6.8.

Solution:

$$m + (-4.03) = 6.8; m = 0.83$$

Averages and Probability

Find the Mean of a Set of Numbers

In the following exercises, find the mean of the numbers.

Exercise:

Problem: 2, 4, 1, 0, 1, and 1

Exercise:

Problem: \$270, \$310.50, \$243.75, and \$252.15

Solution:

\$269.10

Exercise:

Problem:

Each workday last week, Yoshie kept track of the number of minutes she had to wait for the bus. She waited 3, 0, 8, 1, and 8 minutes. Find the mean

Exercise:

Problem:

In the last three months, Raul's water bills were \$31.45, \$48.76, and \$42.60. Find the mean.

Solution:

\$40.94

Find the Median of a Set of Numbers

In the following exercises, find the median.

Exercise:

Problem: 41, 45, 32, 60, 58

Exercise:

Problem: 25, 23, 24, 26, 29, 19, 18, 32

Solution:

24.5

Exercise:

Problem:

The ages of the eight men in Jerry's model train club are 52, 63, 45, 51, 55, 75, 60, and 59. Find the median age.

Exercise:

Problem:

The number of clients at Miranda's beauty salon each weekday last week were 18, 7, 12, 16, and 20. Find the median number of clients.

Solution:

16 clients

Find the Mode of a Set of Numbers

In the following exercises, identify the mode of the numbers.

Exercise:

Problem: 6, 4, 4, 5, 6, 6, 4, 4, 4, 3, 5

Exercise:

Problem:

The number of siblings of a group of students: 2, 0, 3, 2, 4, 1, 6, 5, 4, 1, 2, 3

Solution:

2

Use the Basic Definition of Probability

In the following exercises, solve. (Round decimals to three places.)

Exercise:

Problem:

The Sustainability Club sells 200 tickets to a raffle, and Albert buys one ticket. One ticket will be selected at random to win the grand prize. Find the probability Albert will win the grand prize. Express your answer as a fraction and as a decimal.

Exercise:

Problem:

Luc has to read 3 novels and 12 short stories for his literature class. The professor will choose one reading at random for the final exam. Find the probability that the professor will choose a novel for the final exam. Express your answer as a fraction and as a decimal.

Solution:

$$\frac{1}{5}; 0.2$$

Ratios and Rate

Write a Ratio as a Fraction

In the following exercises, write each ratio as a fraction. Simplify the answer if possible.

Exercise:

Problem: 28 to 40

Exercise:

Problem: 56 to 32

Solution:

$$\frac{7}{4}$$

Exercise:

Problem: 3.5 to 0.5

Exercise:

Problem: 1.2 to 1.8

Solution:

$$\frac{2}{3}$$

Exercise:

Problem: $1\frac{3}{4}$ to $1\frac{5}{8}$

Exercise:

Problem: $2\frac{1}{3}$ to $5\frac{1}{4}$

Solution:

$$\frac{4}{9}$$

Exercise:

Problem: 64 ounces to 30 ounces

Exercise:

Problem: 28 inches to 3 feet

Solution:

$$\frac{7}{9}$$

Write a Rate as a Fraction

In the following exercises, write each rate as a fraction. Simplify the answer if possible.

Exercise:

Problem: 180 calories per 8 ounces

Exercise:

Problem: 90 pounds per 7.5 square inches

Solution:

$$\frac{90 \text{ pounds}}{7.5 \text{ square inches}}$$

Exercise:

Problem: 126 miles in 4 hours

Exercise:

Problem: \$612.50 for 35 hours

Solution:

$$\frac{\$612.50}{35 \text{ hours}}$$

Find Unit Rates

In the following exercises, find the unit rate.

Exercise:

Problem: 180 calories per 8 ounces

Exercise:

Problem: 90 pounds per 7.5 square inches

Solution:

12 pounds/sq.in.

Exercise:

Problem: 126 miles in 4 hours

Exercise:

Problem: \$612.50 for 35 hours

Solution:

\$17.50/hour

Find Unit Price

In the following exercises, find the unit price.

Exercise:

Problem: t-shirts: 3 for \$8.97

Exercise:

Problem: Highlighters: 6 for \$2.52

Solution:

\$0.42

Exercise:

Problem:

An office supply store sells a box of pens for \$11. The box contains 12 pens.
How much does each pen cost?

Exercise:

Problem:

Anna bought a pack of 8 kitchen towels for \$13.20. How much did each towel cost? Round to the nearest cent if necessary.

Solution:

\$1.65

In the following exercises, find each unit price and then determine the better buy.

Exercise:

Problem: Shampoo: 12 ounces for \$4.29 or 22 ounces for \$7.29?

Exercise:

Problem: Vitamins: 60 tablets for \$6.49 or 100 for \$11.99?

Solution:

\$0.11, \$0.12; 60 tablets for \$6.49

Translate Phrases to Expressions with Fractions

In the following exercises, translate the English phrase into an algebraic expression.

Exercise:

Problem: 535 miles per h hours

Exercise:

Problem: a adults to 45 children

Solution:

$$\frac{a \text{ adults}}{45 \text{ children}}$$

Exercise:

Problem: the ratio of $4y$ and the difference of x and 10

Exercise:

Problem: the ratio of 19 and the sum of 3 and n

Solution:

$$\frac{19}{3+n}$$

Simplify and Use Square Roots

Simplify Expressions with Square Roots

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{64}$

Exercise:

Problem: $\sqrt{144}$

Solution:

12

Exercise:

Problem: $-\sqrt{25}$

Exercise:

Problem: $-\sqrt{81}$

Solution:

-9

Exercise:

Problem: $\sqrt{-9}$

Exercise:

Problem: $\sqrt{-36}$

Solution:

not a real number

Exercise:

Problem: $\sqrt{64} + \sqrt{225}$

Exercise:

Problem: $\sqrt{64 + 225}$

Solution:

$$17$$

Estimate Square Roots

In the following exercises, estimate each square root between two consecutive whole numbers.

Exercise:

Problem: $\sqrt{28}$

Exercise:

Problem: $\sqrt{155}$

Solution:

$$12 < \sqrt{155} < 13$$

Approximate Square Roots

In the following exercises, approximate each square root and round to two decimal places.

Exercise:

Problem: $\sqrt{15}$

Exercise:

Problem: $\sqrt{57}$

Solution:

$$7.55$$

Simplify Variable Expressions with Square Roots

In the following exercises, simplify. (Assume all variables are greater than or equal to zero.)

Exercise:

Problem: $\sqrt{q^2}$

Exercise:

Problem: $\sqrt{64b^2}$

Solution:

$$8b$$

Exercise:

Problem: $-\sqrt{121a^2}$

Exercise:

Problem: $\sqrt{225m^2n^2}$

Solution:

$$15mn$$

Exercise:

Problem: $-\sqrt{100q^2}$

Exercise:

Problem: $\sqrt{49y^2}$

Solution:

$$7y$$

Exercise:

Problem: $\sqrt{4a^2b^2}$

Exercise:

Problem: $\sqrt{121c^2d^2}$

Solution:

$11cd$

Use Square Roots in Applications

In the following exercises, solve. Round to one decimal place.

Exercise:

Problem:

Art Diego has 225 square inch tiles. He wants to use them to make a square mosaic. How long can each side of the mosaic be?

Exercise:

Problem:

Landscaping Janet wants to plant a square flower garden in her yard. She has enough topsoil to cover an area of 30 square feet. How long can a side of the flower garden be?

Solution:

5.5 feet

Exercise:

Problem:

Gravity A hiker dropped a granola bar from a lookout spot 576 feet above a valley. How long did it take the granola bar to reach the valley floor?

Exercise:

Problem:

Accident investigation The skid marks of a car involved in an accident were 216 feet. How fast had the car been going before applying the brakes?

Solution:

72 mph

Chapter Practice Test

Exercise:

Problem: Write six and thirty-four thousandths as a decimal.

Exercise:

Problem: Write 1.73 as a fraction.

Solution:

$$1 \frac{73}{100}$$

Exercise:

Problem: Write $\frac{5}{8}$ as a decimal.

Exercise:

Problem: Round 16.749 to the nearest (a) tenth (b) hundredth (c) whole number

Solution:

- (a) 16.7
- (b) 16.75
- (c) 17

Exercise:

Problem:

Write the numbers $\frac{4}{5}$, -0.1 , 0.804 , $\frac{2}{9}$, -7.4 , 0.21 in order from smallest to largest.

In the following exercises, simplify each expression.

Exercise:

Problem: $15.4 + 3.02$

Solution:

18.42

Exercise:

Problem: $20 - 5.71$

Exercise:

Problem: $(0.64)(0.3)$

Solution:

0.192

Exercise:

Problem: $(-4.2)(100)$

Exercise:

Problem: $0.96 \div (-12)$

Solution:

-0.08

Exercise:

Problem: $-5 \div 0.025$

Exercise:

Problem: $-0.6 \div (-0.3)$

Solution:

2

Exercise:

Problem: $(0.7)^2$

Exercise:

Problem: $24 \div (0.1 + 0.02)$

Solution:

200

Exercise:

Problem: $4(10.3 - 5.8)$

Exercise:

Problem: $1.6 + \frac{3}{8}$

Solution:

1.975

Exercise:

Problem: $\frac{2}{3}(14.65 - 4.6)$

In the following exercises, solve.

Exercise:

Problem: $m + 3.7 = 2.5$

Solution:

-1.2

Exercise:

Problem: $\frac{h}{0.5} = 4.38$

Exercise:

Problem: $-6.5y = -57.2$

Solution:

8.8

Exercise:

Problem: $1.94 = a - 2.6$

Exercise:

Problem:

Three friends went out to dinner and agreed to split the bill evenly. The bill was \$79.35. How much should each person pay?

Solution:

\$26.45

Exercise:

Problem:

A circle has radius 12. Find the (a) circumference and (b) area. [Use 3.14 for π .]

Exercise:

The ages, in months, of 10 children in a preschool class are:
55, 55, 50, 51, 52, 50, 53, 51, 55, 49

Problem: Find the (a) mean (b) median (c) mode

Solution:

- Ⓐ 59
- Ⓑ 51.5
- Ⓒ 55

Exercise:

Problem:

Of the 16 nurses in Doreen's department, 12 are women and 4 are men. One of the nurses will be assigned at random to work an extra shift next week. Ⓐ Find the probability a woman nurse will be assigned the extra shift. Ⓑ Convert the fraction to a decimal.

Exercise:

Find each unit price and then the better buy.

Problem: Laundry detergent: 64 ounces for \$10.99 or 48 ounces for \$8.49

Solution:

64 ounces for \$10.99 is the better buy

In the following exercises, simplify.

Exercise:

Problem: $\sqrt{36 + 64}$

Exercise:

Problem: $\sqrt{144n^2}$

Solution:

$12n$

Exercise:

Problem: Estimate $\sqrt{54}$ to between two whole numbers.

Exercise:

Problem:

Yanet wants a square patio in her backyard. She has 225 square feet of tile. How long can a side of the patio be?

Solution:

15 feet

Use Properties of Angles, Triangles, and the Pythagorean Theorem
By the end of this section, you will be able to:

- Use the properties of angles
- Use the properties of triangles
- Use the Pythagorean Theorem

Note:

Before you get started, take this readiness quiz.

1. Solve: $x + 3 + 6 = 11$.

If you missed this problem, review [\[link\]](#).

2. Solve: $\frac{a}{45} = \frac{4}{3}$.

If you missed this problem, review [\[link\]](#).

3. Simplify: $\sqrt{36 + 64}$.

If you missed this problem, review [\[link\]](#).

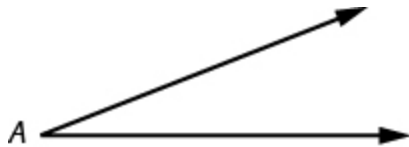
So far in this chapter, we have focused on solving word problems, which are similar to many real-world applications of algebra. In the next few sections, we will apply our problem-solving strategies to some common geometry problems.

Use the Properties of Angles

Are you familiar with the phrase ‘do a 180’? It means to make a full turn so that you face the opposite direction. It comes from the fact that the measure of an angle that makes a straight line is 180 degrees. See [\[link\]](#).



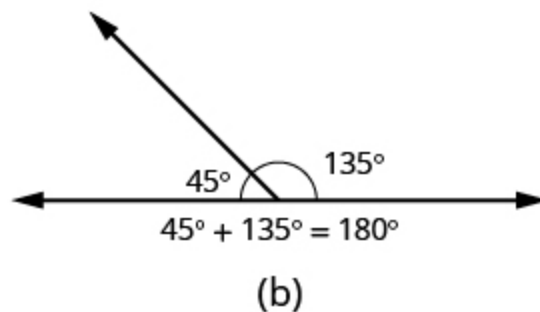
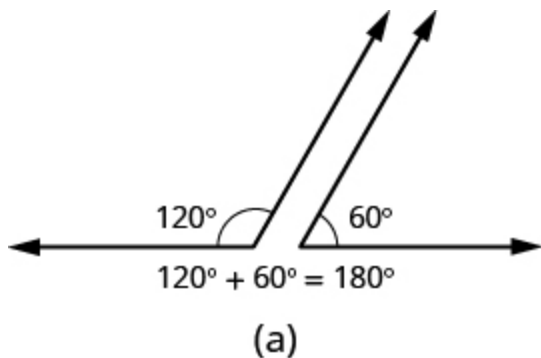
An **angle** is formed by two rays that share a common endpoint. Each ray is called a side of the angle and the common endpoint is called the **vertex**. An angle is named by its vertex. In [\[link\]](#), $\angle A$ is the angle with vertex at point A . The measure of $\angle A$ is written $m\angle A$.



$\angle A$ is the angle
with vertex at
point A .

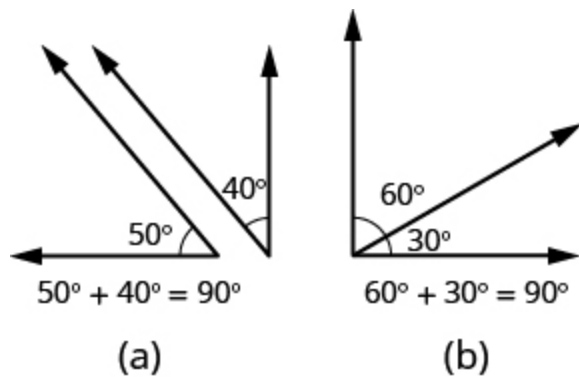
We measure angles in degrees, and use the symbol $^\circ$ to represent degrees. We use the abbreviation m to for the *measure* of an angle. So if $\angle A$ is 27° , we would write $m\angle A = 27$.

If the sum of the measures of two angles is 180° , then they are called **supplementary angles**. In [\[link\]](#), each pair of angles is supplementary because their measures add to 180° . Each angle is the *supplement* of the other.



The sum of the measures of supplementary angles is 180° .

If the sum of the measures of two angles is 90° , then the angles are **complementary angles**. In [\[link\]](#), each pair of angles is complementary, because their measures add to 90° . Each angle is the *complement* of the other.



The sum of the measures of complementary angles is 90° .

Note:

Supplementary and Complementary Angles

If the sum of the measures of two angles is 180° , then the angles are supplementary.

If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180^\circ$.

If the sum of the measures of two angles is 90° , then the angles are complementary.

If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90^\circ$.

In this section and the next, you will be introduced to some common geometry formulas. We will adapt our Problem Solving Strategy for Geometry Applications. The geometry formula will name the variables and give us the equation to solve.

In addition, since these applications will all involve geometric shapes, it will be helpful to draw a figure and then label it with the information from the problem. We will include this step in the Problem Solving Strategy for Geometry Applications.

Note:

Use a Problem Solving Strategy for Geometry Applications.

Read the problem and make sure you understand all the words and ideas.

Draw a figure and label it with the given information.

Identify what you are looking for.

Name what you are looking for and choose a variable to represent it.

Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

Answer the question with a complete sentence.

The next example will show how you can use the Problem Solving Strategy for Geometry Applications to answer questions about supplementary and complementary angles.

Example:

Exercise:

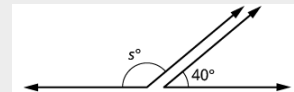
Problem:

An angle measures 40° . Find (a) its supplement, and (b) its complement.

Solution:**Solution**

(a)

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the supplement of a 40° angle.

Step 3. **Name.** Choose a variable to represent it.

let s = the measure of the supplement

Step 4. **Translate.**
Write the appropriate formula for the situation and substitute in the given information.

$$m\angle A + m\angle B = 180$$

$$s + 40 = 180$$

Step 5. **Solve** the equation.

$$s = 140$$

Step 6. **Check:**

$$140 + 40 \stackrel{?}{=} 180$$

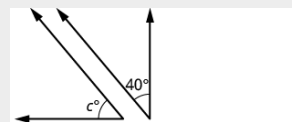
$$180 = 180 \checkmark$$

Step 7. **Answer** the question.

The supplement of the 40° angle is 140° .

ⓑ

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the complement of a 40° angle.

Step 3. **Name.** Choose a variable to represent it.

let c = the measure of the complement

Step 4. **Translate.**
Write the appropriate formula for the situation and substitute in the given information.

$$m\angle A + m\angle B = 90$$

Step 5. **Solve** the equation.

$$c + 40 = 90$$

	$c = 50$
Step 6. Check: $50 + 40 \stackrel{?}{=} 90$ $90 = 90 \checkmark$	
Step 7. Answer the question.	The complement of the 40° angle is 50° .

Note:

Exercise:

Problem:

An angle measures 25° . Find its: (a) supplement (b) complement.

Solution:

- (a) 155°
- (b) 65°

Note:

Exercise:

Problem:

An angle measures 77° . Find its: (a) supplement (b) complement.

Solution:

(a) 103°

(b) 13°

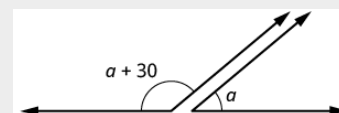
Did you notice that the words complementary and supplementary are in alphabetical order just like 90 and 180 are in numerical order?

Example:**Exercise:****Problem:**

Two angles are supplementary. The larger angle is 30° more than the smaller angle. Find the measure of both angles.

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. Identify what you are looking for.	the measures of both angles
Step 3. Name. Choose a variable to represent it. The larger angle is 30° more than the smaller angle.	let a = measure of smaller angle $a + 30$ = measure of larger angle
Step 4. Translate. Write the appropriate formula and substitute.	$m\angle A + m\angle B = 180$
Step 5. Solve the equation.	$(a + 30) + a = 180$
	$2a + 30 = 180$
	$2a = 150$
	$a = 75$ measure of smaller angle
	$a + 30$ measure of larger angle
Step 6. Check:	$75 + 30$
	105

$$m\angle A + m\angle B = 180$$

$$75 + 105 \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

Step 7. **Answer** the question.

The measures of the angles are 75° and 105° .

Note:

Exercise:

Problem:

Two angles are supplementary. The larger angle is 100° more than the smaller angle. Find the measures of both angles.

Solution:

40° , 140°

Note:

Exercise:

Problem:

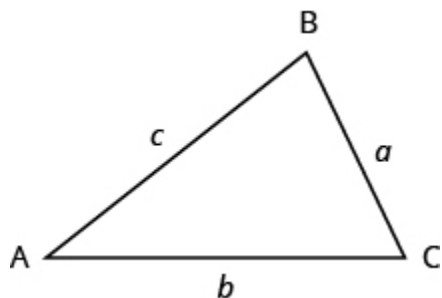
Two angles are complementary. The larger angle is 40° more than the smaller angle. Find the measures of both angles.

Solution:

$25^\circ, 65^\circ$

Use the Properties of Triangles

What do you already know about triangles? Triangles have three sides and three angles. Triangles are named by their vertices. The **triangle** in [\[link\]](#) is called $\triangle ABC$, read ‘triangle ABC’. We label each side with a lower case letter to match the upper case letter of the opposite vertex.



$\triangle ABC$ has vertices A , B , and C and sides a , b , and c .

The three angles of a triangle are related in a special way. The sum of their measures is 180° .

Equation:

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Note:

Sum of the Measures of the Angles of a Triangle

For any $\triangle ABC$, the sum of the measures of the angles is 180° .

Equation:

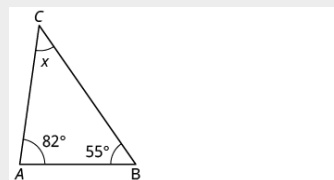
$$m\angle A + m\angle B + m\angle C = 180^\circ$$

Example:**Exercise:****Problem:**

The measures of two angles of a triangle are 55° and 82° . Find the measure of the third angle.

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the measure of the third angle in a triangle

Step 3. **Name.** Choose a variable to represent it.

let x = the measure of the angle

Step 4. Translate.

Write the appropriate formula and substitute.

$$m\angle A + m\angle B + m\angle C = 180$$

Step 5. Solve the equation.

$$55 + 82 + x = 180$$

$$137 + x = 180$$

$$x = 43$$

Step 6. Check:

$$55 + 82 + 43 \stackrel{?}{=} 180$$

$$180 = 180 \checkmark$$

Step 7. Answer the question.

The measure of the third angle is 43 degrees.

Note:

Exercise:

Problem:

The measures of two angles of a triangle are 31° and 128° . Find the measure of the third angle.

Solution:

21°

Note:

Exercise:

Problem:

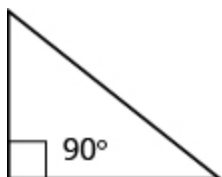
A triangle has angles of 49° and 75° . Find the measure of the third angle.

Solution:

56°

Right Triangles

Some triangles have special names. We will look first at the **right triangle**. A right triangle has one 90° angle, which is often marked with the symbol shown in [\[link\]](#).



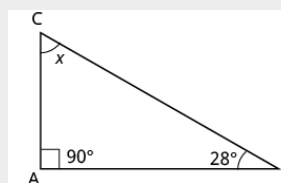
If we know that a triangle is a right triangle, we know that one angle measures 90° so we only need the measure of one of the other angles in order to determine the measure of the third angle.

Example:**Exercise:****Problem:**

One angle of a right triangle measures 28° . What is the measure of the third angle?

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the measure of an angle

Step 3. **Name.** Choose a variable to represent it.

let x = the measure of the angle

Step 4. **Translate.**
Write the appropriate formula and substitute.

$$m\angle A + m\angle B + m\angle C = 180$$

Step 5. **Solve** the equation.

$$x + 90 + 28 = 180$$

$$x + 118 = 180$$

	$x = 62$
Step 6. Check: $180 \stackrel{?}{=} 90 + 28 + 62$ $180 = 180 \checkmark$	
Step 7. Answer the question.	The measure of the third angle is 62° .

Note:

Exercise:

Problem:

One angle of a right triangle measures 56° . What is the measure of the other angle?

Solution:

34°

Note:

Exercise:

Problem:

One angle of a right triangle measures 45° . What is the measure of the other angle?

Solution:

45°

In the examples so far, we could draw a figure and label it directly after reading the problem. In the next example, we will have to define one angle in terms of another. So we will wait to draw the figure until we write expressions for all the angles we are looking for.

Example:**Exercise:****Problem:**

The measure of one angle of a right triangle is 20° more than the measure of the smallest angle. Find the measures of all three angles.

Solution:**Solution**

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the measures of all three angles

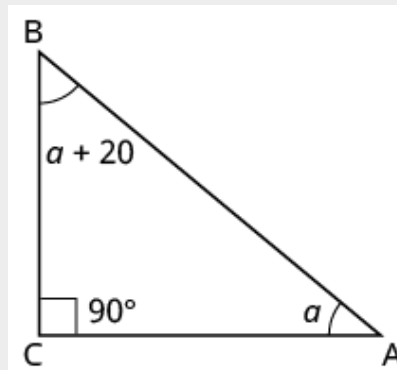
Step 3. **Name.** Choose a variable to represent it.

Now draw the figure and label it with the given information.

Let $a = 1^{\text{st}}$ angle

$a + 20 = 2^{\text{nd}}$ angle

$90 = 3^{\text{rd}}$ angle (the right angle)



Step 4. **Translate.**
Write the appropriate formula and substitute into the formula.

$$m\angle A + m\angle B + m\angle C = 180$$

$$a + (a + 20) + 90 = 180$$

Step 5. **Solve** the equation.

$$2a + 110 = 180$$

$$2a = 70$$

$$a = 35 \text{ first angle}$$

	$a + 20$ second angle $35 + 20$ 55 90 third angle
Step 6. Check: $35 + 55 + 90 \stackrel{?}{=} 180$ $180 = 180 \checkmark$	
Step 7. Answer the question.	The three angles measure 35° , 55° , and 90° .

Note:

Exercise:

Problem:

The measure of one angle of a right triangle is 50° more than the measure of the smallest angle. Find the measures of all three angles.

Solution:

$20^\circ, 70^\circ, 90^\circ$

Note:**Exercise:****Problem:**

The measure of one angle of a right triangle is 30° more than the measure of the smallest angle. Find the measures of all three angles.

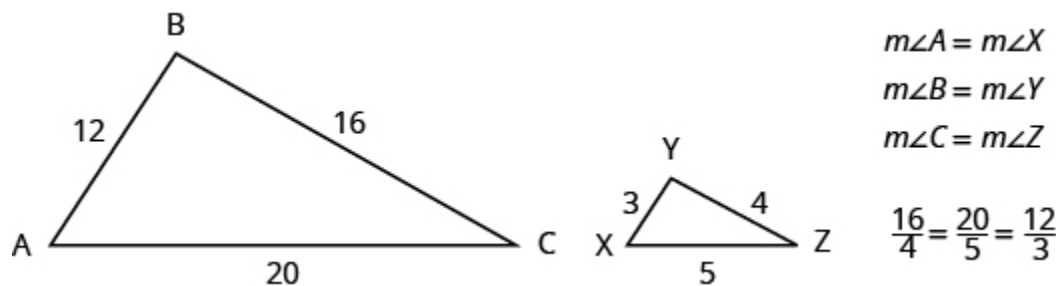
Solution:

$30^\circ, 60^\circ, 90^\circ$

Similar Triangles

When we use a map to plan a trip, a sketch to build a bookcase, or a pattern to sew a dress, we are working with similar figures. In geometry, if two figures have exactly the same shape but different sizes, we say they are **similar figures**. One is a scale model of the other. The corresponding sides of the two figures have the same ratio, and all their corresponding angles are have the same measures.

The two triangles in [\[link\]](#) are similar. Each side of $\triangle ABC$ is four times the length of the corresponding side of $\triangle XYZ$ and their corresponding angles have equal measures.

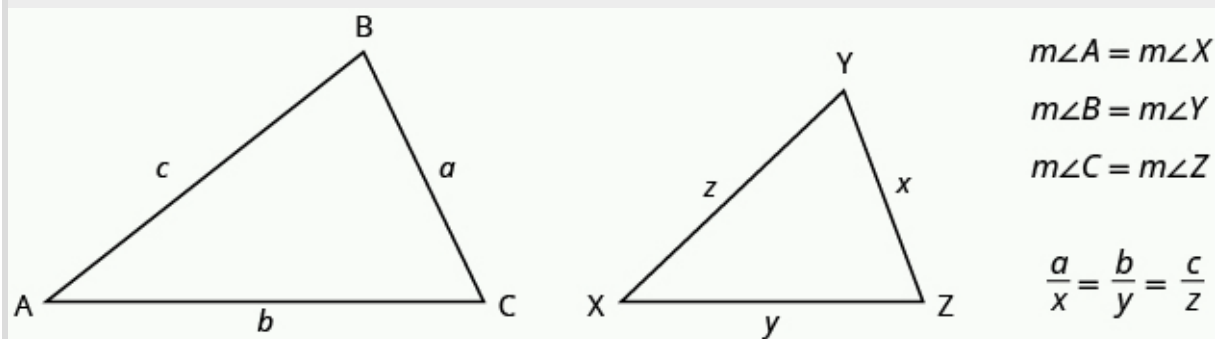


$\triangle ABC$ and $\triangle XYZ$ are similar triangles. Their corresponding sides have the same ratio and the corresponding angles have the same measure.

Note:

Properties of Similar Triangles

If two triangles are similar, then their corresponding angle measures are equal and their corresponding side lengths are in the same ratio.



The length of a side of a triangle may be referred to by its endpoints, two vertices of the triangle. For example, in $\triangle ABC$:

the length a can also be written BC

the length b can also be written AC

the length c can also be written AB

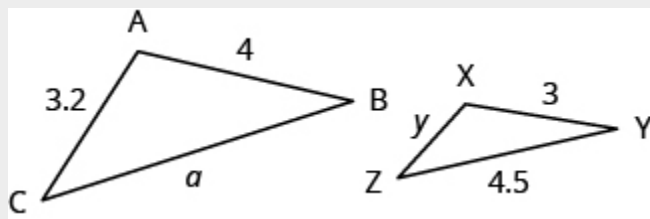
We will often use this notation when we solve similar triangles because it will help us match up the corresponding side lengths.

Example:

Exercise:

Problem:

$\triangle ABC$ and $\triangle XYZ$ are similar triangles. The lengths of two sides of each triangle are shown. Find the lengths of the third side of each triangle.



Solution:

Solution

Step 1. Read the problem. Draw the figure and label it with the given information.	The figure is provided.
Step 2. Identify what you are looking for.	The length of the sides of similar triangles
Step 3. Name . Choose a variable to represent it.	Let a = length of the third side of $\triangle ABC$

y = length of the third side $\triangle XYZ$

Step 4. Translate.

The triangles are similar, so the corresponding sides are in the same ratio. So

Equation:

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

Since the side $AB = 4$ corresponds to the side $XY = 3$, we will use the ratio $\frac{AB}{XY} = \frac{4}{3}$ to find the other sides.

Be careful to match up corresponding sides correctly.

	To find a :	To find y :
sides of large triangle $\longrightarrow$	$\frac{AB}{XY} = \frac{BC}{YZ}$	$\frac{AB}{XY} = \frac{AC}{XZ}$
sides of small triangle $\longrightarrow$	$\frac{4}{3} = \frac{a}{4.5}$	$\frac{4}{3} = \frac{3.2}{y}$

Step 5. Solve the equation.

$3a = 4(4.5)$	$4y = 3(3.2)$
$3a = 18$	$4y = 9.6$
$a = 6$	$y = 2.4$

Step 6. Check:

$\frac{4}{3} \stackrel{?}{=} \frac{6}{4.5}$	$\frac{4}{3} \stackrel{?}{=} \frac{3.2}{2.4}$
$4(4.5) \stackrel{?}{=} 6(3)$	$4(2.4) \stackrel{?}{=} 3.2(3)$
$18 = 18 \checkmark$	$9.6 = 9.6 \checkmark$

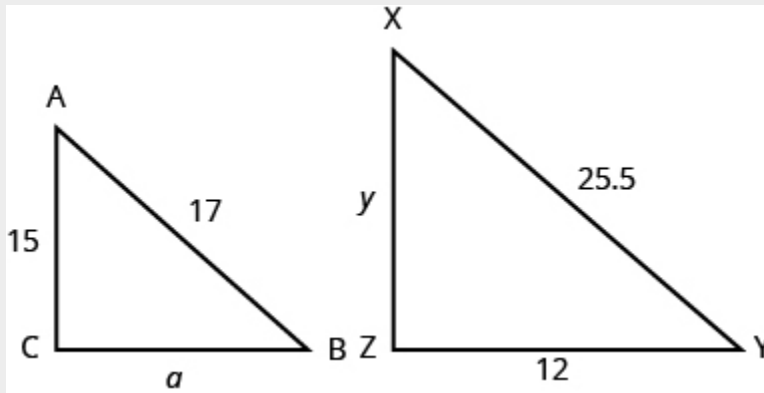
Step 7. **Answer** the question.

The third side of $\triangle ABC$ is 6 and the third side of $\triangle XYZ$ is 2.4.

Note:

Exercise:

Problem: $\triangle ABC$ is similar to $\triangle XYZ$. Find a .



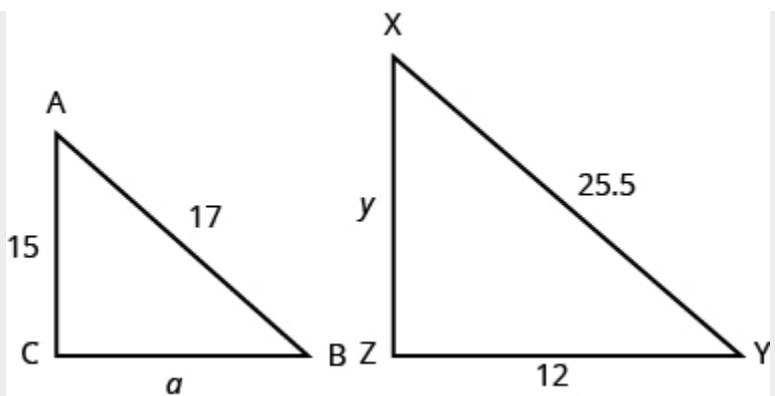
Solution:

8

Note:

Exercise:

Problem: $\triangle ABC$ is similar to $\triangle XYZ$. Find y .



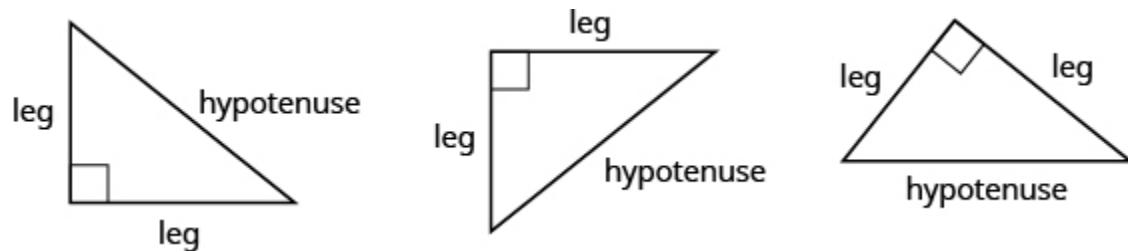
Solution:

22.5

Use the Pythagorean Theorem

The **Pythagorean Theorem** is a special property of right triangles that has been used since ancient times. It is named after the Greek philosopher and mathematician Pythagoras who lived around 500 BCE.

Remember that a right triangle has a 90° angle, which we usually mark with a small square in the corner. The side of the triangle opposite the 90° angle is called the **hypotenuse**, and the other two sides are called the **legs**. See [\[link\]](#).



In a right triangle, the side opposite the 90° angle is called the hypotenuse and each of the other sides is called a leg.

The Pythagorean Theorem tells how the lengths of the three sides of a right triangle relate to each other. It states that in any right triangle, the sum of the squares of the two legs equals the square of the hypotenuse.

Note:

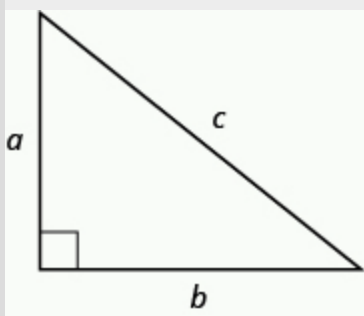
The Pythagorean Theorem

In any right triangle $\triangle ABC$,

Equation:

$$a^2 + b^2 = c^2$$

where c is the length of the hypotenuse a and b are the lengths of the legs.



To solve problems that use the Pythagorean Theorem, we will need to find square roots. In [Simplify and Use Square Roots](#) we introduced the notation $\sqrt{m}$ and defined it in this way:

Equation:

$$\text{If } m = n^2, \text{ then } \sqrt{m} = n \text{ for } n \geq 0$$

For example, we found that $\sqrt{25}$ is 5 because $5^2 = 25$.

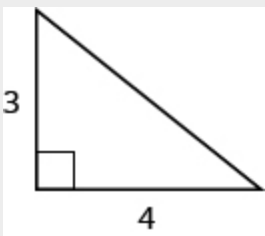
We will use this definition of square roots to solve for the length of a side in a right triangle.

Example:

Exercise:

Problem:

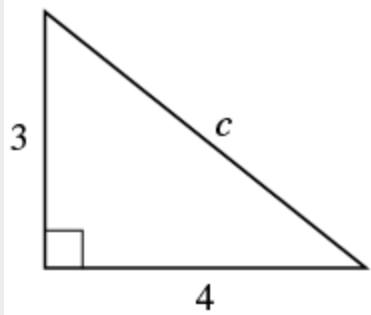
Use the Pythagorean Theorem to find the length of the hypotenuse.



Solution:

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length of the hypotenuse of the triangle
Step 3. Name. Choose a variable to represent it.	Let c = the length of the hypotenuse



Step 4. **Translate.**

Write the appropriate formula.

Substitute.

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$

Step 5. **Solve** the equation.

$$9 + 16 = c^2$$

$$25 = c^2$$

$$\sqrt{25} = c$$

$$5 = c$$

Step 6. **Check:**

$$3^2 + 4^2 = 5^2$$

$$9 + 16 \stackrel{?}{=} 25$$

$$25 = 25 \checkmark$$

Step 7. **Answer** the question.

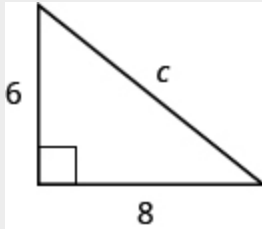
The length of the hypotenuse is 5.

Note:

Exercise:

Problem:

Use the Pythagorean Theorem to find the length of the hypotenuse.



Solution:

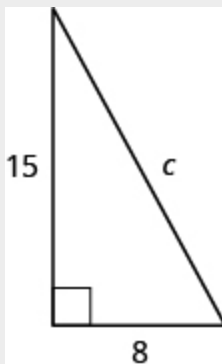
10

Note:

Exercise:

Problem:

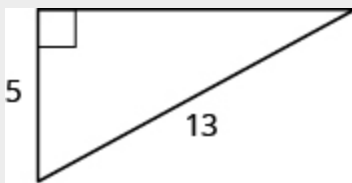
Use the Pythagorean Theorem to find the length of the hypotenuse.



Solution:

Example:**Exercise:****Problem:**

Use the Pythagorean Theorem to find the length of the longer leg.

**Solution:****Solution**

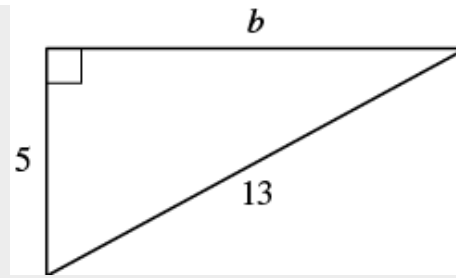
Step 1. **Read** the problem.

Step 2. **Identify** what you are looking for.

The length of the leg of the triangle

Step 3. **Name.** Choose a variable to represent it.

Let
 b = the leg of the triangle
Label side b



Step 4. **Translate.**
Write the appropriate formula.
Substitute.

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 13^2$$

Step 5. **Solve** the equation.
Isolate the variable term. Use
the definition of the square
root.
Simplify.

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b^2 = \sqrt{144}$$

$$b = 12$$

Step 6. **Check:**

$$5^2 + 12^2 \stackrel{?}{=} 13^2$$

$$25 + 144 \stackrel{?}{=} 169$$

$$169 = 169 \checkmark$$

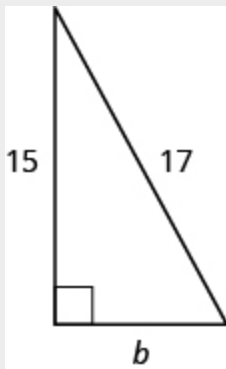
Step 7. **Answer** the question.

The length of the leg is 12.

Note:

Exercise:

Problem: Use the Pythagorean Theorem to find the length of the leg.



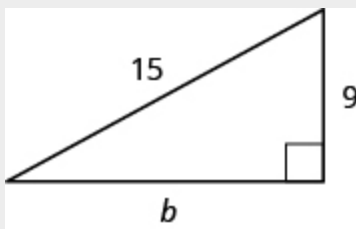
Solution:

8

Note:

Exercise:

Problem: Use the Pythagorean Theorem to find the length of the leg.

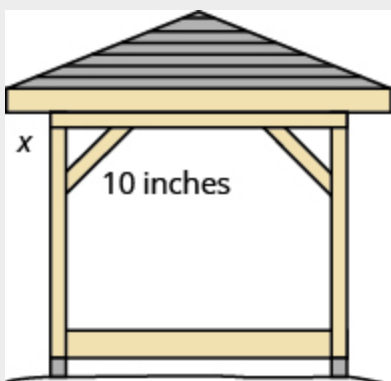


Solution:

12

Example:**Exercise:****Problem:**

Kelvin is building a gazebo and wants to brace each corner by placing a 10-inch wooden bracket diagonally as shown. How far below the corner should he fasten the bracket if he wants the distances from the corner to each end of the bracket to be equal? Approximate to the nearest tenth of an inch.

**Solution:****Solution**

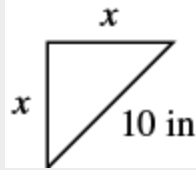
Step 1. **Read** the problem.

Step 2. **Identify** what you are looking for.

the distance from the corner that the bracket should be attached

Step 3. **Name.**
Choose a variable to represent it.

Let x = the distance from the corner



Step 4. Translate.

Write the appropriate formula.

Substitute.

$$a^2 + b^2 = c^2$$

$$x^2 + x^2 = 10^2$$

Step 5. Solve the equation.

Isolate the variable.

Use the definition of the square root.

Simplify.

Approximate to the nearest tenth.

$$2x^2 = 100$$

$$x^2 = 50$$

$$x = \sqrt{50}$$

$$b \approx 7.1$$

Step 6. Check:

$$a^2 + b^2 = c^2$$

$$(7.1)^2 + (7.1)^2 \stackrel{?}{\approx} 10^2$$

Yes.

Step 7. Answer the question.

Kelvin should fasten each piece of wood approximately 7.1" from the corner.

Note:

Exercise:

Problem:

John puts the base of a 13-ft ladder 5 feet from the wall of his house. How far up the wall does the ladder reach?



Solution:

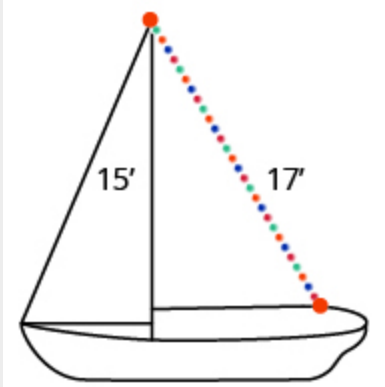
12 feet

Note:

Exercise:

Problem:

Randy wants to attach a 17-ft string of lights to the top of the 15-ft mast of his sailboat. How far from the base of the mast should he attach the end of the light string?



Solution:

8 feet

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Animation: The Sum of the Interior Angles of a Triangle](#)
- [Similar Polygons](#)
- [Example: Determine the Length of the Hypotenuse of a Right Triangle](#)

Key Concepts

- **Supplementary and Complementary Angles**
 - If the sum of the measures of two angles is 180° , then the angles are supplementary.
 - If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180$.
 - If the sum of the measures of two angles is 90° , then the angles are complementary.
 - If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90$.

- **Solve Geometry Applications**

Read the problem and make sure you understand all the words and ideas. Draw a figure and label it with the given information.

Identify what you are looking for.

Name what you are looking for and choose a variable to represent it.

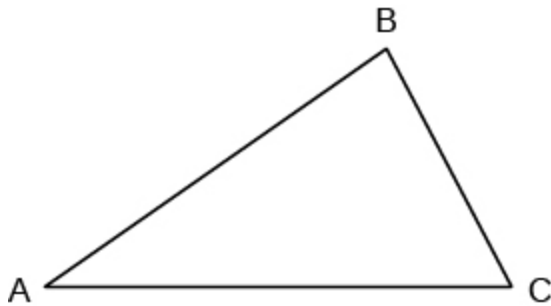
Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

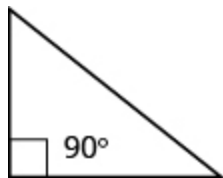
Answer the question with a complete sentence.

- **Sum of the Measures of the Angles of a Triangle**



- For any $\triangle ABC$, the sum of the measures is 180°
- $m\angle A + m\angle B + m\angle C = 180$

- **Right Triangle**



- A right triangle is a triangle that has one 90° angle, which is often marked with a $\square$ symbol.

- **Properties of Similar Triangles**

- If two triangles are similar, then their corresponding angle measures are equal and their corresponding side lengths have the

same ratio.

Practice Makes Perfect

Use the Properties of Angles

In the following exercises, find (a) the supplement and (b) the complement of the given angle.

Exercise:

Problem: 53°

Solution:

- (a) 127°
- (b) 37°

Exercise:

Problem: 16°

Exercise:

Problem: 29°

Solution:

- (a) 151°
- (b) 61°

Exercise:

Problem: 72°

In the following exercises, use the properties of angles to solve.

Exercise:

Problem: Find the supplement of a 135° angle.

Solution:

45°

Exercise:

Problem: Find the complement of a 38° angle.

Exercise:

Problem: Find the complement of a 27.5° angle.

Solution:

62.5°

Exercise:

Problem: Find the supplement of a 109.5° angle.

Exercise:

Problem:

Two angles are supplementary. The larger angle is 56° more than the smaller angle. Find the measures of both angles.

Solution:

$62^\circ, 118^\circ$

Exercise:

Problem:

Two angles are supplementary. The smaller angle is 36° less than the larger angle. Find the measures of both angles.

Exercise:**Problem:**

Two angles are complementary. The smaller angle is 34° less than the larger angle. Find the measures of both angles.

Solution:

$62^\circ, 28^\circ$

Exercise:**Problem:**

Two angles are complementary. The larger angle is 52° more than the smaller angle. Find the measures of both angles.

Use the Properties of Triangles

In the following exercises, solve using properties of triangles.

Exercise:**Problem:**

The measures of two angles of a triangle are 26° and 98° . Find the measure of the third angle.

Solution:

56°

Exercise:

Problem:

The measures of two angles of a triangle are 61° and 84° . Find the measure of the third angle.

Exercise:**Problem:**

The measures of two angles of a triangle are 105° and 31° . Find the measure of the third angle.

Solution:

44°

Exercise:**Problem:**

The measures of two angles of a triangle are 47° and 72° . Find the measure of the third angle.

Exercise:**Problem:**

One angle of a right triangle measures 33° . What is the measure of the other angle?

Solution:

57°

Exercise:**Problem:**

One angle of a right triangle measures 51° . What is the measure of the other angle?

Exercise:

Problem:

One angle of a right triangle measures 22.5° . What is the measure of the other angle?

Solution:

67.5°

Exercise:**Problem:**

One angle of a right triangle measures 36.5° . What is the measure of the other angle?

Exercise:**Problem:**

The two smaller angles of a right triangle have equal measures. Find the measures of all three angles.

Solution:

$45^\circ, 45^\circ, 90^\circ$

Exercise:**Problem:**

The measure of the smallest angle of a right triangle is 20° less than the measure of the other small angle. Find the measures of all three angles.

Exercise:

Problem:

The angles in a triangle are such that the measure of one angle is twice the measure of the smallest angle, while the measure of the third angle is three times the measure of the smallest angle. Find the measures of all three angles.

Solution:

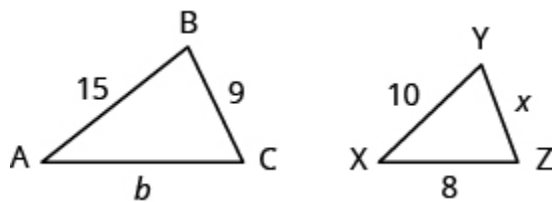
$30^\circ, 60^\circ, 90^\circ$

Exercise:**Problem:**

The angles in a triangle are such that the measure of one angle is 20° more than the measure of the smallest angle, while the measure of the third angle is three times the measure of the smallest angle. Find the measures of all three angles.

Find the Length of the Missing Side

In the following exercises, $\triangle ABC$ is similar to $\triangle XYZ$. Find the length of the indicated side.

**Exercise:**

Problem: side b

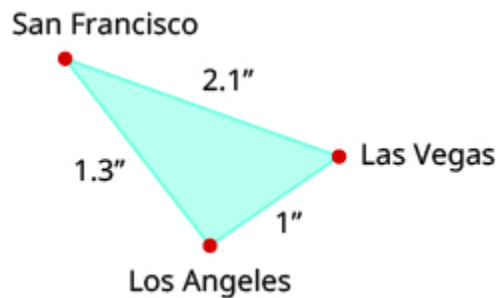
Solution:

12

Exercise:

Problem: side x

On a map, San Francisco, Las Vegas, and Los Angeles form a triangle whose sides are shown in the figure below. The actual distance from Los Angeles to Las Vegas is 270 miles.



Exercise:

Problem: Find the distance from Los Angeles to San Francisco.

Solution:

351 miles

Exercise:

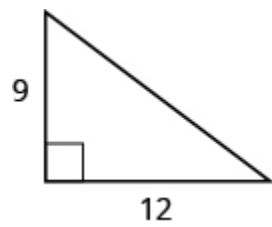
Problem: Find the distance from San Francisco to Las Vegas.

Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the hypotenuse.

Exercise:

Problem:

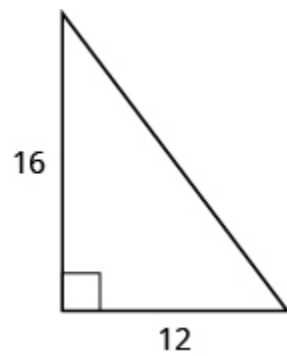


Solution:

15

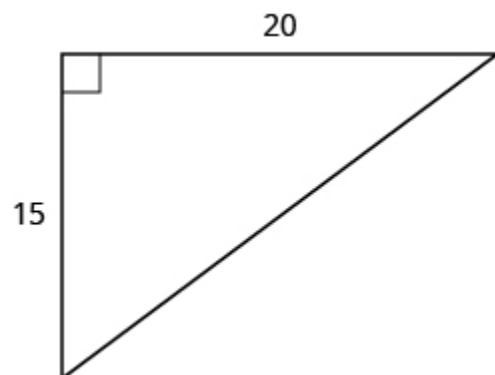
Exercise:

Problem:



Exercise:

Problem:

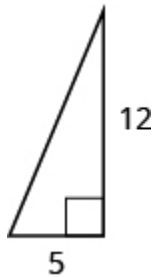


Solution:

25

Exercise:

Problem:

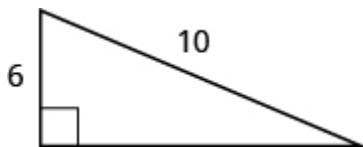


Find the Length of the Missing Side

In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

Exercise:

Problem:

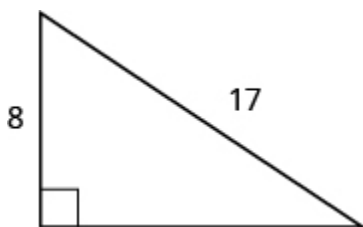


Solution:

8

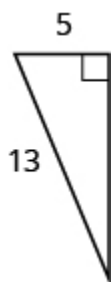
Exercise:

Problem:



Exercise:

Problem:

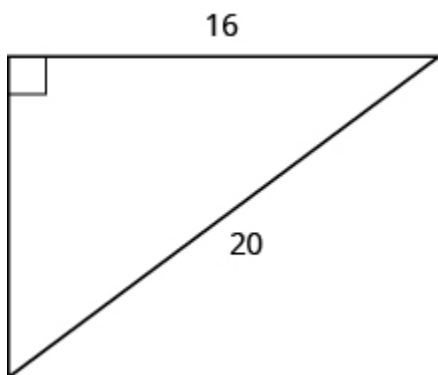


Solution:

12

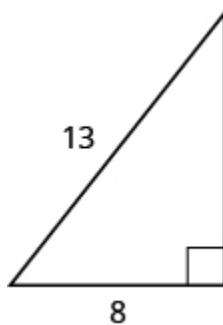
Exercise:

Problem:



Exercise:

Problem:

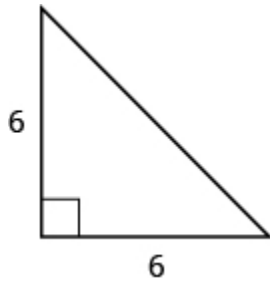


Solution:

10.2

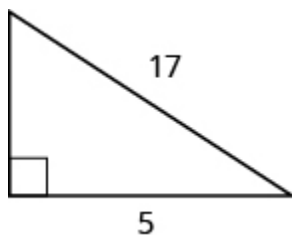
Exercise:

Problem:



Exercise:

Problem:

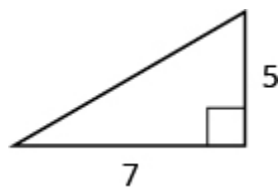


Solution:

8

Exercise:

Problem:

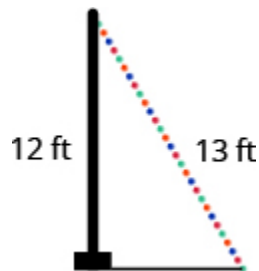


In the following exercises, solve. Approximate to the nearest tenth, if necessary.

Exercise:

Problem:

A 13-foot string of lights will be attached to the top of a 12-foot pole for a holiday display. How far from the base of the pole should the end of the string of lights be anchored?



Solution:

5 feet

Exercise:

Problem:

Pam wants to put a banner across her garage door to congratulate her son on his college graduation. The garage door is 12 feet high and 16 feet wide. How long should the banner be to fit the garage door?



Exercise:

Problem:

Chi is planning to put a path of paving stones through her flower garden. The flower garden is a square with sides of 10 feet. What will the length of the path be?



Solution:

14.1 feet

Exercise:

Problem:

Brian borrowed a 20-foot extension ladder to paint his house. If he sets the base of the ladder 6 feet from the house, how far up will the top of the ladder reach?

**Everyday Math****Exercise:****Problem:**

Building a scale model Joe wants to build a doll house for his daughter. He wants the doll house to look just like his house. His house is 30 feet wide and 35 feet tall at the highest point of the roof. If the dollhouse will be 2.5 feet wide, how tall will its highest point be?

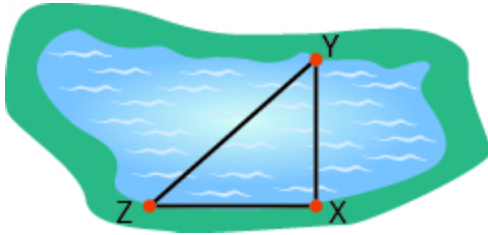
Solution:

2.9 feet

Exercise:

Problem:

Measurement A city engineer plans to build a footbridge across a lake from point X to point Y, as shown in the picture below. To find the length of the footbridge, she draws a right triangle XYZ, with right angle at X. She measures the distance from X to Z, 800 feet, and from Y to Z, 1,000 feet. How long will the bridge be?

**Writing Exercises****Exercise:****Problem:**

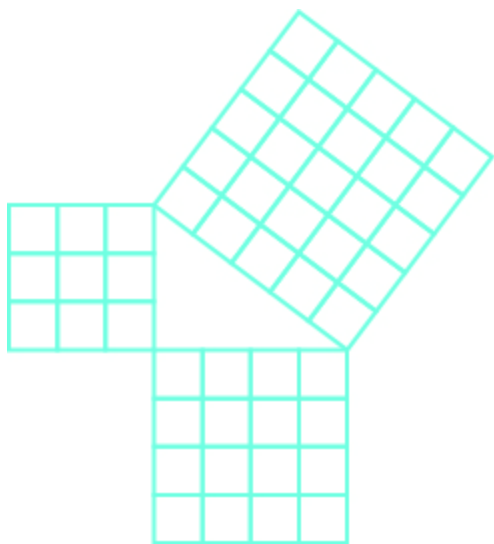
Write three of the properties of triangles from this section and then explain each in your own words.

Solution:

Answers will vary.

Exercise:**Problem:**

Explain how the figure below illustrates the Pythagorean Theorem for a triangle with legs of length 3 and 4.



Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the properties of angles.			
use the properties of triangles.			
use the Pythagorean Theorem.			

Ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

angle

An angle is formed by two rays that share a common endpoint. Each ray is called a side of the angle.

complementary angles

If the sum of the measures of two angles is 90° , then they are called complementary angles.

hypotenuse

The side of the triangle opposite the 90° angle is called the hypotenuse.

legs of a right triangle

The sides of a right triangle adjacent to the right angle are called the legs.

right triangle

A right triangle is a triangle that has one 90° angle.

similar figures

In geometry, if two figures have exactly the same shape but different sizes, we say they are similar figures.

supplementary angles

If the sum of the measures of two angles is 180° , then they are called supplementary angles.

triangle

A triangle is a geometric figure with three sides and three angles.

vertex of an angle

When two rays meet to form an angle, the common endpoint is called the vertex of the angle.

Use Properties of Rectangles, Triangles, and Trapezoids
By the end of this section, you will be able to:

- Understand linear, square, and cubic measure
- Use properties of rectangles
- Use properties of triangles
- Use properties of trapezoids

Note:

Before you get started, take this readiness quiz.

1. The length of a rectangle is 3 less than the width. Let w represent the width. Write an expression for the length of the rectangle.
If you missed this problem, review [\[link\]](#).
2. Simplify: $\frac{1}{2}(6h)$.
If you missed this problem, review [\[link\]](#).
3. Simplify: $\frac{5}{2}(10.3 - 7.9)$.
If you missed this problem, review [\[link\]](#).

In this section, we'll continue working with geometry applications. We will add some more properties of triangles, and we'll learn about the properties of rectangles and trapezoids.

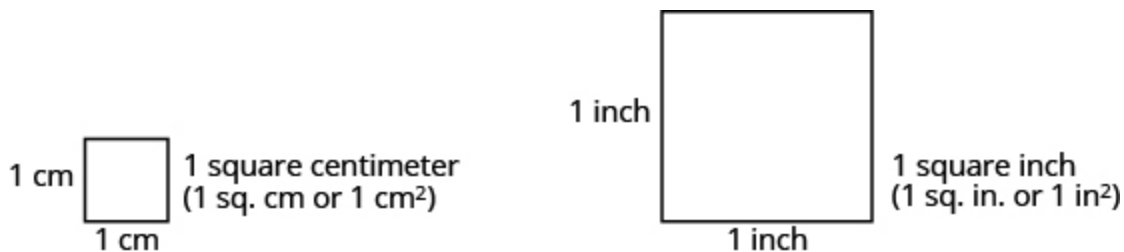
Understand Linear, Square, and Cubic Measure

When you measure your height or the length of a garden hose, you use a ruler or tape measure ([\[link\]](#)). A tape measure might remind you of a line—you use it for **linear measure**, which measures length. Inch, foot, yard, mile, centimeter and meter are units of linear measure.



This tape measure measures inches along the top and centimeters along the bottom.

When you want to know how much tile is needed to cover a floor, or the size of a wall to be painted, you need to know the **area**, a measure of the region needed to cover a surface. Area is measured in **square units**. We often use square inches, square feet, square centimeters, or square miles to measure area. A square centimeter is a square that is one centimeter (cm) on each side. A square inch is a square that is one inch on each side ([link](#)).



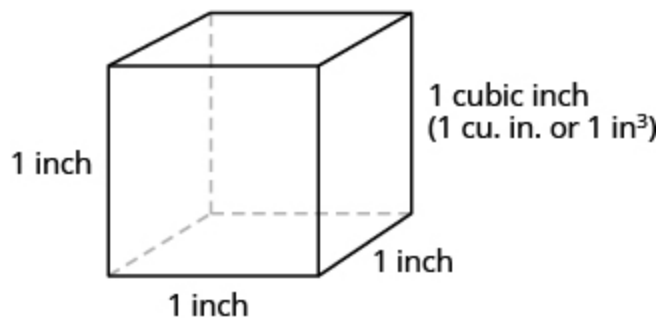
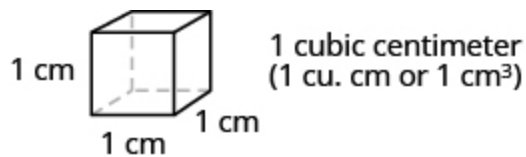
Square measures have sides that are each 1 unit in length.

[link](#) shows a rectangular rug that is 2 feet long by 3 feet wide. Each square is 1 foot wide by 1 foot long, or 1 square foot. The rug is made of 6 squares. The area of the rug is 6 square feet.



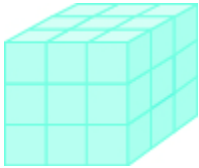
The rug contains six squares of 1 square foot each, so the total area of the rug is 6 square feet.

When you measure how much it takes to fill a container, such as the amount of gasoline that can fit in a tank, or the amount of medicine in a syringe, you are measuring **volume**. Volume is measured in **cubic units** such as cubic inches or cubic centimeters. When measuring the volume of a rectangular solid, you measure how many cubes fill the container. We often use cubic centimeters, cubic inches, and cubic feet. A cubic centimeter is a cube that measures one centimeter on each side, while a cubic inch is a cube that measures one inch on each side ([link](#)).



Cubic measures have sides that are
1 unit in length.

Suppose the cube in [\[link\]](#) measures 3 inches on each side and is cut on the lines shown. How many little cubes does it contain? If we were to take the big cube apart, we would find 27 little cubes, with each one measuring one inch on all sides. So each little cube has a volume of 1 cubic inch, and the volume of the big cube is 27 cubic inches.



A cube
that
measures 3
inches
on each
side is
made up
of 27
one-inch
cubes, or
27 cubic
inches.

Note: Doing the Manipulative Mathematics activity Visualizing Area and Perimeter will help you develop a better understanding of the difference between the area of a figure and its perimeter.

Example:

Exercise:

Problem:

For each item, state whether you would use linear, square, or cubic measure:

- Ⓐ amount of carpeting needed in a room
- Ⓑ extension cord length
- Ⓒ amount of sand in a sandbox
- Ⓓ length of a curtain rod
- Ⓔ amount of flour in a canister
- Ⓕ size of the roof of a doghouse.

Solution:

Solution

Ⓐ You are measuring how much surface the carpet covers, which is the area.

square
measure

Ⓑ You are measuring how long the extension cord is, which is the length.

linear
measure

③ You are measuring the volume of the sand.	cubic measure
④ You are measuring the length of the curtain rod.	linear measure
⑤ You are measuring the volume of the flour.	cubic measure
⑥ You are measuring the area of the roof.	square measure

Note:

Exercise:

Problem:

Determine whether you would use linear, square, or cubic measure for each item.

① amount of paint in a can ② height of a tree ③ floor of your bedroom ④ diameter of bike wheel ⑤ size of a piece of sod ⑥ amount of water in a swimming pool

Solution:

- ① cubic
- ② linear
- ③ square
- ④ linear
- ⑤ square
- ⑥ cubic

Note:**Exercise:****Problem:**

Determine whether you would use linear, square, or cubic measure for each item.

Ⓐ volume of a packing box Ⓑ size of patio Ⓒ amount of medicine in a syringe Ⓓ length of a piece of yarn Ⓔ size of housing lot Ⓕ height of a flagpole

Solution:

- Ⓐ cubic
- Ⓑ square
- Ⓒ cubic
- Ⓓ linear
- Ⓔ square
- Ⓕ linear

Many geometry applications will involve finding the perimeter or the area of a figure. There are also many applications of perimeter and area in everyday life, so it is important to make sure you understand what they each mean.

Picture a room that needs new floor tiles. The tiles come in squares that are a foot on each side—one square foot. How many of those squares are needed to cover the floor? This is the area of the floor.

Next, think about putting new baseboard around the room, once the tiles have been laid. To figure out how many strips are needed, you must know the distance around the room. You would use a tape measure to measure the number of feet around the room. This distance is the perimeter.

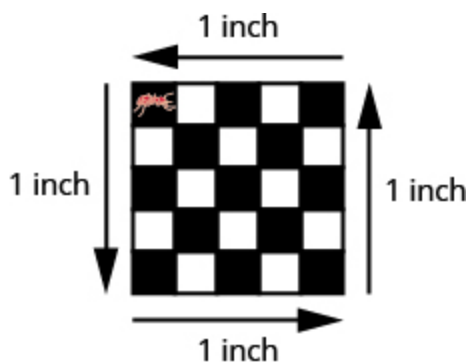
Note:**Perimeter and Area**

The **perimeter** is a measure of the distance around a figure.

The **area** is a measure of the surface covered by a figure.

[\[link\]](#) shows a square tile that is 1 inch on each side. If an ant walked around the edge of the tile, it would walk 4 inches. This distance is the perimeter of the tile.

Since the tile is a square that is 1 inch on each side, its area is one square inch. The area of a shape is measured by determining how many square units cover the shape.



Perimeter = 4 inches

Area = 1 square inch

When the ant walks completely around the tile on its edge, it is tracing the perimeter of the tile. The area of the tile is 1 square inch.

Note: Doing the Manipulative Mathematics activity Measuring Area and Perimeter will help you develop a better understanding of how to measure the area and perimeter of a figure.

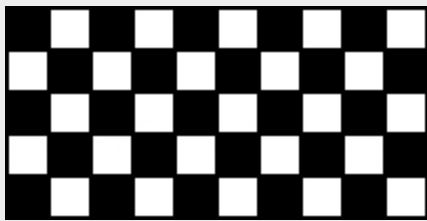
Example:

Exercise:

Problem:

Each of two square tiles is 1 square inch. Two tiles are shown together.

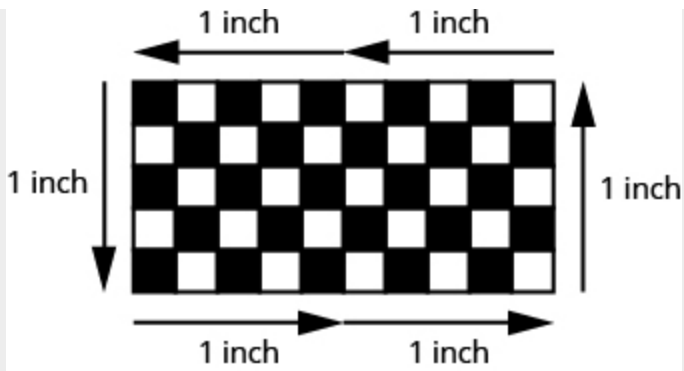
- Ⓐ What is the perimeter of the figure?
- Ⓑ What is the area?



Solution:

Solution

- Ⓐ The perimeter is the distance around the figure. The perimeter is 6 inches.
- Ⓑ The area is the surface covered by the figure. There are 2 square inch tiles so the area is 2 square inches.



Note:

Exercise:

Problem: Find the (a) perimeter and (b) area of the figure:



Solution:

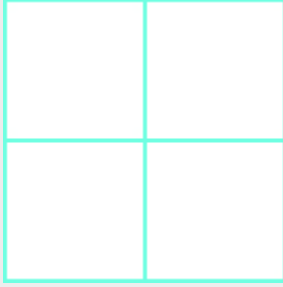
(a) 8 inches

(b) 3 sq. inches

Note:

Exercise:

Problem: Find the (a) perimeter and (b) area of the figure:

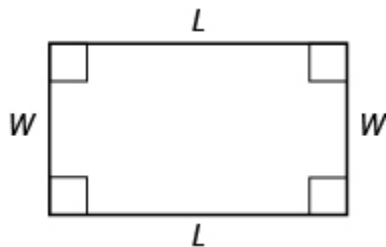


Solution:

- Ⓐ 8 centimeters
- Ⓑ 4 sq. centimeters

Use the Properties of Rectangles

A **rectangle** has four sides and four right angles. The opposite sides of a rectangle are the same length. We refer to one side of the rectangle as the length, L , and the adjacent side as the width, W . See [\[link\]](#).



A rectangle has four sides, and four right angles. The sides are labeled L for length and W for width.

The perimeter, P , of the rectangle is the distance around the rectangle. If you started at one corner and walked around the rectangle, you would walk $L + W + L + W$ units, or two lengths and two widths. The perimeter then is

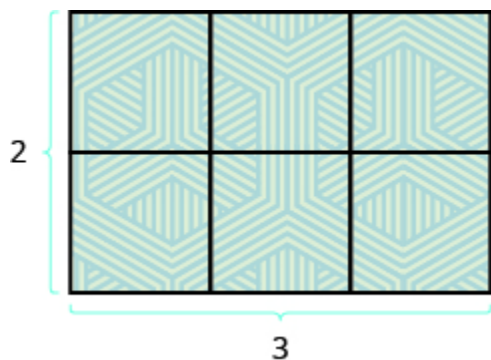
Equation:

$$P = L + W + L + W$$

or

$$P = 2L + 2W$$

What about the area of a rectangle? Remember the rectangular rug from the beginning of this section. It was 2 feet long by 3 feet wide, and its area was 6 square feet. See [\[link\]](#). Since $A = 2 \cdot 3$, we see that the area, A , is the length, L , times the width, W , so the area of a rectangle is $A = L \cdot W$.



The area of this rectangular rug is 6 square feet, its length times its width.

Note:**Properties of Rectangles**

- Rectangles have four sides and four right (90°) angles.
- The lengths of opposite sides are equal.
- The perimeter, P , of a rectangle is the sum of twice the length and twice the width. See [\[link\]](#).

Equation:

$$P = 2L + 2W$$

- The area, A , of a rectangle is the length times the width.

Equation:

$$A = L \cdot W$$

For easy reference as we work the examples in this section, we will restate the Problem Solving Strategy for Geometry Applications here.

Note:**Use a Problem Solving Strategy for Geometry Applications**

Read the problem and make sure you understand all the words and ideas.

Draw the figure and label it with the given information.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

Answer the question with a complete sentence.

Example:

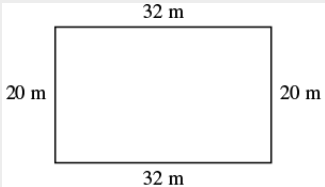
Exercise:

Problem:

The length of a rectangle is 32 meters and the width is 20 meters.
Find (a) the perimeter, and (b) the area.

Solution:

Solution

(a)	
Step 1. Read the problem. Draw the figure and label it with the given information.	
Step 2. Identify what you are looking for.	the perimeter of a rectangle
Step 3. Name. Choose a variable to represent it.	Let P = the perimeter
Step 4. Translate. Write the appropriate formula. Substitute.	$\underbrace{P}_{P} = \underbrace{2L}_{2(32)} + \underbrace{2W}_{2(20)}$
Step 5. Solve the equation.	$P = 64 + 40$ $P = 104$

Step 6. **Check:**

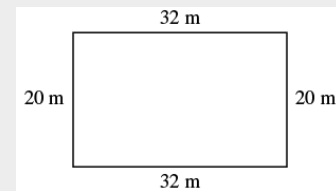
$$\begin{aligned}P &\stackrel{?}{=} 104 \\20 + 32 + 20 + 32 &\stackrel{?}{=} 104 \\104 &= 104 \checkmark\end{aligned}$$

Step 7. **Answer** the question.

The perimeter of the rectangle is 104 meters.

⑥

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the area of a rectangle

Step 3. **Name.** Choose a variable to represent it.

Let A = the area

Step 4. **Translate.**
Write the appropriate formula.
Substitute.

$$\begin{array}{ccccccc}A & = & L & \cdot & W \\ \underbrace{A} & = & \underbrace{L} & \cdot & \underbrace{W} \\ A & = & 32 \text{ m} & \cdot & 20 \text{ m}\end{array}$$

Step 5. Solve the equation.	$A = 640$
Step 6. Check: $A \stackrel{?}{=} 640$ $32 \cdot 20 \stackrel{?}{=} 640$ $640 = 640 \checkmark$	
Step 7. Answer the question.	The area of the rectangle is 60 square meters.

Note:

Exercise:

Problem:

The length of a rectangle is 120 yards and the width is 50 yards. Find
 Ⓐ the perimeter and Ⓑ the area.

Solution:

- Ⓐ 340 yd
- Ⓑ 6000 sq. yd

Note:

Exercise:

Problem:

The length of a rectangle is 62 feet and the width is 48 feet. Find (a) the perimeter and (b) the area.

Solution:

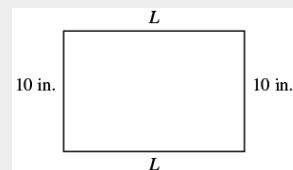
- (a) 220 ft
- (b) 2976 sq. ft

Example:**Exercise:****Problem:**

Find the length of a rectangle with perimeter 50 inches and width 10 inches.

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the length of the rectangle

Step 3. Name. Choose a variable to represent it.	Let L = the length
Step 4. Translate. Write the appropriate formula. Substitute.	$\begin{array}{ccccccc} P & = & 2L & + & 2W \\ \underbrace{50} & = & \underbrace{2L} & + & \underbrace{2(10)} \end{array}$
Step 5. Solve the equation.	$\begin{array}{l} 50 - 20 = 2L + 20 - 20 \\ 30 = 2L \\ \frac{30}{2} = \frac{2L}{2} \\ 15 = L \end{array}$
Step 6. Check: $\begin{array}{l} P = 50 \\ 15 + 10 + 15 + 10 \stackrel{?}{=} 50 \\ 50 = 50 \checkmark \end{array}$	
Step 7. Answer the question.	The length is 15 inches.

Note:

Exercise:

Problem:

Find the length of a rectangle with a perimeter of 80 inches and width of 25 inches.

Solution:

15 in.

Note:

Exercise:

Problem:

Find the length of a rectangle with a perimeter of 30 yards and width of 6 yards.

Solution:

9 yd

In the next example, the width is defined in terms of the length. We'll wait to draw the figure until we write an expression for the width so that we can label one side with that expression.

Example:

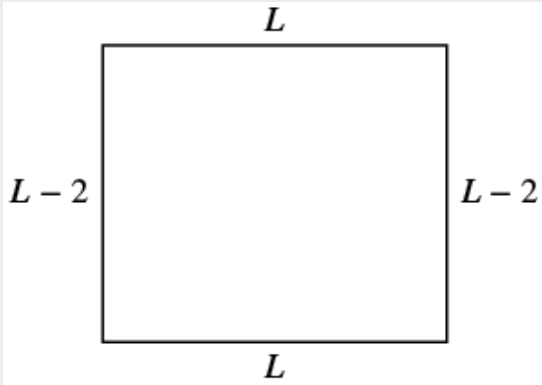
Exercise:

Problem:

The width of a rectangle is two inches less than the length. The perimeter is 52 inches. Find the length and width.

Solution:

Solution

Step 1. Read the problem.	
Step 2. Identify what you are looking for.	the length and width of the rectangle
Step 3. Name. Choose a variable to represent it. Now we can draw a figure using these expressions for the length and width.	Since the width is defined in terms of the length, we let $L =$ length. The width is two feet less than the length, so we let $L - 2 =$ width  The diagram shows a rectangle. The top side is labeled L, the bottom side is labeled L, the left side is labeled $L - 2$, and the right side is labeled $L - 2$.
Step 4. Translate. Write the appropriate formula. The formula for the perimeter of a rectangle relates all the information. Substitute in the given information.	$\underbrace{P}_{52} = \underbrace{2L}_{2L} + \underbrace{2W}_{2(L-2)}$
Step 5. Solve the equation.	$52 = 2L + 2L - 4$
Combine like terms.	$52 = 4L - 4$

Add 4 to each side.	$56 = 4L$
Divide by 4.	$\frac{56}{4} = \frac{4L}{4}$
	$14 = L$
	The length is 14 inches.
Now we need to find the width.	
The width is $L - 2$.	$L - 2$ $14 - 2$ 12 The width is 12 inches.
Step 6. Check: Since $14 + 12 + 14 + 12 = 52$, this works!	
Step 7. Answer the question.	The length is 14 feet and the width is 12 feet.

Note:

Exercise:

Problem:

The width of a rectangle is seven meters less than the length. The perimeter is 58 meters. Find the length and width.

Solution:

18 m, 11 m

Note:

Exercise:

Problem:

The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

Solution:

11 ft , 19 ft

Example:

Exercise:

Problem:

The length of a rectangle is four centimeters more than twice the width. The perimeter is 32 centimeters. Find the length and width.

Solution:

Solution

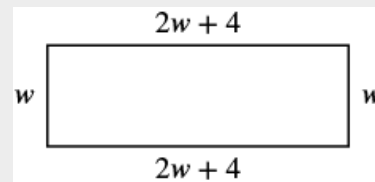
Step 1. Read the problem.	

Step 2. **Identify** what you are looking for.

the length and width

Step 3. **Name.** Choose a variable to represent it.

let W = width
The length is four more than twice the width.
 $2w + 4$ = length



Step 4. **Translate.**
Write the appropriate formula and substitute in the given information.

$$\underbrace{P}_{32} = \underbrace{2L}_{2(2w+4)} + \underbrace{2W}_{2w}$$

Step 5. **Solve** the equation.

$$\begin{aligned} 32 &= 4w + 8 + 2w \\ 32 &= 6w + 8 \\ 24 &= 6w \\ 4 &= w \text{ width} \\ 2w + 4 &\text{ length} \\ 2(4) + 4 & \\ 12 &\text{ The length is 12 cm.} \end{aligned}$$

Step 6. **Check:**

$$\begin{aligned} p &= 2L + 2W \\ 32 &\stackrel{?}{=} 2 \cdot 12 + 2 \cdot 4 \\ 32 &= 32 \end{aligned}$$

Step 7. **Answer** the question.

The length is 12 cm
and the width is 4 cm.

Note:

Exercise:

Problem:

The length of a rectangle is eight more than twice the width. The perimeter is 64 feet. Find the length and width.

Solution:

8 ft, 24 ft

Note:

Exercise:

Problem:

The width of a rectangle is six less than twice the length. The perimeter is 18 centimeters. Find the length and width.

Solution:

5 cm, 4 cm

Example:

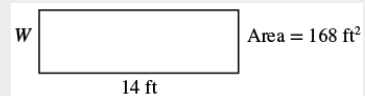
Exercise:

Problem:

The area of a rectangular room is 168 square feet. The length is 14 feet. What is the width?

Solution:
Solution

Step 1. **Read** the problem.



Step 2. **Identify** what you are looking for.

the width of a rectangular room

Step 3. **Name.** Choose a variable to represent it.

Let W = width

Step 4. **Translate.**
Write the appropriate formula and substitute in the given information.

$$A = LW$$
$$168 = 14W$$

Step 5. **Solve** the equation.

$$\frac{168}{14} = \frac{14W}{14}$$
$$12 = W$$

Step 6. **Check:**

$$A = LW$$

$$168 \stackrel{?}{=} 14 \cdot 12$$

$$168 = 168 \checkmark$$

Step 7. **Answer** the question.

The width of the room is 12 feet.

Note:

Exercise:

Problem:

The area of a rectangle is 598 square feet. The length is 23 feet. What is the width?

Solution:

26 ft

Note:

Exercise:

Problem:

The width of a rectangle is 21 meters. The area is 609 square meters. What is the length?

Solution:

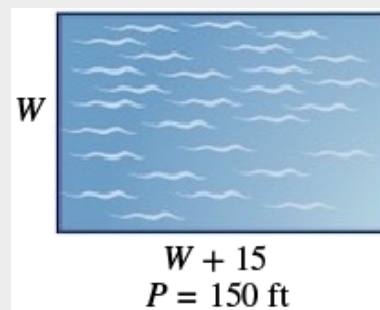
29 m

Example:**Exercise:****Problem:**

The perimeter of a rectangular swimming pool is 150 feet. The length is 15 feet more than the width. Find the length and width.

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the length and width of the pool

Step 3. **Name.** Choose a variable to represent it.
The length is 15 feet more than the width.

Let $W = \text{width}$
 $W + 15 = \text{length}$

Step 4. **Translate.**
Write the appropriate formula and substitute.

$$\underbrace{P}_{150} = \underbrace{2L}_{2(w+15)} + \underbrace{2W}_{2w}$$

Step 5. **Solve** the equation.

$$150 = 2w + 30 + 2w$$

$$150 = 4w + 30$$

$$120 = 4w$$

$$30 = w \text{ the width of the pool}$$

$$w + 15 \text{ the length of the pool}$$

$$30 + 15$$

$$45$$

Step 6. **Check:**

$$p = 2L + 2W$$

$$150 \stackrel{?}{=} 2(45) + 2(30)$$

$$150 = 150$$

Step 7. **Answer** the question.

The length of the pool is 45 feet and the width is 30 feet.

Note:

Exercise:

Problem:

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Solution:

30 ft, 70 ft

Note:

Exercise:

Problem:

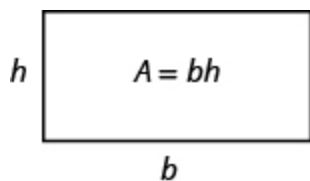
The length of a rectangular garden is 30 yards more than the width.
The perimeter is 300 yards. Find the length and width.

Solution:

60 yd, 90 yd

Use the Properties of Triangles

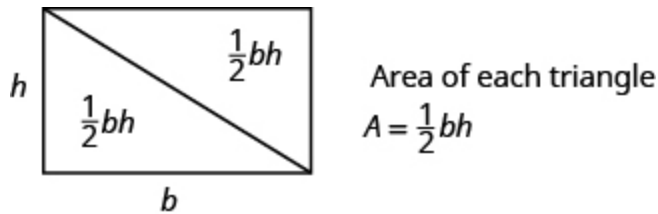
We now know how to find the area of a rectangle. We can use this fact to help us visualize the formula for the area of a triangle. In the rectangle in [\[link\]](#), we've labeled the length b and the width h , so its area is bh .



The area of a rectangle is the base, b , times the height, h .

We can divide this rectangle into two **congruent** triangles ([\[link\]](#)). Triangles that are congruent have identical side lengths and angles, and so their areas are equal. The area of each triangle is one-half the area of the rectangle, or

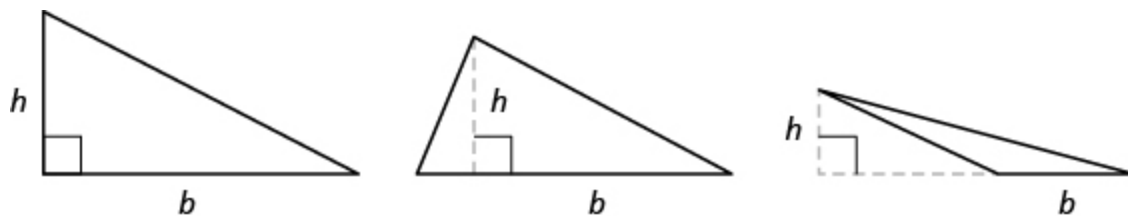
$\frac{1}{2}bh$. This example helps us see why the formula for the area of a triangle is $A = \frac{1}{2}bh$.



A rectangle can be divided into two triangles of equal area. The area of each triangle is one-half the area of the rectangle.

The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base and h is the height.

To find the area of the triangle, you need to know its base and height. The base is the length of one side of the triangle, usually the side at the bottom. The height is the length of the line that connects the base to the opposite vertex, and makes a 90° angle with the base. [\[link\]](#) shows three triangles with the base and height of each marked.



The height h of a triangle is the length of a line segment that connects the the base to the opposite vertex and makes a 90° angle with the base.

Note:**Triangle Properties**

For any triangle $\triangle ABC$, the sum of the measures of the angles is 180° .

Equation:

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

The perimeter of a triangle is the sum of the lengths of the sides.

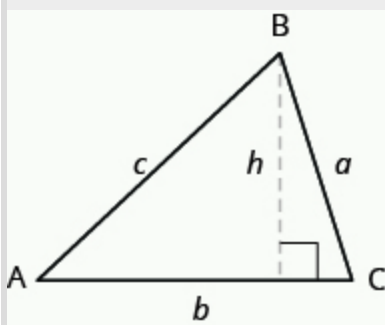
Equation:

$$P = a + b + c$$

The area of a triangle is one-half the base, b , times the height, h .

Equation:

$$A = \frac{1}{2}bh$$

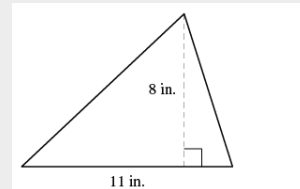
**Example:****Exercise:**

Problem:

Find the area of a triangle whose base is 11 inches and whose height is 8 inches.

Solution:
Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the area of the triangle

Step 3. **Name.** Choose a variable to represent it.

let A = area of the triangle

Step 4. **Translate.**
Write the appropriate formula.
Substitute.

$$\begin{array}{ccccccc} A & = & \frac{1}{2} & \cdot & b & \cdot & h \\ \underbrace{A} & & \underbrace{\frac{1}{2}} & & \underbrace{b} & & \underbrace{h} \\ A & = & \frac{1}{2} & \cdot & 11 & \cdot & 8 \end{array}$$

Step 5. **Solve** the equation.

$$A = 44 \text{ square inches}$$

Step 6. **Check:**

$$A = \frac{1}{2} bh$$

$$44 \stackrel{?}{=} \frac{1}{2} (11) 8$$

$$44 = 44 \checkmark$$

Step 7. **Answer** the question.

The area is 44 square inches.

Note:

Exercise:

Problem:

Find the area of a triangle with base 13 inches and height 2 inches.

Solution:

13 sq. in.

Note:

Exercise:

Problem:

Find the area of a triangle with base 14 inches and height 7 inches.

Solution:

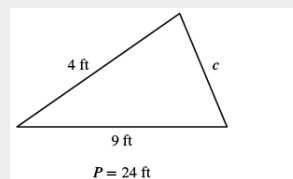
49 sq. in.

Example:**Exercise:****Problem:**

The perimeter of a triangular garden is 24 feet. The lengths of two sides are 4 feet and 9 feet. How long is the third side?

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

length of the third side of a triangle

Step 3. **Name.** Choose a variable to represent it.

Let c = the third side

Step 4. **Translate.**
Write the appropriate formula.
Substitute in the given information.

$$\begin{array}{ccccccc} P & = & a & + & b & + & c \\ \underbrace{24} & = & \underbrace{4} & + & \underbrace{9} & + & \underbrace{c} \end{array}$$

Step 5. **Solve** the equation.

$$\begin{array}{l} 24 = 13 + c \\ 11 = c \end{array}$$

Step 6. **Check:**

$$P = a + b + c$$

$$24 \stackrel{?}{=} 4 + 9 + 11$$

$$24 = 24 \checkmark$$

Step 7. **Answer** the question.

The third side is
11 feet long.

Note:

Exercise:

Problem:

The perimeter of a triangular garden is 24 feet. The lengths of two sides are 18 feet and 22 feet. How long is the third side?

Solution:

8 ft

Note:

Exercise:

Problem:

The lengths of two sides of a triangular window are 7 feet and 5 feet. The perimeter is 18 feet. How long is the third side?

Solution:

6 ft

Example:

Exercise:

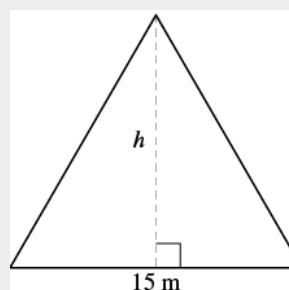
Problem:

The area of a triangular church window is 90 square meters. The base of the window is 15 meters. What is the window's height?

Solution:

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

height of a triangle

Step 3. **Name.** Choose a variable to represent it.

Let h = the height

Step 4. **Translate.**
Write the appropriate formula.
Substitute in the given information.

$$\begin{array}{ccccccc} A & = & \frac{1}{2} & \cdot & b & \cdot & h \\ \underbrace{} & \underbrace{} & \underbrace{\frac{1}{2}} & \cdot & \underbrace{b} & \cdot & \underbrace{h} \\ 90 & = & \frac{1}{2} & \cdot & 15 & \cdot & h \end{array}$$

Step 5. **Solve** the equation.

	$90 = \frac{15}{2}h$ $12 = h$
Step 6. Check: $A = \frac{1}{2}bh$ $90 \stackrel{?}{=} \frac{1}{2} \cdot 15 \cdot 12$ $90 = 90 \checkmark$	
Step 7. Answer the question.	The height of the triangle is 12 meters.

Note:

Exercise:

Problem:

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Solution:

14 in.

Note:

Exercise:

Problem:

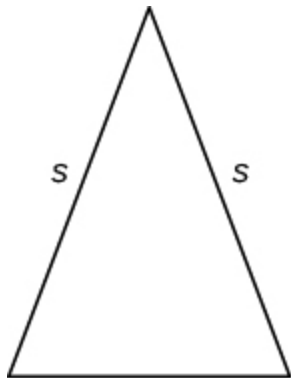
A triangular tent door has an area of 15 square feet. The height is 5 feet. What is the base?

Solution:

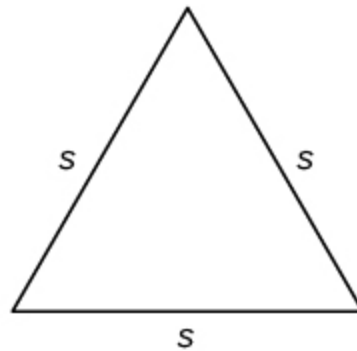
6 ft

Isosceles and Equilateral Triangles

Besides the right triangle, some other triangles have special names. A triangle with two sides of equal length is called an **isosceles triangle**. A triangle that has three sides of equal length is called an **equilateral triangle**. [\[link\]](#) shows both types of triangles.



isosceles triangle



equilateral triangle

In an isosceles triangle, two sides have the same length, and the third side is the base. In an equilateral triangle, all three sides have the same length.

Note:**Isosceles and Equilateral Triangles**

An **isosceles** triangle has two sides the same length.

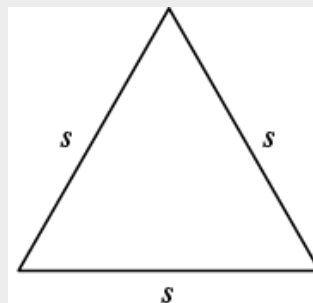
An **equilateral** triangle has three sides of equal length.

Example:**Exercise:****Problem:**

The perimeter of an equilateral triangle is 93 inches. Find the length of each side.

Solution:**Solution**

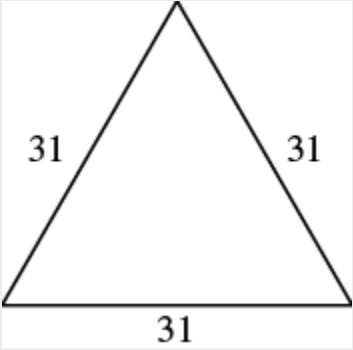
Step 1. **Read** the problem. Draw the figure and label it with the given information.



Perimeter = 93 in.

Step 2. **Identify** what you are looking for.

length of the sides
of an equilateral
triangle

Step 3. Name. Choose a variable to represent it.	Let s = length of each side
Step 4. Translate. Write the appropriate formula. Substitute.	$\underbrace{P}_{93} = \underbrace{a}_s + \underbrace{b}_s + \underbrace{c}_s$
Step 5. Solve the equation.	$93 = 3s$ $31 = s$
Step 6. Check:  $93 \stackrel{?}{=} 31 + 31 + 31$ $93 = 93 \checkmark$	
Step 7. Answer the question.	Each side is 31 inches.

Note:

Exercise:**Problem:**

Find the length of each side of an equilateral triangle with perimeter 39 inches.

Solution:

13 in.

Note:**Exercise:****Problem:**

Find the length of each side of an equilateral triangle with perimeter 51 centimeters.

Solution:

17 cm

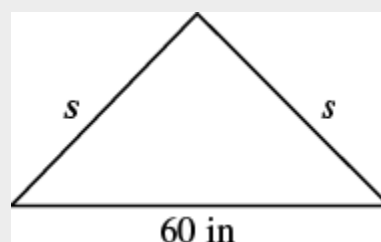
Example:**Exercise:****Problem:**

Arianna has 156 inches of beading to use as trim around a scarf. The scarf will be an isosceles triangle with a base of 60 inches. How long can she make the two equal sides?

Solution:

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



$$P = 156 \text{ in.}$$

Step 2. **Identify** what you are looking for.

the lengths of the two equal sides

Step 3. **Name.** Choose a variable to represent it.

Let s = the length of each side

Step 4. **Translate.**
Write the appropriate formula.
Substitute in the given information.

$$\underbrace{P}_{156} = \underbrace{a}_s + \underbrace{b}_{60} + \underbrace{c}_s$$

Step 5. **Solve** the equation.

$$156 = 2s + 60$$

$$96 = 2s$$

$$48 = s$$

Step 6. **Check:**

$$p = a + b + c$$

$$156 \stackrel{?}{=} 48 + 60 + 48$$

$$156 = 156 \checkmark$$

Step 7. **Answer** the question.

Arianna can make each of the two equal sides 48

inches long.

Note:

Exercise:

Problem:

A backyard deck is in the shape of an isosceles triangle with a base of 20 feet. The perimeter of the deck is 48 feet. How long is each of the equal sides of the deck?

Solution:

14 ft

Note:

Exercise:

Problem:

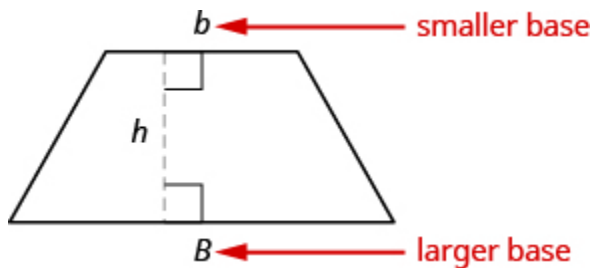
A boat's sail is an isosceles triangle with base of 8 meters. The perimeter is 22 meters. How long is each of the equal sides of the sail?

Solution:

7 m

Use the Properties of Trapezoids

A **trapezoid** is four-sided figure, a *quadrilateral*, with two sides that are parallel and two sides that are not. The parallel sides are called the bases. We call the length of the smaller base b , and the length of the bigger base B . The height, h , of a trapezoid is the distance between the two bases as shown in [\[link\]](#).



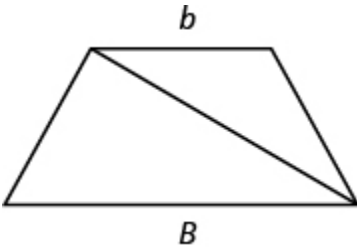
A trapezoid has a larger base, B , and a smaller base, b . The height h is the distance between the bases.

The formula for the area of a trapezoid is:

Equation:

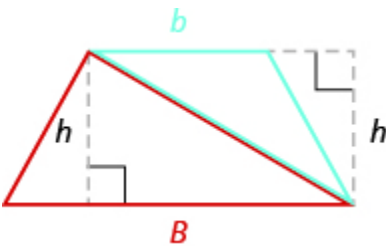
$$\text{Area}_{\text{trapezoid}} = \frac{1}{2}h(b + B)$$

Splitting the trapezoid into two triangles may help us understand the formula. The area of the trapezoid is the sum of the areas of the two triangles. See [\[link\]](#).



Splitting a
trapezoid into
two triangles
may help you
understand the
formula for its
area.

The height of the trapezoid is also the height of each of the two triangles.
See [\[link\]](#).



The formula for the area of a trapezoid is

$$\text{Area}_{\text{trapezoid}} = \frac{1}{2} h(b + B)$$

If we distribute, we get,

$$\text{Area}_{\text{trapezoid}} = \frac{1}{2}bh + \frac{1}{2}Bh$$

$$\text{Area}_{\text{trapezoid}} = A_{\text{blue}\Delta} + A_{\text{red}\Delta}$$

Note:

Properties of Trapezoids

- A trapezoid has four sides. See [\[link\]](#).
- Two of its sides are parallel and two sides are not.
- The area, A , of a trapezoid is $A = \frac{1}{2}h(b + B)$.

Example:

Exercise:

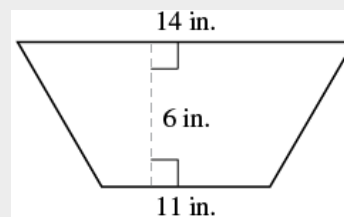
Problem:

Find the area of a trapezoid whose height is 6 inches and whose bases are 14 and 11 inches.

Solution:

Solution

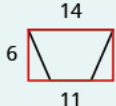
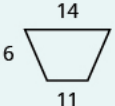
Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. Identify what you are looking for.	the area of the trapezoid
Step 3. Name. Choose a variable to represent it.	Let A = the area
Step 4. Translate. Write the appropriate formula. Substitute.	$\underbrace{A}_{A} = \underbrace{\frac{1}{2}}_{\frac{1}{2}} \cdot \underbrace{h}_{6} \cdot \underbrace{(b+B)}_{(11+14)}$
Step 5. Solve the equation.	$A = \frac{1}{2} \cdot 6(25)$ $A = 3(25)$ $A = 75 \text{ square inches}$
Step 6. Check: Is this answer reasonable?	

If we draw a rectangle around the trapezoid that has the same big base B and a height h , its area should be greater than that of the trapezoid.

If we draw a rectangle inside the trapezoid that has the same little base b and a height h , its area should be smaller than that of the trapezoid.

		
$A_{\text{rectangle}} = bh$	$A_{\text{trapezoid}} = \frac{1}{2}h(b+B)$	$A_{\text{rectangle}} = bh$
$A_{\text{rectangle}} = 14 \cdot 6$	$A_{\text{trapezoid}} = \frac{1}{2} \cdot 6(11+14)$	$A_{\text{rectangle}} = 11 \cdot 6$
$A_{\text{rectangle}} = 84 \text{ sq. in.}$	$A_{\text{trapezoid}} = 75 \text{ sq. in.}$	$A_{\text{rectangle}} = 66 \text{ sq. in.}$

The area of the larger rectangle is 84 square inches and the area of the smaller rectangle is 66 square inches. So it makes sense that the area of the trapezoid is between 84 and 66 square inches

Step 7. **Answer** the question. The area of the trapezoid is 75 square inches.

Note:

Exercise:

Problem:

The height of a trapezoid is 14 yards and the bases are 7 and 16 yards. What is the area?

Solution:

161 sq. yd

Note:

Exercise:

Problem:

The height of a trapezoid is 18 centimeters and the bases are 17 and 8 centimeters. What is the area?

Solution:

225 sq. cm

Example:

Exercise:

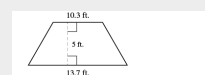
Problem:

Find the area of a trapezoid whose height is 5 feet and whose bases are 10.3 and 13.7 feet.

Solution:

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the area of
the
trapezoid

Step 3. **Name.** Choose a variable to represent it.

Let A = the
area

Step 4. **Translate.**
Write the appropriate formula.
Substitute.

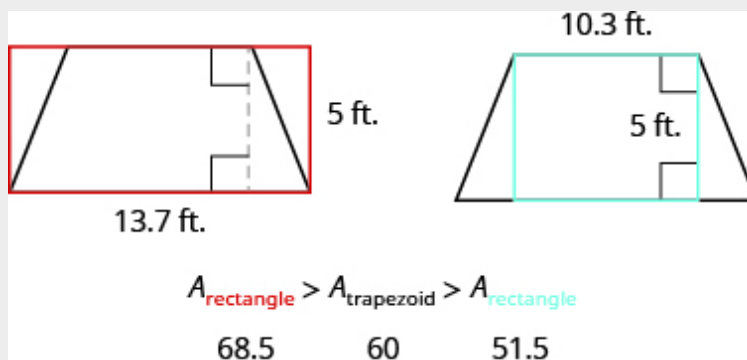
$$A = \frac{1}{2} \cdot h \cdot (b + B)$$
$$A = \frac{1}{2} \cdot 5 \cdot (10.3 + 13.7)$$

Step 5. **Solve** the equation.

$$A = \frac{1}{2} \cdot 5(24)$$
$$A = 12 \cdot 5$$
$$A = 60 \text{ square feet}$$

Step 6. **Check:** Is this answer reasonable?
The area of the trapezoid should be less than
the area of a rectangle with base 13.7 and

height 5, but more than the area of a rectangle with base 10.3 and height 5.



Step 7. **Answer** the question.

The area of the trapezoid is 60 square feet.

Note:

Exercise:

Problem:

The height of a trapezoid is 7 centimeters and the bases are 4.6 and 7.4 centimeters. What is the area?

Solution:

42 sq. cm

Note:

Exercise:**Problem:**

The height of a trapezoid is 9 meters and the bases are 6.2 and 7.8 meters. What is the area?

Solution:

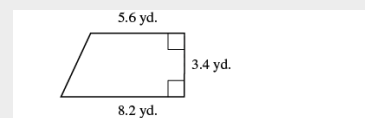
63 sq. m

Example:**Exercise:****Problem:**

Vinny has a garden that is shaped like a trapezoid. The trapezoid has a height of 3.4 yards and the bases are 8.2 and 5.6 yards. How many square yards will be available to plant?

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the area of a trapezoid

Step 3. **Name.** Choose a variable to represent it.

Let A = the area

Step 4. **Translate.**
Write the appropriate formula.
Substitute.

$$\underbrace{A}_{A} = \underbrace{\frac{1}{2}}_{\frac{1}{2}} \cdot \underbrace{h}_{3.4} \cdot \underbrace{(b + B)}_{(5.6 + 8.2)}$$

Step 5. **Solve** the equation.

$$A = \frac{1}{2}(3.4)(13.8)$$

$$A = 23.46 \text{ square yards}$$

Step 6. **Check:** Is this answer reasonable?

Yes. The area of the trapezoid is less than the area of a rectangle with a base of 8.2 yd and height 3.4 yd, but more than the area of a rectangle with base 5.6 yd and height 3.4 yd.

$A_{\text{rectangle}} = Bh$ $= (8.2)(3.4)$ $= 27.88 \text{ yd}^2$	$A_{\text{trapezoid}} = \frac{1}{2}(3.4 \text{ yd})(5.6 + 8.2)$ $= 23.46 \text{ yd}^2$	$A_{\text{rectangle}} = bh$ $= (5.6)(3.4)$ $= 19.04 \text{ yd}^2$
$A_{\text{rectangle}} > A_{\text{trapezoid}} > A_{\text{rectangle}}$ $27.88 \quad 23.46 \quad 19.04$		

Step 7. **Answer** the question.

Vinny has 23.46 square yards in which he can plant.

Note:

Exercise:

Problem:

Lin wants to sod his lawn, which is shaped like a trapezoid. The bases are 10.8 yards and 6.7 yards, and the height is 4.6 yards. How many square yards of sod does he need?

Solution:

40.25 sq. yd

Note:**Exercise:****Problem:**

Kira wants cover his patio with concrete pavers. If the patio is shaped like a trapezoid whose bases are 18 feet and 14 feet and whose height is 15 feet, how many square feet of pavers will he need?

Solution:

240 sq. ft

Note:The Links to Literacy activity *Spaghetti and Meatballs for All* will provide you with another view of the topics covered in this section."

Note:**ACCESS ADDITIONAL ONLINE RESOURCES**

- [Perimeter of a Rectangle](#)
- [Area of a Rectangle](#)

- [Perimeter and Area Formulas](#)
- [Area of a Triangle](#)
- [Area of a Triangle with Fractions](#)
- [Area of a Trapezoid](#)

Key Concepts

- **Properties of Rectangles**

- Rectangles have four sides and four right (90°) angles.
- The lengths of opposite sides are equal.
- The perimeter, P , of a rectangle is the sum of twice the length and twice the width.

- $P = 2L + 2W$

- The area, A , of a rectangle is the length times the width.

- $A = L \cdot W$

- **Triangle Properties**

- For any triangle $\triangle ABC$, the sum of the measures of the angles is 180° .

- $m\angle A + m\angle B + m\angle C = 180^\circ$

- The perimeter of a triangle is the sum of the lengths of the sides.

- $P = a + b + c$

- The area of a triangle is one-half the base, b , times the height, h .

- $A = \frac{1}{2}bh$

Practice Makes Perfect

Understand Linear, Square, and Cubic Measure

In the following exercises, determine whether you would measure each item using linear, square, or cubic units.

Exercise:

Problem: amount of water in a fish tank

Solution:

cubic

Exercise:

Problem: length of dental floss

Exercise:

Problem: living area of an apartment

Solution:

square

Exercise:

Problem: floor space of a bathroom tile

Exercise:

Problem: height of a doorway

Solution:

linear

Exercise:

Problem: capacity of a truck trailer

In the following exercises, find the (a) perimeter and (b) area of each figure.
Assume each side of the square is 1 cm.

Exercise:

Problem:

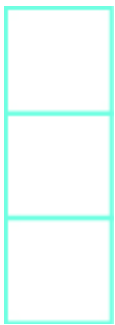


Solution:

- (a) 10 cm
- (b) 4 sq. cm

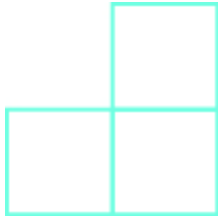
Exercise:

Problem:



Exercise:

Problem:

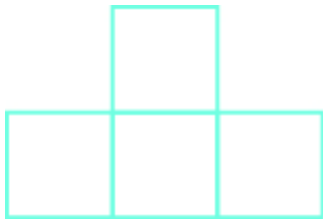


Solution:

- Ⓐ 8 cm
- Ⓑ 3 sq. cm

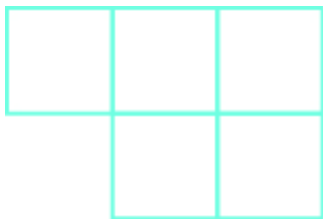
Exercise:

Problem:



Exercise:

Problem:

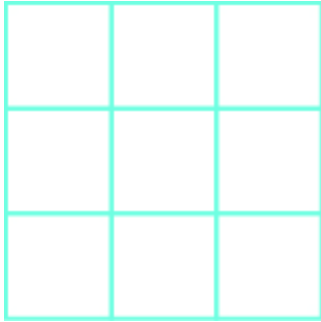


Solution:

- Ⓐ 10 cm
- Ⓑ 5 sq. cm

Exercise:

Problem:



Use the Properties of Rectangles

In the following exercises, find the (a) perimeter and (b) area of each rectangle.

Exercise:

Problem: The length of a rectangle is 85 feet and the width is 45 feet.

Solution:

(a) 260 ft

(b) 3825 sq. ft

Exercise:

Problem:

The length of a rectangle is 26 inches and the width is 58 inches.

Exercise:

Problem: A rectangular room is 15 feet wide by 14 feet long.

Solution:

(a) 58 ft

ⓑ 210 sq. ft

Exercise:

Problem:

A driveway is in the shape of a rectangle 20 feet wide by 35 feet long.

In the following exercises, solve.

Exercise:

Problem:

Find the length of a rectangle with perimeter 124 inches and width 38 inches.

Solution:

24 inches

Exercise:

Problem:

Find the length of a rectangle with perimeter 20.2 yards and width of 7.8 yards.

Exercise:

Problem:

Find the width of a rectangle with perimeter 92 meters and length 19 meters.

Solution:

27 meters

Exercise:

Problem:

Find the width of a rectangle with perimeter 16.2 meters and length 3.2 meters.

Exercise:**Problem:**

The area of a rectangle is 414 square meters. The length is 18 meters. What is the width?

Solution:

23 m

Exercise:**Problem:**

The area of a rectangle is 782 square centimeters. The width is 17 centimeters. What is the length?

Exercise:**Problem:**

The length of a rectangle is 9 inches more than the width. The perimeter is 46 inches. Find the length and the width.

Solution:

7 in., 16 in.

Exercise:**Problem:**

The width of a rectangle is 8 inches more than the length. The perimeter is 52 inches. Find the length and the width.

Exercise:

Problem:

The perimeter of a rectangle is 58 meters. The width of the rectangle is 5 meters less than the length. Find the length and the width of the rectangle.

Solution:

17 m, 12 m

Exercise:**Problem:**

The perimeter of a rectangle is 62 feet. The width is 7 feet less than the length. Find the length and the width.

Exercise:**Problem:**

The width of the rectangle is 0.7 meters less than the length. The perimeter of a rectangle is 52.6 meters. Find the dimensions of the rectangle.

Solution:

13.5 m, 12.8 m

Exercise:**Problem:**

The length of the rectangle is 1.1 meters less than the width. The perimeter of a rectangle is 49.4 meters. Find the dimensions of the rectangle.

Exercise:

Problem:

The perimeter of a rectangle is 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.

Solution:

25 ft, 50 ft

Exercise:**Problem:**

The length of a rectangle is three times the width. The perimeter is 72 feet. Find the length and width of the rectangle.

Exercise:**Problem:**

The length of a rectangle is 3 meters less than twice the width. The perimeter is 36 meters. Find the length and width.

Solution:

7 m, 11 m

Exercise:**Problem:**

The length of a rectangle is 5 inches more than twice the width. The perimeter is 34 inches. Find the length and width.

Exercise:**Problem:**

The width of a rectangular window is 24 inches. The area is 624 square inches. What is the length?

Solution:

26 in.

Exercise:

Problem:

The length of a rectangular poster is 28 inches. The area is 1316 square inches. What is the width?

Exercise:

Problem:

The area of a rectangular roof is 2310 square meters. The length is 42 meters. What is the width?

Solution:

55 m

Exercise:

Problem:

The area of a rectangular tarp is 132 square feet. The width is 12 feet. What is the length?

Exercise:

Problem:

The perimeter of a rectangular courtyard is 160 feet. The length is 10 feet more than the width. Find the length and the width.

Solution:

35 ft, 45 ft

Exercise:

Problem:

The perimeter of a rectangular painting is 306 centimeters. The length is 17 centimeters more than the width. Find the length and the width.

Exercise:**Problem:**

The width of a rectangular window is 40 inches less than the height. The perimeter of the doorway is 224 inches. Find the length and the width.

Solution:

76 in., 36 in.

Exercise:**Problem:**

The width of a rectangular playground is 7 meters less than the length. The perimeter of the playground is 46 meters. Find the length and the width.

Use the Properties of Triangles

In the following exercises, solve using the properties of triangles.

Exercise:**Problem:**

Find the area of a triangle with base 12 inches and height 5 inches.

Solution:

60 sq. in.

Exercise:**Problem:**

Find the area of a triangle with base 45 centimeters and height 30 centimeters.

Exercise:

Problem:

Find the area of a triangle with base 8.3 meters and height 6.1 meters.

Solution:

25.315 sq. m

Exercise:**Problem:**

Find the area of a triangle with base 24.2 feet and height 20.5 feet.

Exercise:**Problem:**

A triangular flag has base of 1 foot and height of 1.5 feet. What is its area?

Solution:

0.75 sq. ft

Exercise:**Problem:**

A triangular window has base of 8 feet and height of 6 feet. What is its area?

Exercise:**Problem:**

If a triangle has sides of 6 feet and 9 feet and the perimeter is 23 feet, how long is the third side?

Solution:

8 ft

Exercise:

Problem:

If a triangle has sides of 14 centimeters and 18 centimeters and the perimeter is 49 centimeters, how long is the third side?

Exercise:

Problem:

What is the base of a triangle with an area of 207 square inches and height of 18 inches?

Solution:

23 in.

Exercise:

Problem:

What is the height of a triangle with an area of 893 square inches and base of 38 inches?

Exercise:

Problem:

The perimeter of a triangular reflecting pool is 36 yards. The lengths of two sides are 10 yards and 15 yards. How long is the third side?

Solution:

11 ft

Exercise:

Problem:

A triangular courtyard has perimeter of 120 meters. The lengths of two sides are 30 meters and 50 meters. How long is the third side?

Exercise:

Problem:

An isosceles triangle has a base of 20 centimeters. If the perimeter is 76 centimeters, find the length of each of the other sides.

Solution:

28 cm

Exercise:**Problem:**

An isosceles triangle has a base of 25 inches. If the perimeter is 95 inches, find the length of each of the other sides.

Exercise:**Problem:**

Find the length of each side of an equilateral triangle with a perimeter of 51 yards.

Solution:

17 ft

Exercise:**Problem:**

Find the length of each side of an equilateral triangle with a perimeter of 54 meters.

Exercise:**Problem:**

The perimeter of an equilateral triangle is 18 meters. Find the length of each side.

Solution:

6 m

Exercise:

Problem:

The perimeter of an equilateral triangle is 42 miles. Find the length of each side.

Exercise:

Problem:

The perimeter of an isosceles triangle is 42 feet. The length of the shortest side is 12 feet. Find the length of the other two sides.

Solution:

15 ft

Exercise:

Problem:

The perimeter of an isosceles triangle is 83 inches. The length of the shortest side is 24 inches. Find the length of the other two sides.

Exercise:

Problem:

A dish is in the shape of an equilateral triangle. Each side is 8 inches long. Find the perimeter.

Solution:

24 in.

Exercise:

Problem:

A floor tile is in the shape of an equilateral triangle. Each side is 1.5 feet long. Find the perimeter.

Exercise:**Problem:**

A road sign in the shape of an isosceles triangle has a base of 36 inches. If the perimeter is 91 inches, find the length of each of the other sides.

Solution:

27.5 in.

Exercise:**Problem:**

A scarf in the shape of an isosceles triangle has a base of 0.75 meters. If the perimeter is 2 meters, find the length of each of the other sides.

Exercise:**Problem:**

The perimeter of a triangle is 39 feet. One side of the triangle is 1 foot longer than the second side. The third side is 2 feet longer than the second side. Find the length of each side.

Solution:

12 ft, 13 ft, 14 ft

Exercise:**Problem:**

The perimeter of a triangle is 35 feet. One side of the triangle is 5 feet longer than the second side. The third side is 3 feet longer than the second side. Find the length of each side.

Exercise:

Problem:

One side of a triangle is twice the smallest side. The third side is 5 feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.

Solution:

3 ft, 6 ft, 8 ft

Exercise:**Problem:**

One side of a triangle is three times the smallest side. The third side is 3 feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

Use the Properties of Trapezoids

In the following exercises, solve using the properties of trapezoids.

Exercise:**Problem:**

The height of a trapezoid is 12 feet and the bases are 9 and 15 feet. What is the area?

Solution:

144 sq. ft

Exercise:**Problem:**

The height of a trapezoid is 24 yards and the bases are 18 and 30 yards. What is the area?

Exercise:

Problem:

Find the area of a trapezoid with a height of 51 meters and bases of 43 and 67 meters.

Solution:

2805 sq. m

Exercise:**Problem:**

Find the area of a trapezoid with a height of 62 inches and bases of 58 and 75 inches.

Exercise:**Problem:**

The height of a trapezoid is 15 centimeters and the bases are 12.5 and 18.3 centimeters. What is the area?

Solution:

231 sq. cm

Exercise:**Problem:**

The height of a trapezoid is 48 feet and the bases are 38.6 and 60.2 feet. What is the area?

Exercise:**Problem:**

Find the area of a trapezoid with a height of 4.2 meters and bases of 8.1 and 5.5 meters.

Solution:

28.56 sq. m

Exercise:

Problem:

Find the area of a trapezoid with a height of 32.5 centimeters and bases of 54.6 and 41.4 centimeters.

Exercise:

Problem:

Laurel is making a banner shaped like a trapezoid. The height of the banner is 3 feet and the bases are 4 and 5 feet. What is the area of the banner?

Solution:

13.5 sq. ft

Exercise:

Problem:

Niko wants to tile the floor of his bathroom. The floor is shaped like a trapezoid with width 5 feet and lengths 5 feet and 8 feet. What is the area of the floor?

Exercise:

Problem:

Theresa needs a new top for her kitchen counter. The counter is shaped like a trapezoid with width 18.5 inches and lengths 62 and 50 inches. What is the area of the counter?

Solution:

1036 sq. in.

Exercise:

Problem:

Elena is knitting a scarf. The scarf will be shaped like a trapezoid with width 8 inches and lengths 48.2 inches and 56.2 inches. What is the area of the scarf?

Everyday Math**Exercise:****Problem:**

Fence Jose just removed the children's playset from his back yard to make room for a rectangular garden. He wants to put a fence around the garden to keep out the dog. He has a 50 foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other side if he wants to use the entire roll of fence?

Solution:

15 ft

Exercise:**Problem:**

Gardening Lupita wants to fence in her tomato garden. The garden is rectangular and the length is twice the width. It will take 48 feet of fencing to enclose the garden. Find the length and width of her garden.

Exercise:

Problem:

Fence Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are 6 feet, 8 feet, and 10 feet. The fence costs \$10 per foot. How much will it cost for Christa to fence in her flowerbed?

Solution:

\$24

Exercise:**Problem:**

Painting Caleb wants to paint one wall of his attic. The wall is shaped like a trapezoid with height 8 feet and bases 20 feet and 12 feet. The cost of the painting one square foot of wall is about \$0.05. About how much will it cost for Caleb to paint the attic wall?

**Writing Exercises****Exercise:****Problem:**

If you need to put tile on your kitchen floor, do you need to know the perimeter or the area of the kitchen? Explain your reasoning.

Solution:

Answers will vary.

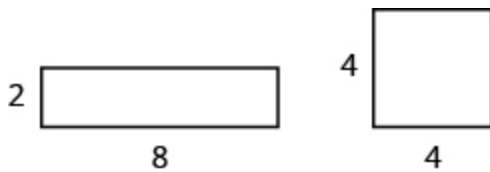
Exercise:

Problem:

If you need to put a fence around your backyard, do you need to know the perimeter or the area of the backyard? Explain your reasoning.

Exercise:

Problem: Look at the two figures.



- Ⓐ Which figure looks like it has the larger area? Which looks like it has the larger perimeter?
 - Ⓑ Now calculate the area and perimeter of each figure. Which has the larger area? Which has the larger perimeter?
-

Solution:

Answers will vary.

Exercise:

Problem:

The length of a rectangle is 5 feet more than the width. The area is 50 square feet. Find the length and the width.

- Ⓐ Write the equation you would use to solve the problem.
- Ⓑ Why can't you solve this equation with the methods you learned in the previous chapter?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
understand linear, square, and cubic measure.			
use the properties of rectangles.			
use the properties of triangles.			
use the properties of trapezoids.			

Ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

area

The area is a measure of the surface covered by a figure.

equilateral triangle

A triangle with all three sides of equal length is called an equilateral triangle.

isosceles triangle

A triangle with two sides of equal length is called an isosceles triangle.

perimeter

The perimeter is a measure of the distance around a figure.

rectangle

A rectangle is a geometric figure that has four sides and four right angles.

trapezoid

A trapezoid is four-sided figure, a quadrilateral, with two sides that are parallel and two sides that are not.

Solve Geometry Applications: Circles and Irregular Figures
By the end of this section, you will be able to:

- Use the properties of circles
- Find the area of irregular figures

Note:

Before you get started, take this readiness quiz.

1. Evaluate x^2 when $x = 5$.
If you missed this problem, review [\[link\]](#).
2. Using 3.14 for π , approximate the (a) circumference and (b) the area of a circle with radius 8 inches.
If you missed this problem, review [\[link\]](#).
3. Simplify $\frac{22}{7}(0.25)^2$ and round to the nearest thousandth.
If you missed this problem, review [\[link\]](#).

In this section, we'll continue working with geometry applications. We will add several new formulas to our collection of formulas. To help you as you do the examples and exercises in this section, we will show the Problem Solving Strategy for Geometry Applications here.

Problem Solving Strategy for Geometry Applications

Read the problem and make sure you understand all the words and ideas.

Draw the figure and label it with the given information.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

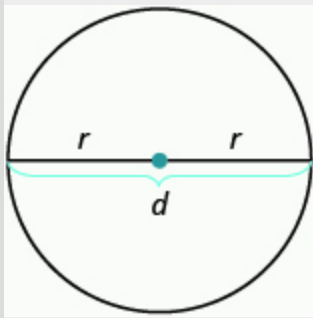
Answer the question with a complete sentence.

Use the Properties of Circles

Do you remember the properties of circles from [Decimals and Fractions Together](#)? We'll show them here again to refer to as we use them to solve applications.

Note:

Properties of Circles



- r is the length of the radius
- d is the length of the diameter
- $d = 2r$
- Circumference is the perimeter of a circle. The formula for **circumference** is

Equation:

$$C = 2\pi r$$

- The formula for area of a circle is

Equation:

$$A = \pi r^2$$

Remember, that we approximate π with 3.14 or $\frac{22}{7}$ depending on whether the radius of the circle is given as a decimal or a fraction. If you use the π

key on your calculator to do the calculations in this section, your answers will be slightly different from the answers shown. That is because the π key uses more than two decimal places.

Example:

Exercise:

Problem:

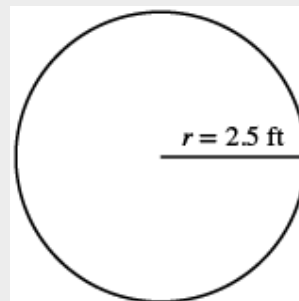
A circular sandbox has a radius of 2.5 feet. Find the (a) circumference and (b) area of the sandbox.

Solution:

Solution

(a)

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the circumference of the circle

Step 3. **Name.** Choose a variable to represent it.

Let $c =$
circumference of
the circle

Step 4. Translate.

Write the appropriate formula

Substitute

$$C = 2\pi r$$

$$C = 2\pi(2.5)$$

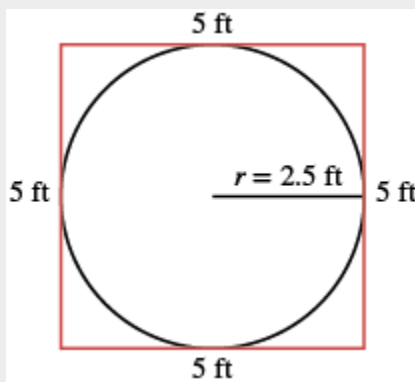
Step 5. Solve the equation.

$$C \approx 2(3.14)(2.5)$$

$$C \approx 15.7$$

Step 6. Check. Does this answer make sense?

Yes. If we draw a square around the circle, its sides would be 5 ft (twice the radius), so its perimeter would be 20 ft. This is slightly more than the circle's circumference, 15.7 ft.



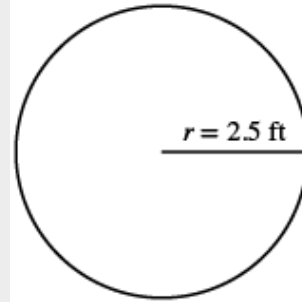
Step 7. Answer the question.

The circumference of the sandbox is 15.7 feet.

ⓑ

Step 1. Read the problem. Draw the

figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the area of the circle

Step 3. **Name.** Choose a variable to represent it.

Let A = the area of the circle

Step 4. **Translate.**
Write the appropriate formula
Substitute

$$A = \pi r^2$$
$$A = \pi(2.5)^2$$

Step 5. **Solve** the equation.

$$A \approx (3.14)(2.5)^2$$
$$A \approx 19.625 \text{ sq. ft}$$

Step 6. **Check.**
Yes. If we draw a square around the circle, its sides would be 5 ft, as shown in part (a). So the area of the square would be 25 sq. ft. This is slightly more than the circle's area, 19.625 sq. ft.

Step 7. **Answer** the question.

The area of the circle is 19.625 square feet.

Note:

Exercise:**Problem:**

A circular mirror has radius of 5 inches. Find the (a) circumference and (b) area of the mirror.

Solution:

- (a) 31.4 in.
- (b) 78.5 sq. in.

Note:**Exercise:****Problem:**

A circular spa has radius of 4.5 feet. Find the (a) circumference and (b) area of the spa.

Solution:

- (a) 28.26 ft
- (b) 63.585 sq. ft

We usually see the formula for circumference in terms of the radius r of the circle:

Equation:

$$C = 2\pi r$$

But since the diameter of a circle is two times the radius, we could write the formula for the circumference in terms of d .

Equation:

Using the commutative property, we get

Then substituting $d = 2r$

So

$$C = 2\pi r$$

$$C = \pi \cdot 2r$$

$$C = \pi \cdot d$$

$$C = \pi d$$

We will use this form of the circumference when we're given the length of the diameter instead of the radius.

Example:

Exercise:

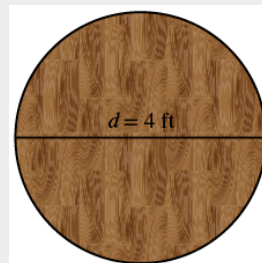
Problem:

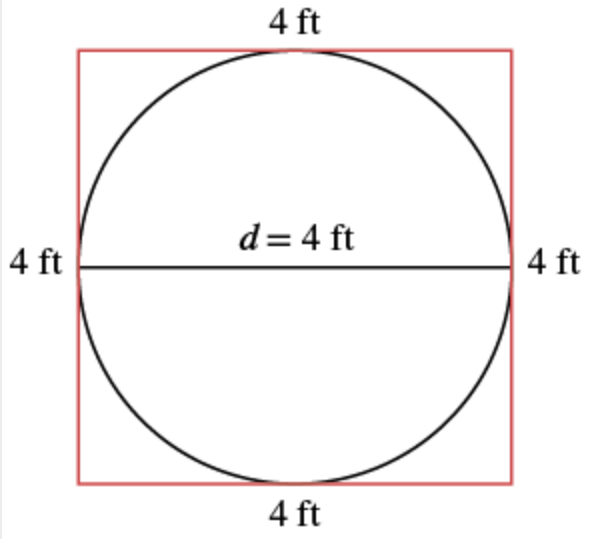
A circular table has a diameter of four feet. What is the circumference of the table?

Solution:

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. Identify what you are looking for.	the circumference of the table
Step 3. Name. Choose a variable to represent it.	Let c = the circumference of the table
Step 4. Translate. Write the appropriate formula for the situation. Substitute.	$C = \pi d$ $C = \pi(4)$
Step 5. Solve the equation, using 3.14 for π .	$C \approx (3.14)(4)$ $C \approx 12.56 \text{ feet}$
Step 6. Check: If we put a square around the circle, its side would be 4. The perimeter would be 16. It makes sense that the circumference of the circle, 12.56, is a little less than 16. 	
Step 7. Answer the question.	The diameter of

the table is
12.56 square
feet.

Note:

Exercise:

Problem:

Find the circumference of a circular fire pit whose diameter is 5.5 feet.

Solution:

17.27 ft

Note:

Exercise:

Problem:

If the diameter of a circular trampoline is 12 feet, what is its circumference?

Solution:

37.68 ft

Example:

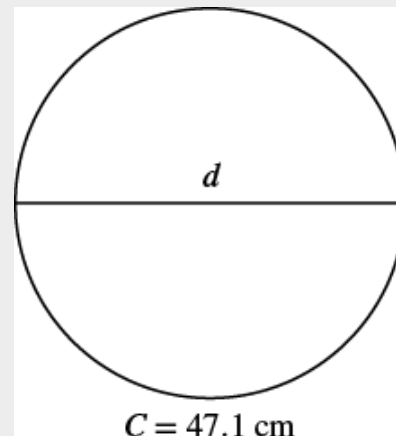
Exercise:

Problem:

Find the diameter of a circle with a circumference of 47.1 centimeters.

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Step 2. **Identify** what you are looking for.

the diameter of the circle

Step 3. **Name.** Choose a variable to represent it.

Let d = the diameter of the circle

Step 4. **Translate.**

Write the formula.
Substitute, using 3.14 to approximate π .

$$C = \pi d$$

	$47.1 \approx 3.14d$
Step 5. Solve.	$\frac{47.1}{3.14} \approx \frac{3.14d}{3.14}$ $15 \approx d$
Step 6. Check: $C = \pi d$ $47.1 \stackrel{?}{=} (3.14)(15)$ $47.1 = 47.1 \checkmark$	
Step 7. Answer the question.	The diameter of the circle is approximately 15 centimeters.

Note:

Exercise:

Problem:

Find the diameter of a circle with circumference of 94.2 centimeters.

Solution:

30 cm

Note:

Exercise:

Problem:

Find the diameter of a circle with circumference of 345.4 feet.

Solution:

110 ft

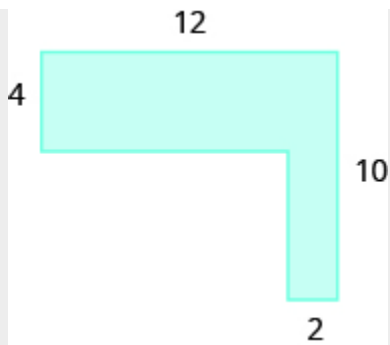
Find the Area of Irregular Figures

So far, we have found area for rectangles, triangles, trapezoids, and circles. An **irregular figure** is a figure that is not a standard geometric shape. Its area cannot be calculated using any of the standard area formulas. But some irregular figures are made up of two or more standard geometric shapes. To find the area of one of these irregular figures, we can split it into figures whose formulas we know and then add the areas of the figures.

Example:

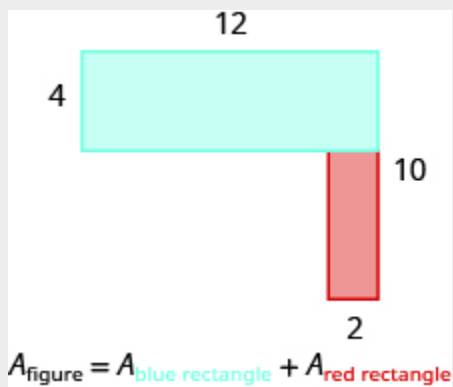
Exercise:

Problem: Find the area of the shaded region.

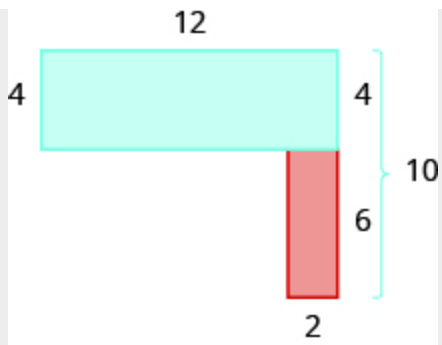


Solution: **Solution**

The given figure is irregular, but we can break it into two rectangles. The area of the shaded region will be the sum of the areas of both rectangles.



The blue rectangle has a width of 12 and a length of 4. The red rectangle has a width of 2, but its length is not labeled. The right side of the figure is the length of the red rectangle plus the length of the blue rectangle. Since the right side of the blue rectangle is 4 units long, the length of the red rectangle must be 6 units.



$$\begin{aligned}
 A_{\text{figure}} &= A_{\text{rectangle}} + A_{\text{rectangle}} \\
 A_{\text{figure}} &= bh + bh \\
 A_{\text{figure}} &= 12 \cdot 4 + 2 \cdot 6 \\
 A_{\text{figure}} &= 48 + 12 \\
 A_{\text{figure}} &= 60
 \end{aligned}$$

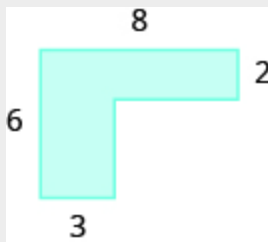
The area of the figure is 60 square units.

Is there another way to split this figure into two rectangles? Try it, and make sure you get the same area.

Note:

Exercise:

Problem: Find the area of each shaded region:



Solution:

28 sq. units

Note:

Exercise:

Problem: Find the area of each shaded region:



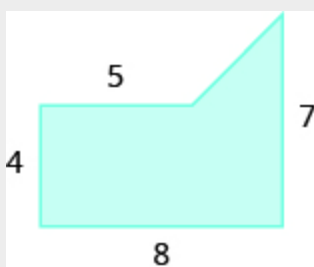
Solution:

110 sq. units

Example:

Exercise:

Problem: Find the area of the shaded region.



Solution:

Solution

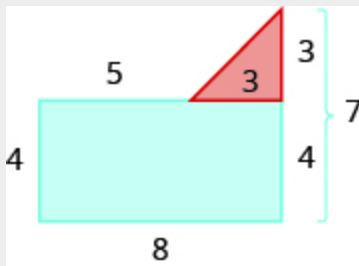
We can break this irregular figure into a triangle and rectangle. The area of the figure will be the sum of the areas of triangle and rectangle.

The rectangle has a length of 8 units and a width of 4 units.

We need to find the base and height of the triangle.

Since both sides of the rectangle are 4, the vertical side of the triangle is 3, which is $7 - 4$.

The length of the rectangle is 8, so the base of the triangle will be 3, which is $8 - 4$.



Now we can add the areas to find the area of the irregular figure.

$$\begin{aligned}A_{\text{figure}} &= A_{\text{rectangle}} + A_{\text{triangle}} \\A_{\text{figure}} &= lw + \frac{1}{2}bh \\A_{\text{figure}} &= 8 \cdot 4 + \frac{1}{2} \cdot 3 \cdot 3 \\A_{\text{figure}} &= 32 + 4.5 \\A_{\text{figure}} &= 36.5 \text{ sq. units}\end{aligned}$$

The area of the figure is 36.5 square units.

Note:

Exercise:

Problem: Find the area of each shaded region.



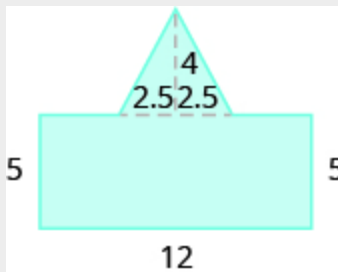
Solution:

36.5 sq. units

Note:

Exercise:

Problem: Find the area of each shaded region.



Solution:

70 sq. units

Example:

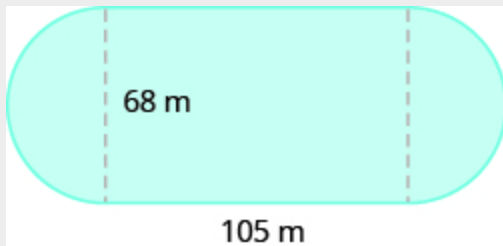
Exercise:

Problem:

A high school track is shaped like a rectangle with a semi-circle (half a circle) on each end. The rectangle has length 105 meters and width 68 meters. Find the area enclosed by the track. Round your answer to the nearest hundredth.

**Solution:**
Solution

We will break the figure into a rectangle and two semi-circles. The area of the figure will be the sum of the areas of the rectangle and the semicircles.



The rectangle has a length of 105 m and a width of 68 m. The semi-circles have a diameter of 68 m, so each has a radius of 34 m.

$$A_{\text{figure}} = A_{\text{rectangle}} + A_{\text{semicircles}}$$

$$A_{\text{figure}} = bh + 2\left(\frac{1}{2}\pi \cdot r^2\right)$$

$$A_{\text{figure}} \approx 105 \cdot 68 + 2\left(\frac{1}{2} \cdot 3.14 \cdot 34^2\right)$$

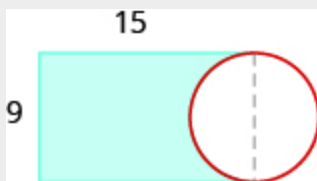
$$A_{\text{figure}} \approx 7140 + 3629.84$$

$$A_{\text{figure}} \approx 10,769.84 \text{ square meters}$$

Note:

Exercise:

Problem: Find the area:



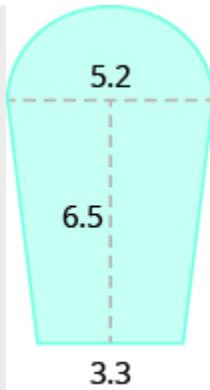
Solution:

103.2 sq. units

Note:

Exercise:

Problem: Find the area:



Solution:

38.24 sq. units

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Circumference of a Circle](#)
- [Area of a Circle](#)
- [Area of an L-shaped polygon](#)
- [Area of an L-shaped polygon with Decimals](#)
- [Perimeter Involving a Rectangle and Circle](#)
- [Area Involving a Rectangle and Circle](#)

Key Concepts

- **Problem Solving Strategy for Geometry Applications**

Read the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that

quantity.

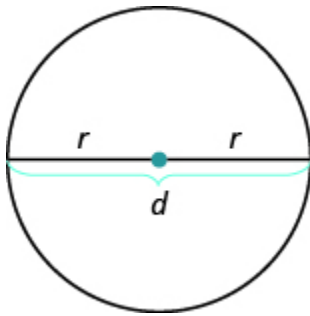
Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

Answer the question with a complete sentence.

- **Properties of Circles**



- $d = 2r$
- **Circumference:** $C = 2\pi r$ or $C = \pi d$
- **Area:** $A = \pi r^2$

Practice Makes Perfect

Use the Properties of Circles

In the following exercises, solve using the properties of circles.

Exercise:

Problem:

The lid of a paint bucket is a circle with radius 7 inches. Find the (a) circumference and (b) area of the lid.

Solution:

- (a) 43.96 in.
- (b) 153.86 sq. in.

Exercise:

Problem:

An extra-large pizza is a circle with radius 8 inches. Find the (a) circumference and (b) area of the pizza.

Exercise:

Problem:

A farm sprinkler spreads water in a circle with radius of 8.5 feet. Find the (a) circumference and (b) area of the watered circle.

Solution:

- (a) 53.38 ft
- (b) 226.865 sq. ft

Exercise:

Problem:

A circular rug has radius of 3.5 feet. Find the (a) circumference and (b) area of the rug.

Exercise:

Problem:

A reflecting pool is in the shape of a circle with diameter of 20 feet. What is the circumference of the pool?

Solution:

62.8 ft

Exercise:

Problem:

A turntable is a circle with diameter of 10 inches. What is the circumference of the turntable?

Exercise:**Problem:**

A circular saw has a diameter of 12 inches. What is the circumference of the saw?

Solution:

37.68 in.

Exercise:**Problem:**

A round coin has a diameter of 3 centimeters. What is the circumference of the coin?

Exercise:**Problem:**

A barbecue grill is a circle with a diameter of 2.2 feet. What is the circumference of the grill?

Solution:

6.908 ft

Exercise:**Problem:**

The top of a pie tin is a circle with a diameter of 9.5 inches. What is the circumference of the top?

Exercise:

Problem:

A circle has a circumference of 163.28 inches. Find the diameter.

Solution:

52 in.

Exercise:

Problem:

A circle has a circumference of 59.66 feet. Find the diameter.

Exercise:

Problem:

A circle has a circumference of 17.27 meters. Find the diameter.

Solution:

5.5 m

Exercise:

Problem:

A circle has a circumference of 80.07 centimeters. Find the diameter.

In the following exercises, find the radius of the circle with given circumference.

Exercise:

Problem: A circle has a circumference of 150.72 feet.

Solution:

24 ft

Exercise:

Problem: A circle has a circumference of 251.2 centimeters.

Exercise:

Problem: A circle has a circumference of 40.82 miles.

Solution:

6.5 mi

Exercise:

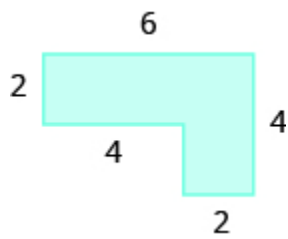
Problem: A circle has a circumference of 78.5 inches.

Find the Area of Irregular Figures

In the following exercises, find the area of the irregular figure. Round your answers to the nearest hundredth.

Exercise:

Problem:

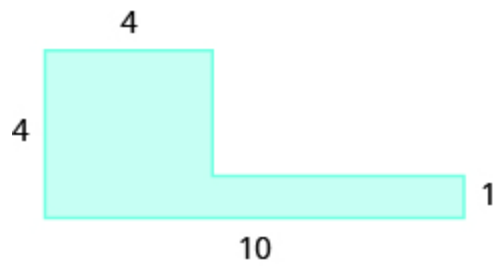


Solution:

16 sq. units

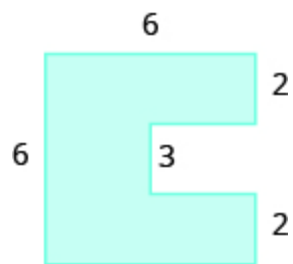
Exercise:

Problem:



Exercise:

Problem:

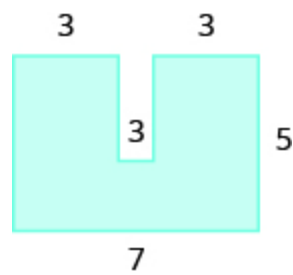


Solution:

30 sq. units

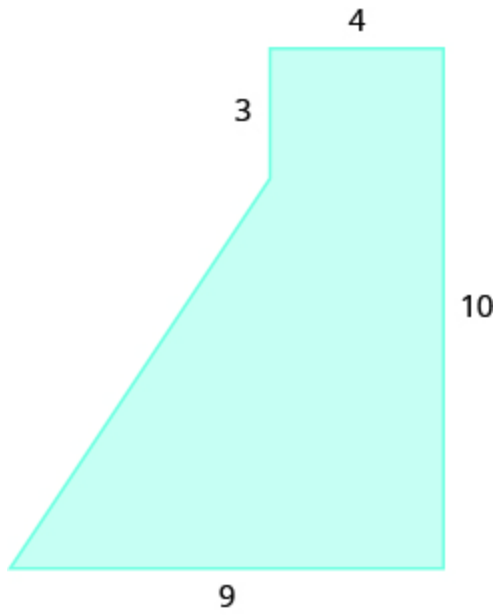
Exercise:

Problem:



Exercise:

Problem:

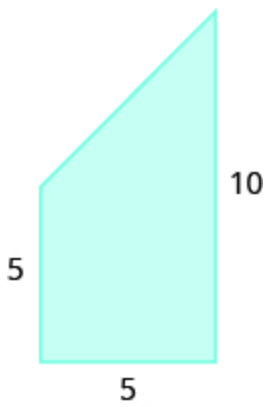


Solution:

57.5 sq. units

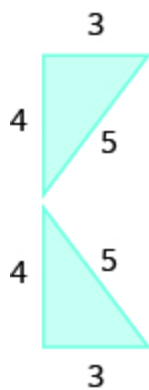
Exercise:

Problem:



Exercise:

Problem:

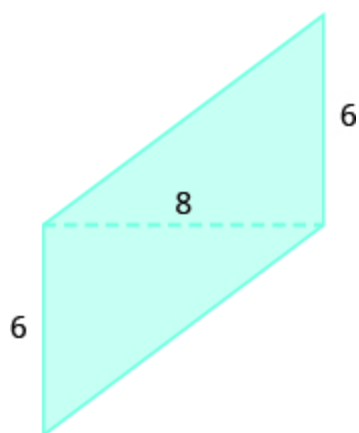


Solution:

12 sq. units

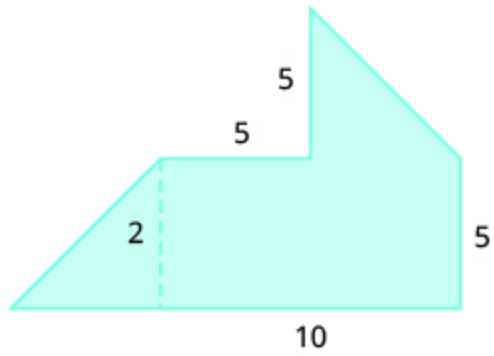
Exercise:

Problem:



Exercise:

Problem:

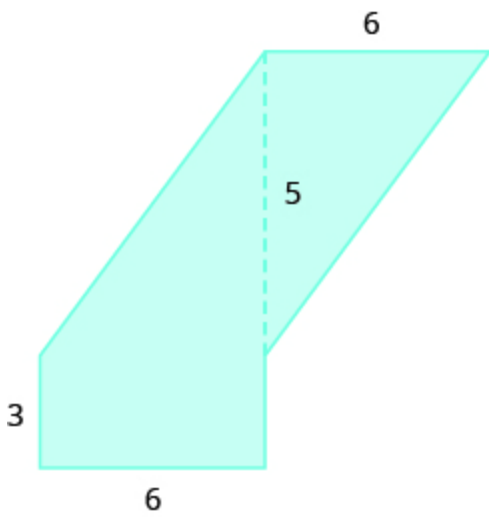


Solution:

67.5 sq. units

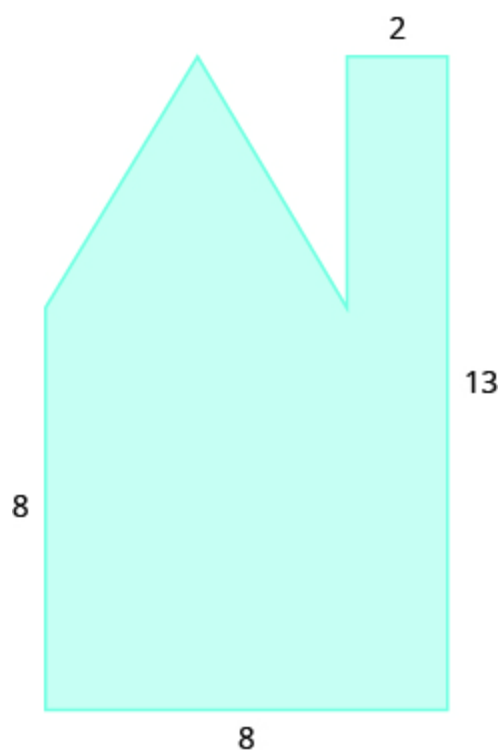
Exercise:

Problem:



Exercise:

Problem:

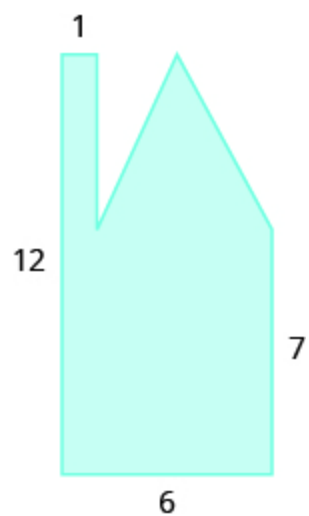


Solution:

89 sq. units

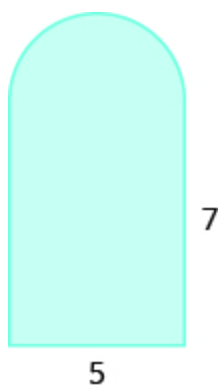
Exercise:

Problem:



Exercise:

Problem:

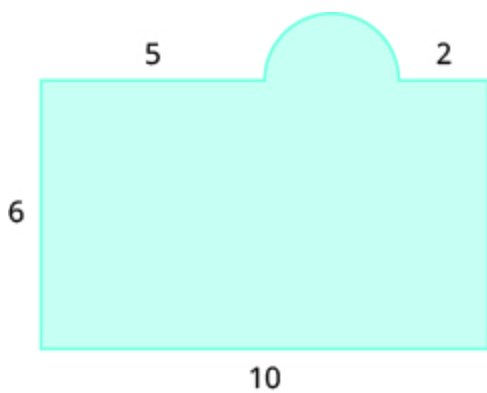


Solution:

44.81 sq. units

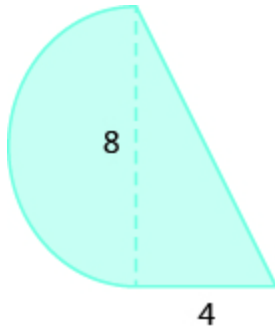
Exercise:

Problem:



Exercise:

Problem:

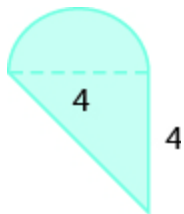


Solution:

41.12 sq. units

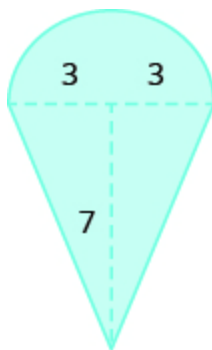
Exercise:

Problem:



Exercise:

Problem:

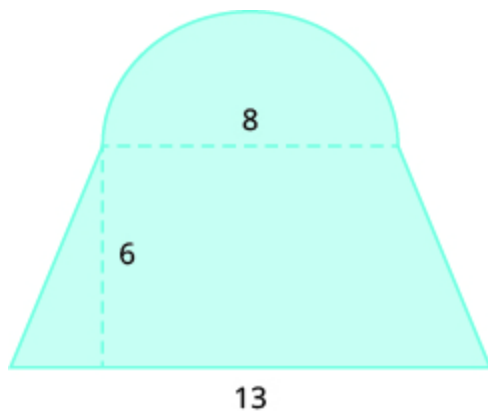


Solution:

35.13 sq. units

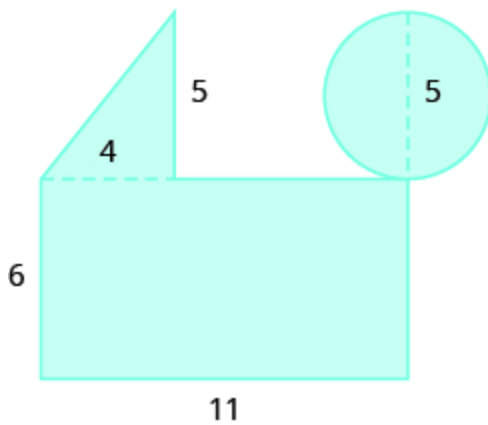
Exercise:

Problem:



Exercise:

Problem:

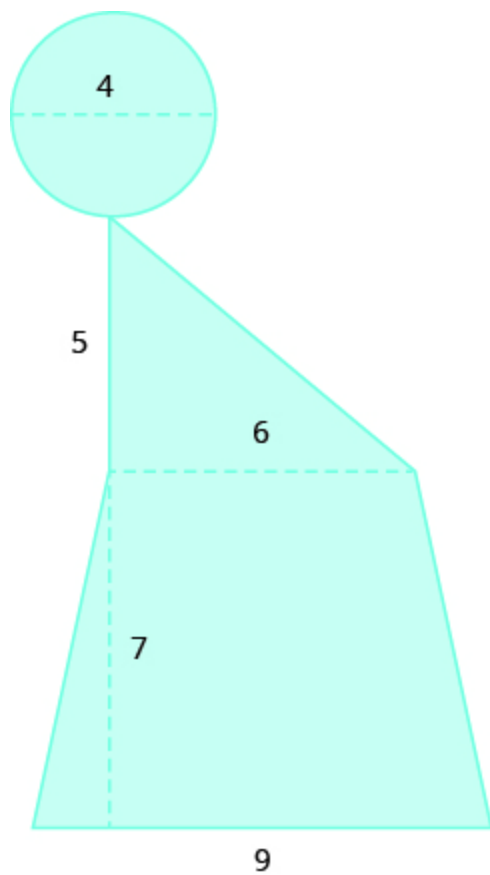


Solution:

95.625 sq. units

Exercise:

Problem:



In the following exercises, solve.

Exercise:

Problem:

A city park covers one block plus parts of four more blocks, as shown. The block is a square with sides 250 feet long, and the triangles are isosceles right triangles. Find the area of the park.

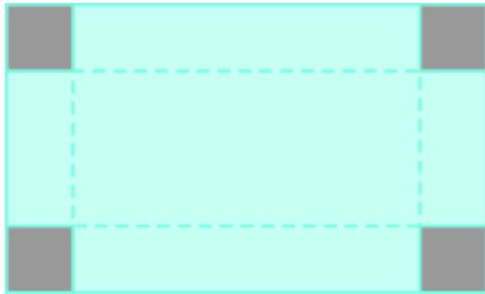


Solution:

187,500 sq. ft

Exercise:**Problem:**

A gift box will be made from a rectangular piece of cardboard measuring 12 inches by 20 inches, with squares cut out of the corners of the sides, as shown. The sides of the squares are 3 inches. Find the area of the cardboard after the corners are cut out.

**Exercise:****Problem:**

Perry needs to put in a new lawn. His lot is a rectangle with a length of 120 feet and a width of 100 feet. The house is rectangular and measures 50 feet by 40 feet. His driveway is rectangular and measures 20 feet by 30 feet, as shown. Find the area of Perry's lawn.

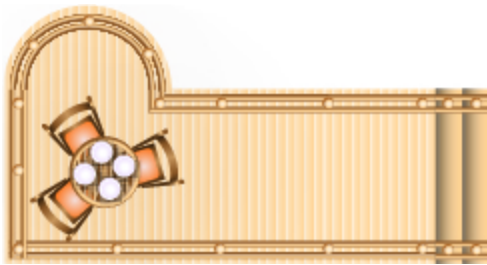


Solution:

9400 sq. ft

Exercise:**Problem:**

Denise is planning to put a deck in her back yard. The deck will be a 20-ft by 12-ft rectangle with a semicircle of diameter 6 feet, as shown below. Find the area of the deck.

**Everyday Math****Exercise:****Problem:**

Area of a Tabletop Yuki bought a drop-leaf kitchen table. The rectangular part of the table is a 1-ft by 3-ft rectangle with a semicircle at each end, as shown. (a) Find the area of the table with one leaf up. (b) Find the area of the table with both leaves up.



Solution:

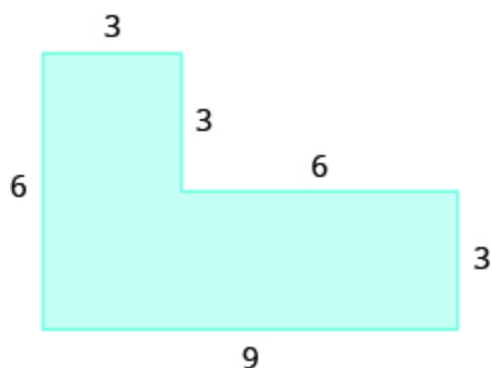
- Ⓐ 6.5325 sq. ft
- Ⓑ 10.065 sq. ft

Exercise:**Problem:**

Painting Leora wants to paint the nursery in her house. The nursery is an 8-ft by 10-ft rectangle, and the ceiling is 8 feet tall. There is a 3-ft by 6.5-ft door on one wall, a 3-ft by 6.5-ft closet door on another wall, and one 4-ft by 3.5-ft window on the third wall. The fourth wall has no doors or windows. If she will only paint the four walls, and not the ceiling or doors, how many square feet will she need to paint?

Writing Exercises**Exercise:****Problem:**

Describe two different ways to find the area of this figure, and then show your work to make sure both ways give the same area.



Solution:

Answers will vary.

Exercise:**Problem:**

A circle has a diameter of 14 feet. Find the area of the circle (a) using 3.14 for π (b) using $\frac{22}{7}$ for π . (c) Which calculation to do prefer? Why?

Self Check

(a) After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the properties of circles.			
find the area of irregular figures.			

(b) After looking at the checklist, do you think you are well prepared for the next section? Why or why not?

Glossary

irregular figure

An irregular figure is a figure that is not a standard geometric shape. Its area cannot be calculated using any of the standard area formulas.

Solve Geometry Applications: Volume and Surface Area

By the end of this section, you will be able to:

- Find volume and surface area of rectangular solids
- Find volume and surface area of spheres
- Find volume and surface area of cylinders
- Find volume of cones

Note:

Before you get started, take this readiness quiz.

1. Evaluate x^3 when $x = 5$.
If you missed this problem, review [\[link\]](#).
2. Evaluate 2^x when $x = 5$.
If you missed this problem, review [\[link\]](#).
3. Find the area of a circle with radius $\frac{7}{2}$.
If you missed this problem, review [\[link\]](#).

In this section, we will finish our study of geometry applications. We find the volume and surface area of some three-dimensional figures. Since we will be solving applications, we will once again show our Problem-Solving Strategy for Geometry Applications.

Problem Solving Strategy for Geometry Applications

Read the problem and make sure you understand all the words and ideas.

Draw the figure and label it with the given information.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

Answer the question with a complete sentence.

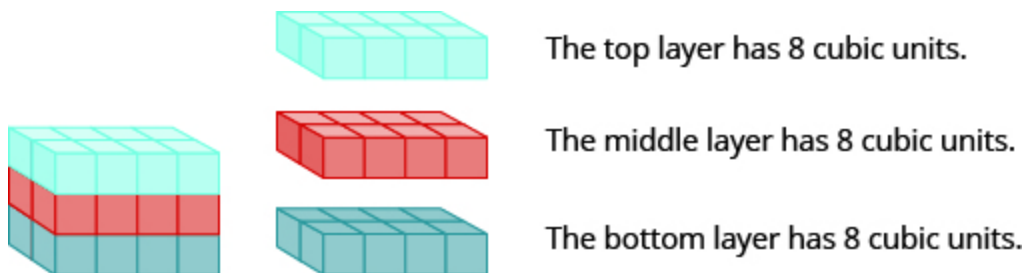
Find Volume and Surface Area of Rectangular Solids

A cheerleading coach is having the squad paint wooden crates with the school colors to stand on at the games. (See [\[link\]](#)). The amount of paint needed to cover the outside of each box is the **surface area**, a square measure of the total area of all the sides. The amount of space inside the crate is the volume, a cubic measure.



This wooden crate is in the shape of a rectangular solid.

Each crate is in the shape of a **rectangular solid**. Its dimensions are the length, width, and height. The rectangular solid shown in [\[link\]](#) has length 4 units, width 2 units, and height 3 units. Can you tell how many cubic units there are altogether? Let's look layer by layer.



Breaking a rectangular solid into layers makes it easier to visualize the number of cubic units it contains. This 4 by 2 by 3 rectangular solid has 24 cubic units.

Altogether there are 24 cubic units. Notice that 24 is the length $\times$ width $\times$ height.

$$\underbrace{V}_{24} = \underbrace{L}_{4} \cdot \underbrace{W}_{2} \cdot \underbrace{H}_{3}$$

The volume, V , of any rectangular solid is the product of the length, width, and height.

Equation:

$$V = LWH$$

We could also write the formula for volume of a rectangular solid in terms of the area of the base. The area of the base, B , is equal to length $\times$ width.

Equation:

$$B = L \cdot W$$

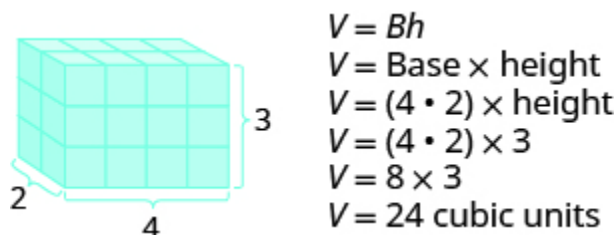
We can substitute B for $L \cdot W$ in the volume formula to get another form of the volume formula.

$$V = L \cdot W \cdot H$$

$$V = (L \cdot W) \cdot H$$

$$V = Bh$$

We now have another version of the volume formula for rectangular solids. Let's see how this works with the $4 \times 2 \times 3$ rectangular solid we started with. See [\[link\]](#).



To find the *surface area* of a rectangular solid, think about finding the area of each of its faces. How many faces does the rectangular solid above have? You can see three of them.

Equation:

$$A_{\text{front}} = L \times W$$

$$A_{\text{front}} = 4 \cdot 3$$

$$A_{\text{front}} = 12$$

$$A_{\text{side}} = L \times W$$

$$A_{\text{side}} = 2 \cdot 3$$

$$A_{\text{side}} = 6$$

$$A_{\text{top}} = L \times W$$

$$A_{\text{top}} = 4 \cdot 2$$

$$A_{\text{top}} = 8$$

Notice for each of the three faces you see, there is an identical opposite face that does not show.

Equation:

$$S = (\text{front} + \text{back}) + (\text{left side} + \text{right side}) + (\text{top} + \text{bottom})$$

$$S = (2 \cdot \text{front}) + (2 \cdot \text{left side}) + (2 \cdot \text{top})$$

$$S = 2 \cdot 12 + 2 \cdot 6 + 2 \cdot 8$$

$$S = 24 + 12 + 16$$

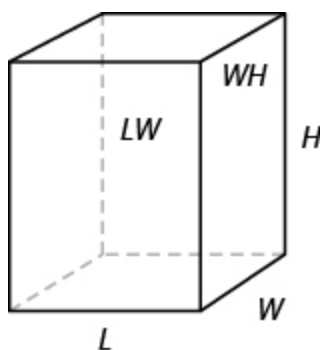
$$S = 52 \text{ sq. units}$$

The surface area S of the rectangular solid shown in [\[link\]](#) is 52 square units.

In general, to find the surface area of a rectangular solid, remember that each face is a rectangle, so its area is the product of its length and its width (see [\[link\]](#)). Find the area of each face that you see and then multiply each area by two to account for the face on the opposite side.

Equation:

$$S = 2LH + 2LW + 2WH$$

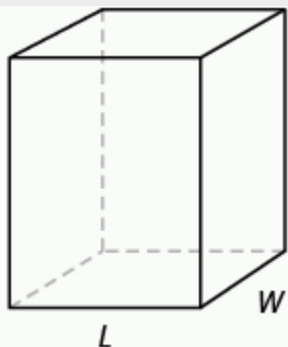


For each face
of the
rectangular
solid facing
you, there is
another face
on the
opposite side.
There are 6
faces in all.

Note:

Volume and Surface Area of a Rectangular Solid

For a rectangular solid with length L , width W , and height H :



$$\text{Volume: } V = LWH$$

$$\text{Surface Area: } S = 2LH + 2LW + 2WH$$

Note: Doing the Manipulative Mathematics activity “Painted Cube” will help you develop a better understanding of volume and surface area.

Example:

Exercise:

Problem:

For a rectangular solid with length 14 cm, height 17 cm, and width 9 cm, find the (a) volume and (b) surface area.

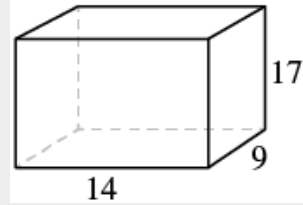
Solution:

Solution

Step 1 is the same for both (a) and (b), so we will show it just once.

Step 1. **Read** the problem. Draw the

figure and
label it with the given information.



(a)

Step 2. **Identify** what you are looking for.

the volume of the rectangular solid

Step 3. **Name.** Choose a variable to represent it.

Let V = volume

Step 4. **Translate.**
Write the appropriate formula.
Substitute.

$$V = LWH$$

$$V = 14 \cdot 9 \cdot 17$$

Step 5. **Solve** the equation.

$$V = 2,142$$

Step 6. **Check**
We leave it to you to check your calculations.

Step 7. **Answer** the question.

The surface area is 1,034 square centimeters.

ⓑ

Step 2. Identify
what you are
looking for.

the surface area of the solid

Step 3. Name.
Choose a variable
to represent it.

Let S = surface area

Step 4. Translate.
Write the
appropriate
formula.
Substitute.

$$S = 2LH + 2LW + 2WH$$
$$S = 2(14 \cdot 17) + 2(14 \cdot 9) + 2(9 \cdot 17)$$

**Step 5. Solve the
equation.**

$$S = 1,034$$

Step 6. Check:
Double-check with
a calculator.

**Step 7. Answer the
question.**

The surface area is 1,034 square
centimeters.

Note:

Exercise:

Problem:

Find the ⓐ volume and ⓑ surface area of rectangular solid with the:
length 8 feet, width 9 feet, and height 11 feet.

Solution:

- Ⓐ 792 cu. ft
- Ⓑ 518 sq. ft

Note:**Exercise:****Problem:**

Find the Ⓐ volume and Ⓑ surface area of rectangular solid with the: length 15 feet, width 12 feet, and height 8 feet.

Solution:

- Ⓐ 1,440 cu. ft
- Ⓑ 792 sq. ft

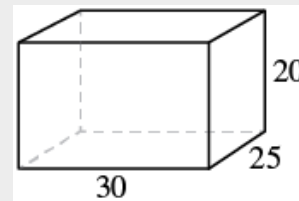
Example:**Exercise:****Problem:**

A rectangular crate has a length of 30 inches, width of 25 inches, and height of 20 inches. Find its Ⓐ volume and Ⓑ surface area.

Solution:**Solution**

Step 1 is the same for both Ⓐ and Ⓑ, so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Ⓐ

Step 2. **Identify** what you are looking for.

the volume of the crate

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.**
Write the appropriate formula.
Substitute.

$$V = LWH$$

$$V = 30 \cdot 25 \cdot 20$$

Step 5. **Solve** the equation.

$$V = 15,000$$

Step 6. **Check:** Double check your math.

Step 7. **Answer** the question.

The volume is 15,000 cubic inches.

ⓑ	
Step 2. Identify what you are looking for.	the surface area of the crate
Step 3. Name. Choose a variable to represent it.	let S = surface area
Step 4. Translate. Write the appropriate formula. Substitute.	$S = 2LH + 2LW + 2WH$ $S = 2(30 \cdot 20) + 2(30 \cdot 25) + 2(25 \cdot 20)$
Step 5. Solve the equation.	$S = 3,700$
Step 6. Check: Check it yourself!	
Step 7. Answer the question.	The surface area is 3,700 square inches.

Note:

Exercise:

Problem:

A rectangular box has length 9 feet, width 4 feet, and height 6 feet. Find its (a) volume and (b) surface area.

Solution:

- (a) 216 cu. ft
- (b) 228 sq. ft

Note:**Exercise:****Problem:**

A rectangular suitcase has length 22 inches, width 14 inches, and height 9 inches. Find its (a) volume and (b) surface area.

Solution:

- (a) 2,772 cu. in.
- (b) 1,264 sq. in.

Volume and Surface Area of a Cube

A **cube** is a rectangular solid whose length, width, and height are equal. See Volume and Surface Area of a Cube, below. Substituting, s for the length, width and height into the formulas for volume and surface area of a rectangular solid, we get:

Equation:

$$V = LWH$$

$$V = s \cdot s \cdot s$$

$$V = s^3$$

$$S = 2LH + 2LW + 2WH$$

$$S = 2s \cdot s + 2s \cdot s + 2s \cdot s$$

$$S = 2s^2 + 2s^2 + 2s^2$$

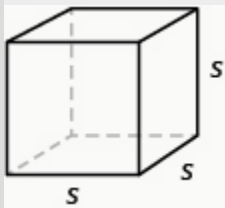
$$S = 6s^2$$

So for a cube, the formulas for volume and surface area are $V = s^3$ and $S = 6s^2$.

Note:

Volume and Surface Area of a Cube

For any cube with sides of length s ,



Volume: $V = s^3$
Surface Area: $S = 6s^2$

Example:

Exercise:

Problem:

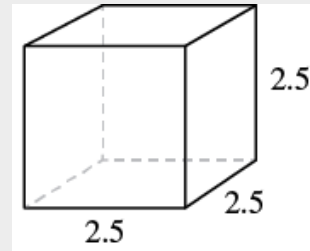
A cube is 2.5 inches on each side. Find its (a) volume and (b) surface area.

Solution:

Solution

Step 1 is the same for both (a) and (b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Ⓐ

Step 2. **Identify** what you are looking for.

the volume of the cube

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.**
Write the appropriate formula.

$$V = s^3$$

Step 5. **Solve.** Substitute and solve.

$$V = (2.5)^3$$

$$V = 15.625$$

Step 6. **Check:** Check your work.

Step 7. **Answer** the question.

The volume is 15.625 cubic inches.

ⓑ	
Step 2. Identify what you are looking for.	the surface area of the cube
Step 3. Name. Choose a variable to represent it.	let S = surface area
Step 4. Translate. Write the appropriate formula.	$S = 6s^2$
Step 5. Solve. Substitute and solve.	$S = 6 \cdot (2.5)^2$ $S = 37.5$
Step 6. Check: The check is left to you.	
Step 7. Answer the question.	The surface area is 37.5 square inches.

Note:

Exercise:

Problem:

For a cube with side 4.5 meters, find the ⓐ volume and ⓑ surface area of the cube.

Solution:

- ⓐ 91.125 cu. m
- ⓑ 121.5 sq. m

Note:

Exercise:

Problem:

For a cube with side 7.3 yards, find the (a) volume and (b) surface area of the cube.

Solution:

(a) 389.017 cu. yd.

(b) 319.74 sq. yd.

Example:

Exercise:

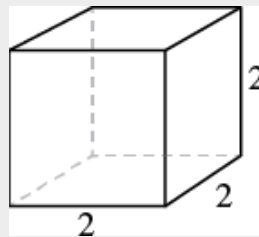
Problem:

A notepad cube measures 2 inches on each side. Find its (a) volume and (b) surface area.

Solution:

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Ⓐ

Step 2. **Identify** what you are looking for.

the volume of the cube

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.**
Write the appropriate formula.

$$V = s^3$$

Step 5. **Solve** the equation.

$$V = 2^3$$
$$V = 8$$

Step 6. **Check:** Check that you did the calculations correctly.

Step 7. **Answer** the question.

The volume is 8 cubic inches.

Ⓑ

Step 2. **Identify** what you are looking for.

the surface area of the cube

Step 3. Name. Choose a variable to represent it.	let S = surface area
Step 4. Translate. Write the appropriate formula.	$S = 6s^2$
Step 5. Solve the equation.	$S = 6 \cdot 2^2$ $S = 24$
Step 6. Check: The check is left to you.	
Step 7. Answer the question.	The surface area is 24 square inches.

Note:

Exercise:

Problem:

A packing box is a cube measuring 4 feet on each side. Find its (a) volume and (b) surface area.

Solution:

- (a) 64 cu. ft
- (b) 96 sq. ft

Note:

Exercise:

Problem:

A wall is made up of cube-shaped bricks. Each cube is 16 inches on each side. Find the (a) volume and (b) surface area of each cube.

Solution:

(a) 4,096 cu. in.

(b) 1536 sq. in.

Find the Volume and Surface Area of Spheres

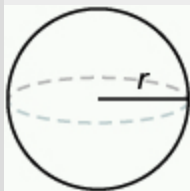
A **sphere** is the shape of a basketball, like a three-dimensional circle. Just like a circle, the size of a sphere is determined by its radius, which is the distance from the center of the sphere to any point on its surface. The formulas for the volume and surface area of a sphere are given below.

Showing where these formulas come from, like we did for a rectangular solid, is beyond the scope of this course. We will approximate π with 3.14.

Note:

Volume and Surface Area of a Sphere

For a sphere with radius r :



$$\text{Volume: } V = \frac{4}{3} \pi r^3$$

$$\text{Surface Area: } S = 4\pi r^2$$

Example:

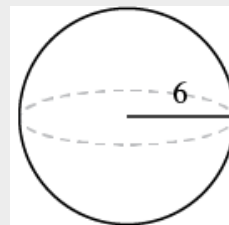
Exercise:**Problem:**

A sphere has a radius 6 inches. Find its (a) volume and (b) surface area.

Solution:**Solution**

Step 1 is the same for both (a) and (b), so we will show it just once.

Step 1. **Read** the problem. Draw the figure and label it with the given information.



(a)

Step 2. **Identify** what you are looking for.

the volume of the sphere

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.**

Write the appropriate formula.

$$V = \frac{4}{3}\pi r^3$$

Step 5. **Solve.**

$$V \approx \frac{4}{3}(3.14)6^3$$

$$V \approx 904.32 \text{ cubic inches}$$

Step 6. **Check:** Double-check your math on a calculator.

Step 7. **Answer** the question.

The volume is approximately 904.32 cubic inches.

ⓑ

Step 2. **Identify** what you are looking for.

the surface area of the cube

Step 3. **Name.** Choose a variable to represent it.

let S = surface area

Step 4. **Translate.**

Write the appropriate formula.

$$S = 4\pi r^2$$

Step 5. **Solve.**

$$S \approx 4(3.14)6^2$$

$$S \approx 452.16$$

Step 6. **Check:** Double-check your math on a calculator

Step 7. **Answer** the question.

The surface area is approximately 452.16 square inches.

Note:

Exercise:

Problem:

Find the (a) volume and (b) surface area of a sphere with radius 3 centimeters.

Solution:

(a) 113.04 cu. cm

(b) 113.04 sq. cm

Note:

Exercise:

Problem:

Find the (a) volume and (b) surface area of each sphere with a radius of 1 foot

Solution:

(a) 4.19 cu. ft

(b) 12.56 sq. ft

Example:**Exercise:****Problem:**

A globe of Earth is in the shape of a sphere with radius 14 centimeters. Find its (a) volume and (b) surface area. Round the answer to the nearest hundredth.

Solution:**Solution**

Step 1. **Read** the problem. Draw a figure with the given information and label it.



(a)

Step 2. **Identify** what you are looking for.

the volume of the sphere

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. Translate.

Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$V = \frac{4}{3}\pi r^3$$

$$V \approx \frac{4}{3}(3.14)14^3$$

Step 5. Solve.

$$V \approx 11,488.21$$

Step 6. Check: We leave it to you to check your calculations.

Step 7. Answer the question.

The volume is approximately 11,488.21 cubic inches.

ⓑ

Step 2. Identify what you are looking for.

the surface area of the sphere

Step 3. Name. Choose a variable to represent it.

let S = surface area

Step 4. Translate.

Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$S = 4\pi r^2$$

$$S \approx 4(3.14)14^2$$

Step 5. Solve.

$$S \approx 2461.76$$

Step 6. Check: We leave it to you to check your calculations.

Step 7. **Answer** the question.

The surface area is approximately 2461.76 square inches.

Note:

Exercise:

Problem:

A beach ball is in the shape of a sphere with radius of 9 inches. Find its (a) volume and (b) surface area.

Solution:

(a) 3052.08 cu. in.

(b) 1017.36 sq. in.

Note:

Exercise:

Problem:

A Roman statue depicts Atlas holding a globe with radius of 1.5 feet. Find the (a) volume and (b) surface area of the globe.

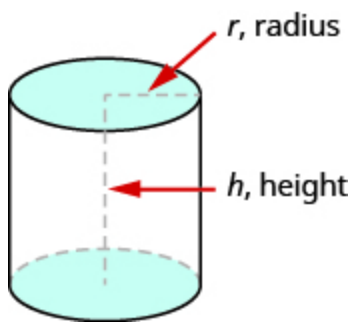
Solution:

(a) 14.13 cu. ft

(b) 28.26 sq. ft

Find the Volume and Surface Area of a Cylinder

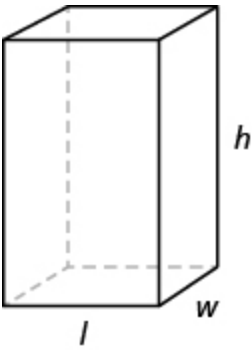
If you have ever seen a can of soda, you know what a cylinder looks like. A **cylinder** is a solid figure with two parallel circles of the same size at the top and bottom. The top and bottom of a cylinder are called the bases. The height h of a cylinder is the distance between the two bases. For all the cylinders we will work with here, the sides and the height, h , will be perpendicular to the bases.



A cylinder has two circular bases of equal size. The height is the distance between the bases.

Rectangular solids and cylinders are somewhat similar because they both have two bases and a height. The formula for the volume of a rectangular solid, $V = Bh$, can also be used to find the volume of a cylinder.

For the rectangular solid, the area of the base, B , is the area of the rectangular base, length $\times$ width. For a cylinder, the area of the base, B , is the area of its circular base, πr^2 . [\[link\]](#) compares how the formula $V = Bh$ is used for rectangular solids and cylinders.



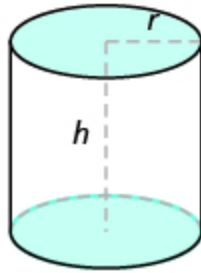
(a)

$$V = Bh$$

$$V = \text{Base} \times h$$

$$V = (lw) \times h$$

$$V = lwh$$



(b)

$$V = Bh$$

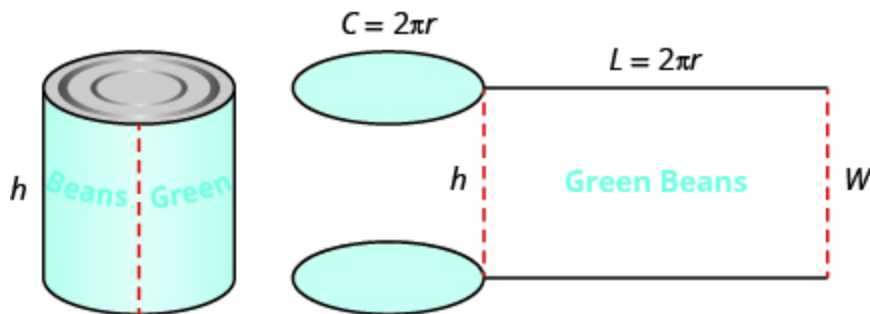
$$V = \text{Base} \times h$$

$$V = (\pi r^2) \times h$$

$$V = \pi r^2 h$$

Seeing how a cylinder is similar to a rectangular solid may make it easier to understand the formula for the volume of a cylinder.

To understand the formula for the surface area of a cylinder, think of a can of vegetables. It has three surfaces: the top, the bottom, and the piece that forms the sides of the can. If you carefully cut the label off the side of the can and unroll it, you will see that it is a rectangle. See [\[link\]](#).



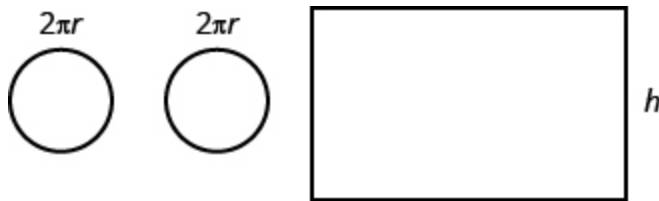
By cutting and unrolling the label of a can of vegetables, we can see that the surface of a cylinder is a rectangle. The length of the rectangle is the circumference of the cylinder's base, and the width is the height of the cylinder.

The distance around the edge of the can is the **circumference** of the cylinder's base it is also the length L of the rectangular label. The height of the cylinder is the width W of the rectangular label. So the area of the label can be represented as

$$A = L \cdot W$$

$$A = 2\pi r \cdot h$$

To find the total surface area of the cylinder, we add the areas of the two circles to the area of the rectangle.



$$S = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{rectangle}}$$

$$S = \underbrace{\pi r^2 + \pi r^2}_{2\pi r^2} + 2\pi r \cdot h$$

$$S = 2 \cdot \pi r^2 + 2\pi r h$$

$$S = 2\pi r^2 + 2\pi r h$$

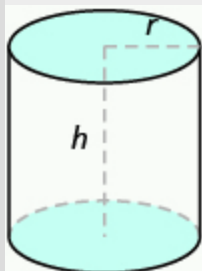
The surface area of a cylinder with radius r and height h , is

Equation:

$$S = 2\pi r^2 + 2\pi r h$$

Note:**Volume and Surface Area of a Cylinder**

For a cylinder with radius r and height h :



Volume: $V = \pi r^2 h$ or $V = Bh$

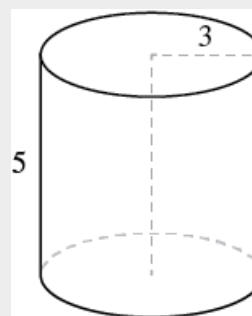
Surface Area: $S = 2\pi r^2 + 2\pi rh$

Example:**Exercise:****Problem:**

A cylinder has height 5 centimeters and radius 3 centimeters. Find the
Ⓐ volume and Ⓑ surface area.

Solution:**Solution**

Step 1. **Read** the problem. Draw the figure and label it with the given information.



Ⓐ

Step 2. **Identify** what you are looking for.

the volume of the cylinder

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.**
Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$V = \pi r^2 h$$
$$V \approx (3.14)3^2 \cdot 5$$

Step 5. **Solve.**

$$V \approx 141.3$$

Step 6. **Check:** We leave it to you to check your calculations.

Step 7. **Answer** the question.

The volume is approximately 141.3 cubic inches.

Ⓑ

Step 2. **Identify** what you are looking for.

the surface area of the cylinder

Step 3. Name. Choose a variable to represent it.	let S = surface area
Step 4. Translate. Write the appropriate formula. Substitute. (Use 3.14 for π)	$S = 2\pi r^2 + 2\pi rh$ $S \approx 2(3.14)3^2 + 2(3.14)(3)5$
Step 5. Solve.	$S \approx 150.72$
Step 6. Check: We leave it to you to check your calculations.	
Step 7. Answer the question.	The surface area is approximately 150.72 square inches.

Note:

Exercise:

Problem:

Find the ① volume and ② surface area of the cylinder with radius 4 cm and height 7cm.

Solution:

- ① 351.68 cu. cm
- ② 276.32 sq. cm

Note:

Exercise:

Problem:

Find the (a) volume and (b) surface area of the cylinder with given radius 2 ft and height 8 ft.

Solution:

(a) 100.48 cu. ft

(b) 125.6 sq. ft

Example:

Exercise:

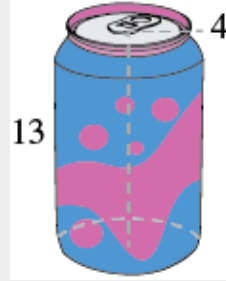
Problem:

Find the (a) volume and (b) surface area of a can of soda. The radius of the base is 4 centimeters and the height is 13 centimeters. Assume the can is shaped exactly like a cylinder.

Solution:

Solution

Step 1. **Read** the problem. Draw the figure and label it with the given information.



(a)

Step 2. **Identify** what you are looking for.

the volume of the cylinder

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.**
Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$V = \pi r^2 h$$

$$V \approx (3.14)4^2 \cdot 13$$

Step 5. **Solve.**

$$V \approx 653.12$$

Step 6. **Check:** We leave it to you to check.

Step 7. **Answer** the question.

The volume is approximately 653.12 cubic centimeters.

ⓑ	
Step 2. Identify what you are looking for.	the surface area of the cylinder
Step 3. Name. Choose a variable to represent it.	let S = surface area
Step 4. Translate. Write the appropriate formula. Substitute. (Use 3.14 for π)	$S = 2\pi r^2 + 2\pi rh$ $S \approx 2(3.14)4^2 + 2(3.14)(4)13$
Step 5. Solve.	$S \approx 427.04$
Step 6. Check: We leave it to you to check your calculations.	
Step 7. Answer the question.	The surface area is approximately 427.04 square centimeters.

Note:

Exercise:

Problem:

Find the ⓐ volume and ⓑ surface area of a can of paint with radius 8 centimeters and height 19 centimeters. Assume the can is shaped exactly like a cylinder.

Solution:

- Ⓐ 3,818.24 cu. cm
- Ⓑ 1,356.48 sq. cm

Note:**Exercise:****Problem:**

Find the Ⓐ volume and Ⓑ surface area of a cylindrical drum with radius 2.7 feet and height 4 feet. Assume the drum is shaped exactly like a cylinder.

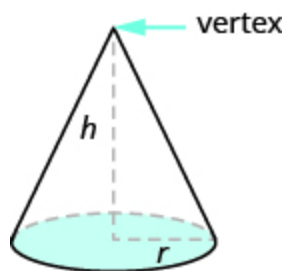
Solution:

- Ⓐ 91.5624 cu. ft
- Ⓑ 113.6052 sq. ft

Find the Volume of Cones

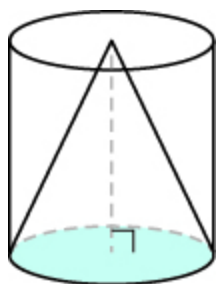
The first image that many of us have when we hear the word ‘cone’ is an ice cream cone. There are many other applications of cones (but most are not as tasty as ice cream cones). In this section, we will see how to find the volume of a cone.

In geometry, a **cone** is a solid figure with one circular base and a vertex. The height of a cone is the distance between its base and the vertex. The cones that we will look at in this section will always have the height perpendicular to the base. See [\[link\]](#).



The height
of a cone is
the distance
between its
base and
the vertex.

Earlier in this section, we saw that the volume of a cylinder is $V = \pi r^2 h$. We can think of a cone as part of a cylinder. [\[link\]](#) shows a cone placed inside a cylinder with the same height and same base. If we compare the volume of the cone and the cylinder, we can see that the volume of the cone is less than that of the cylinder.



The
volume
of a
cone is
less
than the

volume
of a
cylinder
with
the
same
base
and
height.

In fact, the volume of a cone is exactly one-third of the volume of a cylinder with the same base and height. The volume of a cone is

$$V = \frac{1}{3} Bh$$

Since the base of a cone is a circle, we can substitute the formula of area of a circle, πr^2 , for B to get the formula for volume of a cone.

$$V = \frac{1}{3} \pi r^2 h$$

In this book, we will only find the volume of a cone, and not its surface area.

Note:

Volume of a Cone

For a cone with radius r and height h .



Example:

Exercise:

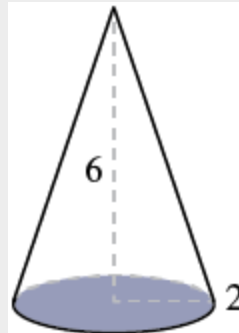
Problem:

Find the volume of a cone with height 6 inches and radius of its base 2 inches.

Solution:

Solution

Step 1. **Read** the problem.
Draw the figure and label it
with the given information.



Step 2. **Identify** what you are
looking for.

the volume of the cone

Step 3. **Name.** Choose a
variable to represent it.

let V = volume

Step 4. **Translate.**
Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$V = \frac{1}{3} \pi r^2 h$$
$$V \approx \frac{1}{3} 3.14 (2)^2 (6)$$

Step 5. Solve.	$V \approx 25.12$
Step 6. Check: We leave it to you to check your calculations.	
Step 7. Answer the question.	The volume is approximately 25.12 cubic inches.

Note:

Exercise:

Problem:

Find the volume of a cone with height 7 inches and radius 3 inches

Solution:

65.94 cu. in.

Note:

Exercise:

Problem:

Find the volume of a cone with height 9 centimeters and radius 5 centimeters

Solution:

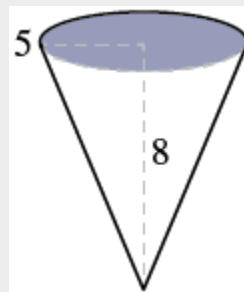
235.5 cu. cm

Example:**Exercise:****Problem:**

Marty's favorite gastro pub serves french fries in a paper wrap shaped like a cone. What is the volume of a conic wrap that is 8 inches tall and 5 inches in diameter? Round the answer to the nearest hundredth.

Solution:**Solution**

Step 1. **Read** the problem.
Draw the figure and label it with the given information.
Notice here that the base is the circle at the top of the cone.



Step 2. **Identify** what you are looking for.

the volume of the cone

Step 3. **Name.** Choose a variable to represent it.

let V = volume

Step 4. **Translate.** Write the appropriate formula.
Substitute. (Use 3.14 for π ,

$$V = \frac{1}{3} \pi r^2 h$$

and notice that we were given the distance across the circle, which is its diameter. The radius is 2.5 inches.)

$$V \approx \frac{1}{3} \quad 3.14 \quad (2.5)^2 \quad (8)$$

Step 5. **Solve.**

$$V \approx 52.33$$

Step 6. **Check:** We leave it to you to check your calculations.

Step 7. **Answer** the question.

The volume of the wrap is approximately 52.33 cubic inches.

Note:

Exercise:

Problem:

How many cubic inches of candy will fit in a cone-shaped piñata that is 18 inches long and 12 inches across its base? Round the answer to the nearest hundredth.

Solution:

678.24 cu. in.

Note:

Exercise:

Problem:

What is the volume of a cone-shaped party hat that is 10 inches tall and 7 inches across at the base? Round the answer to the nearest hundredth.

Solution:

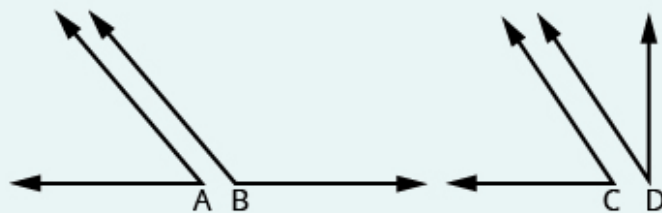
128.2 cu. in.

Summary of Geometry Formulas

The following charts summarize all of the formulas covered in this chapter.

Supplementary and Complementary Angles

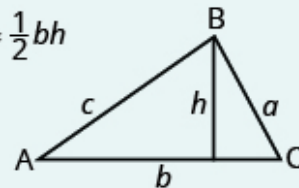
$m\angle A + m\angle B = 180^\circ$ for supplementary angles A and B
 $m\angle C + m\angle D = 90^\circ$ for complementary angles C and D



Triangle

For $\triangle ABC$, angle measures.
 $m\angle A + m\angle B + m\angle C = 180^\circ$
Perimeter. $P = a + b + c$

$$A = \frac{1}{2}bh$$



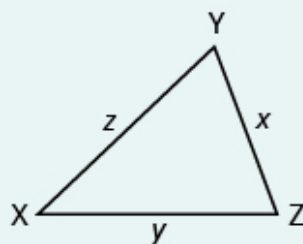
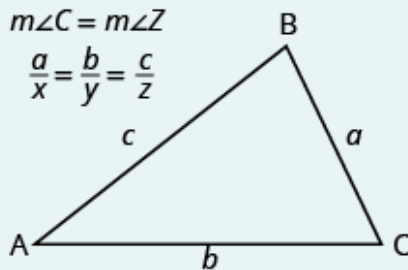
Similar Triangles

$$m\angle A = m\angle X$$

$$m\angle B = m\angle Y$$

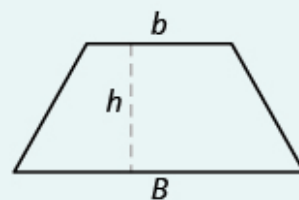
$$m\angle C = m\angle Z$$

$$\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$$



Trapezoid

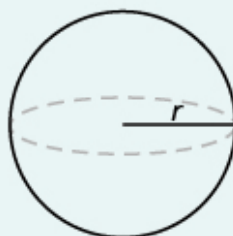
$$\text{Area. } A = \frac{1}{2}h(b + B)$$



Sphere

$$\text{Volume: } V = \frac{4}{3}\pi r^3$$

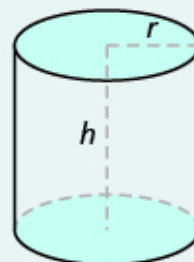
$$\text{Surface Area: } S = 4\pi r^2$$



Cylinder

$$\text{Volume: } V = \pi r^2 h \text{ or } V = Bh$$

$$\text{Surface Area: } S = 2\pi r^2 + 2\pi rh$$

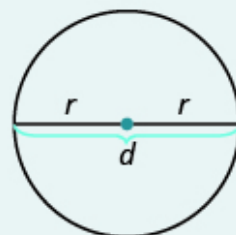


Circle

$$\text{Circumference: } C = 2\pi r$$

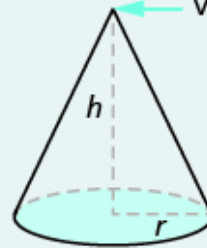
$$\text{or } C = \pi d$$

$$\text{Area: } A = \pi r^2$$

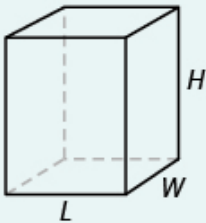
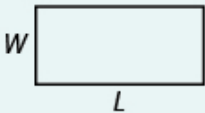
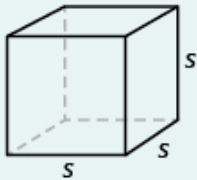


Cone

vertex



$$V = \frac{1}{3}\pi r^2 h$$

Rectangular Solid Volume: $V = LWH$ Surface Area: $S = 2LH + 2LW + 2WH$ 	Rectangle Perimeter: $P = 2L + 2W$ Area: $A = LW$ 	Cube Volume: $V = s^3$ Surface Area: $S = 6s^2$ 
---	---	---

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Volume of a Cone](#)

Key Concepts

- **Volume and Surface Area of a Rectangular Solid**

- $V = LWH$
- $S = 2LH + 2LW + 2WH$

- **Volume and Surface Area of a Cube**

- $V = s^3$
- $S = 6s^2$

- **Volume and Surface Area of a Sphere**

- $V = \frac{4}{3}\pi r^3$
- $S = 4\pi r^2$

- **Volume and Surface Area of a Cylinder**

- $V = \pi r^2 h$
- $S = 2\pi r^2 + 2\pi r h$

- **Volume of a Cone**

- For a cone with radius r and height h :
Volume: $V = \frac{1}{3}\pi r^2 h$

Practice Makes Perfect

Find Volume and Surface Area of Rectangular Solids

In the following exercises, find (a) the volume and (b) the surface area of the rectangular solid with the given dimensions.

Exercise:

Problem: length 2 meters, width 1.5 meters, height 3 meters

Solution:

- (a) 9 cu. m
- (b) 27 sq. m

Exercise:

Problem: length 5 feet, width 8 feet, height 2.5 feet

Exercise:

Problem: length 3.5 yards, width 2.1 yards, height 2.4 yards

Solution:

- (a) 17.64 cu. yd.
- (b) 41.58 sq. yd.

Exercise:

Problem:

length 8.8 centimeters, width 6.5 centimeters, height 4.2 centimeters

In the following exercises, solve.

Exercise:

Problem:

Moving van A rectangular moving van has length 16 feet, width 8 feet, and height 8 feet. Find its (a) volume and (b) surface area.

Solution:

- (a) 1,024 cu. ft
- (b) 640 sq. ft

Exercise:

Problem:

Gift box A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches. Find its (a) volume and (b) surface area.

Exercise:

Problem:

Carton A rectangular carton has length 21.3 cm, width 24.2 cm, and height 6.5 cm. Find its (a) volume and (b) surface area.

Solution:

- (a) 3,350.49 cu. cm
- (b) 1,622.42 sq. cm

Exercise:

Problem:

Shipping container A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet. Find its (a) volume and (b) surface area.

In the following exercises, find (a) the volume and (b) the surface area of the cube with the given side length.

Exercise:

Problem: 5 centimeters

Solution:

- (a) 125 cu. cm
- (b) 150 sq. cm

Exercise:

Problem: 6 inches

Exercise:

Problem: 10.4 feet

Solution:

- (a) 1124.864 cu. ft.
- (b) 648.96 sq. ft

Exercise:

Problem: 12.5 meters

In the following exercises, solve.

Exercise:

Problem:

Science center Each side of the cube at the Discovery Science Center in Santa Ana is 64 feet long. Find its (a) volume and (b) surface area.

Solution:

(a) 262,144 cu. ft

(b) 24,576 sq. ft

Exercise:

Problem:

Museum A cube-shaped museum has sides 45 meters long. Find its (a) volume and (b) surface area.

Exercise:

Problem:

Base of statue The base of a statue is a cube with sides 2.8 meters long. Find its (a) volume and (b) surface area.

Solution:

(a) 21.952 cu. m

(b) 47.04 sq. m

Exercise:

Problem:

Tissue box A box of tissues is a cube with sides 4.5 inches long. Find its (a) volume and (b) surface area.

Find the Volume and Surface Area of Spheres

In the following exercises, find (a) the volume and (b) the surface area of the sphere with the given radius. Round answers to the nearest hundredth.

Exercise:

Problem: 3 centimeters

Solution:

(a) 113.04 cu. cm

(b) 113.04 sq. cm

Exercise:

Problem: 9 inches

Exercise:

Problem: 7.5 feet

Solution:

(a) 1,766.25 cu. ft

(b) 706.5 sq. ft

Exercise:

Problem: 2.1 yards

In the following exercises, solve. Round answers to the nearest hundredth.

Exercise:

Problem:

Exercise ball An exercise ball has a radius of 15 inches. Find its (a) volume and (b) surface area.

Solution:

- Ⓐ 14,130 cu. in.
- Ⓑ 2,826 sq. in.

Exercise:

Problem:

Balloon ride The Great Park Balloon is a big orange sphere with a radius of 36 feet . Find its Ⓐ volume and Ⓑ surface area.

Exercise:

Problem:

Golf ball A golf ball has a radius of 4.5 centimeters. Find its Ⓐ volume and Ⓑ surface area.

Solution:

- Ⓐ 381.51 cu. cm
- Ⓑ 254.34 sq. cm

Exercise:

Problem:

Baseball A baseball has a radius of 2.9 inches. Find its Ⓐ volume and Ⓑ surface area.

Find the Volume and Surface Area of a Cylinder

In the following exercises, find Ⓐ the volume and Ⓑ the surface area of the cylinder with the given radius and height. Round answers to the nearest hundredth.

Exercise:

Problem: radius 3 feet, height 9 feet

Solution:

- Ⓐ 254.34 cu. ft
- Ⓑ 226.08 sq. ft

Exercise:

Problem: radius 5 centimeters, height 15 centimeters

Exercise:

Problem: radius 1.5 meters, height 4.2 meters

Solution:

- Ⓐ 29.673 cu. m
- Ⓑ 53.694 sq. m

Exercise:

Problem: radius 1.3 yards, height 2.8 yards

In the following exercises, solve. Round answers to the nearest hundredth.

Exercise:

Problem:

Coffee can A can of coffee has a radius of 5 cm and a height of 13 cm. Find its Ⓐ volume and Ⓑ surface area.

Solution:

- Ⓐ 1,020.5 cu. cm
- Ⓑ 565.2 sq. cm

Exercise:

Problem:

Snack pack A snack pack of cookies is shaped like a cylinder with radius 4 cm and height 3 cm. Find its (a) volume and (b) surface area.

Exercise:

Problem:

Barber shop pole A cylindrical barber shop pole has a diameter of 6 inches and height of 24 inches. Find its (a) volume and (b) surface area.

Solution:

- (a) 678.24 cu. in.
- (b) 508.68 sq. in.

Exercise:

Problem:

Architecture A cylindrical column has a diameter of 8 feet and a height of 28 feet. Find its (a) volume and (b) surface area.

Find the Volume of Cones

In the following exercises, find the volume of the cone with the given dimensions. Round answers to the nearest hundredth.

Exercise:

Problem: height 9 feet and radius 2 feet

Solution:

37.68 cu. ft

Exercise:

Problem: height 8 inches and radius 6 inches

Exercise:

Problem: height 12.4 centimeters and radius 5 cm

Solution:

324.47 cu. cm

Exercise:

Problem: height 15.2 meters and radius 4 meters

In the following exercises, solve. Round answers to the nearest hundredth.

Exercise:

Problem:

Teepee What is the volume of a cone-shaped teepee tent that is 10 feet tall and 10 feet across at the base?

Solution:

261.67 cu. ft

Exercise:

Problem:

Popcorn cup What is the volume of a cone-shaped popcorn cup that is 8 inches tall and 6 inches across at the base?

Exercise:

Problem:

Silo What is the volume of a cone-shaped silo that is 50 feet tall and 70 feet across at the base?

Solution:

64,108.33 cu. ft

Exercise:

Problem:

Sand pile What is the volume of a cone-shaped pile of sand that is 12 meters tall and 30 meters across at the base?

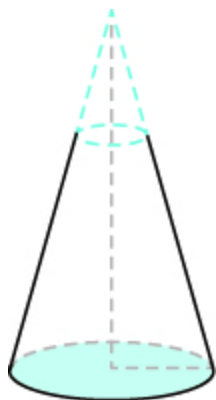
Everyday Math

Exercise:

Problem:

Street light post The post of a street light is shaped like a truncated cone, as shown in the picture below. It is a large cone minus a smaller top cone. The large cone is 30 feet tall with base radius 1 foot. The smaller cone is 10 feet tall with base radius of 0.5 feet. To the nearest tenth,

- Ⓐ find the volume of the large cone.
- Ⓑ find the volume of the small cone.
- Ⓒ find the volume of the post by subtracting the volume of the small cone from the volume of the large cone.



Solution:

- Ⓐ 31.4 cu. ft
- Ⓑ 2.6 cu. ft
- Ⓒ 28.8 cu. ft

Exercise:

Problem:

Ice cream cones A regular ice cream cone is 4 inches tall and has a diameter of 2.5 inches. A waffle cone is 7 inches tall and has a diameter of 3.25 inches. To the nearest hundredth,

- Ⓐ find the volume of the regular ice cream cone.
- Ⓑ find the volume of the waffle cone.
- Ⓒ how much more ice cream fits in the waffle cone compared to the regular cone?

Writing Exercises

Exercise:

Problem:

The formulas for the volume of a cylinder and a cone are similar.
Explain how you can remember which formula goes with which shape.

Solution:

Answers will vary.

Exercise:**Problem:**

Which has a larger volume, a cube of sides of 8 feet or a sphere with a diameter of 8 feet? Explain your reasoning.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find volume and surface area of rectangular solids.			
find volume and surface area of spheres.			
find volume and surface area of cylinders.			
find volume of cones.			

Ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

cone

A cone is a solid figure with one circular base and a vertex.

cube

A cube is a rectangular solid whose length, width, and height are equal.

cylinder

A cylinder is a solid figure with two parallel circles of the same size at the top and bottom.

Solve a Formula for a Specific Variable

By the end of this section, you will be able to:

- Use the distance, rate, and time formula
- Solve a formula for a specific variable

Note:

Before you get started, take this readiness quiz.

1. Write 35 miles per gallon as a unit rate.
If you missed this problem, review [\[link\]](#).
2. Solve $6x + 24 = 96$.
If you missed this problem, review [\[link\]](#).
3. Find the simple interest earned after 5 years on \$1,000 at an interest rate of 4%.
If you missed this problem, review [\[link\]](#).

Use the Distance, Rate, and Time Formula

One formula you'll use often in algebra and in everyday life is the formula for distance traveled by an object moving at a constant speed. The basic idea is probably already familiar to you. Do you know what distance you travel if you drove at a steady rate of 60 miles per hour for 2 hours? (This might happen if you use your car's cruise control while driving on the Interstate.) If you said 120 miles, you already know how to use this formula!

The math to calculate the distance might look like this:

Equation:

$$\begin{aligned}\text{distance} &= \left(\frac{60 \text{ miles}}{1 \text{ hour}} \right) (2 \text{ hours}) \\ \text{distance} &= 120 \text{ miles}\end{aligned}$$

In general, the formula relating distance, rate, and time is

Equation:

$$\text{distance} = \text{rate} \cdot \text{time}$$

Note:

Distance, Rate and Time

For an object moving in at a uniform (constant) rate, the distance traveled, the elapsed time, and the rate are related by the formula

Equation:

$$d = rt$$

where d = distance, r = rate, and t = time.

Notice that the units we used above for the rate were miles per hour, which we can write as a ratio $\frac{\text{miles}}{\text{hour}}$. Then when we multiplied by the time, in hours, the common units ‘hour’ divided out. The answer was in miles.

Example:

Exercise:

Problem:

Jamal rides his bike at a uniform rate of 12 miles per hour for $3\frac{1}{2}$ hours. How much distance has he traveled?

Solution:

Solution

Step 1. Read the problem. You may want to create a mini-chart to summarize the information in the problem.	$d = ?$ $r = 12 \text{ mph}$ $t = 3\frac{1}{2} \text{ hours}$
Step 2. Identify what you are looking for.	distance traveled
Step 3. Name. Choose a variable to represent it.	let d = distance
Step 4. Translate. Write the appropriate formula for the situation. Substitute in the given information.	$d = rt$ $d = 12 \cdot 3\frac{1}{2}$
Step 5. Solve the equation.	$d = 42 \text{ miles}$
Step 6. Check: Does 42 miles make sense? Jamal rides 12 miles in 1 hour, 24 miles in 2 hours, 36 miles in 3 hours, 48 miles in 4 hours, 42 miles in $3\frac{1}{2}$ hours is reasonable	
Step 7. Answer the question with a complete sentence.	Jamal rode 42 miles.

Note:

Exercise:**Problem:**

Lindsay drove for $5\frac{1}{2}$ hours at 60 miles per hour. How much distance did she travel?

Solution:

330 mi

Note:**Exercise:****Problem:**

Trinh walked for $2\frac{1}{3}$ hours at 3 miles per hour. How far did she walk?

Solution:

7 mi

Example:**Exercise:****Problem:**

Rey is planning to drive from his house in San Diego to visit his grandmother in Sacramento, a distance of 520 miles. If he can drive at a steady rate of 65 miles per hour, how many hours will the trip take?

Solution:

Solution

Step 1. Read the problem. Summarize the information in the problem.	$d = 520$ miles $r = 65$ mph $t = ?$
Step 2. Identify what you are looking for.	how many hours (time)
Step 3. Name: Choose a variable to represent it.	let $t =$ time
Step 4. Translate. Write the appropriate formula. Substitute in the given information.	$d = rt$ $520 = 65t$
Step 5. Solve the equation.	$t = 8$
Step 6. Check: Substitute the numbers into the formula and make sure the result is a true statement. $d = rt$ $\quad \quad ?$ $520 = 65 \cdot 8$ $520 = 520 > \checkmark$	
Step 7. Answer the question with a complete sentence. We know the units of time will be hours because we divided miles by miles per hour.	Rey's trip will take 8 hours.

Note:

Exercise:

Problem:

Lee wants to drive from Phoenix to his brother's apartment in San Francisco, a distance of 770 miles. If he drives at a steady rate of 70 miles per hour, how many hours will the trip take?

Solution:

11 hours

Note:**Exercise:****Problem:**

Yesenia is 168 miles from Chicago. If she needs to be in Chicago in 3 hours, at what rate does she need to drive?

Solution:

56 mph

Solve a Formula for a Specific Variable

In this chapter, you became familiar with some formulas used in geometry. Formulas are also very useful in the sciences and social sciences—fields such as chemistry, physics, biology, psychology, sociology, and criminal justice. Healthcare workers use formulas, too, even for something as routine as dispensing medicine. The widely used spreadsheet program Microsoft ExcelTM relies on formulas to do its calculations. Many teachers use spreadsheets to apply formulas to compute student grades. It is important to be familiar with formulas and be able to manipulate them easily.

In [\[link\]](#) and [\[link\]](#), we used the formula $d = rt$. This formula gives the value of d when you substitute in the values of r and t . But in [\[link\]](#), we had to find the value of t . We substituted in values of d and r and then used algebra to solve for t . If you had to do this often, you might wonder why there isn't a formula that gives the value of t when you substitute in the values of d and r . We can get a formula like this by solving the formula $d = rt$ for t .

To solve a formula for a specific variable means to get that variable by itself with a coefficient of 1 on one side of the equation and all the other variables and constants on the other side. We will call this solving an equation for a specific variable *in general*. This process is also called *solving a literal equation*. The result is another formula, made up only of variables. The formula contains letters, or *literals*.

Let's try a few examples, starting with the distance, rate, and time formula we used above.

Example:

Exercise:

Problem: Solve the formula $d = rt$ for t :

- Ⓐ when $d = 520$ and $r = 65$
- Ⓑ in general.

Solution:

Solution

We'll write the solutions side-by-side so you can see that solving a formula in general uses the same steps as when we have numbers to substitute.

	Ⓐ when $d = 520$ and $r = 65$	Ⓑ in general
Write the formula.	$d = rt$	$d = rt$
Substitute any given values.	$520 = 65t$	
Divide to isolate t .	$\frac{520}{65} = \frac{65t}{65}$	$\frac{d}{r} = \frac{rt}{r}$
Simplify.	$8 = t$ $t = 8$	$\frac{d}{r} = t$ $t = \frac{d}{r}$

Notice that the solution for Ⓐ is the same as that in [\[link\]](#). We say the formula $t = \frac{d}{r}$ is solved for t . We can use this version of the formula anytime we are given the distance and rate and need to find the time.

Note:

Exercise:

Problem: Solve the formula $d = rt$ for r :

Ⓐ when $d = 180$ and $t = 4$

Ⓐ in general

Solution:

Ⓐ $r = 45$

Ⓑ $r = \frac{d}{t}$

Note:

Exercise:

Problem: Solve the formula $d = rt$ for r :

Ⓐ when $d = 780$ and $t = 12$

Ⓑ in general

Solution:

Ⓐ $r = 65$

Ⓑ $r = \frac{d}{t}$

We used the formula $A = \frac{1}{2}bh$ in [Use Properties of Rectangles, Triangles, and Trapezoids](#) to find the area of a triangle when we were given the base and height. In the next example, we will solve this formula for the height.

Example:

Exercise:

Problem:

The formula for area of a triangle is $A = \frac{1}{2}bh$. Solve this formula for h :

- Ⓐ when $A = 90$ and $b = 15$
- Ⓑ in general

Solution:
Solution

	Ⓐ when $A = 90$ and $b = 15$	Ⓑ in general
Write the formula.	$A = \frac{1}{2}bh$	$A = \frac{1}{2}bh$
Substitute any given values.	$90 = \frac{1}{2} \cdot 15 \cdot h$	
Clear the fractions.	$2 \cdot 90 = 2 \cdot \frac{1}{2} \cdot 15 \cdot h$	$2 \cdot A = 2 \cdot \frac{1}{2} \cdot b \cdot h$
Simplify.	$180 = 15h$	$2A = bh$
Solve for h .	$12 = h$	$\frac{2A}{b} = h$

We can now find the height of a triangle, if we know the area and the base, by using the formula

Equation:

$$h = \frac{2A}{b}$$

Note:

Exercise:

Problem: Use the formula $A = \frac{1}{2}bh$ to solve for h:

- Ⓐ when $A = 170$ and $b = 17$
- Ⓑ in general

Solution:

Ⓐ $h = 20$ Ⓑ $h = \frac{2A}{b}$

Note:

Exercise:

Problem: Use the formula $A = \frac{1}{2}bh$ to solve for b:

- Ⓐ when $A = 62$ and $h = 31$
- Ⓑ in general

Solution:

- Ⓐ $b = 4$
- Ⓑ $b = \frac{2A}{h}$

In [Solve Simple Interest Applications](#), we used the formula $I = Prt$ to calculate simple interest, where I is interest, P is principal, r is rate as a decimal, and t is time in years.

Example:

Exercise:

Problem: Solve the formula $I = Prt$ to find the principal, P :

- Ⓐ when $I = \$5,600$, $r = 4\%$, $t = 7$ years
- Ⓑ in general

Solution:

Solution

	$I = \$5600$, $r = 4\%$, $t = 7$ years	in general
Write the formula.	$I = Prt$	$I = Prt$
Substitute any given values.	$5600 = P(0.04)(7)$	$I = Prt$

Multiply $r \cdot t$.	$5600 = P(0.28)$	$I = P(rt)$
Divide to isolate P .	$\frac{5600}{0.28} = \frac{P(0.28)}{0.28}$	$\frac{I}{rt} = \frac{P(rt)}{rt}$
Simplify.	$20,000 = P$	$\frac{I}{rt} = P$
State the answer.	The principal is \$20,000.	$P = \frac{I}{rt}$

Note:

Exercise:

Problem: Use the formula $I = Prt$.

Find t : ① when $I = \$2,160$, $r = 6\%$, $P = \$12,000$; ② in general

Solution:

① $t = 3$ years

② $t = \frac{I}{Pr}$

Note:

Exercise:

Problem: Use the formula $I = Prt$.

Find r : ① when $I = \$5,400$, $P = \$9,000$, $t = 5$ years ② in general

Solution:

① $r = 0.12 = 12\%$

② $r = \frac{I}{Pt}$

Later in this class, and in future algebra classes, you'll encounter equations that relate two variables, usually x and y . You might be given an equation that is solved for y and need to solve it for x , or vice versa. In the following example, we're given an equation with both x and y on the same side and we'll solve it for y . To do this, we will follow the same steps that we used to solve a formula for a specific variable.

Example:

Exercise:

Problem: Solve the formula $3x + 2y = 18$ for y :

① when $x = 4$

② in general

Solution:

Solution

	when $x = 4$	in general
Write the equation.	$3x + 2y = 18$	$3x + 2y = 18$
Substitute any given values.	$3(4) + 2y = 18$	$3x + 2y = 18$
Simplify if possible.	$12 + 2y = 18$	$3x + 2y = 18$
Subtract to isolate the y -term.	$12 - 12 + 2y = 18 - 12$	$3x - 3x + 2y = 18 - 3x$
Simplify.	$2y = 6$	$2y = 18 - 3x$
Divide.	$\frac{2y}{2} = \frac{6}{2}$	$\frac{2y}{2} = \frac{18 - 3x}{2}$
Simplify.	$y = 3$	$y = \frac{18 - 3x}{2}$

Note:

Exercise:

Problem: Solve the formula $3x + 4y = 10$ for y :

- Ⓐ when $x = 2$
- Ⓑ in general

Solution:

- Ⓐ $y = 1$
- Ⓑ $y = \frac{10-3x}{4}$

Note:

Exercise:

Problem: Solve the formula $5x + 2y = 18$ for y :

- Ⓐ when $x = 4$
- Ⓑ in general

Solution:

- Ⓐ $y = -1$
- Ⓑ $y = \frac{18-5x}{2}$

In the previous examples, we used the numbers in part (a) as a guide to solving in general in part (b). Do you think you're ready to solve a formula in general without using numbers as a guide?

Example:

Exercise:

Problem: Solve the formula $P = a + b + c$ for a .

Solution:
Solution

We will isolate a on one side of the equation.

We will isolate a on one side of the equation.		
Write the equation.		$P = a + b + c$
Subtract b and c from both sides to isolate a .		$P - b - c = a + b + c - b - c$
Simplify.		$P - b - c = a$

So, $a = P - b - c$

Note:
Exercise:

Problem: Solve the formula $P = a + b + c$ for b .

Solution:

$$b = P - a - c$$

Note:

Exercise:

Problem: Solve the formula $P = a + b + c$ for c .

Solution:

$$c = P - a - b$$

Example:

Exercise:

Problem: Solve the equation $3x + y = 10$ for y .

Solution:

Solution

We will isolate y on one side of the equation.

We will isolate y on one side of the equation.		
Write the equation.		$3x + y = 10$
Subtract $3x$ from both sides to isolate y .		$3x - 3x + y = 10 - 3x$
Simplify.		$y = 10 - 3x$

Note:

Exercise:

Problem: Solve the formula $7x + y = 11$ for y .

Solution:

$$y = 11 - 7x$$

Note:

Exercise:

Problem: Solve the formula $11x + y = 8$ for y .

Solution:

$$y = 8 - 11x$$

Example:

Exercise:

Problem: Solve the equation $6x + 5y = 13$ for y .

Solution:
Solution

We will isolate y on one side of the equation.

We will isolate y on one side of the equation.	
Write the equation.	$6x + 5y = 13$
Subtract to isolate the term with y .	$6x + 5y - 6x = 13 - 6x$
Simplify.	$5y = 13 - 6x$
Divide 5 to make the coefficient 1.	$\frac{5y}{5} = \frac{13 - 6x}{5}$
Simplify.	$y = \frac{13 - 6x}{5}$

Note:

Exercise:

Problem: Solve the formula $4x + 7y = 9$ for y .

Solution:

$$y = \frac{9-4x}{7}$$

Note:

Exercise:

Problem: Solve the formula $5x + 8y = 1$ for y .

Solution:

$$y = \frac{1-5x}{8}$$

Note: The Links to Literacy activity *What's Faster than a Speeding Cheetah?* will provide you with another view of the topics covered in this section.

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Distance = Rate x Time](#)
- [Distance, Rate, Time](#)
- [Simple Interest](#)
- [Solving a Formula for a Specific Variable](#)
- [Solving a Formula for a Specific Variable](#)

Key Concepts

- Distance, Rate, and Time

- $d = rt$

Section Exercises

Practice Makes Perfect

Use the Distance, Rate, and Time Formula

In the following exercises, solve.

Exercise:

Problem:

Steve drove for $8\frac{1}{2}$ hours at 72 miles per hour. How much distance did he travel?

Solution:

612 mi

Exercise:

Problem:

Socorro drove for $4\frac{5}{6}$ hours at 60 miles per hour. How much distance did she travel?

Exercise:

Problem:

Yuki walked for $1\frac{3}{4}$ hours at 4 miles per hour. How far did she walk?

Solution:

7 mi

Exercise:

Problem:

Francie rode her bike for $2\frac{1}{2}$ hours at 12 miles per hour. How far did she ride?

Exercise:

Problem:

Connor wants to drive from Tucson to the Grand Canyon, a distance of 338 miles. If he drives at a steady rate of 52 miles per hour, how many hours will the trip take?

Solution:

6.5 hours

Exercise:**Problem:**

Megan is taking the bus from New York City to Montreal. The distance is 384 miles and the bus travels at a steady rate of 64 miles per hour. How long will the bus ride be?

Exercise:**Problem:**

Aurelia is driving from Miami to Orlando at a rate of 65 miles per hour. The distance is 235 miles. To the nearest tenth of an hour, how long will the trip take?

Solution:

3.6 hours

Exercise:**Problem:**

Kareem wants to ride his bike from St. Louis, Missouri to Champaign, Illinois. The distance is 180 miles. If he rides at a steady rate of 16 miles per hour, how many hours will the trip take?

Exercise:

Problem:

Javier is driving to Bangor, Maine, which is 240 miles away from his current location. If he needs to be in Bangor in 4 hours, at what rate does he need to drive?

Solution:

60 mph

Exercise:**Problem:**

Alejandra is driving to Cincinnati, Ohio, 450 miles away. If she wants to be there in 6 hours, at what rate does she need to drive?

Exercise:**Problem:**

Aisha took the train from Spokane to Seattle. The distance is 280 miles, and the trip took 3.5 hours. What was the speed of the train?

Solution:

80 mph

Exercise:**Problem:**

Philip got a ride with a friend from Denver to Las Vegas, a distance of 750 miles. If the trip took 10 hours, how fast was the friend driving?

Solve a Formula for a Specific Variable

In the following exercises, use the formula. $d = rt$.

Exercise:

Problem: Solve for t :

- Ⓐ when $d = 350$ and $r = 70$
- Ⓑ in general

Solution:

- Ⓐ $t = 5$
- Ⓑ $t = \frac{d}{r}$

Exercise:

Problem: Solve for t :

- Ⓐ when $d = 240$ and $r = 60$
- Ⓑ in general

Exercise:

Problem: Solve for t :

- Ⓐ when $d = 510$ and $r = 60$
- Ⓑ in general

Solution:

- Ⓐ $t = 8.5$
- Ⓑ $t = \frac{d}{r}$

Exercise:

Problem: Solve for t :

- Ⓐ when $d = 175$ and $r = 50$
- Ⓑ in general

Exercise:

Problem: Solve for r :

- Ⓐ when $d = 204$ and $t = 3$
- Ⓑ in general

Solution:

- Ⓐ $r = 68$
- Ⓑ $r = \frac{d}{t}$

Exercise:

Problem: Solve for r :

- Ⓐ when $d = 420$ and $t = 6$
- Ⓑ in general

Exercise:

Problem: Solve for r :

- Ⓐ when $d = 160$ and $t = 2.5$
- Ⓑ in general

Solution:

- Ⓐ $r = 64$
- Ⓑ $r = \frac{d}{t}$

Exercise:

Problem: Solve for r :

- Ⓐ when $d = 180$ and $t = 4.5$
- Ⓑ in general.

In the following exercises, use the formula $A = \frac{1}{2}bh$.

Exercise:

Problem: Solve for b :

- Ⓐ when $A = 126$ and $h = 18$
- Ⓑ in general

Solution:

- Ⓐ $b = 14$
- Ⓑ $b = \frac{2A}{h}$

Exercise:

Problem: Solve for h :

- Ⓐ when $A = 176$ and $b = 22$
- Ⓑ in general

Exercise:

Problem: Solve for h :

- Ⓐ when $A = 375$ and $b = 25$
- Ⓑ in general

Solution:

- Ⓐ $h = 30$
- Ⓑ $h = \frac{2A}{b}$

Exercise:

Problem: Solve for b :

- Ⓐ when $A = 65$ and $h = 13$
- Ⓑ in general

In the following exercises, use the formula $I = Prt$.

Exercise:

Problem: Solve for the principal, P for:

- Ⓐ $I = \$5,480$, $r = 4\%$, $t = 7$ years
- Ⓑ in general

Solution:

- Ⓐ $P = \$19,571.43$
- Ⓑ $P = \frac{I}{rt}$

Exercise:

Problem: Solve for the principal, P for:

- Ⓐ $I = \$3,950$, $r = 6\%$, $t = 5$ years
- Ⓑ in general

Exercise:

Problem: Solve for the time, t for:

- Ⓐ $I = \$2,376, P = \$9,000, r = 4.4\%$
 - Ⓑ in general
-

Solution:

- Ⓐ $t = 6$ years
- Ⓑ $t = \frac{I}{Pr}$

Exercise:

Problem: Solve for the time, t for:

- Ⓐ $I = \$624, P = \$6,000, r = 5.2\%$
- Ⓑ in general

In the following exercises, solve.

Exercise:

Problem: Solve the formula $2x + 3y = 12$ for y :

- Ⓐ when $x = 3$
 - Ⓑ in general
-

Solution:

- Ⓐ $y = 2$
- Ⓑ $y = \frac{12-2x}{3}$

Exercise:

Problem: Solve the formula $5x + 2y = 10$ for y :

- Ⓐ when $x = 4$
- Ⓑ in general

Exercise:

Problem: Solve the formula $3x + y = 7$ for y :

- Ⓐ when $x = -2$
- Ⓑ in general

Solution:

- Ⓐ $y = 13$
- Ⓑ $y = 7 - 3x$

Exercise:

Problem: Solve the formula $4x + y = 5$ for y :

- Ⓐ when $x = -3$
- Ⓑ in general

Exercise:

Problem: Solve $a + b = 90$ for b .

Solution:

- Ⓐ $b = 90 - a$
- Ⓑ $a = 90 - b$

Exercise:

Problem: Solve $a + b = 90$ for a .

Exercise:

Problem: Solve $180 = a + b + c$ for a .

Solution:

$$a = 180 - b - c$$

Exercise:

Problem: Solve $180 = a + b + c$ for c .

Exercise:

Problem: Solve the formula $8x + y = 15$ for y .

Solution:

$$y = 15 - 8x$$

Exercise:

Problem: Solve the formula $9x + y = 13$ for y .

Exercise:

Problem: Solve the formula $-4x + y = -6$ for y .

Solution:

$$y = -6 + 4x$$

Exercise:

Problem: Solve the formula $-5x + y = -1$ for y .

Exercise:

Problem: Solve the formula $4x + 3y = 7$ for y .

Solution:

$$y = \frac{7-4x}{3}$$

Exercise:

Problem: Solve the formula $3x + 2y = 11$ for y .

Exercise:

Problem: Solve the formula $x - y = -4$ for y .

Solution:

$$y = 4 + x$$

Exercise:

Problem: Solve the formula $x - y = -3$ for y .

Exercise:

Problem: Solve the formula $P = 2L + 2W$ for L .

Solution:

$$L = \frac{P-2W}{2}$$

Exercise:

Problem: Solve the formula $P = 2L + 2W$ for W .

Exercise:

Problem: Solve the formula $C = \pi d$ for d .

Solution:

$$d = \frac{C}{\pi}$$

Exercise:

Problem: Solve the formula $C = \pi d$ for π .

Exercise:

Problem: Solve the formula $V = LWH$ for L .

Solution:

$$L = \frac{V}{WH}$$

Exercise:

Problem: Solve the formula $V = LWH$ for H .

Everyday Math

Exercise:

Problem:

Converting temperature While on a tour in Greece, Tatyana saw that the temperature was 40° Celsius. Solve for F in the formula $C = \frac{5}{9}(F - 32)$ to find the temperature in Fahrenheit.

Solution:

104° F

Exercise:**Problem:**

Converting temperature Yon was visiting the United States and he saw that the temperature in Seattle was 50° Fahrenheit. Solve for C in the formula $F = \frac{9}{5}C + 32$ to find the temperature in Celsius.

Writing Exercises**Exercise:**

Problem: Solve the equation $2x + 3y = 6$ for y :

- Ⓐ when $x = -3$
- Ⓑ in general
- Ⓒ Which solution is easier for you? Explain why.

Solution:

Answers will vary

Exercise:

Problem: Solve the equation $5x - 2y = 10$ for x :

- Ⓐ when $y = 10$
- Ⓑ in general
- Ⓒ Which solution is easier for you? Explain why.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

Ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Use a Problem Solving Strategy.

Approach Word Problems with a Positive Attitude

In the following exercises, solve.

Exercise:

Problem:

How has your attitude towards solving word problems changed as a result of working through this chapter? Explain.

Solution:

Answers will vary.

Exercise:

Problem:

Did the Problem Solving Strategy help you solve word problems in this chapter? Explain.

Use a Problem Solving Strategy for Word Problems

In the following exercises, solve using the problem-solving strategy for word problems. Remember to write a complete sentence to answer each question.

Exercise:

Problem:

Three-fourths of the people at a concert are children. If there are 87 children, what is the total number of people at the concert?

Solution:

There are 116 people at the concert.

Exercise:

Problem:

There are 9 saxophone players in the band. The number of saxophone players is one less than twice the number of tuba players. Find the number of tuba players.

Exercise:

Problem:

Reza was very sick and lost 15% of his original weight. He lost 27 pounds. What was his original weight?

Solution:

His original weight was 180 pounds.

Exercise:

Problem:

Dolores bought a crib on sale for \$350. The sale price was 40% of the original price. What was the original price of the crib?

Solve Number Problems

In the following exercises, solve each number word problem.

Exercise:

Problem:

The sum of a number and three is forty-one. Find the number.

Solution:

38

Exercise:

Problem:

Twice the difference of a number and ten is fifty-four. Find the number.

Exercise:

Problem:

One number is nine less than another. Their sum is twenty-seven. Find the numbers.

Solution:

18, 9

Exercise:

Problem:

The sum of two consecutive integers is -135 . Find the numbers.

[Solve Money Applications](#)

Solve Coin Word Problems

In the following exercises, solve each coin word problem.

Exercise:

Problem:

Francie has \$4.35 in dimes and quarters. The number of dimes is 5 more than the number of quarters. How many of each coin does she have?

Solution:

16 dimes, 11 quarters

Exercise:

Problem:

Scott has \$0.39 in pennies and nickels. The number of pennies is 8 times the number of nickels. How many of each coin does he have?

Exercise:

Problem:

Paulette has \$140 in \$5 and \$10 bills. The number of \$10 bills is one less than twice the number of \$5 bills. How many of each does she have?

Solution:

6 of \$5 bills, 11 of \$10 bills

Exercise:

Problem:

Lenny has \$3.69 in pennies, dimes, and quarters. The number of pennies is 3 more than the number of dimes. The number of quarters is twice the number of dimes. How many of each coin does he have?

Solve Ticket and Stamp Word Problems

In the following exercises, solve each ticket or stamp word problem.

Exercise:

Problem:

A church luncheon made \$842. Adult tickets cost \$10 each and children's tickets cost \$6 each. The number of children was 12 more than twice the number of adults. How many of each ticket were sold?

Solution:

35 adults, 82 children

Exercise:

Problem:

Tickets for a basketball game cost \$2 for students and \$5 for adults. The number of students was 3 less than 10 times the number of adults. The total amount of money from ticket sales was \$619. How many of each ticket were sold?

Exercise:

Problem:

Ana spent \$4.06 buying stamps. The number of \$0.41 stamps she bought was 5 more than the number of \$0.26 stamps. How many of each did she buy?

Solution:

3 of 26 -cent stamps, 8 of 41 -cent stamps

Exercise:

Problem:

Yumi spent \$34.15 buying stamps. The number of \$0.56 stamps she bought was 10 less than 4 times the number of \$0.41 stamps. How many of each did she buy?

Use Properties of Angles, Triangles, and the Pythagorean Theorem

Use Properties of Angles

In the following exercises, solve using properties of angles.

Exercise:

Problem: What is the supplement of a 48° angle?

Solution:

132°

Exercise:

Problem: What is the complement of a 61° angle?

Exercise:

Problem:

Two angles are complementary. The smaller angle is 24° less than the larger angle. Find the measures of both angles.

Solution:

$33^\circ, 57^\circ$

Exercise:

Problem:

Two angles are supplementary. The larger angle is 45° more than the smaller angle. Find the measures of both angles.

Use Properties of Triangles

In the following exercises, solve using properties of triangles.

Exercise:

Problem:

The measures of two angles of a triangle are 22 and 85 degrees. Find the measure of the third angle.

Solution:

73°

Exercise:**Problem:**

One angle of a right triangle measures 41.5 degrees. What is the measure of the other small angle?

Exercise:**Problem:**

One angle of a triangle is 30° more than the smallest angle. The largest angle is the sum of the other angles. Find the measures of all three angles.

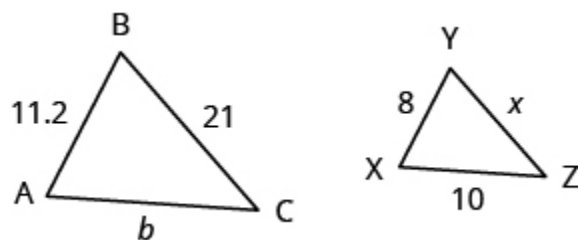
Solution:

$30^\circ, 60^\circ, 90^\circ$

Exercise:**Problem:**

One angle of a triangle is twice the measure of the smallest angle. The third angle is 60° more than the measure of the smallest angle. Find the measures of all three angles.

In the following exercises, $\triangle ABC$ is similar to $\triangle XYZ$. Find the length of the indicated side.



Exercise:

Problem: side x

Solution:

15

Exercise:

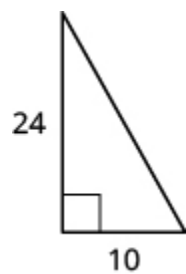
Problem: side b

Use the Pythagorean Theorem

In the following exercises, use the Pythagorean Theorem to find the length of the missing side. Round to the nearest tenth, if necessary.

Exercise:

Problem:

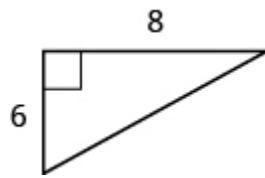


Solution:

26

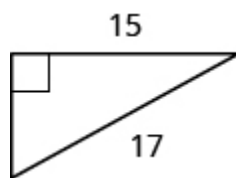
Exercise:

Problem:



Exercise:

Problem:

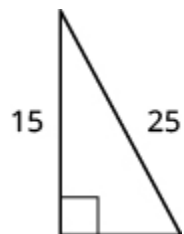


Solution:

8

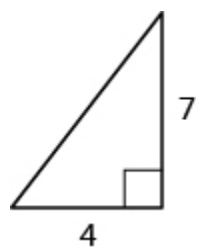
Exercise:

Problem:



Exercise:

Problem:

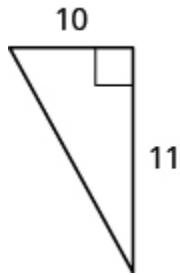


Solution:

8.1

Exercise:

Problem:

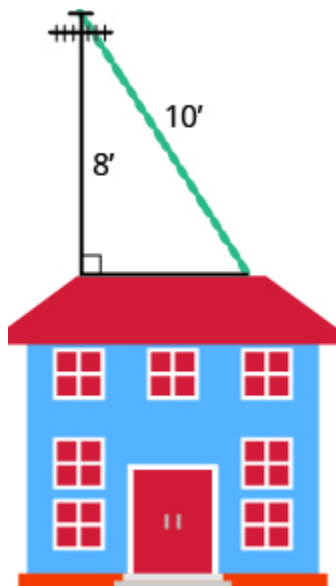


In the following exercises, solve. Approximate to the nearest tenth, if necessary.

Exercise:

Problem:

Sergio needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is 8 feet tall and Sergio has 10 feet of wire. How far from the base of the antenna can he attach the wire?



Solution:

6 feet

Exercise:

Problem:

Seong is building shelving in his garage. The shelves are 36 inches wide and 15 inches tall. He wants to put a diagonal brace across the back to stabilize the shelves, as shown. How long should the brace be?



[Use Properties of Rectangles, Triangles, and Trapezoids](#)

Understand Linear, Square, Cubic Measure

In the following exercises, would you measure each item using linear, square, or cubic measure?

Exercise:

Problem: amount of sand in a sandbag

Solution:

cubic

Exercise:

Problem: height of a tree

Exercise:

Problem: size of a patio

Solution:

square

Exercise:

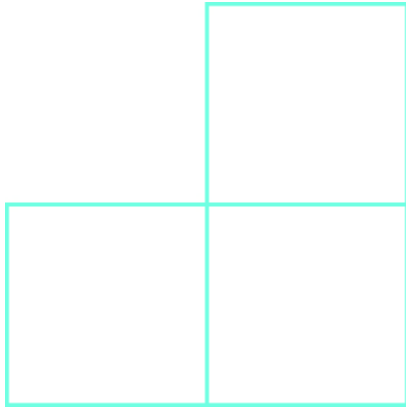
Problem: length of a highway

In the following exercises, find

- Ⓐ the perimeter
- Ⓑ the area of each figure

Exercise:

Problem:

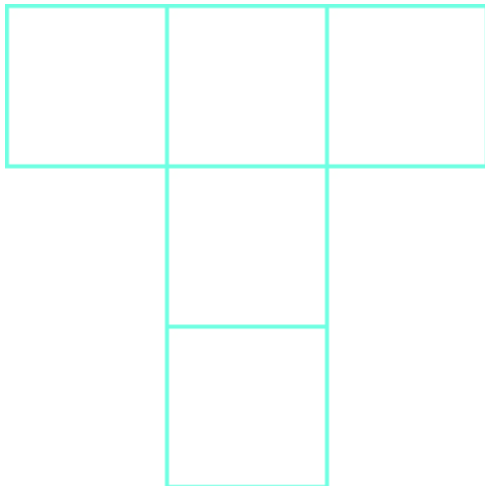


Solution:

- Ⓐ 8 units
- Ⓑ 3 sq. units

Exercise:

Problem:



Use Properties of Rectangles

In the following exercises, find the Ⓐ perimeter Ⓑ area of each rectangle

Exercise:

Problem:

The length of a rectangle is 42 meters and the width is 28 meters.

Solution:

- Ⓐ 140 m
- Ⓑ 1176 sq. m

Exercise:

Problem: The length of a rectangle is 36 feet and the width is 19 feet.

Exercise:**Problem:**

A sidewalk in front of Kathy's house is in the shape of a rectangle 4 feet wide by 45 feet long.

Solution:

- Ⓐ 98 ft.
- Ⓑ 180 sq. ft.

Exercise:

Problem: A rectangular room is 16 feet wide by 12 feet long.

In the following exercises, solve.

Exercise:**Problem:**

Find the length of a rectangle with perimeter of 220 centimeters and width of 85 centimeters.

Solution:

25 cm

Exercise:

Problem:

Find the width of a rectangle with perimeter 39 and length 11.

Exercise:

Problem:

The area of a rectangle is 2356 square meters. The length is 38 meters. What is the width?

Solution:

62 m

Exercise:

Problem:

The width of a rectangle is 45 centimeters. The area is 2700 square centimeters. What is the length?

Exercise:

Problem:

The length of a rectangle is 12 centimeters more than the width. The perimeter is 74 centimeters. Find the length and the width.

Solution:

24.5 in., 12.5 in.

Exercise:

Problem:

The width of a rectangle is 3 more than twice the length. The perimeter is 96 inches. Find the length and the width.

Use Properties of Triangles

In the following exercises, solve using the properties of triangles.

Exercise:**Problem:**

Find the area of a triangle with base 18 inches and height 15 inches.

Solution:

135 sq. in.

Exercise:**Problem:**

Find the area of a triangle with base 33 centimeters and height 21 centimeters.

Exercise:**Problem:**

A triangular road sign has base 30 inches and height 40 inches. What is its area?

Solution:

600 sq. in.

Exercise:

Problem:

If a triangular courtyard has sides 9 feet and 12 feet and the perimeter is 32 feet, how long is the third side?

Exercise:**Problem:**

A tile in the shape of an isosceles triangle has a base of 6 inches. If the perimeter is 20 inches, find the length of each of the other sides.

Solution:

7 in., 7 in.

Exercise:**Problem:**

Find the length of each side of an equilateral triangle with perimeter of 81 yards.

Exercise:**Problem:**

The perimeter of a triangle is 59 feet. One side of the triangle is 3 feet longer than the shortest side. The third side is 5 feet longer than the shortest side. Find the length of each side.

Solution:

17 ft., 20 ft., 22 ft.

Exercise:**Problem:**

One side of a triangle is three times the smallest side. The third side is 9 feet more than the shortest side. The perimeter is 39 feet. Find the lengths of all three sides.

Use Properties of Trapezoids

In the following exercises, solve using the properties of trapezoids.

Exercise:

Problem:

The height of a trapezoid is 8 feet and the bases are 11 and 14 feet.
What is the area?

Solution:

100 sq. ft.

Exercise:

Problem:

The height of a trapezoid is 5 yards and the bases are 7 and 10 yards.
What is the area?

Exercise:

Problem:

Find the area of the trapezoid with height 25 meters and bases 32.5 and 21.5 meters.

Solution:

675 sq. m

Exercise:

Problem:

A flag is shaped like a trapezoid with height 62 centimeters and the bases are 91.5 and 78.1 centimeters. What is the area of the flag?

[Solve Geometry Applications: Circles and Irregular Figures](#)

Use Properties of Circles

In the following exercises, solve using the properties of circles. Round answers to the nearest hundredth.

Exercise:

Problem: A circular mosaic has radius 3 meters. Find the

- Ⓐ circumference
- Ⓑ area of the mosaic

Solution:

- Ⓐ 18.84 m
- Ⓑ 28.26 sq. m

Exercise:

Problem: A circular fountain has radius 8 feet. Find the

- Ⓐ circumference
- Ⓑ area of the fountain

Exercise:

Problem:

Find the diameter of a circle with circumference 150.72 inches.

Solution:

48 in.

Exercise:

Problem:

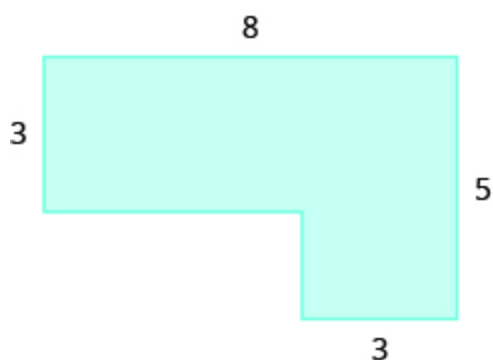
Find the radius of a circle with circumference 345.4 centimeters

Find the Area of Irregular Figures

In the following exercises, find the area of each shaded region.

Exercise:

Problem:

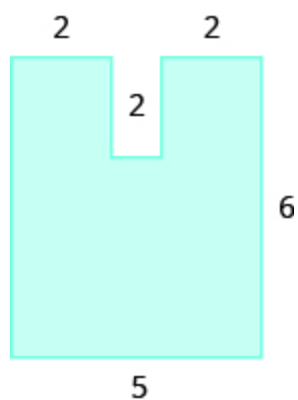


Solution:

30 sq. units

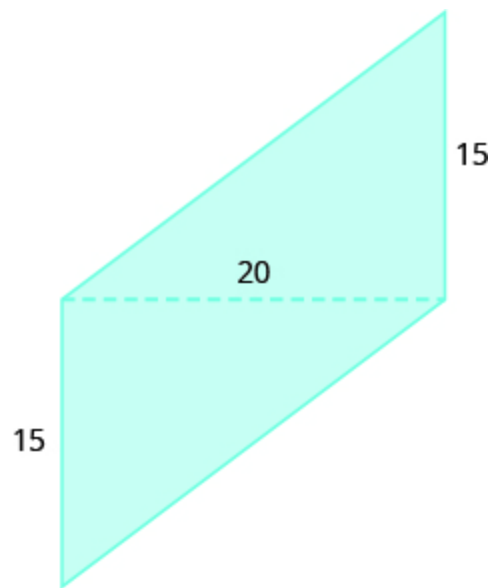
Exercise:

Problem:



Exercise:

Problem:

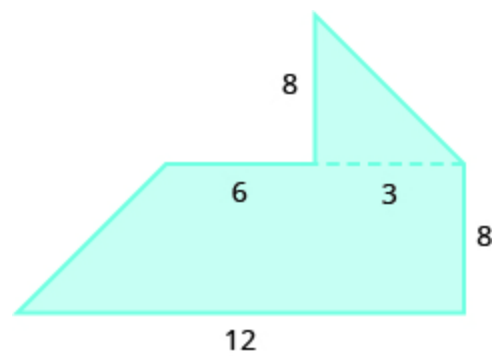


Solution:

300 sq. units

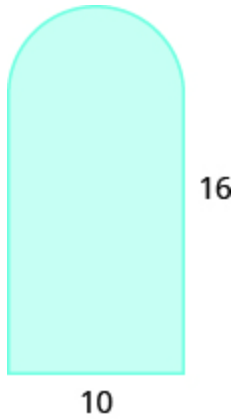
Exercise:

Problem:



Exercise:

Problem:

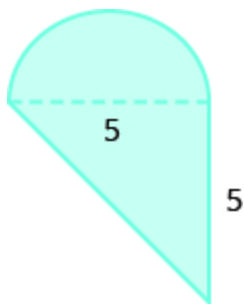


Solution:

199.25 sq. units

Exercise:

Problem:



Solve Geometry Applications: Volume and Surface Area

Find Volume and Surface Area of Rectangular Solids

In the following exercises, find the

- Ⓐ volume
- Ⓑ surface area of the rectangular solid

Exercise:

Problem:

a rectangular solid with length 14 centimeters, width 4.5 centimeters, and height 10 centimeters

Solution:

- Ⓐ 630 cu. cm
- Ⓑ 496 sq. cm

Exercise:

Problem: a cube with sides that are 3 feet long

Exercise:

Problem: a cube of tofu with sides 2.5 inches

Solution:

- Ⓐ 15.625 cu. in.
- Ⓑ 37.5 sq. in.

Exercise:**Problem:**

a rectangular carton with length 32 inches, width 18 inches, and height 10 inches

Find Volume and Surface Area of Spheres

In the following exercises, find the

- Ⓐ volume
- Ⓑ surface area of the sphere.

Exercise:

Problem: a sphere with radius 4 yards

Solution:

- Ⓐ 267.95 cu. yd.
- Ⓑ 200.96 sq. yd.

Exercise:

Problem: a sphere with radius 12 meters

Exercise:

Problem: a baseball with radius 1.45 inches

Solution:

- Ⓐ 12.76 cu. in.
- Ⓑ 26.41 sq. in.

Exercise:

Problem: a soccer ball with radius 22 centimeters

Find Volume and Surface Area of Cylinders

In the following exercises, find the

- Ⓐ volume
- Ⓑ surface area of the cylinder

Exercise:

Problem: a cylinder with radius 2 yards and height 6 yards

Solution:

- Ⓐ 75.36 cu. yd.
- Ⓑ 100.48 sq. yd.

Exercise:

Problem: a cylinder with diameter 18 inches and height 40 inches

Exercise:

Problem:

a juice can with diameter 8 centimeters and height 15 centimeters

Solution:

- Ⓐ 753.6 cu. cm
- Ⓑ 477.28 sq. cm

Exercise:

Problem:

a cylindrical pylon with diameter 0.8 feet and height 2.5 feet

Find Volume of Cones

In the following exercises, find the volume of the cone.

Exercise:

Problem: a cone with height 5 meters and radius 1 meter

Solution:

5.233 cu. m

Exercise:

Problem: a cone with height 24 feet and radius 8 feet

Exercise:

Problem:

a cone-shaped water cup with diameter 2.6 inches and height 2.6 inches

Solution:

4.599 cu. in.

Exercise:

Problem:

a cone-shaped pile of gravel with diameter 6 yards and height 5 yards

Solve a Formula for a Specific Variable

Use the Distance, Rate, and Time Formula

In the following exercises, solve using the formula for distance, rate, and time.

Exercise:

Problem:

A plane flew 4 hours at 380 miles per hour. What distance was covered?

Solution:

1520 miles

Exercise:

Problem:

Gus rode his bike for $1\frac{1}{2}$ hours at 8 miles per hour. How far did he ride?

Exercise:**Problem:**

Jack is driving from Bangor to Portland at a rate of 68 miles per hour. The distance is 107 miles. To the nearest tenth of an hour, how long will the trip take?

Solution:

1.6 hours

Exercise:**Problem:**

Jasmine took the bus from Pittsburgh to Philadelphia. The distance is 305 miles and the trip took 5 hours. What was the speed of the bus?

Solve a Formula for a Specific Variable

In the following exercises, use the formula $d = rt$.

Exercise:

Problem: Solve for t :

- Ⓐ when $d = 403$ and $r = 65$
 - Ⓑ in general
-

Solution:

- Ⓐ $t = 6.2$
- Ⓑ $t = \frac{d}{r}$

Exercise:

Problem: Solve for r :

- Ⓐ when $d = 750$ and $t = 15$
- Ⓑ in general

In the following exercises, use the formula $A = \frac{1}{2}bh$.

Exercise:

Problem: Solve for b :

- Ⓐ when $A = 416$ and $h = 32$
- Ⓑ in general

Solution:

- Ⓐ $b = 26$
- Ⓑ $b = \frac{2A}{h}$

Exercise:

Problem: Solve for h :

- Ⓐ when $A = 48$ and $b = 8$
- Ⓑ in general

In the following exercises, use the formula $I = Prt$.

Exercise:

Problem: Solve for the principal, P , for:

- Ⓐ $I = \$720, r = 4\%, t = 3$ years
 - Ⓑ in general
-

Solution:

- Ⓐ $P = \$6000$
- Ⓑ $P = \frac{I}{(r \cdot t)}$

Exercise:

Problem: Solve for the time, t for:

- Ⓐ $I = \$3630, P = \$11,000, r = 5.5\%$
- Ⓑ in general

In the following exercises, solve.

Exercise:

Problem: Solve the formula $6x + 5y = 20$ for y :

- Ⓐ when $x = 0$
 - Ⓑ in general
-

Solution:

- Ⓐ $y = 4$
- Ⓑ $y = \frac{20-6x}{5}$

Exercise:

Problem: Solve the formula $2x + y = 15$ for y :

- Ⓐ when $x = -5$

ⓑ in general

Exercise:

Problem: Solve $a + b = 90$ for a .

Solution:

$$a = 90 - b$$

Exercise:

Problem: Solve $180 = a + b + c$ for a .

Exercise:

Problem: Solve the formula $4x + y = 17$ for y .

Solution:

$$y = 17 - 4x$$

Exercise:

Problem: Solve the formula $-3x + y = -6$ for y .

Exercise:

Problem: Solve the formula $P = 2L + 2W$ for W .

Solution:

$$W = \frac{P-2L}{2}$$

Exercise:

Problem: Solve the formula $V = LWH$ for H .

Exercise:

Problem:

Describe how you have used two topics from this chapter in your life outside of math class during the past month.

Chapter Practice Test**Exercise:****Problem:**

Four-fifths of the people on a hike are children. If there are 12 children, what is the total number of people on the hike?

Exercise:**Problem:**

The sum of 13 and twice a number is -19 . Find the number.

Solution:

-16

Exercise:**Problem:**

One number is 3 less than another number. Their sum is 65. Find the numbers.

Exercise:**Problem:**

Bonita has \$2.95 in dimes and quarters in her pocket. If she has 5 more dimes than quarters, how many of each coin does she have?

Solution:

7 quarters, 12 dimes

Exercise:

Problem:

At a concert, \$1600 in tickets were sold. Adult tickets were \$9 each and children's tickets were \$4 each. If the number of adult tickets was 30 fewer than twice the number of children's tickets, how many of each kind were sold?

Exercise:

Problem: Find the complement of a 52° angle.

Solution:

38°

Exercise:

Problem:

The measure of one angle of a triangle is twice the measure of the smallest angle. The measure of the third angle is 14 more than the measure of the smallest angle. Find the measures of all three angles.

Exercise:

Problem:

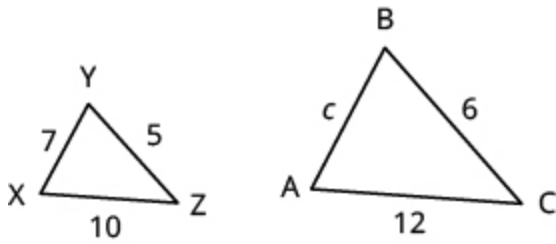
The perimeter of an equilateral triangle is 145 feet. Find the length of each side.

Solution:

48.3

Exercise:

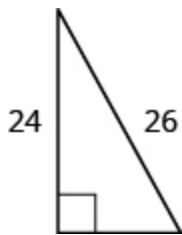
Problem: $\triangle ABC$ is similar to $\triangle XYZ$. Find the length of side c .



Exercise:

Problem:

Find the length of the missing side. Round to the nearest tenth, if necessary.



Solution:

10

Exercise:

Problem:

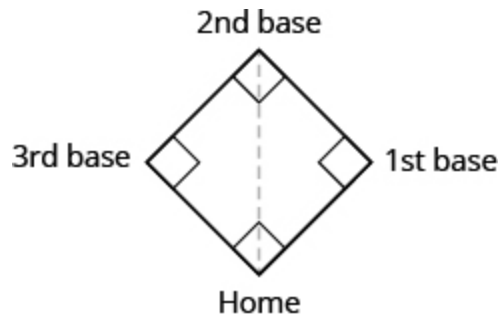
Find the length of the missing side. Round to the nearest tenth, if necessary.



Exercise:

Problem:

A baseball diamond is shaped like a square with sides 90 feet long. How far is it from home plate to second base, as shown?



Solution:

127.3 ft

Exercise:**Problem:**

The length of a rectangle is 2 feet more than five times the width. The perimeter is 40 feet. Find the dimensions of the rectangle.

Exercise:**Problem:**

A triangular poster has base 80 centimeters and height 55 centimeters. Find the area of the poster.

Solution:

2200 square centimeters

Exercise:**Problem:**

A trapezoid has height 14 inches and bases 20 inches and 23 inches. Find the area of the trapezoid.

Exercise:

Problem:

A circular pool has diameter 90 inches. What is its circumference?
Round to the nearest *tenth*.

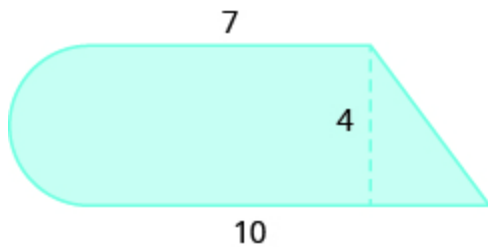
Solution:

282.6 inches

Exercise:

Problem:

Find the area of the shaded region. Round to the nearest tenth.



Exercise:

Problem:

Find the volume of a rectangular room with width 12 feet, length 15 feet, and height 8 feet.

Solution:

1440

Exercise:

Problem:

A coffee can is shaped like a cylinder with height 7 inches and radius 5 inches. Find (a) the surface area and (b) the volume of the can. Round to the nearest tenth.

Exercise:**Problem:**

A traffic cone has height 75 centimeters. The radius of the base is 20 centimeters. Find the volume of the cone. Round to the nearest tenth.

Solution:

31,400 cubic inches

Exercise:**Problem:**

Leon drove from his house in Cincinnati to his sister's house in Cleveland. He drove at a uniform rate of 63 miles per hour and the trip took 4 hours. What was the distance?

Exercise:**Problem:**

The Catalina Express takes $1\frac{1}{2}$ hours to travel from Long Beach to Catalina Island, a distance of 22 miles. To the nearest tenth, what is the speed of the boat?

Solution:

14.7 miles per hour

Exercise:

Problem: Use the formula $I = Prt$ to solve for the principal, P , for:

- Ⓐ $I = \$1380$, $r = 5\%$, $t = 3$ years
- Ⓑ in general

Exercise:

Problem: Solve the formula $A = \frac{1}{2}bh$ for h :

- Ⓐ when $A = 1716$ and $b = 66$
 - Ⓑ in general
-

Solution:

- Ⓐ height = 52
- Ⓑ $h = \frac{2A}{b}$

Exercise:

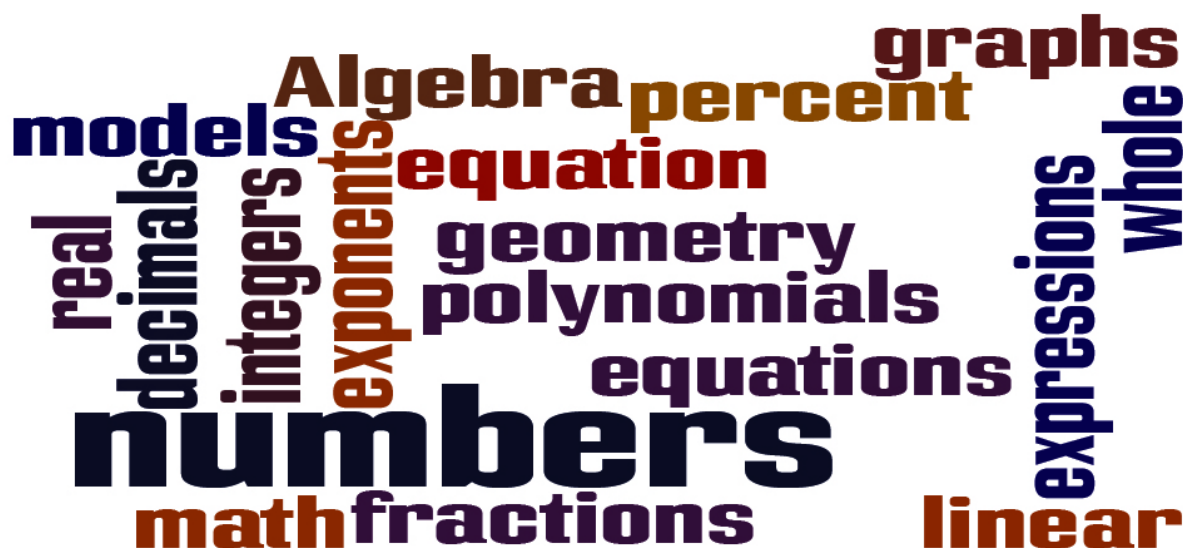
Problem: Solve $x + 5y = 14$ for y .

Introduction to the Language of Algebra

class="introduction"

--

Algebra
has a
language
of its own.
The
picture
shows just
some of
the words
you may
see and
use in your
study of
Prealgebra



You may not realize it, but you already use algebra every day. Perhaps you figure out how much to tip a server in a restaurant. Maybe you calculate the amount of change you should get when you pay for something. It could

even be when you compare batting averages of your favorite players. You can describe the algebra you use in specific words, and follow an orderly process. In this chapter, you will explore the words used to describe algebra and start on your path to solving algebraic problems easily, both in class and in your everyday life.

Use the Language of Algebra

By the end of this section, you will be able to:

- Use variables and algebraic symbols
- Identify expressions and equations
- Simplify expressions with exponents
- Simplify expressions using the order of operations

Note:

Before you get started, take this readiness quiz.

1. Add: $43 + 69$.

If you missed this problem, review [\[link\]](#).

2. Multiply: $(896)201$.

If you missed this problem, review [\[link\]](#).

3. Divide: $7,263 \div 9$.

If you missed this problem, review [\[link\]](#).

Use Variables and Algebraic Symbols

Greg and Alex have the same birthday, but they were born in different years. This year Greg is 20 years old and Alex is 23, so Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right?

In the language of algebra, we say that Greg's age and Alex's age are variable and the three is a constant. The ages change, or vary, so age is a variable. The 3 years between them always stays the same, so the age difference is the constant.

In algebra, letters of the alphabet are used to represent variables. Suppose we call Greg's age g . Then we could use $g + 3$ to represent Alex's age. See [\[link\]](#).

Greg's age	Alex's age
12	15

Greg's age	Alex's age
20	23
35	38
g	$g + 3$

Letters are used to represent variables. Letters often used for variables are x , y , a , b , and c .

Note:

Variables and Constants

A variable is a letter that represents a number or quantity whose value may change.

A constant is a number whose value always stays the same.

To write algebraically, we need some symbols as well as numbers and variables. There are several types of symbols we will be using. In [Whole Numbers](#), we introduced the symbols for the four basic arithmetic operations: addition, subtraction, multiplication, and division. We will summarize them here, along with words we use for the operations and the result.

Operation	Notation	Say:	The result is...
Addition	$a + b$	a plus b	the sum of a and b
Subtraction	$a - b$	a minus b	the difference of a and b
Multiplication	$a \cdot b$, $(a)(b)$, $(a)b$, $a(b)$	a times b	The product of a and b
Division	$a \div b$, a/b , $\frac{a}{b}$, $b \overline{)a}$	a divided by b	The quotient of a and b

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ (three times y) or $3 \cdot x \cdot y$ (three times x times y)?

To make it clear, use $\cdot$ or parentheses for multiplication.

We perform these operations on two numbers. When translating from symbolic form to words, or from words to symbolic form, pay attention to the words *of* or *and* to help you find the numbers.

- The *sum of 5 and 3* means add 5 plus 3, which we write as $5 + 3$.
- The *difference of 9 and 2* means subtract 9 minus 2, which we write as $9 - 2$.
- The *product of 4 and 8* means multiply 4 times 8, which we can write as $4 \cdot 8$.
- The *quotient of 20 and 5* means divide 20 by 5, which we can write as $20 \div 5$.

Example:

Exercise:

Problem: Translate from algebra to words:

- (a) $12 + 14$
- (b) $(30)(5)$
- (c) $64 \div 8$
- (d) $x - y$

Solution:

Solution

(a)

$12 + 14$

12 plus 14

the sum of twelve and fourteen

⑥

$$(30)(5)$$

30 times 5

the product of thirty and five

⑦

$$64 \div 8$$

64 divided by 8

the quotient of sixty-four and eight

⑧

$$x - y$$

x minus y

the difference of x and y

Note:

Exercise:

Problem: Translate from algebra to words.

⑨ $18 + 11$

- ⓑ $(27)(9)$
- ⓒ $84 \div 7$
- ⓓ $p - q$

Solution:

- ⓐ 18 plus 11; the sum of eighteen and eleven
- ⓑ 27 times 9; the product of twenty-seven and nine
- ⓒ 84 divided by 7; the quotient of eighty-four and seven
- ⓓ p minus q ; the difference of p and q

Note:

Exercise:

Problem: Translate from algebra to words.

- ⓐ $47 - 19$
- ⓑ $72 \div 9$
- ⓒ $m + n$
- ⓓ $(13)(7)$

Solution:

- ⓐ 47 minus 19; the difference of forty-seven and nineteen
- ⓑ 72 divided by 9; the quotient of seventy-two and nine
- ⓒ m plus n ; the sum of m and n
- ⓓ 13 times 7; the product of thirteen and seven

When two quantities have the same value, we say they are equal and connect them with an *equal sign*.

Note:

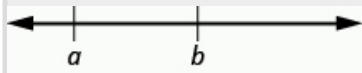
Equality Symbol

$a = b$ is read a is equal to b

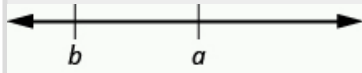
The symbol $=$ is called the equal sign.

An inequality is used in algebra to compare two quantities that may have different values. The number line can help you understand inequalities. Remember that on the number line the numbers get larger as they go from left to right. So if we know that b is greater than a , it means that b is to the right of a on the number line. We use the symbols “ $<$ ” and “ $>$ ” for inequalities.

Note:
 Inequality
 $a < b$ is read a is less than b
 a is to the left of b on the number line



$a > b$ is read a is greater than b
 a is to the right of b on the number line



The expressions $a < b$ and $a > b$ can be read from left-to-right or right-to-left, though in English we usually read from left-to-right. In general,

Equation:
 $a < b$ is equivalent to $b > a$.
 $a > b$ is equivalent to $b < a$.

For example, $7 < 11$ is equivalent to $11 > 7$.
 For example, $17 > 4$ is equivalent to $4 < 17$.

When we write an inequality symbol with a line under it, such as $a \leq b$, it means $a < b$ or $a = b$. We read this a is less than or equal to b . Also, if we put a slash through an equal sign, $\neq$, it means not equal.

We summarize the symbols of equality and inequality in [\[link\]](#).

Algebraic Notation	Say
$a = b$	a is equal to b

Algebraic Notation	Say
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b

Note:

Symbols $<$ and $>$

The symbols $<$ and $>$ each have a smaller side and a larger side.

smaller side $<$ larger side

larger side $>$ smaller side

The smaller side of the symbol faces the smaller number and the larger faces the larger number.

Example:

Exercise:

Problem: Translate from algebra to words:

(a) $20 \leq 35$

(b) $11 \neq 15 - 3$

(c) $9 > 10 \div 2$

(d) $x + 2 < 10$

Solution:

Solution

(a)

$$20 \leq 35$$

20 is less than or equal to 35

⑥

$$11 \neq 15 - 3$$

11 is not equal to 15 minus 3

⑦

$$9 > 10 \div 2$$

9 is greater than 10 divided by 2

⑧

$$x + 2 < 10$$

x plus 2 is less than 10

Note:

Exercise:

Problem: Translate from algebra to words.

- Ⓐ $14 \leq 27$
- Ⓑ $19 - 2 \neq 8$
- Ⓒ $12 > 4 \div 2$
- Ⓓ $x - 7 < 1$

Solution:

- Ⓐ fourteen is less than or equal to twenty-seven
- Ⓑ nineteen minus two is not equal to eight
- Ⓒ twelve is greater than four divided by two
- Ⓓ x minus seven is less than one

Note:

Exercise:

Problem: Translate from algebra to words.

- Ⓐ $19 \geq 15$
- Ⓑ $7 = 12 - 5$
- Ⓒ $15 \div 3 < 8$
- Ⓓ $y - 3 > 6$

Solution:

- Ⓐ nineteen is greater than or equal to fifteen
- Ⓑ seven is equal to twelve minus five
- Ⓒ fifteen divided by three is less than eight
- Ⓓ y minus three is greater than six

Example:

Exercise:

Problem:

The information in [\[link\]](#) compares the fuel economy in miles-per-gallon (mpg) of several cars. Write the appropriate symbol =, <, or > in each expression to compare the fuel economy of the cars.

Car					
Fuel economy (mpg)	48	27	28	26	27

(credit: modification of work by Bernard Goldbach, Wikimedia Commons)

- (a) MPG of Prius_____ MPG of Mini Cooper
- (b) MPG of Versa_____ MPG of Fit
- (c) MPG of Mini Cooper_____ MPG of Fit
- (d) MPG of Corolla_____ MPG of Versa
- (e) MPG of Corolla_____ MPG of Prius

Solution:
Solution

(a)	
	MPG of Prius_____MPG of Mini Cooper
Find the values in the chart.	48_____27
Compare.	48 > 27
	MPG of Prius > MPG of Mini Cooper

⑥	
	MPG of Versa____MPG of Fit
Find the values in the chart.	26____27
Compare.	$26 < 27$
	MPG of Versa < MPG of Fit

⑦	
	MPG of Mini Cooper____MPG of Fit
Find the values in the chart.	27____27
Compare.	$27 = 27$
	MPG of Mini Cooper = MPG of Fit

⑧	
	MPG of Corolla____MPG of Versa
Find the values in the chart.	28____26
Compare.	$28 > 26$
	MPG of Corolla > MPG of Versa

④	
	MPG of Corolla____MPG of Prius
Find the values in the chart.	28____48
Compare.	$28 < 48$
	MPG of Corolla < MPG of Prius

Note:

Exercise:

Problem: Use [\[link\]](#) to fill in the appropriate symbol, =, <, or >.

- ④ MPG of Prius____MPG of Versa
- ④ MPG of Mini Cooper____ MPG of Corolla

Solution:

- ④ >
- ④ >

Note:

Exercise:

Problem: Use [\[link\]](#) to fill in the appropriate symbol, =, <, or >.

- ④ MPG of Fit____ MPG of Prius
- ④ MPG of Corolla _____ MPG of Fit

Solution:

- ④ <
- ④ <

Grouping symbols in algebra are much like the commas, colons, and other punctuation marks in written language. They indicate which expressions are to be kept together and separate from other expressions. [\[link\]](#) lists three of the most commonly used grouping symbols in algebra.

Common Grouping Symbols	
parentheses	()
brackets	[]
braces	{ }

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.

Equation:

8(14 − 8)

21 − 3[2 + 4(9 − 8)]

24 ÷ {13 − 2[1(6 − 5) + 4]}

Identify Expressions and Equations

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb.

In algebra, we have *expressions* and *equations*. An expression is like a phrase. Here are some examples of expressions and how they relate to word phrases:

Expression	Words	Phrase
------------	-------	--------

Expression	Words	Phrase
$3 + 5$	3 plus 5	the sum of three and five
$n - 1$	n minus one	the difference of n and one
$6 \cdot 7$	6 times 7	the product of six and seven
$\frac{x}{y}$	x divided by y	the quotient of x and y

Notice that the phrases do not form a complete sentence because the phrase does not have a verb. An equation is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb. Here are some examples of equations:

Equation	Sentence
$3 + 5 = 8$	The sum of three and five is equal to eight.
$n - 1 = 14$	n minus one equals fourteen.
$6 \cdot 7 = 42$	The product of six and seven is equal to forty-two.
$x = 53$	x is equal to fifty-three.
$y + 9 = 2y - 3$	y plus nine is equal to two y minus three.

Note:

Expressions and Equations

An **expression** is a number, a variable, or a combination of numbers and variables and operation symbols.

An **equation** is made up of two expressions connected by an equal sign.

Example:

Exercise:

Problem: Determine if each is an expression or an equation:

- Ⓐ $16 - 6 = 10$
- Ⓑ $4 \cdot 2 + 1$
- Ⓒ $x \div 25$
- Ⓓ $y + 8 = 40$

Solution:
Solution

Ⓐ $16 - 6 = 10$	This is an equation—two expressions are connected with an equal sign.
Ⓑ $4 \cdot 2 + 1$	This is an expression—no equal sign.
Ⓒ $x \div 25$	This is an expression—no equal sign.
Ⓓ $y + 8 = 40$	This is an equation—two expressions are connected with an equal sign.

Note:
Exercise:

Problem: Determine if each is an expression or an equation:

- Ⓐ $23 + 6 = 29$
- Ⓑ $7 \cdot 3 - 7$

Solution:

- Ⓐ equation
- Ⓑ expression

Note:

Exercise:

Problem: Determine if each is an expression or an equation:

$$y \div 14$$

$$x - 6 = 21$$

Solution:

Ⓐ expression

Ⓑ equation

Simplify Expressions with Exponents

To simplify a numerical expression means to do all the math possible. For example, to simplify $4 \cdot 2 + 1$ we'd first multiply $4 \cdot 2$ to get 8 and then add the 1 to get 9. A good habit to develop is to work down the page, writing each step of the process below the previous step. The example just described would look like this:

Equation:

$$4 \cdot 2 + 1$$

Equation:

$$8 + 1$$

Equation:

$$9$$

Suppose we have the expression $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. We could write this more compactly using exponential notation. Exponential notation is used in algebra to represent a quantity multiplied by itself several times. We write $2 \cdot 2 \cdot 2$ as 2^3 and $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as 2^9 . In expressions such as 2^3 , the 2 is called the **base** and the 3 is called the exponent. The exponent tells us how many factors of the base we have to multiply.

base $\rightarrow 2^3 \leftarrow$ exponent

Equation:

means multiply three factors of 2

We say 2^3 is in exponential notation and $2 \cdot 2 \cdot 2$ is in expanded notation.

Note:

Exponential Notation

For any expression a^n , a is a factor multiplied by itself n times if n is a positive integer.

Equation:

a^n means multiply n factors of a

base $\rightarrow a^n \leftarrow$ exponent

$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ factors}}$

The expression a^n is read a to the n^{th} power.

For powers of $n = 2$ and $n = 3$, we have special names.

Equation:

a^2 is read as " a squared"

a^3 is read as " a cubed"

[\[link\]](#) lists some examples of expressions written in exponential notation.

Exponential Notation	In Words
7^2	7 to the second power, or 7 squared
5^3	5 to the third power, or 5 cubed
9^4	9 to the fourth power

Exponential Notation	In Words
12^5	12 to the fifth power

Example:

Exercise:

Problem: Write each expression in exponential form:

- (a) $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$
- (b) $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
- (c) $x \cdot x \cdot x \cdot x$
- (d) $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

Solution:

Solution

(a) The base 16 is a factor 7 times.	16^7
(b) The base 9 is a factor 5 times.	9^5
(c) The base x is a factor 4 times.	x^4
(d) The base a is a factor 8 times.	a^8

Note:

Exercise:

Problem: Write each expression in exponential form:

$$41 \cdot 41 \cdot 41 \cdot 41 \cdot 41$$

Solution:

$$41^5$$

Note:

Exercise:

Problem: Write each expression in exponential form:

$$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$$

Solution:

$$7^9$$

Example:

Exercise:

Problem: Write each exponential expression in expanded form:

Ⓐ 8^6

Ⓑ x^5

Solution:

Solution

Ⓐ The base is 8 and the exponent is 6, so 8^6 means $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

Ⓑ The base is x and the exponent is 5, so x^5 means $x \cdot x \cdot x \cdot x \cdot x$

Note:

Exercise:

Problem: Write each exponential expression in expanded form:

Ⓐ 4^8

Ⓑ a^7

Solution:

- Ⓐ $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
- Ⓑ $a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a$

Note:

Exercise:

Problem: Write each exponential expression in expanded form:

- Ⓐ 8^8
- Ⓑ b^6

Solution:

- Ⓐ $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$
- Ⓑ $b \cdot b \cdot b \cdot b \cdot b \cdot b$

To simplify an exponential expression without using a calculator, we write it in expanded form and then multiply the factors.

Example:

Exercise:

Problem: Simplify: 3^4 .

Solution:
Solution

3^4

Expand the expression.	$3 \cdot 3 \cdot 3 \cdot 3$
Multiply left to right.	$9 \cdot 3 \cdot 3$
	$27 \cdot 3$
Multiply.	81

Note:

Exercise:

Problem: Simplify:

- Ⓐ 5^3
- Ⓑ 1^7

Solution:

- Ⓐ 125
- Ⓑ 1

Note:

Exercise:

Problem: Simplify:

- Ⓐ 7^2
- Ⓑ 0^5

Solution:

- Ⓐ 49
- Ⓑ 0

Simplify Expressions Using the Order of Operations

We've introduced most of the symbols and notation used in algebra, but now we need to clarify the order of operations. Otherwise, expressions may have different meanings, and they may result in different values.

For example, consider the expression:

Equation:

$$4 + 3 \cdot 7$$

Equation:

Some students say it simplifies to 49.

$$4 + 3 \cdot 7$$

Since $4 + 3$ gives 7.

$$7 \cdot 7$$

And $7 \cdot 7$ is 49.

$$49$$

Some students say it simplifies to 25.

$$4 + 3 \cdot 7$$

Since $3 \cdot 7$ is 21.

$$4 + 21$$

And $21 + 4$ makes 25.

$$25$$

Imagine the confusion that could result if every problem had several different correct answers. The same expression should give the same result. So mathematicians established some guidelines called the order of operations, which outlines the order in which parts of an expression must be simplified.

Note:

Order of Operations

When simplifying mathematical expressions perform the operations in the following order:

1. Parentheses and other Grouping Symbols

- Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.

2. Exponents

- Simplify all expressions with exponents.

3. Multiplication and Division

- Perform all multiplication and division in order from left to right. These operations have equal priority.

4. Addition and Subtraction

- Perform all addition and subtraction in order from left to right. These operations have equal priority.

Students often ask, “How will I remember the order?” Here is a way to help you remember: Take the first letter of each key word and substitute the silly phrase. **P**lease **E**xclude **M**y **D**ear **A**unt **S**ally.

Order of Operations	
Please	Parentheses
Excuse	Exponents
My Dear	Multiplication and Division
Aunt Sally	Addition and Subtraction

It’s good that ‘**My Dear**’ goes together, as this reminds us that **m**ultiplication and **d**ivision have equal priority. We do not always do multiplication before division or always do division before multiplication. We do them in order from left to right.

Similarly, ‘**Aunt Sally**’ goes together and so reminds us that **a**ddition and **s**ubtraction also have equal priority and we do them in order from left to right.

Note: Doing the Manipulative Mathematics activity Game of 24 will give you practice using the order of operations.

Example:
Exercise:

Problem: Simplify the expressions:

- Ⓐ $4 + 3 \cdot 7$
- Ⓑ $(4 + 3) \cdot 7$

Solution:
Solution

Ⓐ	
	$4 + 3 \cdot 7$
Are there any p arentheses? No.	
Are there any e xponents? No.	
Is there any m ultiplication or d ivision? Yes.	
Multiply first.	$4 + 3 \cdot 7$
Add.	$4 + 21$
	25

Ⓑ	
	$(4 + 3) \cdot 7$
Are there any p arentheses? Yes.	$(4 + 3) \cdot 7$
Simplify inside the parentheses.	

	(7)7
Are there any exponents ? No.	
Is there any multiplication or division ? Yes.	
Multiply.	49

Note:

Exercise:

Problem: Simplify the expressions:

- Ⓐ $12 - 5 \cdot 2$
- Ⓑ $(12 - 5) \cdot 2$

Solution:

- Ⓐ 2
- Ⓑ 14

Note:

Exercise:

Problem: Simplify the expressions:

- Ⓐ $8 + 3 \cdot 9$
- Ⓑ $(8 + 3) \cdot 9$

Solution:

- Ⓐ 35
- Ⓑ 99

Example:

Exercise:

Problem: Simplify:

Ⓐ $18 \div 9 \cdot 2$

Ⓑ $18 \cdot 9 \div 2$

Solution:

Solution

Ⓐ	
	$18 \div 9 \cdot 2$
Are there any p arentheses? No.	
Are there any e xponents? No.	
Is there any m ultiplication or d ivision? Yes.	
Multiply and divide from left to right. Divide.	$2 \cdot 2$
Multiply.	4

Ⓑ	
	$18 \cdot 9 \div 2$

Are there any p arentheses? No.	
Are there any e xponents? No.	
Is there any m ultiplication or d ivision? Yes.	
Multiply and divide from left to right.	
Multiply.	$162 \div 2$
Divide.	81

Note:

Exercise:

Problem: Simplify:

$$42 \div 7 \cdot 3$$

Solution:

18

Note:

Exercise:

Problem: Simplify:

$$12 \cdot 3 \div 4$$

Solution:

9

Example:

Exercise:

Problem: Simplify: $18 \div 6 + 4(5 - 2)$.

Solution:

Solution

	$18 \div 6 + 4(5 - 2)$
Parentheses? Yes, subtract first.	$18 \div 6 + 4(3)$
Exponents? No.	
Multiplication or division? Yes.	
Divide first because we multiply and divide left to right.	$3 + 4(3)$
Any other multiplication or division? Yes.	
Multiply.	$3 + 12$
Any other multiplication or division? No.	
Any addition or subtraction? Yes.	15

Note:

Exercise:

Problem: Simplify:

$$30 \div 5 + 10(3 - 2)$$

Solution:

16

Note:

Exercise:

Problem: Simplify:

$$70 \div 10 + 4(6 - 2)$$

Solution:

23

When there are multiple grouping symbols, we simplify the innermost parentheses first and work outward.

Example:

Exercise:

Problem: Simplify: $5 + 2^3 + 3[6 - 3(4 - 2)]$.

Solution:

Solution

$$5 + 2^3 + 3[6 - 3(4 - 2)]$$

Are there any parentheses (or other grouping symbol)? Yes.	
Focus on the parentheses that are inside the brackets.	$5 + 2^3 + 3[6 - 3(4 - 2)]$
Subtract.	$5 + 2^3 + 3[6 - 3(2)]$
Continue inside the brackets and multiply.	$5 + 2^3 + 3[6 - 6]$
Continue inside the brackets and subtract.	$5 + 2^3 + 3[0]$
The expression inside the brackets requires no further simplification.	
Are there any exponents? Yes.	
Simplify exponents.	$5 + 2^3 + 3[0]$
Is there any multiplication or division? Yes.	
Multiply.	$5 + 8 + 3[0]$
Is there any addition or subtraction? Yes.	
Add.	$5 + 8 + 0$
Add.	$13 + 0$
	13

Note:
Exercise:

Problem: Simplify:

$$9 + 5^3 - [4(9 + 3)]$$

Solution:

86

Note:
Exercise:

Problem: Simplify:

$$7^2 - 2[4(5 + 1)]$$

Solution:

1

Example:
Exercise:

Problem: Simplify: $2^3 + 3^4 \div 3 - 5^2$.

Solution:
Solution

	$2^3 + 3^4 \div 3 - 5^2$
If an expression has several exponents, they may be simplified in the same step.	

Simplify exponents.	$2^3 + 3^4 \div 3 - 5^2$
Divide.	$8 + 81 \div 3 - 25$
Add.	$8 + 27 - 25$
Subtract.	$35 - 25$
	10

Note:

Exercise:

Problem: Simplify:

$$3^2 + 2^4 \div 2 + 4^3$$

Solution:

81

Note:

Exercise:

Problem: Simplify:

$$6^2 - 5^3 \div 5 + 8^2$$

Solution:

75

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Order of Operations](#)
- [Order of Operations – The Basics](#)
- [Ex: Evaluate an Expression Using the Order of Operations](#)
- [Example 3: Evaluate an Expression Using The Order of Operations](#)

Key Concepts

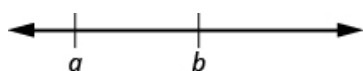
Operation	Notation	Say:	The result is...
Addition	$a + b$	a plus b	the sum of a and b
Multiplication	$a \cdot b$, $(a)(b)$, $(a)b$, $a(b)$	a times b	The product of a and b
Subtraction	$a - b$	a minus b	the difference of a and b
Division	$a \div b$, a/b , $\frac{a}{b}$, $b \overline{)a}$	a divided by b	The quotient of a and b

- **Equality Symbol**

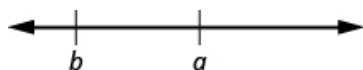
- $a = b$ is read as a is equal to b
- The symbol $=$ is called the equal sign.

- **Inequality**

- $a < b$ is read a is less than b
- a is to the left of b on the number line



- $a > b$ is read a is greater than b
- a is to the right of b on the number line



Algebraic Notation	Say
$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b

• Exponential Notation

- For any expression a^n is a factor multiplied by itself n times, if n is a positive integer.
- a^n means multiply n factors of a

base $\rightarrow a^n \leftarrow$ exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ factors}}$$

- The expression of a^n is read a to the n th power.

Order of Operations When simplifying mathematical expressions perform the operations in the following order:

- Parentheses and other Grouping Symbols: Simplify all expressions inside the parentheses or other grouping symbols, working on the innermost parentheses first.
- Exponents: Simplify all expressions with exponents.
- Multiplication and Division: Perform all multiplication and division in order from left to right. These operations have equal priority.
- Addition and Subtraction: Perform all addition and subtraction in order from left to right. These operations have equal priority.

Practice Makes Perfect

Use Variables and Algebraic Symbols

In the following exercises, translate from algebraic notation to words.

Exercise:

Problem: $16 - 9$

Solution:

16 minus 9, the difference of sixteen and nine

Exercise:

Problem: $25 - 7$

Exercise:

Problem: $5 \cdot 6$

Solution:

5 times 6, the product of five and six

Exercise:

Problem: $3 \cdot 9$

Exercise:

Problem: $28 \div 4$

Solution:

28 divided by 4, the quotient of twenty-eight and four

Exercise:

Problem: $45 \div 5$

Exercise:

Problem: $x + 8$

Solution:

x plus 8, the sum of x and eight

Exercise:

Problem: $x + 11$

Exercise:

Problem: $(2)(7)$

Solution:

2 times 7, the product of two and seven

Exercise:

Problem: $(4)(8)$

Exercise:

Problem: $14 < 21$

Solution:

fourteen is less than twenty-one

Exercise:

Problem: $17 < 35$

Exercise:

Problem: $36 \geq 19$

Solution:

thirty-six is greater than or equal to nineteen

Exercise:

Problem: $42 \geq 27$

Exercise:

Problem: $3n = 24$

Solution:

3 times n equals 24, the product of three and n equals twenty-four

Exercise:

Problem: $6n = 36$

Exercise:

Problem: $y - 1 > 6$

Solution:

y minus 1 is greater than 6, the difference of y and one is greater than six

Exercise:

Problem: $y - 4 > 8$

Exercise:

Problem: $2 \leq 18 \div 6$

Solution:

2 is less than or equal to 18 divided by 6; 2 is less than or equal to the quotient of eighteen and six

Exercise:

Problem: $3 \leq 20 \div 4$

Exercise:

Problem: $a \neq 7 \cdot 4$

Solution:

a is not equal to 7 times 4, a is not equal to the product of seven and four

Exercise:

Problem: $a \neq 1 \cdot 12$

Identify Expressions and Equations

In the following exercises, determine if each is an expression or an equation.

Exercise:

Problem: $9 \cdot 6 = 54$

Solution:

equation

Exercise:

Problem: $7 \cdot 9 = 63$

Exercise:

Problem: $5 \cdot 4 + 3$

Solution:

expression

Exercise:

Problem: $6 \cdot 3 + 5$

Exercise:

Problem: $x + 7$

Solution:

expression

Exercise:

Problem: $x + 9$

Exercise:

Problem: $y - 5 = 25$

Solution:

equation

Exercise:

Problem: $y - 8 = 32$

Simplify Expressions with Exponents

In the following exercises, write in exponential form.

Exercise:

Problem: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

Solution:

$$3^7$$

Exercise:

Problem: $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Exercise:

Problem: $x \cdot x \cdot x \cdot x \cdot x$

Solution:

$$x^5$$

Exercise:

Problem: $y \cdot y \cdot y \cdot y \cdot y \cdot y$

In the following exercises, write in expanded form.

Exercise:

Problem: 5^3

Solution:

$$125$$

Exercise:

Problem: 8^3

Exercise:

Problem: 2^8

Solution:

Exercise:**Problem:** 10^5 **Simplify Expressions Using the Order of Operations**

In the following exercises, simplify.

Exercise:**Problem:**

Ⓐ $3 + 8 \cdot 5$

Ⓑ $(3+8) \cdot 5$

Solution:

Ⓐ 43

Ⓑ 55

Exercise:**Problem:**

Ⓐ $2 + 6 \cdot 3$

Ⓑ $(2+6) \cdot 3$

Exercise:**Problem:** $2^3 - 12 \div (9 - 5)$

Solution:

5

Exercise:**Problem:** $3^2 - 18 \div (11 - 5)$ **Exercise:****Problem:** $3 \cdot 8 + 5 \cdot 2$

Solution:

34

Exercise:

Problem: $4 \cdot 7 + 3 \cdot 5$

Exercise:

Problem: $2 + 8(6 + 1)$

Solution:

58

Exercise:

Problem: $4 + 6(3 + 6)$

Exercise:

Problem: $4 \cdot 12/8$

Solution:

6

Exercise:

Problem: $2 \cdot 36/6$

Exercise:

Problem: $6 + 10/2 + 2$

Solution:

13

Exercise:

Problem: $9 + 12/3 + 4$

Exercise:

Problem: $(6 + 10) \div (2 + 2)$

Solution:

4

Exercise:

Problem: $(9 + 12) \div (3 + 4)$

Exercise:

Problem: $20 \div 4 + 6 \cdot 5$

Solution:

35

Exercise:

Problem: $33 \div 3 + 8 \cdot 2$

Exercise:

Problem: $20 \div (4 + 6) \cdot 5$

Solution:

10

Exercise:

Problem: $33 \div (3 + 8) \cdot 2$

Exercise:

Problem: $4^2 + 5^2$

Solution:

41

Exercise:

Problem: $3^2 + 7^2$

Exercise:

Problem: $(4 + 5)^2$

Solution:

81

Exercise:

Problem: $(3 + 7)^2$

Exercise:

Problem: $3(1 + 9 \cdot 6) - 4^2$

Solution:

149

Exercise:

Problem: $5(2 + 8 \cdot 4) - 7^2$

Exercise:

Problem: $2[1 + 3(10 - 2)]$

Solution:

50

Exercise:

Problem: $5[2 + 4(3 - 2)]$

Everyday Math

Exercise:

Problem:

Basketball In the 2014 NBA playoffs, the San Antonio Spurs beat the Miami Heat. The table below shows the heights of the starters on each team. Use this table to fill in the appropriate symbol ($=$, $<$, $>$).

Spurs	Height		Heat	Height
Tim Duncan	83"		Rashard Lewis	82"
Boris Diaw	80"		LeBron James	80"
Kawhi Leonard	79"		Chris Bosh	83"
Tony Parker	74"		Dwyane Wade	76"
Danny Green	78"		Ray Allen	77"

- (a) Height of Tim Duncan ____ Height of Rashard Lewis
 (b) Height of Boris Diaw ____ Height of LeBron James
 (c) Height of Kawhi Leonard ____ Height of Chris Bosh
 (d) Height of Tony Parker ____ Height of Dwyane Wade
 (e) Height of Danny Green ____ Height of Ray Allen

Exercise:

Problem:

Elevation In Colorado there are more than 50 mountains with an elevation of over 14,000 feet. The table shows the ten tallest. Use this table to fill in the appropriate inequality symbol.

Mountain	Elevation
Mt. Elbert	14,433'
Mt. Massive	14,421'
Mt. Harvard	14,420'
Blanca Peak	14,345'
La Plata Peak	14,336'
Uncompahgre Peak	14,309'

Mountain	Elevation
Crestone Peak	14,294'
Mt. Lincoln	14,286'
Grays Peak	14,270'
Mt. Antero	14,269'

- Ⓐ Elevation of La Plata Peak ____ Elevation of Mt. Antero
- Ⓑ Elevation of Blanca Peak ____ Elevation of Mt. Elbert
- Ⓒ Elevation of Gray's Peak ____ Elevation of Mt. Lincoln
- Ⓓ Elevation of Mt. Massive ____ Elevation of Crestone Peak
- Ⓔ Elevation of Mt. Harvard ____ Elevation of Uncompahgre Peak

Writing Exercises

Exercise:

Problem: Explain the difference between an expression and an equation.

Exercise:

Problem: Why is it important to use the order of operations to simplify an expression?

Self Check

- Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use variables and algebraic symbols.			
identify expressions and equations.			
simplify expressions with exponents.			
simplify expressions using the order of operations.			

⑥ If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

...no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

expressions

An expression is a number, a variable, or a combination of numbers and variables and operation symbols.

equation

An equation is made up of two expressions connected by an equal sign.

Evaluate, Simplify, and Translate Expressions

By the end of this section, you will be able to:

- Evaluate algebraic expressions
- Identify terms, coefficients, and like terms
- Simplify expressions by combining like terms
- Translate word phrases to algebraic expressions

Note:

Before you get started, take this readiness quiz.

1. Is $n \div 5$ an expression or an equation?
If you missed this problem, review [\[link\]](#).
2. Simplify 4^5 .
If you missed this problem, review [\[link\]](#).
3. Simplify $1 + 8 \cdot 9$.
If you missed this problem, review [\[link\]](#).

Evaluate Algebraic Expressions

In the last section, we simplified expressions using the order of operations. In this section, we'll evaluate expressions—again following the order of operations.

To **evaluate** an algebraic expression means to find the value of the expression when the variable is replaced by a given number. To evaluate an expression, we substitute the given number for the variable in the expression and then simplify the expression using the order of operations.

Example:

Exercise:

Problem: Evaluate $x + 7$ when

- Ⓐ $x = 3$
- Ⓑ $x = 12$

Solution:
Solution

- Ⓐ To evaluate, substitute 3 for x in the expression, and then simplify.

	$x + 7$
Substitute.	$3 + 7$
Add.	10

When $x = 3$, the expression $x + 7$ has a value of 10.

- Ⓑ To evaluate, substitute 12 for x in the expression, and then simplify.

	$x + 7$
Substitute.	$12 + 7$
Add.	19

When $x = 12$, the expression $x + 7$ has a value of 19.

Notice that we got different results for parts (a) and (b) even though we started with the same expression. This is because the values used for x were different. When we evaluate an expression, the value varies depending on the value used for the variable.

Note:

Exercise:

Problem: Evaluate:

$y + 4$ when

- (a) $y = 6$
- (b) $y = 15$

Solution:

- (a) 10
- (b) 19

Note:

Exercise:

Problem: Evaluate:

$a - 5$ when

- Ⓐ $a = 9$
- Ⓑ $a = 17$

Solution:

- Ⓐ 4
- Ⓑ 12

Example:

Exercise:

Problem: Evaluate $9x - 2$, when

- Ⓐ $x = 5$
- Ⓑ $x = 1$

Solution:

Solution

Remember ab means a times b , so $9x$ means 9 times x .

- Ⓐ To evaluate the expression when $x = 5$, we substitute 5 for x , and then simplify.

	$9x - 2$
Substitute 5 for x .	$9 \cdot 5 - 2$
Multiply.	$45 - 2$
Subtract.	43

⑥ To evaluate the expression when $x = 1$, we substitute 1 for x , and then simplify.

	$9x - 2$
Substitute 1 for x .	$9(1) - 2$
Multiply.	$9 - 2$
Subtract.	7

Notice that in part (a) that we wrote $9 \cdot 5$ and in part (b) we wrote $9(1)$. Both the dot and the parentheses tell us to multiply.

Note:

Exercise:

Problem: Evaluate:

$8x - 3$, when

(a) $x = 2$

(b) $x = 1$

Solution:

(a) 13

(b) 5

Note:

Exercise:

Problem: Evaluate:

$4y - 4$, when

(a) $y = 3$

(b) $y = 5$

Solution:

- Ⓐ 8
- Ⓑ 16

Example:

Exercise:

Problem: Evaluate x^2 when $x = 10$.

Solution:

Solution

We substitute 10 for x , and then simplify the expression.

	x^2
Substitute 10 for x .	10^2
Use the definition of exponent.	$10 \cdot 10$
Multiply.	100

When $x = 10$, the expression x^2 has a value of 100.

Note:

Exercise:

Problem: Evaluate:

x^2 when $x = 8$.

Solution:

64

Note:

Exercise:

Problem: Evaluate:

x^3 when $x = 6$.

Solution:

216

Example:

Exercise:

Problem: Evaluate 2^x when $x = 5$.

Solution:
Solution

In this expression, the variable is an exponent.

	2^x
Substitute 5 for x.	2^5
Use the definition of exponent.	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
Multiply.	32

When $x = 5$, the expression 2^x has a value of 32.

Note:

Exercise:

Problem: Evaluate:

2^x when $x = 6$.

Solution:

64

Note:

Exercise:

Problem: Evaluate:

3^x when $x = 4$.

Solution:

81

Example:

Exercise:

Problem: Evaluate $3x + 4y - 6$ when $x = 10$ and $y = 2$.

Solution:

Solution

This expression contains two variables, so we must make two substitutions.

	$3x + 4y - 6$
Substitute 10 for x and 2 for y .	$3(10) + 4(2) - 6$
Multiply.	$30 + 8 - 6$
Add and subtract left to right.	32

When $x = 10$ and $y = 2$, the expression $3x + 4y - 6$ has a value of 32.

Note:

Exercise:

Problem: Evaluate:

$2x + 5y - 4$ when $x = 11$ and $y = 3$

Solution:

33

Note:

Exercise:

Problem: Evaluate:

$5x - 2y - 9$ when $x = 7$ and $y = 8$

Solution:

10

Example:

Exercise:

Problem: Evaluate $2x^2 + 3x + 8$ when $x = 4$.

Solution:
Solution

We need to be careful when an expression has a variable with an exponent. In this expression, $2x^2$ means $2 \cdot x \cdot x$ and is different from the expression $(2x)^2$, which means $2x \cdot 2x$.

	$2x^2 + 3x + 8$
Substitute 4 for each x .	$2(4)^2 + 3(4) + 8$
Simplify 4^2 .	$2(16) + 3(4) + 8$
Multiply.	$32 + 12 + 8$
Add.	52

Note:

Exercise:

Problem: Evaluate:

$$3x^2 + 4x + 1 \text{ when } x = 3.$$

Solution:

40

Note:

Exercise:

Problem: Evaluate:

$$6x^2 - 4x - 7 \text{ when } x = 2.$$

Solution:

9

Identify Terms, Coefficients, and Like Terms

Algebraic expressions are made up of *terms*. A **term** is a constant or the product of a constant and one or more variables. Some examples of terms are 7, y , $5x^2$, $9a$, and $13xy$.

The constant that multiplies the variable(s) in a term is called the **coefficient**. We can think of the coefficient as the number *in front of* the variable. The coefficient of the term $3x$ is 3. When we write x , the

coefficient is 1, since $x = 1 \cdot x$. [\[link\]](#) gives the coefficients for each of the terms in the left column.

Term	Coefficient
7	7
$9a$	9
y	1
$5x^2$	5

An algebraic expression may consist of one or more terms added or subtracted. In this chapter, we will only work with terms that are added together. [\[link\]](#) gives some examples of algebraic expressions with various numbers of terms. Notice that we include the operation before a term with it.

Expression	Terms
7	7
y	y
$x + 7$	$x, 7$

Expression	Terms
$2x + 7y + 4$	$2x, 7y, 4$
$3x^2 + 4x^2 + 5y + 3$	$3x^2, 4x^2, 5y, 3$

Example:

Exercise:

Problem:

Identify each term in the expression $9b + 15x^2 + a + 6$. Then identify the coefficient of each term.

Solution:

Solution

The expression has four terms. They are $9b$, $15x^2$, a , and 6 .

The coefficient of $9b$ is 9.

The coefficient of $15x^2$ is 15.

Remember that if no number is written before a variable, the coefficient is 1. So the coefficient of a is 1.

The coefficient of a constant is the constant, so the coefficient of 6 is 6.

Note:

Exercise:

Problem:

Identify all terms in the given expression, and their coefficients:

$$4x + 3b + 2$$

Solution:

The terms are $4x$, $3b$, and 2 . The coefficients are 4 , 3 , and 2 .

Note:**Exercise:****Problem:**

Identify all terms in the given expression, and their coefficients:

$$9a + 13a^2 + a^3$$

Solution:

The terms are $9a$, $13a^2$, and a^3 , The coefficients are 9 , 13 , and 1 .

Some terms share common traits. Look at the following terms. Which ones seem to have traits in common?

Equation:

$$5x, 7, n^2, 4, 3x, 9n^2$$

Which of these terms are like terms?

- The terms 7 and 4 are both constant terms.
- The terms $5x$ and $3x$ are both terms with x .

- The terms n^2 and $9n^2$ both have n^2 .

Terms are called **like terms** if they have the same variables and exponents. All constant terms are also like terms. So among the terms

$5x, 7, n^2, 4, 3x, 9n^2,$

Equation:

7 and 4 are like terms.

Equation:

$5x$ and $3x$ are like terms.

Equation:

n^2 and $9n^2$ are like terms.

Note:

Like Terms

Terms that are either constants or have the same variables with the same exponents are like terms.

Example:

Exercise:

Problem: Identify the like terms:

- Ⓐ $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$
 Ⓑ $4x^2 + 2x + 5x^2 + 6x + 40x + 8xy$

Solution:

Solution

Ⓐ $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$

Look at the variables and exponents. The expression contains y^3 , x^2 , x , and constants.

The terms y^3 and $4y^3$ are like terms because they both have y^3 .

The terms $7x^2$ and $5x^2$ are like terms because they both have x^2 .

The terms 14 and 23 are like terms because they are both constants.

The term $9x$ does not have any like terms in this list since no other terms have the variable x raised to the power of 1.

Ⓑ $4x^2 + 2x + 5x^2 + 6x + 40x + 8xy$

Look at the variables and exponents. The expression contains the terms $4x^2$, $2x$, $5x^2$, $6x$, $40x$, and $8xy$

The terms $4x^2$ and $5x^2$ are like terms because they both have x^2 .

The terms $2x$, $6x$, and $40x$ are like terms because they all have x .

The term $8xy$ has no like terms in the given expression because no other terms contain the two variables xy .

Note:

Exercise:

Problem: Identify the like terms in the list or the expression:

$9, 2x^3, y^2, 8x^3, 15, 9y, 11y^2$

Solution:

$9, 15; 2x^3$ and $8x^3, y^2$, and $11y^2$

Note:

Exercise:

Problem: Identify the like terms in the list or the expression:

$$4x^3 + 8x^2 + 19 + 3x^2 + 24 + 6x^3$$

Solution:

$$4x^3 \text{ and } 6x^3; 8x^2 \text{ and } 3x^2; 19 \text{ and } 24$$

Simplify Expressions by Combining Like Terms

We can simplify an expression by combining the like terms. What do you think $3x + 6x$ would simplify to? If you thought $9x$, you would be right!

We can see why this works by writing both terms as addition problems.

$$\begin{array}{rcccl} & 3x & & + & 6x \\ \underbrace{x + x + x} & & & + & \underbrace{x + x + x + x + x + x} \\ & 9x & & & \end{array}$$

Add the coefficients and keep the same variable. It doesn't matter what x is. If you have 3 of something and add 6 more of the same thing, the result is 9 of them. For example, 3 oranges plus 6 oranges is 9 oranges. We will discuss the mathematical properties behind this later.

The expression $3x + 6x$ has only two terms. When an expression contains more terms, it may be helpful to rearrange the terms so that like terms are together. The Commutative Property of Addition says that we can change the order of addends without changing the sum. So we could rearrange the following expression before combining like terms.

$$3x + 4y - 2x + 6y$$

$$\underbrace{3x - 2x} + \underbrace{4y + 6y}$$

Now it is easier to see the like terms to be combined.

Note:

Combine like terms.

Identify like terms.

Rearrange the expression so like terms are together.

Add the coefficients of the like terms.

Example:

Exercise:

Problem: Simplify the expression: $3x + 7 + 4x + 5$.

Solution:

Solution

$$3x + 7 + 4x + 5$$

Identify the like terms.

$$3x + 7 + 4x + 5$$

Rearrange the expression, so the like terms are together.

$$3x + 4x + 7 + 5$$

Add the coefficients of the like terms.

$$\underbrace{3x + 4x}_{7x} + \underbrace{7 + 5}_{12}$$

The original expression is simplified to...

$$7x + 12$$

Note:

Exercise:

Problem: Simplify:

$$7x + 9 + 9x + 8$$

Solution:

$$16x + 17$$

Note:

Exercise:

Problem: Simplify:

$$5y + 2 + 8y + 4y + 5$$

Solution:

$$17y + 7$$

Example:

Exercise:

Problem: Simplify the expression: $7x^2 + 8x + x^2 + 4x$.

Solution:

Solution

	$7x^2 + 8x + x^2 + 4x$
Identify the like terms.	$7x^2 + 8x + x^2 + 4x$
Rearrange the expression so like terms are together.	$7x^2 + x^2 + 8x + 4x$
Add the coefficients of the like terms.	$8x^2 + 12x$

These are not like terms and cannot be combined. So $8x^2 + 12x$ is in simplest form.

Note:

Exercise:

Problem: Simplify:

$$3x^2 + 9x + x^2 + 5x$$

Solution:

$$4x^2 + 14x$$

Note:

Exercise:

Problem: Simplify:

$$11y^2 + 8y + y^2 + 7y$$

Solution:

$$12y^2 + 15y$$

Translate Words to Algebraic Expressions

In the previous section, we listed many operation symbols that are used in algebra, and then we translated expressions and equations into word phrases and sentences. Now we'll reverse the process and translate word phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. They are summarized in [\[link\]](#).

Operation	Phrase	Expression
Addition	a plus b the sum of a and b a increased by b b more than a the total of a and b b added to a	$a + b$
Subtraction	a minus b the difference of a and b b subtracted from a a decreased by b b less than a	$a - b$
Multiplication	a times b the product of a and b	$a \cdot b$, ab , $a(b)$, $(a)(b)$
Division	a divided by b the quotient of a and b the ratio of a and b b divided into a	$a \div b$, a/b , $\frac{a}{b}$, $b \overline{)a}$

Look closely at these phrases using the four operations:

- the sum *of a and b*
- the difference *of a and b*
- the product *of a and b*
- the quotient *of a and b*

Each phrase tells you to operate on two numbers. Look for the words ***of*** and ***and*** to find the numbers.

Example:

Exercise:

Problem: Translate each word phrase into an algebraic expression:

- Ⓐ the difference of 20 and 4
- Ⓑ the quotient of $10x$ and 3

Solution:**Solution**

Ⓐ The key word is *difference*, which tells us the operation is subtraction. Look for the words *of* and *and* to find the numbers to subtract.

the difference of 20 *and* 4

20 minus 4

$20 - 4$

Ⓑ The key word is *quotient*, which tells us the operation is division.

the quotient of $10x$ and 3

divide $10x$ by 3

$10x \div 3$

This can also be written as $10x/3$ or $\frac{10x}{3}$

Note:**Exercise:****Problem:**

Translate the given word phrase into an algebraic expression:

- Ⓐ the difference of 47 and 41
- Ⓑ the quotient of $5x$ and 2

Solution:

- Ⓐ $47 - 41$
- Ⓑ $5x \div 2$

Note:

Exercise:

Problem:

Translate the given word phrase into an algebraic expression:

- Ⓐ the sum of 17 and 19
- Ⓑ the product of 7 and x

Solution:

- Ⓐ $17 + 19$
- Ⓑ $7x$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight *more than* means eight added to your present age.

How old were you seven years ago? This is seven years less than your age now. You subtract 7 from your present age. Seven *less than* means seven subtracted from your present age.

Example:**Exercise:**

Problem: Translate each word phrase into an algebraic expression:

- Ⓐ Eight more than y
- Ⓑ Seven less than $9z$

Solution:**Solution**

Ⓐ The key words are *more than*. They tell us the operation is addition. *More than* means “added to”.

Eight more than y

Eight added to y

$$y + 8$$

Ⓑ The key words are *less than*. They tell us the operation is subtraction. *Less than* means “subtracted from”.

Seven less than $9z$

Seven subtracted from $9z$

$$9z - 7$$

Note:**Exercise:**

Problem: Translate each word phrase into an algebraic expression:

- Ⓐ Eleven more than x
- Ⓑ Fourteen less than $11a$

Solution:

- Ⓐ $x + 11$
- Ⓑ $11a - 14$

Note:

Exercise:

Problem: Translate each word phrase into an algebraic expression:

- Ⓐ 19 more than j
- Ⓑ 21 less than $2x$

Solution:

- Ⓐ $j + 19$
- Ⓑ $2x - 21$

Example:

Exercise:

Problem: Translate each word phrase into an algebraic expression:

- Ⓐ five times the sum of m and n
- Ⓑ the sum of five times m and n

Solution:

Solution

Ⓐ There are two operation words: *times* tells us to multiply and *sum* tells us to add. Because we are multiplying 5 times the sum, we need parentheses around the sum of m and n .

five times the sum of m and n

$$5(m + n)$$

Ⓑ To take a sum, we look for the words *of* and *and* to see what is being added. Here we are taking the sum of five times m and n .

the sum of five times m and n

$$5m + n$$

Notice how the use of parentheses changes the result. In part Ⓐ, we add first and in part Ⓑ, we multiply first.

Note:

Exercise:

Problem: Translate the word phrase into an algebraic expression:

- Ⓐ four times the sum of p and q
- Ⓑ the sum of four times p and q

Solution:

- Ⓐ $4(p + q)$
- Ⓑ $4p + q$

Note:

Exercise:

Problem: Translate the word phrase into an algebraic expression:

- Ⓐ the difference of two times x and 8
- Ⓑ two times the difference of x and 8

Solution:

- Ⓐ $2x - 8$
- Ⓑ $2(x - 8)$

Later in this course, we'll apply our skills in algebra to solving equations. We'll usually start by translating a word phrase to an algebraic expression. We'll need to be clear about what the expression will represent. We'll see how to do this in the next two examples.

Example:**Exercise:****Problem:**

The height of a rectangular window is 6 inches less than the width. Let w represent the width of the window. Write an expression for the height of the window.

Solution:**Solution**

Write a phrase about the height.	6 less than the width
Substitute w for the width.	6 less than w
Rewrite 'less than' as 'subtracted from'.	6 subtracted from w
Translate the phrase into algebra.	$w - 6$

Note:

Exercise:

Problem:

The length of a rectangle is 5 inches less than the width. Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Solution:

$$w - 5$$

Note:

Exercise:

Problem:

The width of a rectangle is 2 meters greater than the length. Let l represent the length of the rectangle. Write an expression for the width of the rectangle.

Solution:

$$l + 2$$

Example:

Exercise:

Problem:

Blanca has dimes and quarters in her purse. The number of dimes is 2 less than 5 times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution:

Solution

Write a phrase about the number of dimes.	two less than five times the number of quarters
Substitute q for the number of quarters.	2 less than five times q
Translate 5 times q .	2 less than $5q$
Translate the phrase into algebra.	$5q - 2$

Note:

Exercise:

Problem:

Geoffrey has dimes and quarters in his pocket. The number of dimes is seven less than six times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Solution:

$$6q - 7$$

Note:**Exercise:****Problem:**

Lauren has dimes and nickels in her purse. The number of dimes is eight more than four times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Solution:

$$4n + 8$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Algebraic Expression Vocabulary](#)

Key Concepts

- Combine like terms.

Identify like terms.

Rearrange the expression so like terms are together.

Add the coefficients of the like terms

Practice Makes Perfect

Evaluate Algebraic Expressions

In the following exercises, evaluate the expression for the given value.

Exercise:

Problem: $7x + 8$ when $x = 2$

Solution:

22

Exercise:

Problem: $9x + 7$ when $x = 3$

Exercise:

Problem: $5x - 4$ when $x = 6$

Solution:

26

Exercise:

Problem: $8x - 6$ when $x = 7$

Exercise:

Problem: x^2 when $x = 12$

Solution:

144

Exercise:

Problem: x^3 when $x = 5$

Exercise:

Problem: x^5 when $x = 2$

Solution:

32

Exercise:

Problem: x^4 when $x = 3$

Exercise:

Problem: 3^x when $x = 3$

Solution:

27

Exercise:

Problem: 4^x when $x = 2$

Exercise:

Problem: $x^2 + 3x - 7$ when $x = 4$

Solution:

21

Exercise:

Problem: $x^2 + 5x - 8$ when $x = 6$

Exercise:

Problem: $2x + 4y - 5$ when $x = 7, y = 8$

Solution:

41

Exercise:

Problem: $6x + 3y - 9$ when $x = 6, y = 9$

Exercise:

Problem: $(x - y)^2$ when $x = 10, y = 7$

Solution:

9

Exercise:

Problem: $(x + y)^2$ when $x = 6, y = 9$

Solution:

225

Exercise:

Problem: $a^2 + b^2$ when $a = 3, b = 8$

Solution:

Exercise:

Problem: $r^2 - s^2$ when $r = 12, s = 5$

Exercise:

Problem: $2l + 2w$ when $l = 15, w = 12$

Solution:

54

Exercise:

Problem: $2l + 2w$ when $l = 18, w = 14$

Identify Terms, Coefficients, and Like Terms

In the following exercises, list the terms in the given expression.

Exercise:

Problem: $15x^2 + 6x + 2$

Solution:

$15x^2, 6x, 2$

Exercise:

Problem: $11x^2 + 8x + 5$

Exercise:

Problem: $10y^3 + y + 2$

Solution:

$$10y^3, y, 2$$

Exercise:

Problem: $9y^3 + y + 5$

In the following exercises, identify the coefficient of the given term.

Exercise:

Problem: $8a$

Solution:

$$8$$

Exercise:

Problem: $13m$

Exercise:

Problem: $5r^2$

Solution:

$$5$$

Exercise:

Problem: $6x^3$

In the following exercises, identify all sets of like terms.

Exercise:

Problem: $x^3, 8x, 14, 8y, 5, 8x^3$

Solution:

x^3 , $8x^3$ and 14, 5

Exercise:

Problem: $6z$, $3w^2$, 1, $6z^2$, $4z$, w^2

Exercise:

Problem: $9a$, a^2 , $16ab$, $16b^2$, $4ab$, $9b^2$

Solution:

$16ab$ and $4ab$; $16b^2$ and $9b^2$

Exercise:

Problem: 3, $25r^2$, $10s$, $10r$, $4r^2$, $3s$

Simplify Expressions by Combining Like Terms

In the following exercises, simplify the given expression by combining like terms.

Exercise:

Problem: $10x + 3x$

Solution:

$13x$

Exercise:

Problem: $15x + 4x$

Exercise:

Problem: $17a + 9a$

Solution:

$$26a$$

Exercise:

Problem: $18z + 9z$

Exercise:

Problem: $4c + 2c + c$

Solution:

$$7c$$

Exercise:

Problem: $6y + 4y + y$

Exercise:

Problem: $9x + 3x + 8$

Solution:

$$12x + 8$$

Exercise:

Problem: $8a + 5a + 9$

Exercise:

Problem: $7u + 2 + 3u + 1$

Solution:

$$10u + 3$$

Exercise:

Problem: $8d + 6 + 2d + 5$

Exercise:

Problem: $7p + 6 + 5p + 4$

Solution:

$$12p + 10$$

Exercise:

Problem: $8x + 7 + 4x - 5$

Exercise:

Problem: $10a + 7 + 5a - 2 + 7a - 4$

Solution:

$$22a + 1$$

Exercise:

Problem: $7c + 4 + 6c - 3 + 9c - 1$

Exercise:

Problem: $3x^2 + 12x + 11 + 14x^2 + 8x + 5$

Solution:

$$17x^2 + 20x + 16$$

Exercise:

Problem: $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

Translate English Phrases into Algebraic Expressions

In the following exercises, translate the given word phrase into an algebraic expression.

Exercise:

Problem: The sum of 8 and 12

Solution:

$$8 + 12$$

Exercise:

Problem: The sum of 9 and 1

Exercise:

Problem: The difference of 14 and 9

Solution:

$$14 - 9$$

Exercise:

Problem: 8 less than 19

Exercise:

Problem: The product of 9 and 7

Solution:

$$9 \cdot 7$$

Exercise:

Problem: The product of 8 and 7

Exercise:

Problem: The quotient of 36 and 9

Solution:

$$36 \div 9$$

Exercise:

Problem: The quotient of 42 and 7

Exercise:

Problem: The difference of x and 4

Solution:

$$x - 4$$

Exercise:

Problem: 3 less than x

Exercise:

Problem: The product of 6 and y

Solution:

$$6y$$

Exercise:

Problem: The product of 9 and y

Exercise:

Problem: The sum of $8x$ and $3x$

Solution:

$$8x + 3x$$

Exercise:

Problem: The sum of $13x$ and $3x$

Exercise:

Problem: The quotient of y and 3

Solution:

$$\frac{y}{3}$$

Exercise:

Problem: The quotient of y and 8

Exercise:

Problem: Eight times the difference of y and nine

Solution:

$$8(y - 9)$$

Exercise:

Problem: Seven times the difference of y and one

Exercise:

Problem: Five times the sum of x and y

Solution:

$$5(x + y)$$

Exercise:

Problem: Nine times five less than twice x

In the following exercises, write an algebraic expression.

Exercise:

Problem:

Adele bought a skirt and a blouse. The skirt cost \$15 more than the blouse. Let b represent the cost of the blouse. Write an expression for the cost of the skirt.

Solution:

$$b + 15$$

Exercise:

Problem:

Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let c represent the number of classical CDs. Write an expression for the number of rock CDs.

Exercise:

Problem:

The number of girls in a second-grade class is 4 less than the number of boys. Let b represent the number of boys. Write an expression for the number of girls.

Solution:

$$b - 4$$

Exercise:**Problem:**

Marcella has 6 fewer male cousins than female cousins. Let f represent the number of female cousins. Write an expression for the number of boy cousins.

Exercise:**Problem:**

Greg has nickels and pennies in his pocket. The number of pennies is seven less than twice the number of nickels. Let n represent the number of nickels. Write an expression for the number of pennies.

Solution:

$$2n - 7$$

Exercise:**Problem:**

Jeannette has \$5 and \$10 bills in her wallet. The number of fives is three more than six times the number of tens. Let t represent the number of tens. Write an expression for the number of fives.

In the following exercises, use algebraic expressions to solve the problem.

Exercise:

Problem:

Car insurance Justin's car insurance has a \$750 deductible per incident. This means that he pays \$750 and his insurance company will pay all costs beyond \$750. If Justin files a claim for \$2,100, how much will he pay, and how much will his insurance company pay?

Solution:

He will pay \$750. His insurance company will pay \$1350.

Exercise:

Problem:

Home insurance Pam and Armando's home insurance has a \$2,500 deductible per incident. This means that they pay \$2,500 and their insurance company will pay all costs beyond \$2,500. If Pam and Armando file a claim for \$19,400, how much will they pay, and how much will their insurance company pay?

Writing Exercises

Exercise:

Problem:

Explain why "the sum of x and y " is the same as "the sum of y and x ," but "the difference of x and y " is not the same as "the difference of y and x ." Try substituting two random numbers for x and y to help you explain.

Exercise:

Problem:

Explain the difference between “4 times the sum of x and y ” and “the sum of 4 times x and y .”

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
evaluate algebraic expressions.			
identify terms, coefficients, and like terms.			
simplify expressions by combining like terms.			
translate word phrases to algebraic expressions.			

Ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

term

A term is a constant or the product of a constant and one or more variables.

coefficient

The constant that multiplies the variable(s) in a term is called the coefficient.

like terms

Terms that are either constants or have the same variables with the same exponents are like terms.

evaluate

To evaluate an algebraic expression means to find the value of the expression when the variable is replaced by a given number.

Solving Equations Using the Subtraction and Addition Properties of Equality

By the end of this section, you will be able to:

- Determine whether a number is a solution of an equation
- Model the Subtraction Property of Equality
- Solve equations using the Subtraction Property of Equality
- Solve equations using the Addition Property of Equality
- Translate word phrases to algebraic equations
- Translate to an equation and solve

Note:

Before you get started, take this readiness quiz.

1. Evaluate $x + 8$ when $x = 11$.
If you missed this problem, review [\[link\]](#).
2. Evaluate $5x - 3$ when $x = 9$.
If you missed this problem, review [\[link\]](#).
3. Translate into algebra: the difference of x and 8.
If you missed this problem, review [\[link\]](#).

When some people hear the word *algebra*, they think of solving equations. The applications of solving equations are limitless and extend to all careers and fields. In this section, we will begin solving equations. We will start by solving basic equations, and then as we proceed through the course we will build up our skills to cover many different forms of equations.

Determine Whether a Number is a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. An algebraic equation states that two algebraic expressions are equal. To solve an equation is to determine the values of the variable that make the equation a

true statement. Any number that makes the equation true is called a **solution** of the equation. It is the answer to the puzzle!

Note:

Solution of an Equation

A **solution to an equation** is a value of a variable that makes a true statement when substituted into the equation.

The process of finding the solution to an equation is called solving the equation.

To find the solution to an equation means to find the value of the variable that makes the equation true. Can you recognize the solution of $x + 2 = 7$? If you said 5, you're right! We say 5 is a solution to the equation $x + 2 = 7$ because when we substitute 5 for x the resulting statement is true.

Equation:

$$x + 2 = 7$$

$$5 + 2 \stackrel{?}{=} 7$$

$$7 = 7 \checkmark$$

Since $5 + 2 = 7$ is a true statement, we know that 5 is indeed a solution to the equation.

The symbol $\stackrel{?}{=}$ asks whether the left side of the equation is equal to the right side. Once we know, we can change to an equal sign ($=$) or not-equal sign ($\neq$).

Note:

Determine whether a number is a solution to an equation.

Substitute the number for the variable in the equation.
Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true.

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example:

Exercise:

Problem:

Determine whether $x = 5$ is a solution of $6x - 17 = 16$.

Solution:

Solution

	$6x - 17 = 16$
Substitute 5 for x .	$6 \cdot 5 - 17 \stackrel{?}{=} 16$
Multiply.	$30 - 17 \stackrel{?}{=} 16$

Subtract.

$$13 \neq 16$$

So $x = 5$ is not a solution to the equation $6x - 17 = 16$.

Note:

Exercise:

Problem: Is $x = 3$ a solution of $4x - 7 = 16$?

Solution:

no

Note:

Exercise:

Problem: Is $x = 2$ a solution of $6x - 2 = 10$?

Solution:

yes

Example:

Exercise:

Problem:

Determine whether $y = 2$ is a solution of $6y - 4 = 5y - 2$.

Solution:
Solution

Here, the variable appears on both sides of the equation. We must substitute 2 for each y .

	$6y - 4 = 5y - 2$
Substitute 2 for y .	$6(2) - 4 \stackrel{?}{=} 5(2) - 2$
Multiply.	$12 - 4 \stackrel{?}{=} 10 - 2$
Subtract.	$8 = 8 \checkmark$

Since $y = 2$ results in a true equation, we know that 2 is a solution to the equation $6y - 4 = 5y - 2$.

Note:
Exercise:

Problem: Is $y = 3$ a solution of $9y - 2 = 8y + 1$?

Solution:

yes

Note:

Exercise:

Problem: Is $y = 4$ a solution of $5y - 3 = 3y + 5$?

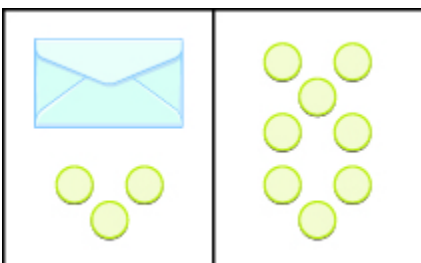
Solution:

yes

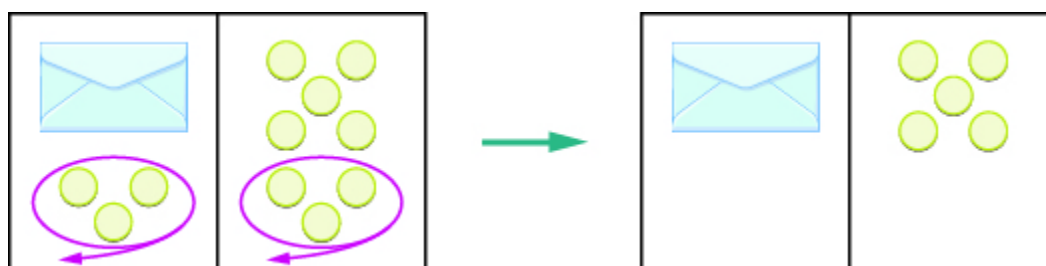
Model the Subtraction Property of Equality

We will use a model to help you understand how the process of solving an equation is like solving a puzzle. An envelope represents the variable – since its contents are unknown – and each counter represents one.

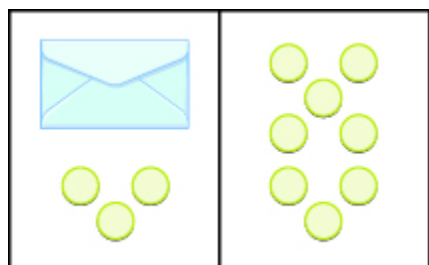
Suppose a desk has an imaginary line dividing it in half. We place three counters and an envelope on the left side of desk, and eight counters on the right side of the desk as in [\[link\]](#). Both sides of the desk have the same number of counters, but some counters are hidden in the envelope. Can you tell how many counters are in the envelope?



What steps are you taking in your mind to figure out how many counters are in the envelope? Perhaps you are thinking “I need to remove the 3 counters from the left side to get the envelope by itself. Those 3 counters on the left match with 3 on the right, so I can take them away from both sides. That leaves five counters on the right, so there must be 5 counters in the envelope.” [\[link\]](#) shows this process.



What algebraic equation is modeled by this situation? Each side of the desk represents an expression and the center line takes the place of the equal sign. We will call the contents of the envelope x , so the number of counters on the left side of the desk is $x + 3$. On the right side of the desk are 8 counters. We are told that $x + 3$ is equal to 8 so our equation is $x + 3 = 8$.



Equation:

$$x + 3 = 8$$

Let's write algebraically the steps we took to discover how many counters were in the envelope.

	$x + 3 = 8$
First, we took away three from each side.	$x + 3 - 3 = 8 - 3$
Then we were left with five.	$x = 5$

Now let's check our solution. We substitute 5 for x in the original equation and see if we get a true statement.

$$x + 3 = 8$$

$$5 + 3 \stackrel{?}{=} 8$$

$$8 = 8 \checkmark$$

Our solution is correct. Five counters in the envelope plus three more equals eight.

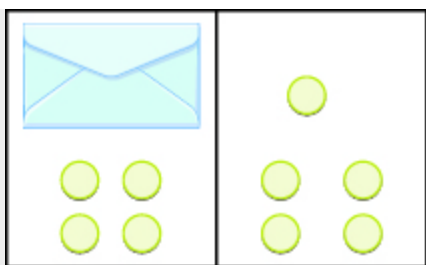
Note: Doing the Manipulative Mathematics activity, “Subtraction Property of Equality” will help you develop a better understanding of how to solve equations by using the Subtraction Property of Equality.

Example:

Exercise:

Problem:

Write an equation modeled by the envelopes and counters, and then solve the equation:



Solution:
Solution

On the left, write x for the contents of the envelope, add the 4 counters, so we have $x + 4$.

$$x + 4$$

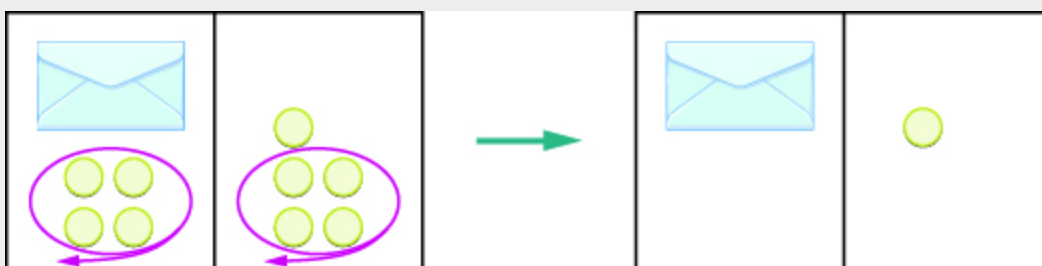
On the right, there are 5 counters.

$$5$$

The two sides are equal.

$$x + 4 = 5$$

Solve the equation by subtracting 4 counters from each side.



We can see that there is one counter in the envelope. This can be shown algebraically as:

$$\begin{aligned}
 x + 4 &= 5 \\
 x + 4 - 4 &= 5 - 4 \\
 x &= 1
 \end{aligned}$$

Substitute 1 for x in the equation to check.

$$\begin{aligned}
 x + 4 &= 5 \\
 1 + 4 &\stackrel{?}{=} 5 \\
 5 &= 5 \checkmark
 \end{aligned}$$

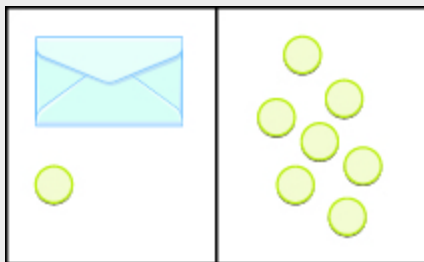
Since $x = 1$ makes the statement true, we know that 1 is indeed a solution.

Note:

Exercise:

Problem:

Write the equation modeled by the envelopes and counters, and then solve the equation:



Solution:

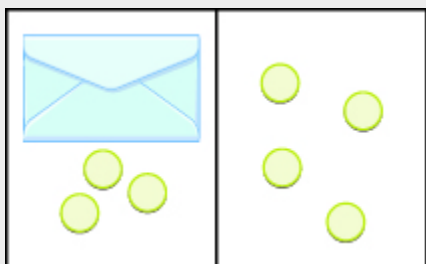
$$x + 1 = 7; x = 6$$

Note:

Exercise:

Problem:

Write the equation modeled by the envelopes and counters, and then solve the equation:



Solution:

$$x + 3 = 4; x = 1$$

Solve Equations Using the Subtraction Property of Equality

Our puzzle has given us an idea of what we need to do to solve an equation. The goal is to isolate the variable by itself on one side of the equations. In the previous examples, we used the Subtraction Property of Equality, which states that when we subtract the same quantity from both sides of an equation, we still have equality.

Note:

Subtraction Property of Equality

For any numbers a , b , and c , if

Equation:

$$a = b$$

then

Equation:

$$a - c = b - c$$

Think about twin brothers Andy and Bobby. They are 17 years old. How old was Andy 3 years ago? He was 3 years less than 17, so his age was $17 - 3$, or 14. What about Bobby's age 3 years ago? Of course, he was 14 also. Their ages are equal now, and subtracting the same quantity from both of them resulted in equal ages 3 years ago.

Equation:

$$a = b$$

$$a - 3 = b - 3$$

Note:

Solve an equation using the Subtraction Property of Equality.

Use the Subtraction Property of Equality to isolate the variable.

Simplify the expressions on both sides of the equation.

Check the solution.

Example:

Exercise:

Problem: Solve: $x + 8 = 17$.

Solution:

Solution

We will use the Subtraction Property of Equality to isolate x .

	$x + 8 = 17$
Subtract 8 from both sides.	$x + 8 - 8 = 17 - 8$
Simplify.	$x = 9$
	$x + 8 = 17$
	$9 + 8 = 17$
	$17 = 17 \checkmark$

Since $x = 9$ makes $x + 8 = 17$ a true statement, we know 9 is the solution to the equation.

Note:

Exercise:

Problem: Solve:

$$x + 6 = 19$$

Solution:

$$x = 13$$

Note:

Exercise:

Problem: Solve:

$$x + 9 = 14$$

Solution:

$$x = 5$$

Example:

Exercise:

Problem: Solve: $100 = y + 74$.

Solution:
Solution

To solve an equation, we must always isolate the variable—it doesn't matter which side it is on. To isolate y , we will subtract 74 from both sides.

	$100 = y + 74$
Subtract 74 from both sides.	$100 - 74 = y + 74 - 74$
Simplify.	$26 = y$
Substitute 26 for y to check. $100 = y + 74$ $100 \stackrel{?}{=} 26 + 74$ $100 = 100 \checkmark$	

Since $y = 26$ makes $100 = y + 74$ a true statement, we have found the solution to this equation.

Note:

Exercise:

Problem: Solve:

$$95 = y + 67$$

Solution:

$$y = 28$$

Note:

Exercise:

Problem: Solve:

$$91 = y + 45$$

Solution:

$$y = 46$$

Solve Equations Using the Addition Property of Equality

In all the equations we have solved so far, a number was added to the variable on one side of the equation. We used subtraction to “undo” the addition in order to isolate the variable.

But suppose we have an equation with a number subtracted from the variable, such as $x - 5 = 8$. We want to isolate the variable, so to “undo” the subtraction we will add the number to both sides.

We use the Addition Property of Equality, which says we can add the same number to both sides of the equation without changing the equality. Notice how it mirrors the Subtraction Property of Equality.

Note:

Addition Property of Equality

For any numbers a , b , and c , if

Equation:

$$a = b$$

then

Equation:

$$a + c = b + c$$

Remember the 17-year-old twins, Andy and Bobby? In ten years, Andy's age will still equal Bobby's age. They will both be 27.

Equation:

$$\begin{aligned}a &= b \\a + 10 &= b + 10\end{aligned}$$

We can add the same number to both sides and still keep the equality.

Note:

Solve an equation using the Addition Property of Equality.

Use the Addition Property of Equality to isolate the variable.
Simplify the expressions on both sides of the equation.
Check the solution.

Example:

Exercise:

Problem: Solve: $x - 5 = 8$.

Solution:

Solution

We will use the Addition Property of Equality to isolate the variable.

	$x - 5 = 8$
Add 5 to both sides.	$x - 5 + 5 = 8 + 5$
Simplify.	$x = 13$
Now we can check. Let $x = 13$.	
$x - 5 = 8$	
$13 - 5 \stackrel{?}{=} 8$	
$8 = 8 \checkmark$	

Note:

Exercise:

Problem: Solve:

$$x - 9 = 13$$

Solution:

$$x = 22$$

Note:

Exercise:

Problem: Solve:

$$y - 1 = 3$$

Solution:

$$y = 4$$

Example:

Exercise:

Problem: Solve: $27 = a - 16$.

Solution:

Solution

We will add 16 to each side to isolate the variable.

$$27 = a - 16$$

Add 16 to each side.

$$27 + 16 = a - 16 + 16$$

Simplify.

$$43 = a$$

Now we can check. Let $a = 43$.

$$27 = a - 16$$

$$27 \stackrel{?}{=} 43 - 16$$

$$27 = 27 \checkmark$$

The solution to $27 = a - 16$ is $a = 43$.

Note:

Exercise:

Problem: Solve:

$$19 = a - 18$$

Solution:

$$a = 37$$

Note:

Exercise:

Problem: Solve:

$$27 = n - 14$$

Solution:

$$n = 41$$

Translate Word Phrases to Algebraic Equations

Remember, an equation has an equal sign between two algebraic expressions. So if we have a sentence that tells us that two phrases are equal, we can translate it into an equation. We look for clue words that mean *equals*. Some words that translate to the equal sign are:

- is equal to
- is the same as
- is
- gives
- was
- will be

It may be helpful to put a box around the *equals* word(s) in the sentence to help you focus separately on each phrase. Then translate each phrase into an expression, and write them on each side of the equal sign.

We will practice translating word sentences into algebraic equations. Some of the sentences will be basic number facts with no variables to solve for. Some sentences will translate into equations with variables. The focus right now is just to translate the words into algebra.

Example:**Exercise:****Problem:**

Translate the sentence into an algebraic equation: The sum of 6 and 9 is 15.

Solution:**Solution**

The word *is* tells us the equal sign goes between 9 and 15.

Locate the “equals” word(s).

The sum of 6 and 9 is 15.

Write the = sign.

The sum of 6 and 9 = 15.

Translate the words to the left of the *equals* word into an algebraic expression.

6 + 9 = ____

Translate the words to the right of the *equals* word into an algebraic expression.

6 + 9 = 15

Note:**Exercise:**

Problem: Translate the sentence into an algebraic equation:

The sum of 7 and 6 gives 13.

Solution:

$$7 + 6 = 13$$

Note:

Exercise:

Problem: Translate the sentence into an algebraic equation:

The sum of 8 and 6 is 14.

Solution:

$$8 + 6 = 14$$

Example:

Exercise:

Problem:

Translate the sentence into an algebraic equation: The product of 8 and 7 is 56.

Solution:

Solution

The location of the word *is* tells us that the equal sign goes between 7 and 56.

Locate the “equals” word(s).	The product of 8 and 7 is 56.	
Write the = sign.	The product of 8 and 7 = 56.	
Translate the words to the left of the <i>equals</i> word into an algebraic expression.	$8 \cdot 7 = \underline{\quad}$	
Translate the words to the right of the <i>equals</i> word into an algebraic expression.	$8 \cdot 7 = \underline{56}$	

Note:

Exercise:

Problem: Translate the sentence into an algebraic equation:

The product of 6 and 9 is 54.

Solution:

$$6 \cdot 9 = 54$$

Note:

Exercise:

Problem: Translate the sentence into an algebraic equation:

The product of 21 and 3 gives 63.

Solution:

$$21 \cdot 3 = 63$$

Example:**Exercise:****Problem:**

Translate the sentence into an algebraic equation: Twice the difference of x and 3 gives 18.

Solution:**Solution**

Locate the “equals” word(s).

Twice the difference of x and 3 gives 18.

Recognize the key words: *twice*;
difference of and

Twice means two times.

Translate.

Twice the difference of x and 3 gives 18.

$$2 \qquad (x - 3) \qquad = \qquad 18$$

Note:

Exercise:

Problem: Translate the given sentence into an algebraic equation:

Twice the difference of x and 5 gives 30.

Solution:

$$2(x - 5) = 30$$

Note:**Exercise:**

Problem: Translate the given sentence into an algebraic equation:

Twice the difference of y and 4 gives 16.

Solution:

$$2(y - 4) = 16$$

Translate to an Equation and Solve

Now let's practice translating sentences into algebraic equations and then solving them. We will solve the equations by using the Subtraction and Addition Properties of Equality.

Example:**Exercise:**

Problem: Translate and solve: Three more than x is equal to 47.

Solution:
Solution

		Three more than x is equal to 47.
Translate.		$x + 3 = 47$
Subtract 3 from both sides of the equation.		$x + 3 - 3 = 47 - 3$
Simplify.		$x = 44$
We can check. Let $x = 44$.	$x + 3 = 47$	
	$44 + 3 \stackrel{?}{=} 47$	
	$47 = 47 \checkmark$	

So $x = 44$ is the solution.

Note:

Exercise:

Problem: Translate and solve:

Seven more than x is equal to 37.

Solution:

$$x + 7 = 37; x = 30$$

Note:

Exercise:

Problem: Translate and solve:

Eleven more than y is equal to 28.

Solution:

$$y + 11 = 28; y = 17$$

Example:

Exercise:

Problem: Translate and solve: The difference of y and 14 is 18.

Solution:
Solution

		The difference of y and 14 is 18.
Translate.		$y - 14 = 18$
Add 14 to both sides.		$y - 14 + 14 = 18 + 14$
Simplify.		$y = 32$
We can check. Let $y = 32$.	$y - 14 = 18$	
	$32 - 14 \stackrel{?}{=} 18$	
	$18 = 18 \checkmark$	
So $y = 32$ is the solution.		

Note:

Exercise:

Problem: Translate and solve:

The difference of z and 17 is equal to 37.

Solution:

$$z - 17 = 37; z = 54$$

Note:**Exercise:**

Problem: Translate and solve:

The difference of x and 19 is equal to 45.

Solution:

$$x - 19 = 45; x = 64$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solving One Step Equations By Addition and Subtraction](#)

Key Concepts

- **Determine whether a number is a solution to an equation.**

Substitute the number for the variable in the equation.

Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true. If it is true, the number is a solution.

If it is not true, the number is not a solution.

- **Subtraction Property of Equality**

- For any numbers a , b , and c ,

if	$a = b$
then	$a - c = b - c$

- **Solve an equation using the Subtraction Property of Equality.**

Use the Subtraction Property of Equality to isolate the variable.
Simplify the expressions on both sides of the equation.
Check the solution.

- **Addition Property of Equality**

- For any numbers a , b , and c ,

if	$a = b$
then	$a + c = b + c$

- **Solve an equation using the Addition Property of Equality.**

Use the Addition Property of Equality to isolate the variable.
Simplify the expressions on both sides of the equation.
Check the solution.

Practice Makes Perfect

Determine Whether a Number is a Solution of an Equation

In the following exercises, determine whether each given value is a solution to the equation.

Exercise:

Problem: $x + 13 = 21$

- Ⓐ $x = 8$
- Ⓑ $x = 34$

Solution:

- Ⓐ yes
- Ⓑ no

Exercise:

Problem: $y + 18 = 25$

- Ⓐ $y = 7$
- Ⓑ $y = 43$

Exercise:

Problem: $m - 4 = 13$

- Ⓐ $m = 9$
- Ⓑ $m = 17$

Solution:

- Ⓐ no

⒃ yes

Exercise:

Problem: $n - 9 = 6$

- Ⓐ $n = 3$
- Ⓑ $n = 15$

Exercise:

Problem: $3p + 6 = 15$

- Ⓐ $p = 3$
- Ⓑ $p = 7$

Solution:

- Ⓐ yes
- Ⓑ no

Exercise:

Problem: $8q + 4 = 20$

- Ⓐ $q = 2$
- Ⓑ $q = 3$

Exercise:

Problem: $18d - 9 = 27$

- Ⓐ $d = 1$
- Ⓑ $d = 2$

Solution:

- Ⓐ no
- Ⓑ yes

Exercise:

Problem: $24f - 12 = 60$

- Ⓐ $f = 2$
- Ⓑ $f = 3$

Exercise:

Problem: $8u - 4 = 4u + 40$

- Ⓐ $u = 3$
- Ⓑ $u = 11$

Solution:

- Ⓐ no
- Ⓑ yes

Exercise:

Problem: $7v - 3 = 4v + 36$

- Ⓐ $v = 3$
- Ⓑ $v = 11$

Exercise:

Problem: $20h - 5 = 15h + 35$

Ⓐ $h = 6$

Ⓑ $h = 8$

Solution:

Ⓐ no

Ⓑ yes

Exercise:

Problem: $18k - 3 = 12k + 33$

Ⓐ $k = 1$

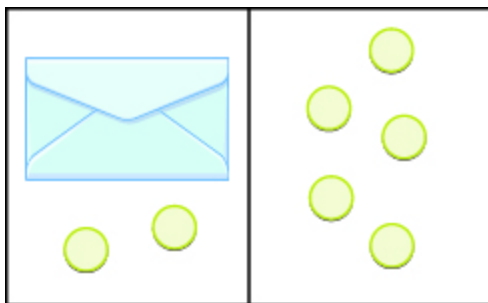
Ⓑ $k = 6$

Model the Subtraction Property of Equality

In the following exercises, write the equation modeled by the envelopes and counters and then solve using the subtraction property of equality.

Exercise:

Problem:

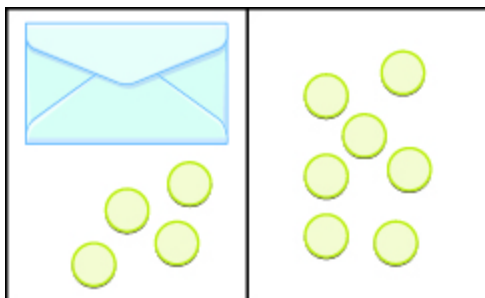


Solution:

$$x + 2 = 5; x = 3$$

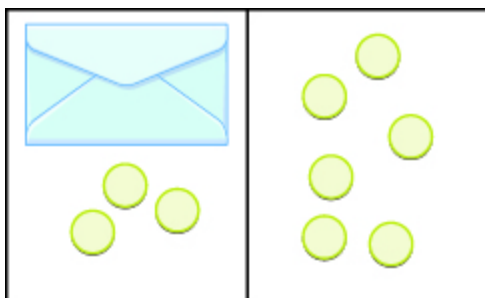
Exercise:

Problem:



Exercise:

Problem:

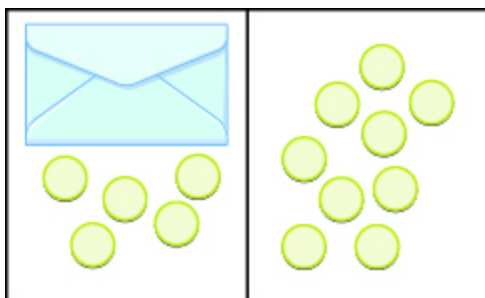


Solution:

$$x + 3 = 6; x = 3$$

Exercise:

Problem:



Solve Equations using the Subtraction Property of Equality

In the following exercises, solve each equation using the subtraction property of equality.

Exercise:

Problem: $a + 2 = 18$

Solution:

$$a = 16$$

Exercise:

Problem: $b + 5 = 13$

Exercise:

Problem: $p + 18 = 23$

Solution:

$$p = 5$$

Exercise:

Problem: $q + 14 = 31$

Exercise:

Problem: $r + 76 = 100$

Solution:

$$r = 24$$

Exercise:

Problem: $s + 62 = 95$

Exercise:

Problem: $16 = x + 9$

Solution:

$$x = 7$$

Exercise:

Problem: $17 = y + 6$

Exercise:

Problem: $93 = p + 24$

Solution:

$$p = 69$$

Exercise:

Problem: $116 = q + 79$

Exercise:

Problem: $465 = d + 398$

Solution:

$$d = 67$$

Exercise:

Problem: $932 = c + 641$

Solve Equations using the Addition Property of Equality

In the following exercises, solve each equation using the addition property of equality.

Exercise:

Problem: $y - 3 = 19$

Solution:

$$y = 22$$

Exercise:

Problem: $x - 4 = 12$

Exercise:

Problem: $u - 6 = 24$

Solution:

$$u = 30$$

Exercise:

Problem: $v - 7 = 35$

Exercise:

Problem: $f - 55 = 123$

Solution:

$$f = 178$$

Exercise:

Problem: $g - 39 = 117$

Exercise:

Problem: $19 = n - 13$

Solution:

$$n = 32$$

Exercise:

Problem: $18 = m - 15$

Exercise:

Problem: $10 = p - 38$

Solution:

$$p = 48$$

Exercise:

Problem: $18 = q - 72$

Exercise:

Problem: $268 = y - 199$

Solution:

$$y = 467$$

Exercise:

Problem: $204 = z - 149$

Translate Word Phrase to Algebraic Equations

In the following exercises, translate the given sentence into an algebraic equation.

Exercise:

Problem: The sum of 8 and 9 is equal to 17.

Solution:

$$8 + 9 = 17$$

Exercise:

Problem: The sum of 7 and 9 is equal to 16.

Exercise:

Problem: The difference of 23 and 19 is equal to 4.

Solution:

$$23 - 19 = 4$$

Exercise:

Problem: The difference of 29 and 12 is equal to 17.

Exercise:

Problem: The product of 3 and 9 is equal to 27.

Solution:

$$3 \cdot 9 = 27$$

Exercise:

Problem: The product of 6 and 8 is equal to 48.

Exercise:

Problem: The quotient of 54 and 6 is equal to 9.

Solution:

$$54 \div 6 = 9$$

Exercise:

Problem: The quotient of 42 and 7 is equal to 6.

Exercise:

Problem: Twice the difference of n and 10 gives 52.

Solution:

$$2(n - 10) = 52$$

Exercise:

Problem: Twice the difference of m and 14 gives 64.

Exercise:

Problem: The sum of three times y and 10 is 100.

Solution:

$$3y + 10 = 100$$

Exercise:

Problem: The sum of eight times x and 4 is 68.

Translate to an Equation and Solve

In the following exercises, translate the given sentence into an algebraic equation and then solve it.

Exercise:

Problem: Five more than p is equal to 21.

Solution:

$$p + 5 = 21; p = 16$$

Exercise:

Problem: Nine more than q is equal to 40.

Exercise:

Problem: The sum of r and 18 is 73.

Solution:

$$r + 18 = 73; r = 55$$

Exercise:

Problem: The sum of s and 13 is 68.

Exercise:

Problem: The difference of d and 30 is equal to 52.

Solution:

$$d - 30 = 52; d = 82$$

Exercise:

Problem: The difference of c and 25 is equal to 75.

Exercise:

Problem: 12 less than u is 89.

Solution:

$$u - 12 = 89; u = 101$$

Exercise:

Problem: 19 less than w is 56.

Exercise:

Problem: 325 less than c gives 799.

Solution:

$$c - 325 = 799; c = 1124$$

Exercise:

Problem: 299 less than d gives 850.

Everyday Math

Exercise:

Problem:

Insurance Vince's car insurance has a \$500 deductible. Find the amount the insurance company will pay, p , for an \$1800 claim by solving the equation $500 + p = 1800$.

Solution:

\$1300

Exercise:**Problem:**

Insurance Marta's homeowner's insurance policy has a \$750 deductible. The insurance company paid \$5800 to repair damages caused by a storm. Find the total cost of the storm damage, d , by solving the equation $d - 750 = 5800$.

Exercise:**Problem:**

Sale purchase Arthur bought a suit that was on sale for \$120 off. He paid \$340 for the suit. Find the original price, p , of the suit by solving the equation $p - 120 = 340$.

Solution:

\$460

Exercise:**Problem:**

Sale purchase Rita bought a sofa that was on sale for \$1299. She paid a total of \$1409, including sales tax. Find the amount of the sales tax, t , by solving the equation $1299 + t = 1409$.

Writing Exercises**Exercise:****Problem:**

Is $x = 1$ a solution to the equation $8x - 2 = 16 - 6x$? How do you know?

Exercise:

Problem:

Write the equation $y - 5 = 21$ in words. Then make up a word problem for this equation.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether a number is a solution of an equation.			
model the subtraction property of equality.			
solve equations using the subtraction property of equality.			
solve equations using the addition property of equality.			
translate word phrases to algebraic equations.			
translate to an equation and solve.			

Ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

solution of an equation

A solution to an equation is a value of a variable that makes a true statement when substituted into the equation. The process of finding the solution to an equation is called solving the equation.

Find Multiples and Factors

By the end of this section, you will be able to:

- Identify multiples of numbers
- Use common divisibility tests
- Find all the factors of a number
- Identify prime and composite numbers

Note:

Before you get started, take this readiness quiz.

1. Which of the following numbers are counting numbers (natural numbers)?

0, 4, 215

If you missed this problem, review [\[link\]](#).

2. Find the sum of 3, 5, and 7.

If you missed the problem, review [\[link\]](#).

Identify Multiples of Numbers

Annie is counting the shoes in her closet. The shoes are matched in pairs, so she doesn't have to count each one. She counts by twos: 2, 4, 6, 8, 10, 12. She has 12 shoes in her closet.

The numbers 2, 4, 6, 8, 10, 12 are called multiples of 2. Multiples of 2 can be written as the product of a counting number and 2. The first six multiples of 2 are given below.

Equation:

$$1 \cdot 2 = 2$$

$$2 \cdot 2 = 4$$

$$3 \cdot 2 = 6$$

$$4 \cdot 2 = 8$$

$$5 \cdot 2 = 10$$

$$6 \cdot 2 = 12$$

A **multiple of a number** is the product of the number and a counting number. So a multiple of 3 would be the product of a counting number and 3. Below are the first six multiples of 3.

Equation:

$$1 \cdot 3 = 3$$

$$2 \cdot 3 = 6$$

$$3 \cdot 3 = 9$$

$$4 \cdot 3 = 12$$

$$5 \cdot 3 = 15$$

$$6 \cdot 3 = 18$$

We can find the multiples of any number by continuing this process. [\[link\]](#) shows the multiples of 2 through 9 for the first twelve counting numbers.

Counting Number	1	2	3	4	5	6	7	8	9	10	11	12
Multiples of 2	2	4	6	8	10	12	14	16	18	20	22	24
Multiples of 3	3	6	9	12	15	18	21	24	27	30	33	36
Multiples of 4	4	8	12	16	20	24	28	32	36	40	44	48
Multiples of 5	5	10	15	20	25	30	35	40	45	50	55	60
Multiples of 6	6	12	18	24	30	36	42	48	54	60	66	72
Multiples of 7	7	14	21	28	35	42	49	56	63	70	77	84
Multiples of 8	8	16	24	32	40	48	56	64	72	80	88	96
Multiples of 9	9	18	27	36	45	54	63	72	81	90	99	108

Note:

Multiple of a Number

A number is a multiple of n if it is the product of a counting number and n .

Recognizing the patterns for multiples of 2, 5, 10, and 3 will be helpful to you as you continue in this course.

Note: Doing the Manipulative Mathematics activity “Multiples” will help you develop a better understanding of multiples.

[\[link\]](#) shows the counting numbers from 1 to 50. Multiples of 2 are highlighted. Do you notice a pattern?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Multiples of 2 between 1 and 50

The last digit of each highlighted number in [\[link\]](#) is either 0, 2, 4, 6, or 8. This is true for the product of 2 and any counting number. So, to tell if any number is a multiple of 2 look at the last digit. If it is 0, 2, 4, 6, or 8, then the number is a multiple of 2.

Example:

Exercise:**Problem:** Determine whether each of the following is a multiple of 2:

- Ⓐ 489
- Ⓑ 3,714

Solution:**Solution**

Ⓐ	
Is 489 a multiple of 2?	
Is the last digit 0, 2, 4, 6, or 8?	No.
	489 is not a multiple of 2.

Ⓑ	
Is 3,714 a multiple of 2?	
Is the last digit 0, 2, 4, 6, or 8?	Yes.
	3,714 is a multiple of 2.

Note:**Exercise:****Problem:** Determine whether each number is a multiple of 2:

- Ⓐ 678
- Ⓑ 21,493

Solution:

- Ⓐ yes
- Ⓑ no

Note:

Exercise:

Problem: Determine whether each number is a multiple of 2:

a

 979

b

 17,780

Solution:

a

 no

b

 yes

Now let’s look at multiples of 5. [link](#) highlights all of the multiples of 5 between 1 and 50. What do you notice about the multiples of 5?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Multiples of 5 between 1 and 50

All multiples of 5 end with either 5 or 0. Just like we identify multiples of 2 by looking at the last digit, we can identify multiples of 5 by looking at the last digit.

Example:

Exercise:

Problem: Determine whether each of the following is a multiple of 5:

a

 579

b

 880

Solution:
Solution

a

Is 579 a multiple of 5?

Is the last digit 5 or 0?	No.
	579 is not a multiple of 5.
ⓑ	
Is 880 a multiple of 5?	
Is the last digit 5 or 0?	Yes.
	880 is a multiple of 5.

Note:

Exercise:

Problem: Determine whether each number is a multiple of 5.

- ⓐ 675
- ⓑ 1,578

Solution:

- ⓐ yes
- ⓑ no

Note:

Exercise:

Problem: Determine whether each number is a multiple of 5.

- ⓐ 421
- ⓑ 2,690

Solution:

- ⓐ no
- ⓑ yes

[\[link\]](#) highlights the multiples of 10 between 1 and 50. All multiples of 10 all end with a zero.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Multiples of 10 between 1 and 50

Example:

Exercise:

Problem: Determine whether each of the following is a multiple of 10:

- Ⓐ 425
- Ⓑ 350

Solution:

Solution

Ⓐ

Is 425 a multiple of 10?

Is the last digit zero?

No.

425 is not a multiple of 10.

Ⓑ

Is 350 a multiple of 10?

Is the last digit zero?

Yes.

350 is a multiple of 10.

Note:

Exercise:

Problem: Determine whether each number is a multiple of 10:

- (a) 179
- (b) 3,540

Solution:

- (a) no
- (b) yes

Note:

Exercise:

Problem: Determine whether each number is a multiple of 10:

- (a) 110
- (b) 7,595

Solution:

- (a) yes
- (b) no

[\[link\]](#) highlights multiples of 3. The pattern for multiples of 3 is not as obvious as the patterns for multiples of 2, 5, and 10.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Multiples of 3 between 1 and 50

Unlike the other patterns we've examined so far, this pattern does not involve the last digit. The pattern for multiples of 3 is based on the sum of the digits. If the sum of the digits of a number is a multiple of 3, then the number itself is a multiple of 3. See [\[link\]](#).

Multiple of 3	3	6	9	12	15	18	21	24
Sum of digits	3	6	9	$\begin{array}{r} 1+2 \\ 3 \end{array}$	$\begin{array}{r} 1+5 \\ 6 \end{array}$	$\begin{array}{r} 1+8 \\ 9 \end{array}$	$\begin{array}{r} 2+1 \\ 3 \end{array}$	$\begin{array}{r} 2+4 \\ 6 \end{array}$

Consider the number 42. The digits are 4 and 2, and their sum is $4 + 2 = 6$. Since 6 is a multiple of 3, we know that 42 is also a multiple of 3.

Example:

Exercise:

Problem: Determine whether each of the given numbers is a multiple of 3:

- Ⓐ 645
- Ⓑ 10,519

Solution:

Solution

- Ⓐ Is 645 a multiple of 3?

Find the sum of the digits.	$6 + 4 + 5 = 15$
Is 15 a multiple of 3?	Yes.
If we're not sure, we could add its digits to find out. We can check it by dividing 645 by 3.	$645 \div 3$
The quotient is 215.	$3 \cdot 215 = 645$

- Ⓑ Is 10,519 a multiple of 3?

Find the sum of the digits.	$1 + 0 + 5 + 1 + 9 = 16$
Is 16 a multiple of 3?	No.
So 10,519 is not a multiple of 3 either..	$645 \div 3$
We can check this by dividing by 10,519 by 3.	$\begin{array}{r} 3,506R1 \\ 3 \overline{)10,519} \end{array}$

When we divide 10,519 by 3, we do not get a counting number, so 10,519 is not the product of a counting number and 3. It is not a multiple of 3.

Note:

Exercise:

Problem: Determine whether each number is a multiple of 3:

- (a) 954
- (b) 3,742

Solution:

- (a) yes
- (b) no

Note:

Exercise:

Problem: Determine whether each number is a multiple of 3:

- (a) 643
- (b) 8,379

Solution:

- (a) no
- (b) yes

Look back at the charts where you highlighted the multiples of 2, of 5, and of 10. Notice that the multiples of 10 are the numbers that are multiples of both 2 and 5. That is because $10 = 2 \cdot 5$. Likewise, since $6 = 2 \cdot 3$, the multiples of 6 are the numbers that are multiples of both 2 and 3.

Use Common Divisibility Tests

Another way to say that 375 is a multiple of 5 is to say that 375 is divisible by 5. In fact, $375 \div 5$ is 75, so 375 is $5 \cdot 75$. Notice in [\[link\]](#) that 10,519 is not a multiple 3. When we divided 10,519 by 3 we did not get a counting number, so 10,519 is not divisible by 3.

Note:

Divisibility

If a number m is a multiple of n , then we say that m is divisible by n .

Since multiplication and division are inverse operations, the patterns of multiples that we found can be used as divisibility tests. [\[link\]](#) summarizes divisibility tests for some of the counting numbers between one and ten.

Divisibility Tests	
A number is divisible by	
2	if the last digit is 0, 2, 4, 6, or 8
3	if the sum of the digits is divisible by 3
5	if the last digit is 5 or 0
6	if divisible by both 2 and 3
10	if the last digit is 0

Example:
Exercise:

Problem: Determine whether 1,290 is divisible by 2, 3, 5, and 10.

Solution:
Solution

[\[link\]](#) applies the divisibility tests to 1,290. In the far right column, we check the results of the divisibility tests by seeing if the quotient is a whole number.

Divisible by...?	Test	Divisible?	Check
2	Is last digit 0, 2, 4, 6, or 8? Yes.	yes	$1290 \div 2 = 645$
3	Is sum of digits divisible by 3? $1 + 2 + 9 + 0 = 12$ Yes.	yes	$1290 \div 3 = 430$
5	Is last digit 5 or 0? Yes.	yes	$1290 \div 5 = 258$
10	Is last digit 0? Yes.	yes	$1290 \div 10 = 129$

Thus, 1,290 is divisible by 2, 3, 5, and 10.

Note:

Exercise:

Problem: Determine whether the given number is divisible by 2, 3, 5, and 10.

6240

Solution:

Divisible by 2, 3, 5, and 10

Note:

Exercise:

Problem: Determine whether the given number is divisible by 2, 3, 5, and 10.

7248

Solution:

Divisible by 2 and 3, not 5 or 10.

Example:

Exercise:

Problem: Determine whether 5,625 is divisible by 2, 3, 5, and 10.

Solution:

Solution

[\[link\]](#) applies the divisibility tests to 5,625 and tests the results by finding the quotients.

Divisible by...?	Test	Divisible?	Check
2	Is last digit 0, 2, 4, 6, or 8? <i>No.</i>	no	$5625 \div 2 = 2812.5$
3	Is sum of digits divisible by 3? $5 + 6 + 2 + 5 = 18$ <i>Yes.</i>	yes	$5625 \div 3 = 1875$
5	Is last digit is 5 or 0? <i>Yes.</i>	yes	$5625 \div 5 = 1125$
10	Is last digit 0? <i>No.</i>	no	$5625 \div 10 = 562.5$

Thus, 5,625 is divisible by 3 and 5, but not 2, or 10.

Note:

Exercise:

Problem: Determine whether the given number is divisible by 2, 3, 5, and 10.

4962

Solution:

Divisible by 2, 3, not 5 or 10.

Note:

Exercise:

Problem: Determine whether the given number is divisible by 2, 3, 5, and 10.

3765

Solution:

Divisible by 3 and 5.

Find All the Factors of a Number

There are often several ways to talk about the same idea. So far, we've seen that if m is a multiple of n , we can say that m is divisible by n . We know that 72 is the product of 8 and 9, so we can say 72 is a multiple of 8 and 72 is a multiple of 9. We can also say 72 is divisible by 8 and by 9. Another way to talk about this is to say that 8 and 9 are factors of 72. When we write $72 = 8 \cdot 9$ we can say that we have factored 72.

$$\underbrace{8 \cdot 9}_{\text{factors}} = \underbrace{72}_{\text{product}}$$

Note:

Factors

If $a \cdot b = m$, then a and b are factors of m , and m is the product of a and b .

In algebra, it can be useful to determine all of the factors of a number. This is called factoring a number, and it can help us solve many kinds of problems.

Note: Doing the Manipulative Mathematics activity “Model Multiplication and Factoring” will help you develop a better understanding of multiplication and factoring.

For example, suppose a choreographer is planning a dance for a ballet recital. There are 24 dancers, and for a certain scene, the choreographer wants to arrange the dancers in groups of equal sizes on stage.

In how many ways can the dancers be put into groups of equal size? Answering this question is the same as identifying the factors of 24. [\[link\]](#) summarizes the different ways that the choreographer can arrange the dancers.

Number of Groups	Dancers per Group	Total Dancers
1	24	$1 \cdot 24 = 24$
2	12	$2 \cdot 12 = 24$
3	8	$3 \cdot 8 = 24$
4	6	$4 \cdot 6 = 24$
6	4	$6 \cdot 4 = 24$
8	3	$8 \cdot 3 = 24$
12	2	$12 \cdot 2 = 24$
24	1	$24 \cdot 1 = 24$

What patterns do you see in [\[link\]](#)? Did you notice that the number of groups times the number of dancers per group is always 24? This makes sense, since there are always 24 dancers.

You may notice another pattern if you look carefully at the first two columns. These two columns contain the exact same set of numbers—but in reverse order. They are mirrors of one another, and in fact, both columns list all of the factors of 24, which are:

Equation:

$$1, 2, 3, 4, 6, 8, 12, 24$$

We can find all the factors of any counting number by systematically dividing the number by each counting number, starting with 1. If the quotient is also a counting number, then the divisor and the quotient are factors of the number. We can stop when the quotient becomes smaller than the divisor.

Note:
Find all the factors of a counting number.

Divide the number by each of the counting numbers, in order, until the quotient is smaller than the divisor.

- If the quotient is a counting number, the divisor and quotient are a pair of factors.
- If the quotient is not a counting number, the divisor is not a factor.

List all the factor pairs.
Write all the factors in order from smallest to largest.

Example:**Exercise:**

Problem: Find all the factors of 72.

Solution:**Solution**

Divide 72 by each of the counting numbers starting with 1. If the quotient is a whole number, the divisor and quotient are a pair of factors.

Dividend	Divisor	Quotient	Factors
72	1	72	1, 72
72	2	36	2, 36
72	3	24	3, 24
72	4	18	4, 18
72	5	14.4	–
72	6	12	6, 12
72	7	~10.29	–
72	8	9	8, 9

The next line would have a divisor of 9 and a quotient of 8. The quotient would be smaller than the divisor, so we stop. If we continued, we would end up only listing the same factors again in reverse order. Listing all the factors from smallest to greatest, we have

1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72

Note:**Exercise:**

Problem: Find all the factors of the given number:

96

Solution:

1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

Note:**Exercise:**

Problem: Find all the factors of the given number:

80

Solution:

1, 2, 4, 5, 8, 10, 16, 20, 40, 80

Identify Prime and Composite Numbers

Some numbers, like 72, have many factors. Other numbers, such as 7, have only two factors: 1 and the number. A number with only two factors is called a **prime number**. A number with more than two factors is called a **composite number**. The number 1 is neither prime nor composite. It has only one factor, itself.

Note:
Prime Numbers and Composite Numbers
A prime number is a counting number greater than 1 whose only factors are 1 and itself.
A composite number is a counting number that is not prime.

[\[link\]](#) lists the counting numbers from 2 through 20 along with their factors. The highlighted numbers are prime, since each has only two factors.

Number	Factors	Prime or Composite?		Number	Factor	Prime or Composite?
2	1,2	Prime		12	1,2,3,4,6,12	Composite
3	1,3	Prime		13	1,13	Prime
4	1,2,4	Composite		14	1,2,7,14	Composite
5	1,5	Prime		15	1,3,5,15	Composite
6	1,2,3,6	Composite		16	1,2,4,8,16	Composite
7	1,7	Prime		17	1,17	Prime
8	1,2,4,8	Composite		18	1,2,3,6,9,18	Composite
9	1,3,9	Composite		19	1,19	Prime
10	1,2,5,10	Composite		20	1,2,4,5,10,20	Composite
11	1,11	Prime				

Factors of the counting numbers from 2 through 20, with prime numbers highlighted

The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19. There are many larger prime numbers too. In order to determine whether a number is prime or composite, we need to see if the number has any factors other than 1 and itself. To do this, we can test each of the smaller prime numbers in order to see if it is a factor of the number. If none of the prime numbers are factors, then that number is also prime.

Note:
Determine if a number is prime.

Test each of the primes, in order, to see if it is a factor of the number.
Start with 2 and stop when the quotient is smaller than the divisor or when a prime factor is found.
If the number has a prime factor, then it is a composite number. If it has no prime factors, then the number is prime.

Example:
Exercise:

Problem: Identify each number as prime or composite:

- Ⓐ 83
Ⓑ 77

Solution:
Solution

Ⓐ Test each prime, in order, to see if it is a factor of 83, starting with 2, as shown. We will stop when the quotient is smaller than the divisor.

Prime	Test	Factor of 83?
2	Last digit of 83 is not 0, 2, 4, 6, or 8.	No.
3	$8 + 3 = 11$, and 11 is not divisible by 3.	No.
5	The last digit of 83 is not 5 or 0.	No.
7	$83 \div 7 = 11.857\dots$	No.
11	$83 \div 11 = 7.545\dots$	No.

We can stop when we get to 11 because the quotient (7.545...) is less than the divisor.

We did not find any prime numbers that are factors of 83, so we know 83 is prime.

Ⓑ Test each prime, in order, to see if it is a factor of 77.

Prime	Test	Factor of 77?
2	Last digit is not 0, 2, 4, 6, or 8.	No.
3	$7 + 7 = 14$, and 14 is not divisible by 3.	No.
5	the last digit is not 5 or 0.	No.
7	$77 \div 11 = 7$	Yes.

Since 77 is divisible by 7, we know it is not a prime number. It is composite.

Note:
Exercise:

Problem: Identify the number as prime or composite:

91

Solution:

composite

Note:

Exercise:

Problem: Identify the number as prime or composite:

137

Solution:

prime

Note:The Links to Literacy activities *One Hundred Hungry Ants*, *Spunky Monkeys on Parade* and *A Remainder of One* will provide you with another view of the topics covered in this section.

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Divisibility Rules](#)
- [Factors](#)
- [Ex 1: Determine Factors of a Number](#)
- [Ex 2: Determine Factors of a Number](#)
- [Ex 3: Determine Factors of a Number](#)

Key Concepts

Divisibility Tests	
A number is divisible by	
2	if the last digit is 0, 2, 4, 6, or 8
3	if the sum of the digits is divisible by 3

Divisibility Tests	
5	if the last digit is 5 or 0
6	if divisible by both 2 and 3
10	if the last digit is 0

- **Factors** If $a \cdot b = m$, then a and b are factors of m , and m is the product of a and b .
- **Find all the factors of a counting number.**

Divide the number by each of the counting numbers, in order, until the quotient is smaller than the divisor.

- If the quotient is a counting number, the divisor and quotient are a pair of factors.
- If the quotient is not a counting number, the divisor is not a factor.

List all the factor pairs.

Write all the factors in order from smallest to largest.

- **Determine if a number is prime.**

Test each of the primes, in order, to see if it is a factor of the number.

Start with 2 and stop when the quotient is smaller than the divisor or when a prime factor is found.

If the number has a prime factor, then it is a composite number. If it has no prime factors, then the number is prime.

Practice Makes Perfect

Identify Multiples of Numbers

In the following exercises, list all the multiples less than 50 for the given number.

Exercise:

Problem: 2

Solution:

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48

Exercise:

Problem: 3

Exercise:

Problem: 4

Solution:

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

Exercise:

Problem: 5

Exercise:

Problem: 6

Solution:

6, 12, 18, 24, 30, 36, 42, 48

Exercise:

Problem: 7

Exercise:

Problem: 8

Solution:

8, 16, 24, 32, 40, 48

Exercise:

Problem: 9

Exercise:

Problem: 10

Solution:

10, 20, 30, 40

Exercise:

Problem: 12

Use Common Divisibility Tests

In the following exercises, use the divisibility tests to determine whether each number is divisible by 2, 3, 4, 5, 6, and 10.

Exercise:

Problem: 84

Solution:

Divisible by 2, 3, 4, 6

Exercise:

Problem: 96

Exercise:

Problem: 75

Solution:

Divisible by 3, 5

Exercise:

Problem: 78

Exercise:

Problem: 168

Solution:

Divisible by 2, 3, 4, 6

Exercise:

Problem: 264

Exercise:

Problem: 900

Solution:

Divisible by 2, 3, 4, 5, 6, 10

Exercise:

Problem: 800

Exercise:

Problem: 896

Solution:

Divisible by 2, 4

Exercise:

Problem: 942

Exercise:

Problem: 375

Solution:

Divisible by 3, 5

Exercise:

Problem: 750

Exercise:

Problem: 350

Solution:

Divisible by 2, 5, 10

Exercise:

Problem: 550

Exercise:

Problem: 1430

Solution:

Divisible by 2, 5, 10

Exercise:

Problem: 1080

Exercise:

Problem: 22,335

Solution:

Divisible by 3, 5

Exercise:

Problem: 39,075

Find All the Factors of a Number

In the following exercises, find all the factors of the given number.

Exercise:

Problem: 36

Solution:

1, 2, 3, 4, 6, 9, 12, 18, 36

Exercise:

Problem: 42

Exercise:

Problem: 60

Solution:

1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Exercise:

Problem: 48

Exercise:

Problem: 144

Solution:

1, 2, 3, 4, 6, 8, 12, 18, 24, 36, 48, 72, 144

Exercise:

Problem: 200

Exercise:

Problem: 588

Solution:

1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 49, 84, 98, 147, 196, 294, 588

Exercise:

Problem: 576

Identify Prime and Composite Numbers

In the following exercises, determine if the given number is prime or composite.

Exercise:

Problem: 43

Solution:

prime

Exercise:

Problem: 67

Exercise:

Problem: 39

Solution:

composite

Exercise:

Problem: 53

Exercise:

Problem: 71

Solution:

prime

Exercise:

Problem: 119

Exercise:

Problem: 481

Solution:

composite

Exercise:

Problem: 221

Exercise:

Problem: 209

Solution:

composite

Exercise:

Problem: 359

Exercise:

Problem: 667

Solution:

composite

Exercise:

Problem: 1771

Everyday Math

Exercise:

Problem:

Banking Frank's grandmother gave him \$100 at his high school graduation. Instead of spending it, Frank opened a bank account. Every week, he added \$15 to the account. The table shows how much money Frank had put in the account by the end of each week. Complete the table by filling in the blanks.

Weeks after graduation	Total number of dollars Frank put in the account	Simplified Total
0	100	100
1	$100 + 15$	115
2	$100 + 15 \cdot 2$	130
3	$100 + 15 \cdot 3$	
4	$100 + 15 \cdot []$	
5	$100 + []$	
6		
20		
x		

Solution:

Weeks after graduation	Total number of dollars Frank put in the account	Simplified Total
0	100	100
1	$100 + 15$	115
2	$100 + 15 \cdot 2$	130
3	$100 + 15 \cdot 3$	145
4	$100 + 15 \cdot 4$	160
5	$100 + 15 \cdot 5$	175
6	$100 + 15 \cdot 6$	190
20	$100 + 15 \cdot 20$	400
x	$100 + 15 \cdot x$	$100 + 15x$

Exercise:

Problem:

Banking In March, Gina opened a Christmas club savings account at her bank. She deposited \$75 to open the account. Every week, she added \$20 to the account. The table shows how much money Gina had put in the account by the end of each week. Complete the table by filling in the blanks.

Weeks after opening the account	Total number of dollars Gina put in the account	Simplified Total
0	75	75

Weeks after opening the account	Total number of dollars Gina put in the account	Simplified Total
1	$75 + 20$	95
2	$75 + 20 \cdot 2$	115
3	$75 + 20 \cdot 3$	
4	$75 + 20 \cdot []$	
5	$75 + []$	
6		
20		
x		

Writing Exercises

Exercise:

Problem: If a number is divisible by 2 and by 3, why is it also divisible by 6?

Exercise:

Problem: What is the difference between prime numbers and composite numbers?

Self Check

- Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify multiples of numbers.			
use common divisibility tests.			
find all the factors of a number.			
identify prime and composite numbers.			

- Ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

multiple of a number

A number is a multiple of n if it is the product of a counting number and n .

divisibility

If a number m is a multiple of n , then we say that m is divisible by n .

prime number

A prime number is a counting number greater than 1 whose only factors are 1 and itself.

composite number

A composite number is a counting number that is not prime.

Prime Factorization and the Least Common Multiple

By the end of this section, you will be able to:

- Find the prime factorization of a composite number
- Find the least common multiple (LCM) of two numbers

Note:

Before you get started, take this readiness quiz.

1. Is 810 divisible by 2, 3, 5, 6, or 10?
If you missed this problem, review [\[link\]](#).
2. Is 127 prime or composite?
If you missed this problem, review [\[link\]](#).
3. Write $2 \cdot 2 \cdot 2 \cdot 2$ in exponential notation.
If you missed this problem, review [\[link\]](#).

Find the Prime Factorization of a Composite Number

In the previous section, we found the factors of a number. Prime numbers have only two factors, the number 1 and the prime number itself. Composite numbers have more than two factors, and every composite number can be written as a unique product of primes. This is called the **prime factorization** of a number. When we write the prime factorization of a number, we are rewriting the number as a product of primes. Finding the prime factorization of a composite number will help you later in this course.

Note:

Prime Factorization

The prime factorization of a number is the product of prime numbers that equals the number.

Note: Doing the Manipulative Mathematics activity “Prime Numbers” will help you develop a better sense of prime numbers.

You may want to refer to the following list of prime numbers less than 50 as you work through this section.

Equation:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

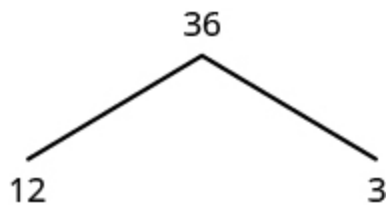
Prime Factorization Using the Factor Tree Method

One way to find the prime factorization of a number is to make a factor tree. We start by writing the number, and then writing it as the product of two factors. We write the factors below the number and connect them to the number with a small line segment—a “branch” of the factor tree.

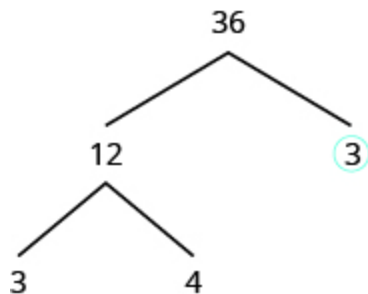
If a factor is prime, we circle it (like a bud on a tree), and do not factor that “branch” any further. If a factor is not prime, we repeat this process, writing it as the product of two factors and adding new branches to the tree.

We continue until all the branches end with a prime. When the factor tree is complete, the circled primes give us the prime factorization.

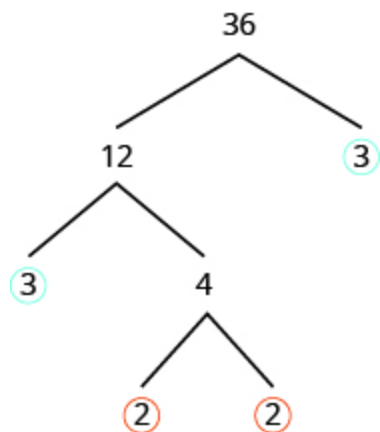
For example, let’s find the prime factorization of 36. We can start with any factor pair such as 3 and 12. We write 3 and 12 below 36 with branches connecting them.



The factor 3 is prime, so we circle it. The factor 12 is composite, so we need to find its factors. Let's use 3 and 4. We write these factors on the tree under the 12.



The factor 3 is prime, so we circle it. The factor 4 is composite, and it factors into $2 \cdot 2$. We write these factors under the 4. Since 2 is prime, we circle both 2s.



The prime factorization is the product of the circled primes. We generally write the prime factorization in order from least to greatest.

Equation:

$$2 \cdot 2 \cdot 3 \cdot 3$$

In cases like this, where some of the prime factors are repeated, we can write prime factorization in exponential form.

Equation:

$$2 \cdot 2 \cdot 3 \cdot 3$$

$$2^2 \cdot 3^2$$

Note that we could have started our factor tree with any factor pair of 36. We chose 12 and 3, but the same result would have been the same if we had started with 2 and 18, 4 and 9, or 6 and 6.

Note:

Find the prime factorization of a composite number using the tree method.

Find any factor pair of the given number, and use these numbers to create two branches.

If a factor is prime, that branch is complete. Circle the prime.

If a factor is not prime, write it as the product of a factor pair and continue the process.

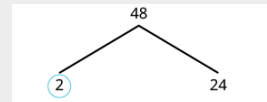
Write the composite number as the product of all the circled primes.

Example:**Exercise:****Problem:**

Find the prime factorization of 48 using the factor tree method.

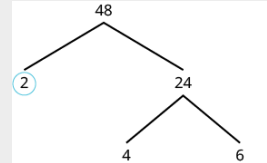
Solution:**Solution**

We can start our tree using any factor pair of 48. Let's use 2 and 24.



We circle the 2 because it is prime and so that branch is complete.

Now we will factor 24. Let's use 4 and 6.

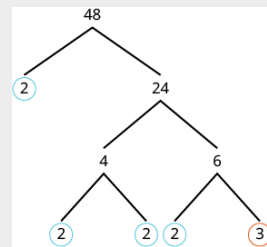


Neither factor is prime, so we do not circle either.

We factor the 4, using 2 and 2.

We factor 6, using 2 and 3.

We circle the 2s and the 3 since they are prime. Now all of the branches end in a prime.



Write the product of the circled numbers.

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$$

Write in exponential form.

$$2^4 \cdot 3$$

Check this on your own by multiplying all the factors together. The result should be 48.

Note:

Exercise:

Problem:

Find the prime factorization using the factor tree method: 80

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^4 \cdot 5$

Note:**Exercise:****Problem:**

Find the prime factorization using the factor tree method: 60

Solution:

$2 \cdot 2 \cdot 3 \cdot 5$, or $2^2 \cdot 3 \cdot 5$

Example:**Exercise:****Problem:**

Find the prime factorization of 84 using the factor tree method.

Solution:

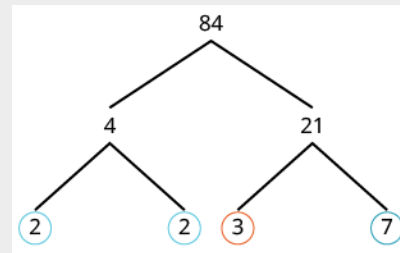
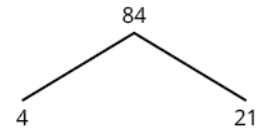
Solution

We start with the factor pair 4 and 21.

Neither factor is prime so we factor them further.

Now the factors are all prime, so we circle them.

Then we write 84 as the product of all circled primes.



$$2 \cdot 2 \cdot 3 \cdot 7$$
$$2^2 \cdot 3 \cdot 7$$

Draw a factor tree of 84.

Note:

Exercise:

Problem:

Find the prime factorization using the factor tree method: 126

Solution:

$$2 \cdot 3 \cdot 3 \cdot 7, \text{ or } 2 \cdot 3^2 \cdot 7$$

Note:

Exercise:

Problem:

Find the prime factorization using the factor tree method: 294

Solution:

$$2 \cdot 3 \cdot 7 \cdot 7, \text{ or } 2 \cdot 3 \cdot 7^2$$

Prime Factorization Using the Ladder Method

The ladder method is another way to find the prime factors of a composite number. It leads to the same result as the factor tree method. Some people prefer the ladder method to the factor tree method, and vice versa.

To begin building the “ladder,” divide the given number by its smallest prime factor. For example, to start the ladder for 36, we divide 36 by 2, the smallest prime factor of 36.

$$\begin{array}{r} 18 \\ 2 \overline{) 36} \end{array}$$

To add a “step” to the ladder, we continue dividing by the same prime until it no longer divides evenly.

$$\begin{array}{r} 9 \\ 2 \overline{) 18} \\ 2 \overline{) 36} \end{array}$$

Then we divide by the next prime; so we divide 9 by 3.

$$\begin{array}{r} 3 \\ 3 \overline{) 9} \\ 2 \overline{) 18} \\ 2 \overline{) 36} \end{array}$$

We continue dividing up the ladder in this way until the quotient is prime. Since the quotient, 3, is prime, we stop here.

Do you see why the ladder method is sometimes called stacked division?

The prime factorization is the product of all the primes on the sides and top of the ladder.

Equation:

$$2 \cdot 2 \cdot 3 \cdot 3$$

$$2^2 \cdot 3^2$$

Notice that the result is the same as we obtained with the factor tree method.

Note:

Find the prime factorization of a composite number using the ladder method.

Divide the number by the smallest prime.

Continue dividing by that prime until it no longer divides evenly.

Divide by the next prime until it no longer divides evenly.

Continue until the quotient is a prime.

Write the composite number as the product of all the primes on the sides and top of the ladder.

Example:

Exercise:

Problem:

Find the prime factorization of 120 using the ladder method.

Solution:**Solution**

Divide the number by the smallest prime, which is 2.

$$\begin{array}{r} 60 \\ 2 \overline{) 120} \end{array}$$

Continue dividing by 2 until it no longer divides evenly.

$$\begin{array}{r} 15 \\ 2 \overline{) 30} \\ 2 \overline{) 60} \\ 2 \overline{) 120} \end{array}$$

Divide by the next prime, 3.

$$\begin{array}{r} 5 \\ 3 \overline{) 15} \\ 2 \overline{) 30} \\ 2 \overline{) 60} \\ 2 \overline{) 120} \end{array}$$

The quotient, 5, is prime, so the ladder is complete. Write the prime factorization of 120.

$$\begin{array}{l} 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\ 2^3 \cdot 3 \cdot 5 \end{array}$$

Check this yourself by multiplying the factors. The result should be 120.

Note:

Exercise:

Problem: Find the prime factorization using the ladder method: 80

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^4 \cdot 5$

Note:

Exercise:

Problem: Find the prime factorization using the ladder method: 60

Solution:

$2 \cdot 2 \cdot 3 \cdot 5$, or $2^2 \cdot 3 \cdot 5$

Example:

Exercise:

Problem: Find the prime factorization of 48 using the ladder method.

Solution:

Solution

Divide the number by the smallest prime, 2.	$\begin{array}{r} 24 \\ 2 \overline{)48} \end{array}$
Continue dividing by 2 until it no longer divides evenly.	$\begin{array}{r} 3 \\ 2 \overline{)6} \\ 2 \overline{)12} \\ 2 \overline{)24} \\ 2 \overline{)48} \end{array}$
The quotient, 3, is prime, so the ladder is complete. Write the prime factorization of 48.	$\begin{array}{l} 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \\ 2^4 \cdot 3 \end{array}$

Note:

Exercise:

Problem: Find the prime factorization using the ladder method. 126

Solution:

$$2 \cdot 3 \cdot 3 \cdot 7, \text{ or } 2 \cdot 3^2 \cdot 7$$

Note:

Exercise:

Problem: Find the prime factorization using the ladder method. 294

Solution:

$$2 \cdot 3 \cdot 7 \cdot 7, \text{ or } 2 \cdot 3 \cdot 7^2$$

Find the Least Common Multiple (LCM) of Two Numbers

One of the reasons we look at multiples and primes is to use these techniques to find the least common multiple of two numbers. This will be useful when we add and subtract fractions with different denominators.

Listing Multiples Method

A common multiple of two numbers is a number that is a multiple of both numbers. Suppose we want to find common multiples of 10 and 25. We can list the first several multiples of each number. Then we look for multiples that are common to both lists—these are the common multiples.

Equation:

10:10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, ...

25:25, 50, 75, 100, 125, ...

We see that 50 and 100 appear in both lists. They are common multiples of 10 and 25. We would find more common multiples if we continued the list of multiples for each.

The smallest number that is a multiple of two numbers is called the **least common multiple** (LCM). So the least LCM of 10 and 25 is 50.

Note:

Find the least common multiple (LCM) of two numbers by listing multiples.

List the first several multiples of each number.

Look for multiples common to both lists. If there are no common multiples in the lists, write out additional multiples for each number.

Look for the smallest number that is common to both lists.

This number is the LCM.

Example:

Exercise:

Problem: Find the LCM of 15 and 20 by listing multiples.

Solution:

Solution

List the first several multiples of 15 and of 20. Identify the first common multiple.

15: 15, 30, 45, 60, 75, 90, 105, 120

20: 20, 40, 60, 80, 100, 120, 140, 160

The smallest number to appear on both lists is 60, so 60 is the least common multiple of 15 and 20.

Notice that 120 is on both lists, too. It is a common multiple, but it is not the least common multiple.

Note:

Exercise:

Problem:

Find the least common multiple (LCM) of the given numbers:
9 and 12

Solution:

36

Note:

Exercise:

Problem:

Find the least common multiple (LCM) of the given numbers:
18 and 24

Solution:

72

Prime Factors Method

Another way to find the least common multiple of two numbers is to use their prime factors. We'll use this method to find the LCM of 12 and 18.

We start by finding the prime factorization of each number.

Equation:

$$12 = 2 \cdot 2 \cdot 3 \qquad 18 = 2 \cdot 3 \cdot 3$$

Then we write each number as a product of primes, matching primes vertically when possible.

Equation:

$$\begin{array}{l} 12 = 2 \cdot 2 \cdot 3 \\ 18 = 2 \cdot \quad 3 \cdot 3 \end{array}$$

Now we bring down the primes in each column. The LCM is the product of these factors.

$$\begin{array}{r} 12 = 2 \cdot 2 \cdot 3 \\ 18 = 2 \cdot \cdot 3 \cdot 3 \\ \hline \text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3 \\ \text{LCM} = 2 \cdot 2 \cdot 3 \cdot 3 = 36 \end{array}$$

Notice that the prime factors of 12 and the prime factors of 18 are included in the LCM. By matching up the common primes, each common prime factor is used only once. This ensures that 36 is the least common multiple.

Note:

Find the LCM using the prime factors method.

Find the prime factorization of each number.

Write each number as a product of primes, matching primes vertically when possible.

Bring down the primes in each column.

Multiply the factors to get the LCM.

Example:

Exercise:

Problem:

Find the LCM of 15 and 18 using the prime factors method.

Solution:

Solution

Write each number as a product of primes.	$15 = 3 \cdot 5$ $18 = 2 \cdot 3 \cdot 3$
Write each number as a product of primes, matching primes vertically when possible.	$ \begin{array}{r} 15 = 3 \cdot 5 \\ 18 = 2 \cdot 3 \cdot 3 \end{array} $
Bring down the primes in each column.	$ \begin{array}{r} 15 = 3 \cdot 5 \\ 18 = 2 \cdot 3 \cdot 3 \\ \hline \text{LCM} = 2 \cdot 3 \cdot 3 \cdot 5 \end{array} $
Multiply the factors to get the LCM.	$\text{LCM} = 2 \cdot 3 \cdot 3 \cdot 5$ The LCM of 15 and 18 is 90.

Note:

Exercise:

Problem: Find the LCM using the prime factors method. 15 and 20

Solution:

60

Note:

Exercise:

Problem: Find the LCM using the prime factors method. 15 and 35

Solution:

105

Example:

Exercise:

Problem:

Find the LCM of 50 and 100 using the prime factors method.

Solution:

Solution

Write the prime factorization of each number.

$$50 = 2 \cdot 5 \cdot 5 \quad 100 = 2 \cdot 2 \cdot 5 \cdot 5$$

Write each number as a product of primes, matching primes vertically when possible.

$$\begin{array}{rcl} 50 & = & 2 \cdot 5 \cdot 5 \\ 100 & = & 2 \cdot 2 \cdot 5 \cdot 5 \end{array}$$

Bring down the primes in each column.

$$\begin{array}{rcl} 50 & = & 2 \cdot 5 \cdot 5 \\ 100 & = & 2 \cdot 2 \cdot 5 \cdot 5 \\ \hline \text{LCM} & = & 2 \cdot 2 \cdot 5 \cdot 5 \end{array}$$

Multiply the factors to get the LCM.

$$\begin{array}{l} \text{LCM} = 2 \cdot 2 \cdot 5 \cdot 5 \\ \text{The LCM of 50 and} \\ \text{100 is 100.} \end{array}$$

Note:

Exercise:

Problem: Find the LCM using the prime factors method: 55, 88

Solution:

440

Note:

Exercise:

Problem: Find the LCM using the prime factors method: 60, 72

Solution:

360

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Ex 1: Prime Factorization](#)
- [Ex 2: Prime Factorization](#)
- [Ex 3: Prime Factorization](#)
- [Ex 1: Prime Factorization Using Stacked Division](#)
- [Ex 2: Prime Factorization Using Stacked Division](#)
- [The Least Common Multiple](#)
- [Example: Determining the Least Common Multiple Using a List of Multiples](#)
- [Example: Determining the Least Common Multiple Using Prime Factorization](#)

Key Concepts

- **Find the prime factorization of a composite number using the tree method.**

Find any factor pair of the given number, and use these numbers to create two branches.

If a factor is prime, that branch is complete. Circle the prime.

If a factor is not prime, write it as the product of a factor pair and continue the process.

Write the composite number as the product of all the circled primes.

- **Find the prime factorization of a composite number using the ladder method.**

Divide the number by the smallest prime.

Continue dividing by that prime until it no longer divides evenly.

Divide by the next prime until it no longer divides evenly.

Continue until the quotient is a prime.

Write the composite number as the product of all the primes on the sides and top of the ladder.

- **Find the LCM by listing multiples.**

List the first several multiples of each number.

Look for multiples common to both lists. If there are no common multiples in the lists, write out additional multiples for each number.

Look for the smallest number that is common to both lists.

This number is the LCM.

- **Find the LCM using the prime factors method.**

Find the prime factorization of each number.

Write each number as a product of primes, matching primes vertically when possible.

Bring down the primes in each column.

Multiply the factors to get the LCM.

Section Exercises

Practice Makes Perfect

Find the Prime Factorization of a Composite Number

In the following exercises, find the prime factorization of each number using the factor tree method.

Exercise:

Problem: 86

Solution:

$$2 \cdot 43$$

Exercise:

Problem: 78

Exercise:

Problem: 132

Solution:

$$2 \cdot 2 \cdot 3 \cdot 11$$

Exercise:

Problem: 455

Exercise:

Problem: 693

Solution:

$$3 \cdot 3 \cdot 7 \cdot 11$$

Exercise:

Problem: 420

Exercise:

Problem: 115

Solution:

$$5 \cdot 23$$

Exercise:

Problem: 225

Exercise:

Problem: 2475

Solution:

$$3 \cdot 3 \cdot 5 \cdot 5 \cdot 11$$

Exercise:

Problem: 1560

In the following exercises, find the prime factorization of each number using the ladder method.

Exercise:

Problem: 56

Solution:

$$2 \cdot 2 \cdot 2 \cdot 7$$

Exercise:

Problem: 72

Exercise:

Problem: 168

Solution:

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 7$$

Exercise:

Problem: 252

Exercise:

Problem: 391

Solution:

$$17 \cdot 23$$

Exercise:

Problem: 400

Exercise:

Problem: 432

Solution:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$$

Exercise:

Problem: 627

Exercise:

Problem: 2160

Solution:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$$

Exercise:

Problem: 2520

In the following exercises, find the prime factorization of each number using any method.

Exercise:

Problem: 150

Solution:

$$2 \cdot 3 \cdot 5 \cdot 5$$

Exercise:

Problem: 180

Exercise:

Problem: 525

Solution:

$$3 \cdot 5 \cdot 5 \cdot 7$$

Exercise:

Problem: 444

Exercise:

Problem: 36

Solution:

$$2 \cdot 2 \cdot 3 \cdot 3$$

Exercise:

Problem: 50

Exercise:

Problem: 350

Solution:

$$2 \cdot 5 \cdot 5 \cdot 7$$

Exercise:

Problem: 144

Find the Least Common Multiple (LCM) of Two Numbers

In the following exercises, find the least common multiple (LCM) by listing multiples.

Exercise:

Problem: 8, 12

Solution:

$$24$$

Exercise:

Problem: 4, 3

Exercise:

Problem: 6, 15

Solution:

30

Exercise:

Problem: 12, 16

Exercise:

Problem: 30, 40

Solution:

120

Exercise:

Problem: 20, 30

Exercise:

Problem: 60, 75

Solution:

300

Exercise:

Problem: 44, 55

In the following exercises, find the least common multiple (LCM) by using the prime factors method.

Exercise:

Problem: 8, 12

Solution:

24

Exercise:

Problem: 12, 16

Exercise:

Problem: 24, 30

Solution:

120

Exercise:

Problem: 28, 40

Exercise:

Problem: 70, 84

Solution:

420

Exercise:

Problem: 84, 90

In the following exercises, find the least common multiple (LCM) using any method.

Exercise:

Problem: 6, 21

Solution:

42

Exercise:

Problem: 9, 15

Exercise:

Problem: 24, 30

Solution:

120

Exercise:

Problem: 32, 40

Everyday Math

Exercise:

Problem:

Grocery shopping Hot dogs are sold in packages of ten, but hot dog buns come in packs of eight. What is the smallest number of hot dogs and buns that can be purchased if you want to have the same number of hot dogs and buns? (Hint: it is the LCM!)

Solution:

40

Exercise:**Problem:**

Grocery shopping Paper plates are sold in packages of 12 and party cups come in packs of 8. What is the smallest number of plates and cups you can purchase if you want to have the same number of each? (Hint: it is the LCM!)

Writing Exercises**Exercise:****Problem:**

Do you prefer to find the prime factorization of a composite number by using the factor tree method or the ladder method? Why?

Exercise:**Problem:**

Do you prefer to find the LCM by listing multiples or by using the prime factors method? Why?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
find the prime factorization of a composite number.			
find the least common multiple (LCM) of two numbers.			

Ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Use the Language of Algebra

Use Variables and Algebraic Symbols

In the following exercises, translate from algebra to English.

Exercise:

Problem: $3 \cdot 8$

Solution:

the product of 3 and 8

Exercise:

Problem: $12 - x$

Exercise:

Problem: $24 \div 6$

Solution:

the quotient of 24 and 6

Exercise:

Problem: $9 + 2a$

Exercise:

Problem: $50 \geq 47$

Solution:

50 is greater than or equal to 47

Exercise:

Problem: $3y < 15$

Exercise:

Problem: $n + 4 = 13$

Solution:

The sum of n and 4 is equal to 13

Exercise:

Problem: $32 - k = 7$

Identify Expressions and Equations

In the following exercises, determine if each is an expression or equation.

Exercise:

Problem: $5 + u = 84$

Solution:

equation

Exercise:

Problem: $36 - 6s$

Exercise:

Problem: $4y - 11$

Solution:

expression

Exercise:

Problem: $10x = 120$

Simplify Expressions with Exponents

In the following exercises, write in exponential form.

Exercise:

Problem: $2 \cdot 2 \cdot 2$

Solution:

2^3

Exercise:

Problem: $a \cdot a \cdot a \cdot a \cdot a$

Exercise:

Problem: $x \cdot x \cdot x \cdot x \cdot x \cdot x$

Solution:

$$x^6$$

Exercise:

Problem: $10 \cdot 10 \cdot 10$

In the following exercises, write in expanded form.

Exercise:

Problem: 8^4

Solution:

$$8 \cdot 8 \cdot 8 \cdot 8$$

Exercise:

Problem: 3^6

Exercise:

Problem: y^5

Solution:

$$y \cdot y \cdot y \cdot y \cdot y$$

Exercise:

Problem: n^4

In the following exercises, simplify each expression.

Exercise:

Problem: 3^4

Solution:

81

Exercise:

Problem: 10^6

Exercise:

Problem: 2^7

Solution:

128

Exercise:

Problem: 4^3

Simplify Expressions Using the Order of Operations

In the following exercises, simplify.

Exercise:

Problem: $10 + 2 \cdot 5$

Solution:

20

Exercise:

Problem: $(10 + 2) \cdot 5$

Exercise:

Problem: $(30 + 6) \div 2$

Solution:

18

Exercise:

Problem: $30 + 6 \div 2$

Exercise:

Problem: $7^2 + 5^2$

Solution:

74

Exercise:

Problem: $(7 + 5)^2$

Exercise:

Problem: $4 + 3(10 - 1)$

Solution:

31

Exercise:

Problem: $(4 + 3)(10 - 1)$

Evaluate, Simplify, and Translate Expressions

Evaluate an Expression

In the following exercises, evaluate the following expressions.

Exercise:

Problem: $9x - 5$ when $x = 7$

Solution:

58

Exercise:

Problem: y^3 when $y = 5$

Exercise:

Problem: $3a - 4b$ when $a = 10, b = 1$

Solution:

26

Exercise:

Problem: bh when $b = 7, h = 8$

Identify Terms, Coefficients and Like Terms

In the following exercises, identify the terms in each expression.

Exercise:

Problem: $12n^2 + 3n + 1$

Solution:

$$12n^2, 3n, 1$$

Exercise:

Problem: $4x^3 + 11x + 3$

In the following exercises, identify the coefficient of each term.

Exercise:

Problem: $6y$

Solution:

$$6$$

Exercise:

Problem: $13x^2$

In the following exercises, identify the like terms.

Exercise:

Problem: $5x^2, 3, 5y^2, 3x, x, 4$

Solution:

$$3, 4, \text{ and } 3x, x$$

Exercise:

Problem: $8, 8r^2, 8r, 3r, r^2, 3s$

Simplify Expressions by Combining Like Terms

In the following exercises, simplify the following expressions by combining like terms.

Exercise:

Problem: $15a + 9a$

Solution:

$$24a$$

Exercise:

Problem: $12y + 3y + y$

Exercise:

Problem: $4x + 7x + 3x$

Solution:

$$14x$$

Exercise:

Problem: $6 + 5c + 3$

Exercise:

Problem: $8n + 2 + 4n + 9$

Solution:

$$12n + 11$$

Exercise:

Problem: $19p + 5 + 4p - 1 + 3p$

Exercise:

Problem: $7y^2 + 2y + 11 + 3y^2 - 8$

Solution:

$$10y^2 + 2y + 3$$

Exercise:

Problem: $13x^2 - x + 6 + 5x^2 + 9x$

Translate English Phrases to Algebraic Expressions

In the following exercises, translate the following phrases into algebraic expressions.

Exercise:

Problem: the difference of x and 6

Solution:

$$x - 6$$

Exercise:

Problem: the sum of 10 and twice a

Exercise:

Problem: the product of $3n$ and 9

Solution:

$$3n \cdot 9$$

Exercise:

Problem: the quotient of s and 4

Exercise:

Problem: 5 times the sum of y and 1

Solution:

$$5(y + 1)$$

Exercise:

Problem: 10 less than the product of 5 and z

Exercise:

Problem:

Jack bought a sandwich and a coffee. The cost of the sandwich was \$3 more than the cost of the coffee. Call the cost of the coffee c . Write an expression for the cost of the sandwich.

Solution:

$$c + 3$$

Exercise:

Problem:

The number of poetry books on Brianna's bookshelf is 5 less than twice the number of novels. Call the number of novels n . Write an expression for the number of poetry books.

[Solve Equations Using the Subtraction and Addition Properties of Equality](#)

Determine Whether a Number is a Solution of an Equation

In the following exercises, determine whether each number is a solution to the equation.

Exercise:

Problem: $y + 16 = 40$

- Ⓐ 24
 - Ⓑ 56
-

Solution:

- Ⓐ yes
- Ⓑ no

Exercise:

Problem: $d - 6 = 21$

- Ⓐ 15
- Ⓑ 27

Exercise:

Problem: $4n + 12 = 36$

- Ⓐ 6
 - Ⓑ 12
-

Solution:

- Ⓐ yes
- Ⓑ no

Exercise:

Problem: $20q - 10 = 70$

- Ⓐ 3
- Ⓑ 4

Exercise:

Problem: $15x - 5 = 10x + 45$

- Ⓐ 2
- Ⓑ 10

Solution:

- Ⓐ no
- Ⓑ yes

Exercise:

Problem: $22p - 6 = 18p + 86$

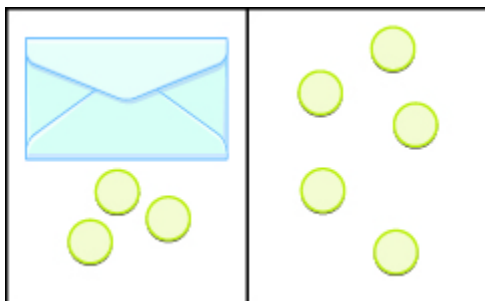
- Ⓐ 4
- Ⓑ 23

Model the Subtraction Property of Equality

In the following exercises, write the equation modeled by the envelopes and counters and then solve the equation using the subtraction property of equality.

Exercise:

Problem:

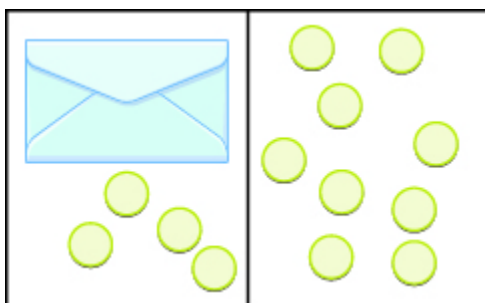


Solution:

$$x + 3 = 5; x = 2$$

Exercise:

Problem:



Solve Equations using the Subtraction Property of Equality

In the following exercises, solve each equation using the subtraction property of equality.

Exercise:

Problem: $c + 8 = 14$

Solution:

6

Exercise:

Problem: $v + 8 = 150$

Exercise:

Problem: $23 = x + 12$

Solution:

11

Exercise:

Problem: $376 = n + 265$

Solve Equations using the Addition Property of Equality

In the following exercises, solve each equation using the addition property of equality.

Exercise:

Problem: $y - 7 = 16$

Solution:

23

Exercise:

Problem: $k - 42 = 113$

Exercise:

Problem: $19 = p - 15$

Solution:

34

Exercise:

Problem: $501 = u - 399$

Translate English Sentences to Algebraic Equations

In the following exercises, translate each English sentence into an algebraic equation.

Exercise:

Problem: The sum of 7 and 33 is equal to 40.

Solution:

$$7 + 33 = 44$$

Exercise:

Problem: The difference of 15 and 3 is equal to 12.

Exercise:

Problem: The product of 4 and 8 is equal to 32.

Solution:

$$4 \cdot 8 = 32$$

Exercise:

Problem: The quotient of 63 and 9 is equal to 7.

Exercise:

Problem: Twice the difference of n and 3 gives 76.

Solution:

$$2(n - 3) = 76$$

Exercise:

Problem: The sum of five times y and 4 is 89.

Translate to an Equation and Solve

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise:

Problem: Eight more than x is equal to 35.

Solution:

$$x + 8 = 35; x = 27$$

Exercise:

Problem: 21 less than a is 11.

Exercise:

Problem: The difference of q and 18 is 57.

Solution:

$$q - 18 = 57; q = 75$$

Exercise:

Problem: The sum of m and 125 is 240.

Mixed Practice

In the following exercises, solve each equation.

Exercise:

Problem: $h - 15 = 27$

Solution:

$$h = 42$$

Exercise:

Problem: $k - 11 = 34$

Exercise:

Problem: $z + 52 = 85$

Solution:

$$z = 33$$

Exercise:

Problem: $x + 93 = 114$

Exercise:

Problem: $27 = q + 19$

Solution:

$$q = 8$$

Exercise:

Problem: $38 = p + 19$

Exercise:

Problem: $31 = v - 25$

Solution:

$$v = 56$$

Exercise:

Problem: $38 = u - 16$

Find Multiples and Factors

Identify Multiples of Numbers

In the following exercises, list all the multiples less than 50 for each of the following.

Exercise:

Problem: 3

Solution:

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48

Exercise:

Problem: 2

Exercise:

Problem: 8

Solution:

8, 16, 24, 32, 40, 48

Exercise:

Problem: 10

Use Common Divisibility Tests

In the following exercises, using the divisibility tests, determine whether each number is divisible by 2, by 3, by 5, by 6, and by 10.

Exercise:

Problem: 96

Solution:

2, 3, 6

Exercise:

Problem: 250

Exercise:

Problem: 420

Solution:

2, 3, 5, 6, 10

Exercise:

Problem: 625

Find All the Factors of a Number

In the following exercises, find all the factors of each number.

Exercise:

Problem: 30

Solution:

1, 2, 3, 5, 6, 10, 15, 30

Exercise:

Problem: 70

Exercise:

Problem: 180

Solution:

1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180

Exercise:

Problem: 378

Identify Prime and Composite Numbers

In the following exercises, identify each number as prime or composite.

Exercise:

Problem: 19

Solution:

prime

Exercise:

Problem: 51

Exercise:

Problem: 121

Solution:

composite

Exercise:

Problem: 219

Prime Factorization and the Least Common Multiple

Find the Prime Factorization of a Composite Number

In the following exercises, find the prime factorization of each number.

Exercise:

Problem: 84

Solution:

$$2 \cdot 2 \cdot 3 \cdot 7$$

Exercise:

Problem: 165

Exercise:

Problem: 350

Solution:

$$2 \cdot 5 \cdot 5 \cdot 7$$

Exercise:

Problem: 572

Find the Least Common Multiple of Two Numbers

In the following exercises, find the least common multiple of each pair of numbers.

Exercise:

Problem: 9, 15

Solution:

45

Exercise:

Problem: 12, 20

Exercise:

Problem: 25, 35

Solution:

175

Exercise:

Problem: 18, 40

Everyday Math

Exercise:

Problem:

Describe how you have used two topics from [The Language of Algebra](#) chapter in your life outside of your math class during the past month.

Solution:

Answers will vary

Chapter Practice Test

In the following exercises, translate from an algebraic equation to English phrases.

Exercise:

Problem: $6 \cdot 4$

Exercise:

Problem: $15 - x$

Solution:

fifteen minus x

In the following exercises, identify each as an expression or equation.

Exercise:

Problem: $5 \cdot 8 + 10$

Exercise:

Problem: $x + 6 = 9$

Solution:

equation

Exercise:

Problem: $3 \cdot 11 = 33$

Exercise:

Problem:

- Ⓐ Write $n \cdot n \cdot n \cdot n \cdot n \cdot n$ in exponential form.
 - Ⓑ Write 3^5 in expanded form and then simplify.
-

Solution:

- Ⓐ n^6
- Ⓑ $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

In the following exercises, simplify, using the order of operations.

Exercise:

Problem: $4 + 3 \cdot 5$

Exercise:

Problem: $(8 + 1) \cdot 4$

Solution:

36

Exercise:

Problem: $1 + 6(3 - 1)$

Exercise:

Problem: $(8 + 4) \div 3 + 1$

Solution:

5

Exercise:

Problem: $(1 + 4)^2$

Exercise:

Problem: $5[2 + 7(9 - 8)]$

Solution:

45

In the following exercises, evaluate each expression.

Exercise:

Problem: $8x - 3$ when $x = 4$

Exercise:

Problem: y^3 when $y = 5$

Solution:

125

Exercise:

Problem: $6a - 2b$ when $a = 5, b = 7$

Exercise:

Problem: hw when $h = 12, w = 3$

Solution:

36

Exercise:

Problem: Simplify by combining like terms.

Ⓐ $6x + 8x$

Ⓑ $9m + 10 + m + 3$

In the following exercises, translate each phrase into an algebraic expression.

Exercise:

Problem: 5 more than x

Solution:

$$x + 5$$

Exercise:

Problem: the quotient of 12 and y

Exercise:

Problem: three times the difference of a and b

Solution:

$$3(a - b)$$

Exercise:

Problem:

Caroline has 3 fewer earrings on her left ear than on her right ear. Call the number of earrings on her right ear, r . Write an expression for the number of earrings on her left ear.

In the following exercises, solve each equation.

Exercise:

Problem: $n - 6 = 25$

Solution:

$$n = 31$$

Exercise:

Problem: $x + 58 = 71$

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise:**Problem:** 15 less than y is 32.

Solution:

$$y - 15 = 32; y = 47$$

Exercise:**Problem:** the sum of a and 129 is 164.**Exercise:****Problem:** List all the multiples of 4, that are less than 50.

Solution:

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

Exercise:

Problem: Find all the factors of 90.

Exercise:

Problem: Find the prime factorization of 1080.

Solution:

$$2^3 \cdot 3^3 \cdot 5$$

Exercise:

Problem: Find the LCM (Least Common Multiple) of 24 and 40.

Glossary

least common multiple

The smallest number that is a multiple of two numbers is called the least common multiple (LCM).

prime factorization

The prime factorization of a number is the product of prime numbers that equals the number.

Introduction to Integers

class="introduction"

The peak
of Mount
Everest.
(credit:
Gunther
Hagleitner
, Flickr)



At over 29,000 feet, Mount Everest stands as the tallest peak on land. Located along the border of Nepal and China, Mount Everest is also known for its extreme climate. Near the summit, temperatures never rise above freezing. Every year, climbers from around the world brave the extreme conditions in an effort to scale the tremendous height. Only some are successful. Describing the drastic change in elevation the climbers experience and the change in temperatures requires using numbers that extend both above and below zero. In this chapter, we will describe these kinds of numbers and operations using them.

Introduction to Integers

By the end of this section, you will be able to:

- Locate positive and negative numbers on the number line
- Order positive and negative numbers
- Find opposites
- Simplify expressions with absolute value
- Translate word phrases to expressions with integers

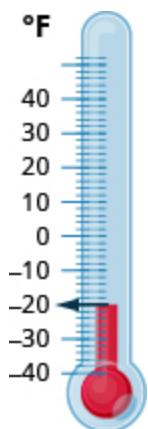
Note:

Before you get started, take this readiness quiz.

1. Plot 0, 1, and 3 on a number line.
If you missed this problem, review [\[link\]](#).
2. Fill in the appropriate symbol: ($=$, $<$, or $>$): 2 ____ 4
If you missed this problem, review [\[link\]](#).

Locate Positive and Negative Numbers on the Number Line

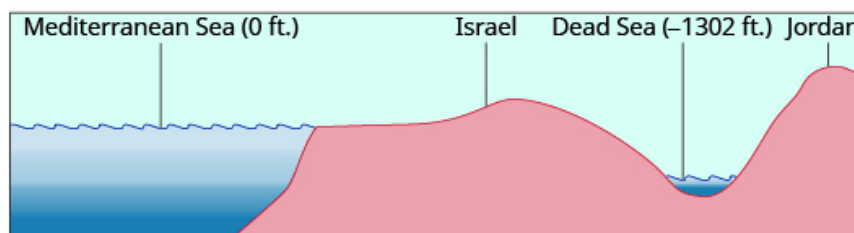
Do you live in a place that has very cold winters? Have you ever experienced a temperature below zero? If so, you are already familiar with negative numbers. A **negative number** is a number that is less than 0. Very cold temperatures are measured in degrees below zero and can be described by negative numbers. For example, -1°F (read as “negative one degree Fahrenheit”) is 1 degree below 0. A minus sign is shown before a number to indicate that it is negative. [\[link\]](#) shows -20°F , which is 20 degrees below 0.



Temperatures below zero are described by negative numbers.

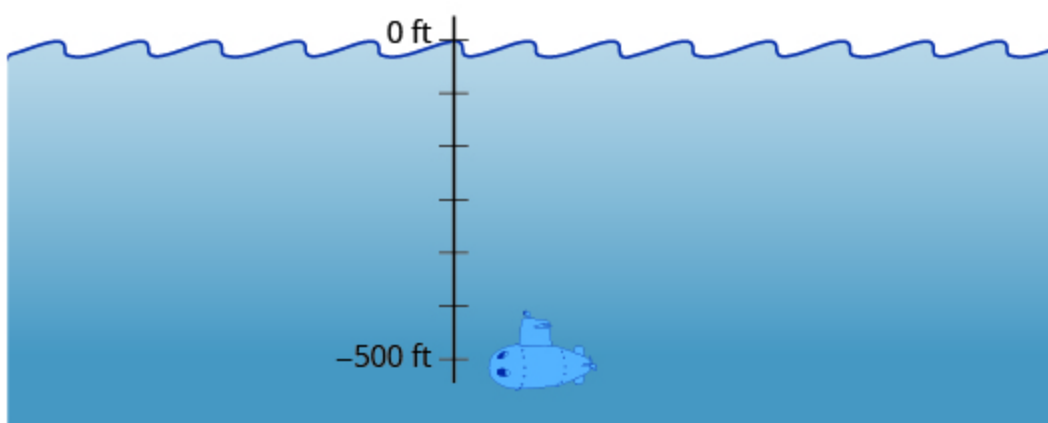
Temperatures are not the only negative numbers. A bank overdraft is another example of a negative number. If a person writes a check for more than he has in his account, his balance will be negative.

Elevations can also be represented by negative numbers. The elevation at sea level is 0 feet. Elevations above sea level are positive and elevations below sea level are negative. The elevation of the Dead Sea, which borders Israel and Jordan, is about 1,302 feet below sea level, so the elevation of the Dead Sea can be represented as $-1,302$ feet. See [\[link\]](#).



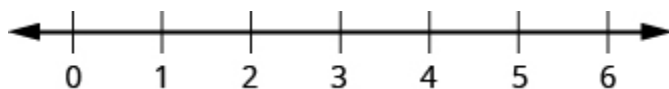
The surface of the Mediterranean Sea has an elevation of 0 ft. The diagram shows that nearby mountains have higher (positive) elevations whereas the Dead Sea has a lower (negative) elevation.

Depths below the ocean surface are also described by negative numbers. A submarine, for example, might descend to a depth of 500 feet. Its position would then be -500 feet as labeled in [\[link\]](#).

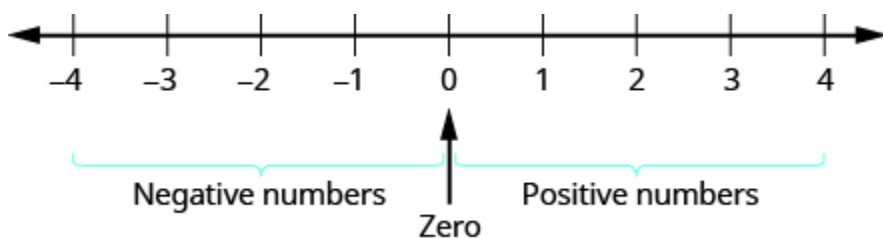


Depths below sea level are described by negative numbers. A submarine 500 ft below sea level is at -500 ft.

Both positive and negative numbers can be represented on a number line. Recall that the number line created in [Add Whole Numbers](#) started at 0 and showed the counting numbers increasing to the right as shown in [\[link\]](#). The counting numbers (1, 2, 3, ...) on the number line are all positive. We could write a plus sign, +, before a positive number such as +2 or +3, but it is customary to omit the plus sign and write only the number. If there is no sign, the number is assumed to be positive.



Now we need to extend the number line to include negative numbers. We mark several units to the left of zero, keeping the intervals the same width as those on the positive side. We label the marks with negative numbers, starting with -1 at the first mark to the left of 0, -2 at the next mark, and so on. See [\[link\]](#).



On a number line, positive numbers are to the right of zero. Negative numbers are to the left of zero. What about zero? Zero is neither positive nor negative.

The arrows at either end of the line indicate that the number line extends forever in each direction. There is no greatest positive number and there is no smallest negative number.

Note:

Doing the Manipulative Mathematics activity "Number Line-part 2" will help you develop a better understanding of integers.

Example:

Exercise:

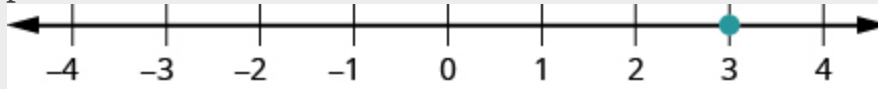
Problem: Plot the numbers on a number line:

- Ⓐ 3
- Ⓑ -3
- Ⓒ -2

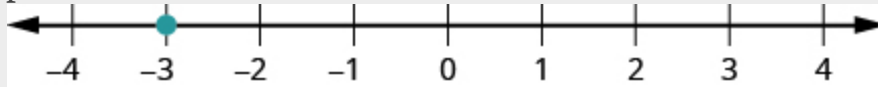
Solution:
Solution

Draw a number line. Mark 0 in the center and label several units to the left and right.

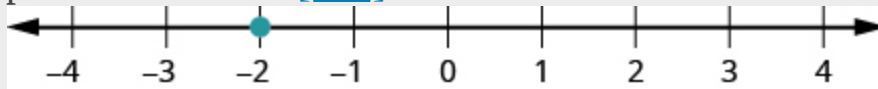
- Ⓐ To plot 3, start at 0 and count three units to the right. Place a point as shown in [\[link\]](#).



- Ⓑ To plot -3 , start at 0 and count three units to the left. Place a point as shown in [\[link\]](#).



- Ⓒ To plot -2 , start at 0 and count two units to the left. Place a point as shown in [\[link\]](#).



Note:

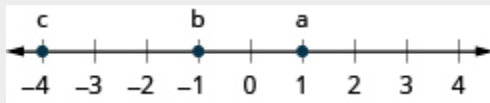
Exercise:

Problem: Plot the numbers on a number line.

- Ⓐ 1
- Ⓑ -1

Ⓒ -4

Solution:



Note:

Exercise:

Problem: Plot the numbers on a number line.

Ⓐ -4

Ⓑ 4

Ⓐ -1

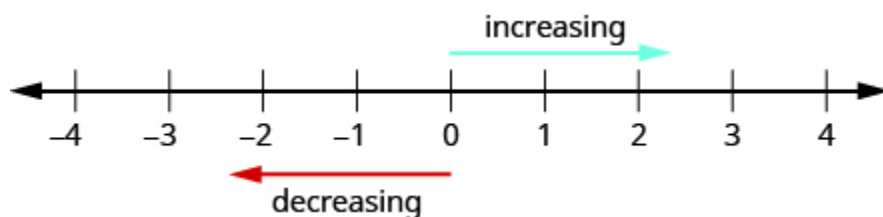
Solution:



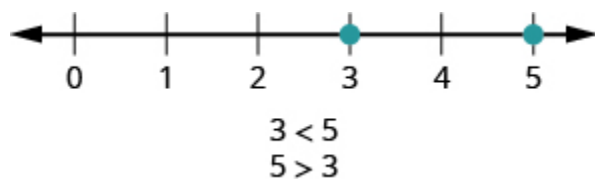
Order Positive and Negative Numbers

We can use the number line to compare and order positive and negative numbers. Going from left to right, numbers increase in value. Going from

right to left, numbers decrease in value. See [\[link\]](#).



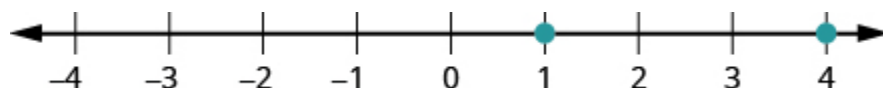
Just as we did with positive numbers, we can use inequality symbols to show the ordering of positive and negative numbers. Remember that we use the notation $a < b$ (read *a is less than b*) when a is to the left of b on the number line. We write $a > b$ (read *a is greater than b*) when a is to the right of b on the number line. This is shown for the numbers 3 and 5 in [\[link\]](#).



The number 3 is to the left of 5 on the number line. So 3 is less than 5, and 5 is greater than 3.

The numbers lines to follow show a few more examples.

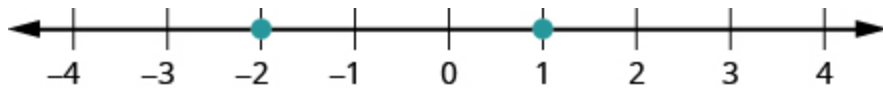
(a)



4 is to the right of 1 on the number line, so $4 > 1$.

1 is to the left of 4 on the number line, so $1 < 4$.

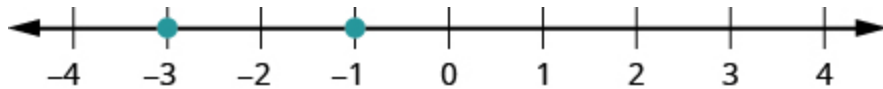
ⓑ



-2 is to the left of 1 on the number line, so $-2 < 1$.

1 is to the right of -2 on the number line, so $1 > -2$.

ⓒ



-1 is to the right of -3 on the number line, so $-1 > -3$.

-3 is to the left of -1 on the number line, so $-3 < -1$.

Example:

Exercise:

Problem: Order each of the following pairs of numbers using $<$ or $>$:

- ⓐ 14 ____ 6
- ⓑ -1 ____ 9
- ⓒ -1 ____ -4
- ⓓ 2 ____ -20

Solution:

Solution

Begin by plotting the numbers on a number line as shown in [\[link\]](#).



Ⓐ Compare 14 and 6.

14 ____ 6

14 is to the right of 6 on the number line.

14 > 6

Ⓑ Compare -1 and 9.

-1 ____ 9

-1 is to the left of 9 on the number line.

-1 < 9

Ⓒ Compare -1 and -4.

-1 ____ -4

-1 is to the right of -4 on the number line.

-1 > -4

Ⓓ Compare 2 and -20.

-2 ____ -20

2 is to the right of -20 on the number line.

$$2 > -20$$

Note:

Exercise:

Problem: Order each of the following pairs of numbers using $<$ or $>$.

- Ⓐ $15 \underline{\hspace{1cm}} 7$
- Ⓑ $-2 \underline{\hspace{1cm}} 5$
- Ⓒ $-3 \underline{\hspace{1cm}} -7$
- Ⓓ $5 \underline{\hspace{1cm}} -17$

Solution:

- Ⓐ $>$
- Ⓑ $<$
- Ⓒ $>$
- Ⓓ $>$

Note:

Exercise:

Problem: Order each of the following pairs of numbers using $<$ or $>$.

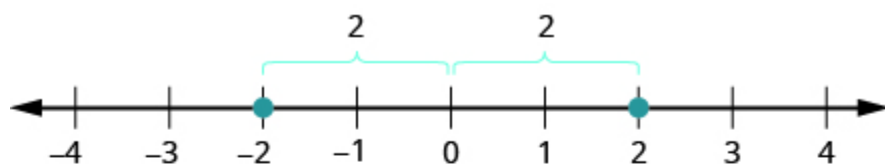
- Ⓐ $8 \underline{\hspace{1cm}} 13$
- Ⓑ $3 \underline{\hspace{1cm}} -4$
- Ⓒ $-5 \underline{\hspace{1cm}} -2$
- Ⓓ $9 \underline{\hspace{1cm}} -21$

Solution:

- (a) $<$
- (b) $>$
- (c) $<$
- (d) $>$

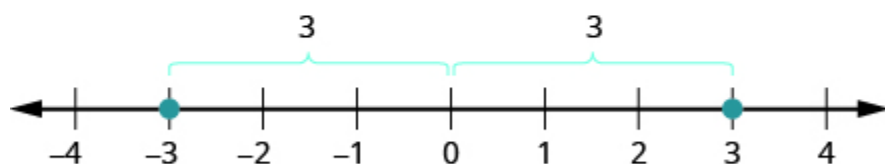
Find Opposites

On the number line, the negative numbers are a mirror image of the positive numbers with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they are called **opposites**. The opposite of 2 is -2 , and the opposite of -2 is 2 as shown in [\[link\]\(a\)](#). Similarly, 3 and -3 are opposites as shown in [\[link\]\(b\)](#).



The numbers -2 and 2 are opposites.

(a)



The numbers -3 and 3 are opposites.

(b)

Note:**Opposite**

The opposite of a number is the number that is the same distance from zero on the number line, but on the opposite side of zero.

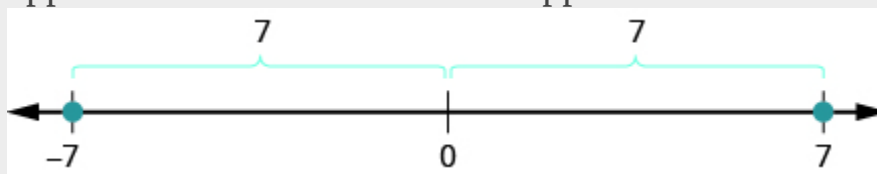
Example:**Exercise:**

Problem: Find the opposite of each number:

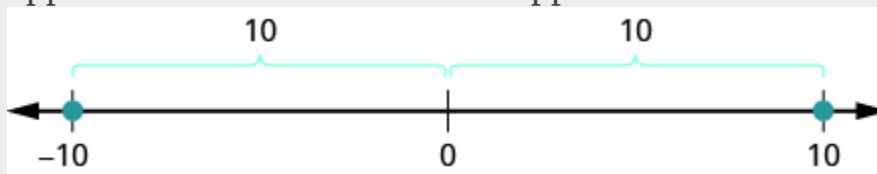
- Ⓐ 7
- Ⓑ -10

Solution:**Solution**

- Ⓐ The number -7 is the same distance from 0 as 7, but on the opposite side of 0. So -7 is the opposite of 7 as shown in [\[link\]](#).



- Ⓑ The number 10 is the same distance from 0 as -10 , but on the opposite side of 0. So 10 is the opposite of -10 as shown in [\[link\]](#).

**Note:****Exercise:**

Problem: Find the opposite of each number:

Ⓐ 4

Ⓑ -3

Solution:

Ⓐ -4

Ⓑ 3

Note:

Exercise:

Problem: Find the opposite of each number:

Ⓐ 8

Ⓑ -5

Solution:

Ⓐ -8

Ⓑ 5

Opposite Notation

Just as the same word in English can have different meanings, the same symbol in algebra can have different meanings. The specific meaning

becomes clear by looking at how it is used. You have seen the symbol “—”, in three different ways.

$10 - 4$	Between two numbers, the symbol indicates the operation of subtraction. We read $10 - 4$ as 10 <i>minus</i> 4.
-8	In front of a number, the symbol indicates a negative number. We read -8 as <i>negative eight</i> .
$-x$	In front of a variable or a number, it indicates the opposite. We read $-x$ as <i>the opposite of x</i> .
$-(-2)$	Here we have two signs. The sign in the parentheses indicates that the number is negative 2. The sign outside the parentheses indicates the opposite. We read $-(-2)$ as <i>the opposite of -2</i> .

Note:

Opposite Notation

$-a$ means the opposite of the number a

The notation $-a$ is read *the opposite of a*.

Example:

Exercise:

Problem: Simplify: $-(-6)$.

Solution:
Solution

	$-(-6)$
The opposite of -6 is 6 .	6

Note:
Exercise:

Problem: Simplify:

$$-(-1)$$

Solution:

$$1$$

Note:
Exercise:

Problem: Simplify:

$$-(-5)$$

Solution:

$$5$$

Integers

The set of counting numbers, their opposites, and 0 is the set of integers.

Note:

Integers

Integers are counting numbers, their opposites, and zero.

Equation:

$$\dots -3, -2, -1, 0, 1, 2, 3 \dots$$

We must be very careful with the signs when evaluating the opposite of a variable.

Example:

Exercise:

Problem: Evaluate $-x$:

Ⓐ when $x = 8$

Ⓑ when $x = -8$.

Solution:

① To evaluate $-x$ when $x = 8$, substitute 8 for x .

$$-x$$

Substitute 8 for x .

$$-(8)$$

Simplify.

$$-8$$

② To evaluate $-x$ when $x = -8$, substitute -8 for x .

$$-x$$

Substitute -8 for x .

$$-(-8)$$

Simplify.

$$8$$

Note:

Exercise:

Problem: Evaluate $-n$:

- Ⓐ when $n = 4$
- Ⓑ when $n = -4$

Solution:

- Ⓐ -4
- Ⓑ 4

Note:

Exercise:

Problem: Evaluate: $-m$:

- Ⓐ when $m = 11$
- Ⓑ when $m = -11$

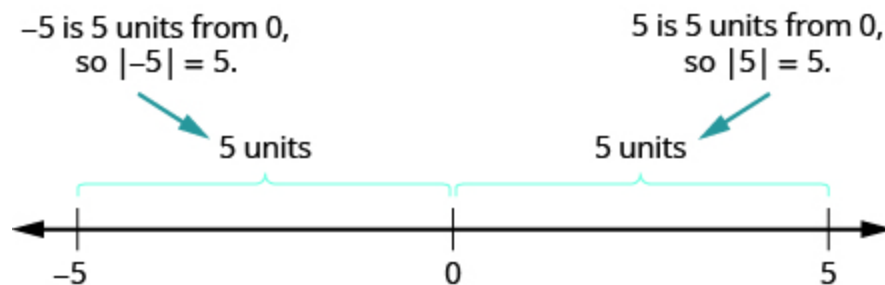
Solution:

- Ⓐ -11
- Ⓑ 11

Simplify Expressions with Absolute Value

We saw that numbers such as 5 and -5 are opposites because they are the same distance from 0 on the number line. They are both five units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number. Because distance is never negative, the absolute value of any number is never negative.

The symbol for absolute value is two vertical lines on either side of a number. So the absolute value of 5 is written as $|5|$, and the absolute value of -5 is written as $|-5|$ as shown in [\[link\]](#).



Note:

Absolute Value

The absolute value of a number is its distance from 0 on the number line. The absolute value of a number n is written as $|n|$.

Equation:

$$|n| \geq 0 \text{ for all numbers}$$

Example:

Exercise:

Problem: Simplify:

- Ⓐ $|3|$
- Ⓑ $|-44|$
- Ⓒ $|0|$

Solution:
Solution

Ⓐ	
	$ 3 $
3 is 3 units from zero.	3

Ⓑ	
	$ -44 $
-44 is 44 units from zero.	44

Ⓒ	

	$ 0 $
0 is already at zero.	0

Note:

Exercise:

Problem: Simplify:

Ⓐ $|12|$

Ⓑ $-|-28|$

Solution:

Ⓐ 12

Ⓑ -28

Note:

Exercise:

Problem: Simplify:

Ⓐ $|9|$

Ⓑ $-(b) - |37|$

Solution:

Ⓐ 9

ⓑ -37

We treat absolute value bars just like we treat parentheses in the order of operations. We simplify the expression inside first.

Example:

Exercise:

Problem: Evaluate:

- ⓐ $|x|$ when $x = -35$
- ⓑ $|-y|$ when $y = -20$
- ⓒ $-|u|$ when $u = 12$
- ⓓ $-|p|$ when $p = -14$

Solution:

Solution

ⓐ To find $|x|$ when $x = -35$:

	$ x $
Substitute -35 for x .	$ -35 $

Take the absolute value.

35

ⓑ To find $|-y|$ when $y = -20$:

$$|-y|$$

Substitute -20 for y .

$$|-(-20)|$$

Simplify.

$$|20|$$

Take the absolute value.

20

ⓒ To find $-|u|$ when $u = 12$:

$$-|u|$$

Substitute 12 for u .

$$-|12|$$

Take the absolute value.

-12

④ To find $-|p|$ when $p = -14$:

$$-|p|$$

Substitute -14 for p .

$$-|-14|$$

Take the absolute value.

$$-14$$

Notice that the result is negative only when there is a negative sign outside the absolute value symbol.

Note:

Exercise:

Problem: Evaluate:

- ① $|x|$ when $x = -17$
- ② $|-y|$ when $y = -39$
- ③ $-|m|$ when $m = 22$
- ④ $-|p|$ when $p = -11$

Solution:

- ① 17
- ② 39
- ③ -22
- ④ -11

Note:

Exercise:

Problem:

- Ⓐ $|y|$ when $y = -23$
- Ⓑ $|-y|$ when $y = -21$
- Ⓒ $-|n|$ when $n = 37$
- Ⓓ $-|q|$ when $q = -49$

Solution:

- Ⓐ 23
- Ⓑ 21
- Ⓒ -37
- Ⓓ -49

Example:

Exercise:

Problem: Fill in $<$, $>$, or $=$ for each of the following:

- Ⓐ $|-5|$ ____ $-|-5|$
- Ⓑ 8 ____ $-|-8|$
- Ⓒ -9 ____ $-|-9|$
- Ⓓ $-|-7|$ ____ -7

Solution:

Solution

To compare two expressions, simplify each one first. Then compare.

Ⓐ

$$|-5| \text{ \_\_\_\_ } -|-5|$$

Simplify.

$$5 \text{ \_\_\_\_ } -5$$

Order.

$$5 > -5$$

Ⓑ

$$8 \text{ \_\_\_\_ } -|-8|$$

Simplify.

$$8 \text{ \_\_\_\_ } -8$$

Order.

$$8 > -8$$

Ⓒ

$$-9 \text{ \_\_\_\_ } -|-9|$$

Simplify.

$$-9 \text{ \_\_\_\_ } -9$$

Order.

$$-9 = -9$$

④	
	$- -7 \text{ \_\_\_\_ } -7$
Simplify.	$-7 \text{ \_\_\_\_ } -7$
Order.	$-7 = -7$

Note:

Exercise:

Problem: Fill in $<$, $>$, or $=$ for each of the following:

① $|-9| \text{ \_\_\_\_ } -|-9|$

② $2 \text{ \_\_\_\_ } -|-2|$

③ $-8 \text{ \_\_\_\_ } |-8|$

④ $-|-5| \text{ \_\_\_\_ } -5$

Solution:

① $>$

② $>$

③ $<$

④ $=$

Note:

Exercise:

Problem: Fill in $<$, $>$, or $=$ for each of the following:

- Ⓐ $7 \underline{\hspace{1cm}} - |-7|$
- Ⓑ $-|-11| \underline{\hspace{1cm}} -11$
- Ⓒ $|-4| \underline{\hspace{1cm}} - |-4|$
- Ⓓ $-1 \underline{\hspace{1cm}} |-1|$

Solution:

- Ⓐ $>$
- Ⓑ $=$
- Ⓒ $>$
- Ⓓ $<$

Absolute value bars act like grouping symbols. First simplify inside the absolute value bars as much as possible. Then take the absolute value of the resulting number, and continue with any operations outside the absolute value symbols.

Example:

Exercise:

Problem: Simplify:

- Ⓐ $|9-3|$
- Ⓑ $4|-2|$

Solution:
Solution

For each expression, follow the order of operations. Begin inside the absolute value symbols just as with parentheses.

Ⓐ	
	$ 9-3 $
Simplify inside the absolute value sign.	$ 6 $
Take the absolute value.	6

Ⓑ	
	$4 -2 $
Take the absolute value.	$4 \cdot 2$
Multiply.	8

Note:

Exercise:

Problem: Simplify:

Ⓐ $|12 - 9|$

Ⓑ $3|-6|$

Solution:

Ⓐ 3

Ⓑ 18

Note:

Exercise:

Problem: Simplify:

Ⓐ $|27 - 16|$

Ⓑ $9|-7|$

Solution:

Ⓐ 11

Ⓑ 63

Example:

Exercise:

Problem: Simplify: $|8 + 7| - |5 + 6|$.

Solution:

Solution

For each expression, follow the order of operations. Begin inside the absolute value symbols just as with parentheses.

	$ 8+7 - 5+6 $
Simplify inside each absolute value sign.	$ 15 - 11 $
Subtract.	4

Note:

Exercise:

Problem: Simplify: $|1 + 8| - |2 + 5|$

Solution:

2

Note:

Exercise:

Problem: Simplify: $|9-5| - |7-6|$

Solution:

3

Example:

Exercise:

Problem: Simplify: $24 - |19 - 3(6 - 2)|$.

Solution:

Solution

We use the order of operations. Remember to simplify grouping symbols first, so parentheses inside absolute value symbols would be first.

	$24 - 19 - 3(6 - 2) $
Simplify in the parentheses first.	$24 - 19 - 3(4) $
Multiply $3(4)$.	$24 - 19 - 12 $
Subtract inside the absolute value sign.	$24 - 7 $
Take the absolute value.	$24 - 7$
Subtract.	17

Note:

Exercise:

Problem: Simplify: $19 - |11 - 4(3 - 1)|$

Solution:

16

Note:

Exercise:

Problem: Simplify: $9 - |8 - 4(7 - 5)|$

Solution:

9

Translate Word Phrases into Expressions with Integers

Now we can translate word phrases into expressions with integers. Look for words that indicate a negative sign. For example, the word *negative* in “negative twenty” indicates -20 . So does the word *opposite* in “the opposite of 20.”

Example:

Exercise:

Problem: Translate each phrase into an expression with integers:

- Ⓐ the opposite of positive fourteen
- Ⓑ the opposite of -11
- Ⓒ negative sixteen
- Ⓓ two minus negative seven

Solution:
Solution

- Ⓐ the opposite of fourteen
 -14
- Ⓑ the opposite of -11
 $-(-11)$
- Ⓒ negative sixteen
 -16
- Ⓓ two minus negative seven
 $2 - (-7)$

Note:
Exercise:

Problem: Translate each phrase into an expression with integers:

- Ⓐ the opposite of positive nine
- Ⓑ the opposite of -15
- Ⓒ negative twenty
- Ⓓ eleven minus negative four

Solution:

- Ⓐ -9
- Ⓑ 15
- Ⓒ -20

Ⓓ $11 - (-4)$

Note:

Exercise:

Problem: Translate each phrase into an expression with integers:

- Ⓐ the opposite of negative nineteen
- Ⓑ the opposite of twenty-two
- Ⓒ negative nine
- Ⓓ negative eight minus negative five

Solution:

- Ⓐ 19
- Ⓑ -22
- Ⓒ -9
- Ⓓ $-8 - (-5)$

As we saw at the start of this section, negative numbers are needed to describe many real-world situations. We'll look at some more applications of negative numbers in the next example.

Example:

Exercise:

Problem: Translate into an expression with integers:

- Ⓐ The temperature is 12 degrees Fahrenheit below zero.
- Ⓑ The football team had a gain of 3 yards.
- Ⓒ The elevation of the Dead Sea is 1,302 feet below sea level.
- Ⓓ A checking account is overdrawn by \$40.

Solution:
Solution

Look for key phrases in each sentence. Then look for words that indicate negative signs. Don't forget to include units of measurement described in the sentence.

Ⓐ	The temperature is 12 degrees Fahrenheit below zero.
<i>Below zero</i> tells us that 12 is a negative number.	−12°F

Ⓑ	The football team had a gain of 3 yards.
<i>A gain</i> tells us that 3 is a positive number.	3 yards

©

The elevation of the Dead Sea is 1,302 feet below sea level.

Below sea level tells us that 1,302 is a negative number.

−1,302 feet

©

A checking account is overdrawn by \$40.

Overdrawn tells us that 40 is a negative number.

−\$40

Note:

Exercise:

Problem: Translate into an expression with integers:

The football team had a gain of 5 yards.

Solution:

5 yards

Note:

Exercise:

Problem: Translate into an expression with integers:

The scuba diver was 30 feet below the surface of the water.

Solution:

−30 feet

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Introduction to Integers](#)
- [Simplifying the Opposites of Negative Integers](#)
- [Comparing Absolute Value of Integers](#)
- [Comparing Integers Using Inequalities](#)

Key Concepts

- Opposite Notation
 - $-a$ means the opposite of the number a
 - The notation $-a$ is read *the opposite of a* .
- Absolute Value Notation
 - The absolute value of a number n is written as $|n|$.
 - $|n| \geq 0$ for all numbers.

Practice Makes Perfect

Locate Positive and Negative Numbers on the Number Line

In the following exercises, locate and label the given points on a number line.

Exercise:

Problem:

- (a) 2
- (b) -2
- (c) -5

Solution:



Exercise:

Problem:

- (a) 5
- (b) -5
- (c) -2

Exercise:

Problem:

- (a) -8
- (b) 8
- (c) -6

Solution:



Exercise:

Problem:

- Ⓐ -7
- Ⓑ 7
- Ⓒ -1

Order Positive and Negative Numbers on the Number Line

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

Exercise:

Problem:

- Ⓐ $9 \underline{\hspace{0.5em}} 4$
- Ⓑ $-3 \underline{\hspace{0.5em}} 6$
- Ⓒ $-8 \underline{\hspace{0.5em}} -2$
- Ⓓ $1 \underline{\hspace{0.5em}} -10$

Solution:

- Ⓐ $>$
- Ⓑ $<$
- Ⓒ $<$
- Ⓓ $>$

Exercise:

Problem:

- Ⓐ $6 \underline{\hspace{1cm}} 2$;
- Ⓑ $-7 \underline{\hspace{1cm}} 4$;
- Ⓒ $-9 \underline{\hspace{1cm}} -1$;
- Ⓓ $9 \underline{\hspace{1cm}} -3$

Exercise:

Problem:

- Ⓐ $-5 \underline{\hspace{1cm}} 1$;
- Ⓑ $-4 \underline{\hspace{1cm}} -9$;
- Ⓒ $6 \underline{\hspace{1cm}} 10$;
- Ⓓ $3 \underline{\hspace{1cm}} -8$

Solution:

- Ⓐ $<$
- Ⓑ $>$
- Ⓒ $<$
- Ⓓ $>$

Exercise:

Problem:

- Ⓐ $-7 \underline{\hspace{1cm}} 3$;
- Ⓑ $-10 \underline{\hspace{1cm}} -5$;
- Ⓒ $2 \underline{\hspace{1cm}} -6$;
- Ⓓ $8 \underline{\hspace{1cm}} 9$

Find Opposites

In the following exercises, find the opposite of each number.

Exercise:

Problem:

- Ⓐ 2
- Ⓑ -6

Solution:

- Ⓐ -2
- Ⓑ 6

Exercise:

Problem:

- Ⓐ 9
- Ⓑ -4

Exercise:

Problem:

- Ⓐ -8
- Ⓑ 1

Solution:

- Ⓐ 8
- Ⓑ -1

Exercise:

Problem:

- Ⓐ -2
- Ⓑ 6

In the following exercises, simplify.

Exercise:

Problem: $-(-4)$

Solution:

4

Exercise:

Problem: $-(-8)$

Exercise:

Problem: $-(-15)$

Solution:

15

Exercise:

Problem: $-(-11)$

In the following exercises, evaluate.

Exercise:

Problem: $-m$ when

Ⓐ $m = 3$

Ⓑ $m = -3$

Solution:

Ⓐ -3

Ⓑ 3

Exercise:

Problem: $-p$ when

Ⓐ $p = 6$

Ⓑ $p = -6$

Exercise:

Problem: $-c$ when

Ⓐ $c = 12$

Ⓑ $c = -12$

Solution:

Ⓐ -12 ;

Ⓑ 12

Exercise:

Problem: $-d$ when

Ⓐ $d = 21$

Ⓑ $d = -21$

Simplify Expressions with Absolute Value

In the following exercises, simplify each absolute value expression.

Exercise:

Problem:

Ⓐ $|7|$

Ⓑ $|-25|$

Ⓒ $|0|$

Solution:

- Ⓐ 7
- Ⓑ 25
- Ⓒ 0

Exercise:

Problem:

- Ⓐ $|5|$
- Ⓑ $|20|$
- Ⓒ $|-19|$

Exercise:

Problem:

- Ⓐ $|-32|$
- Ⓑ $|-18|$
- Ⓒ $|16|$

Solution:

- Ⓐ 32
- Ⓑ 18
- Ⓒ 16

Exercise:

Problem:

- Ⓐ $|-41|$
- Ⓑ $|-40|$

Ⓒ $|22|$

In the following exercises, evaluate each absolute value expression.

Exercise:

Problem:

- Ⓐ $|x|$ when $x = -28$
- Ⓑ $|-u|$ when $u = -15$

Solution:

- Ⓐ 28
- Ⓑ 15

Exercise:

Problem:

- Ⓐ $|y|$ when $y = -37$
- Ⓑ $|-z|$ when $z = -24$

Exercise:

Problem:

- Ⓐ $-|p|$ when $p = 19$
- Ⓑ $-|q|$ when $q = -33$

Solution:

- Ⓐ -19
- Ⓑ -33

Exercise:

Problem:

- Ⓐ $-|a|$ when $a = 60$
Ⓑ $-|b|$ when $b = -12$

In the following exercises, fill in $<$, $>$, or $=$ to compare each expression.

Exercise:**Problem:**

- Ⓐ $-6 \underline{\hspace{1cm}} |-6|$
Ⓑ $-|-3| \underline{\hspace{1cm}} -3$

Solution:

- Ⓐ $<$
Ⓑ $=$

Exercise:**Problem:**

- Ⓐ $-8 \underline{\hspace{1cm}} |-8|$
Ⓑ $-|-2| \underline{\hspace{1cm}} -2$

Exercise:**Problem:**

- Ⓐ $|-3| \underline{\hspace{1cm}} -|-3|$
Ⓑ $4 \underline{\hspace{1cm}} -|-4|$

Solution:

- Ⓐ $>$

ⓑ $>$

Exercise:

Problem:

ⓐ $|-5| \underline{\hspace{1cm}} - |-5|$

ⓑ $9 \underline{\hspace{1cm}} - |-9|$

In the following exercises, simplify each expression.

Exercise:

Problem: $|8 - 4|$

Solution:

4

Exercise:

Problem: $|9 - 6|$

Exercise:

Problem: $8|-7|$

Solution:

56

Exercise:

Problem: $5|-5|$

Exercise:

Problem: $|15 - 7| - |14 - 6|$

Solution:

0

Exercise:

Problem: $|17 - 8| - |13 - 4|$

Exercise:

Problem: $18 - |2(8 - 3)|$

Solution:

8

Exercise:

Problem: $15 - |3(8 - 5)|$

Exercise:

Problem: $8(14 - 2|-2|)$

Solution:

80

Exercise:

Problem: $6(13 - 4|-2|)$

Translate Word Phrases into Expressions with Integers

Translate each phrase into an expression with integers. *Do not simplify.*

Exercise:

Problem:

- Ⓐ the opposite of 8
 - Ⓑ the opposite of -6
 - Ⓒ negative three
 - Ⓓ 4 minus negative 3
-

Solution:

- Ⓐ -8
- Ⓑ $-(-6)$, or 6
- Ⓒ -3
- Ⓓ $4 - (-3)$

Exercise:

Problem:

- Ⓐ the opposite of 11
- Ⓑ the opposite of -4
- Ⓒ negative nine
- Ⓓ 8 minus negative 2

Exercise:

Problem:

- Ⓐ the opposite of 20
 - Ⓑ the opposite of -5
 - Ⓒ negative twelve
 - Ⓓ 18 minus negative 7
-

Solution:

- Ⓐ -20
- Ⓑ $-(-5)$, or 5
- Ⓒ -12
- Ⓓ $18 - (-7)$

Exercise:

Problem:

- Ⓐ the opposite of 15
- Ⓑ the opposite of -9
- Ⓒ negative sixty
- Ⓓ 12 minus 5

Exercise:

Problem: a temperature of 6 degrees below zero

Solution:

-6 degrees

Exercise:

Problem: a temperature of 14 degrees below zero

Exercise:

Problem: an elevation of 40 feet below sea level

Solution:

-40 feet

Exercise:

Problem: an elevation of 65 feet below sea level

Exercise:

Problem: a football play loss of 12 yards

Solution:

-12 yards

Exercise:

Problem: a football play gain of 4 yards

Exercise:

Problem: a stock gain of \$3

Solution:

\$3

Exercise:

Problem: a stock loss of \$5

Exercise:

Problem: a golf score one above par

Solution:

+1

Exercise:

Problem: a golf score of 3 below par

Everyday Math

Exercise:

Problem:

Elevation The highest elevation in the United States is Mount McKinley, Alaska, at 20,320 feet above sea level. The lowest elevation is Death Valley, California, at 282 feet below sea level. Use integers to write the elevation of:

- Ⓐ Mount McKinley
- Ⓑ Death Valley

Solution:

- Ⓐ 20,320 feet
- Ⓑ -282 feet

Exercise:**Problem:**

Extreme temperatures The highest recorded temperature on Earth is 58° Celsius, recorded in the Sahara Desert in 1922. The lowest recorded temperature is 90° below 0° Celsius, recorded in Antarctica in 1983. Use integers to write the:

- Ⓐ highest recorded temperature
- Ⓑ lowest recorded temperature

Exercise:**Problem:**

State budgets In June, 2011, the state of Pennsylvania estimated it would have a budget surplus of \$540 million. That same month, Texas estimated it would have a budget deficit of \$27 billion. Use integers to write the budget:

- Ⓐ surplus

ⓑ deficit

Solution:

ⓐ \$540 million

ⓑ −\$27 billion

Exercise:

Problem:

College enrollments Across the United States, community college enrollment grew by 1,400,000 students from 2007 to 2010. In California, community college enrollment declined by 110,171 students from 2009 to 2010. Use integers to write the change in enrollment:

ⓐ growth

ⓑ decline

Writing Exercises

Exercise:

Problem:

Give an example of a negative number from your life experience.

Solution:

Sample answer: I have experienced negative temperatures.

Exercise:

Problem:

What are the three uses of the “–” sign in algebra? Explain how they differ.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
locate positive and negative numbers on the number line.			
order positive and negative numbers.			
find opposites.			
simplify expressions with absolute value.			
translate word phrases to expressions with integers.			

Ⓑ If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

...no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

absolute value

The absolute value of a number is its distance from 0 on the number line.

integers

Integers are counting numbers, their opposites, and zero ... -3 , -2 , -1 , 0 , 1 , 2 , 3 ...

negative number

A negative number is less than zero.

opposites

The opposite of a number is the number that is the same distance from zero on the number line, but on the opposite side of zero.

Add Integers

By the end of this section, you will be able to:

- Model addition of integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate word phrases to algebraic expressions
- Add integers in applications

Note:

Before you get started, take this readiness quiz.

1. Evaluate $x + 8$ when $x = 6$.
If you missed this problem, review [\[link\]](#).
2. Simplify: $8 + 2(5 + 1)$.
If you missed this problem, review [\[link\]](#).
3. Translate *the sum of 3 and negative 7* into an algebraic expression.
If you missed this problem, review [\[link\]](#)

Model Addition of Integers

Now that we have located positive and negative numbers on the number line, it is time to discuss arithmetic operations with integers.

Most students are comfortable with the addition and subtraction facts for positive numbers. But doing addition or subtraction with both positive and negative numbers may be more difficult. This difficulty relates to the way the brain learns.

The brain learns best by working with objects in the real world and then generalizing to abstract concepts. Toddlers learn quickly that if they have two cookies and their older brother steals one, they have only one left. This is a concrete example of $2 - 1$. Children learn their basic addition and

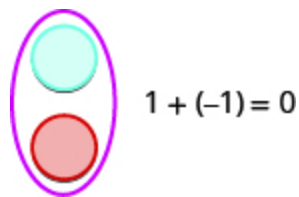
subtraction facts from experiences in their everyday lives. Eventually, they know the number facts without relying on cookies.

Addition and subtraction of negative numbers have fewer real world examples that are meaningful to us. Math teachers have several different approaches, such as number lines, banking, temperatures, and so on, to make these concepts real.

We will model addition and subtraction of negatives with two color counters. We let a blue counter represent a positive and a red counter will represent a negative.



If we have one positive and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero as summarized in [\[link\]](#).



A blue counter represents $+1$. A red counter represents -1 . Together they add to zero.

Note: Doing the Manipulative Mathematics activity "Addition of signed Numbers" will help you develop a better understanding of adding integers.

We will model four addition facts using the numbers 5, -5 and 3, -3 .

Equation:

$$5 + 3 \quad -5 + (-3) \quad -5 + 3 \quad 5 + (-3)$$

Example:

Exercise:

Problem: Model: $5 + 3$.

Solution:

Solution

Interpret the expression.

$5 + 3$ means the sum of 5 and 3.

Model the first number. Start with 5 positives.



Model the second number. Add 3

positives.



Count the total number of counters.



The sum of 5 and 3 is 8.

$$5 + 3 = 8$$

Note:

Exercise:

Problem: Model the expression.

$$2 + 4$$

Solution:



6

Note:

Exercise:

Problem: Model the expression.

$$2 + 5$$

Solution:



7

Example:

Exercise:

Problem: Model: $-5 + (-3)$.

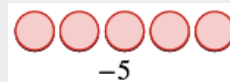
Solution:

Solution

Interpret the expression.

$-5 + (-3)$ means the sum of -5 and -3 .

Model the first number. Start with 5 negatives.



Model the second number. Add 3 negatives.



Count the total number of counters.



The sum of -5 and -3 is -8 .

$$-5 + -3 = -8$$

Note:

Exercise:

Problem: Model the expression.

$$-2 + (-4)$$

Solution:



$$-6$$

Note:

Exercise:

Problem: Model the expression.

$$-2 + (-5)$$

Solution:



-7

[\[link\]](#) and [\[link\]](#) are very similar. The first example adds 5 positives and 3 positives—both positives. The second example adds 5 negatives and 3 negatives—both negatives. In each case, we got a result of 8—either 8 positives or 8 negatives. When the signs are the same, the counters are all the same color.

Now let's see what happens when the signs are different.

Example:

Exercise:

Problem: Model: $-5 + 3$.

Solution:
Solution

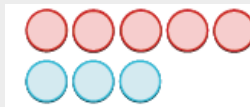
Interpret the expression.

$-5 + 3$ means the sum of -5 and 3.

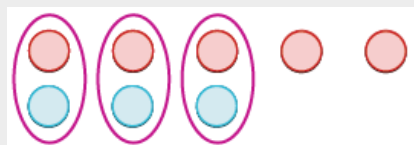
Model the first number. Start with 5 negatives.



Model the second number. Add 3 positives.



Remove any neutral pairs.



Count the result.



The sum of -5 and 3 is -2 .

$$-5 + 3 = -2$$

Notice that there were more negatives than positives, so the result is negative.

Note:

Exercise:

Problem: Model the expression, and then simplify:

$$2 + (-4)$$

Solution:



-2

Note:

Exercise:

Problem: Model the expression, and then simplify:

$$2 + (-5)$$

Solution:



-3

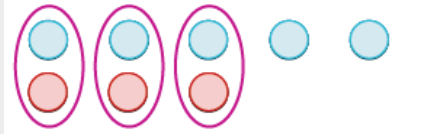
Example:

Exercise:

Problem: Model: $5 + (-3)$.

Solution:

Solution

Interpret the expression.	$5 + (-3)$ means the sum of 5 and -3 .
Model the first number. Start with 5 positives.	
Model the second number. Add 3 negatives.	
Remove any neutral pairs.	
Count the result.	
The sum of 5 and -3 is 2.	$5 + (-3) = 2$

Note:

Exercise:

Problem: Model the expression, and then simplify:

$$(-2) + 4$$

Solution:



-2

Note:

Exercise:

Problem: Model the expression:

$$(-2) + 5$$

Solution:



3

Example:

Modeling Addition of Positive and Negative Integers

Model each addition.

Exercise:

Problem:

Ⓐ $4 + 2$

- ⓑ $-3 + 6$
- ⓒ $4 + (-5)$
- ⓓ $-2 + (-3)$

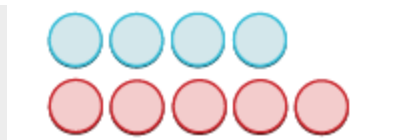
Solution:

ⓐ	
	$4 + 2$
Start with 4 positives.	
Add two positives.	
How many do you have?	6. $4 + 2 = 6$

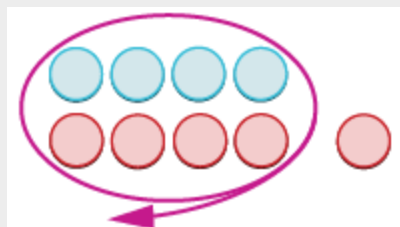
ⓑ	
	$-3 + 6$
Start with 3 negatives.	

	
Add 6 positives.	
Remove neutral pairs.	
How many are left?	
	3. $-3 + 6 = 3$
©	
	$4 + (-5)$
Start with 4 positives.	

Add 5 negatives.



Remove neutral pairs.



How many are left?



$$-1.4 + (-5) = -1$$

④

$$-2 + (-3)$$

Start with 2 negatives.



Add 3 negatives.



How many do you have?

$$-5. -2 + (-3) = -5$$

Note:

Exercise:

Problem: Model each addition.

- (a) $3 + 4$
- (b) $-1 + 4$
- (c) $4 + (-6)$
- (d) $-2 + (-2)$

Solution:

(a)



(b)



(c)



(d)



Note:

Exercise:

Problem:

- (a) $5 + 1$
- (b) $-3 + 7$
- (c) $2 + (-8)$
- (d) $-3 + (-4)$

Solution:

(a)



(b)



(c)



(d)



Simplify Expressions with Integers

Now that you have modeled adding small positive and negative integers, you can visualize the model in your mind to simplify expressions with any integers.

For example, if you want to add $37 + (-53)$, you don't have to count out 37 blue counters and 53 red counters.

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more negative counters than positive counters, the sum would be negative. Because $53 - 37 = 16$, there are 16 more negative counters.

Equation:

$$37 + (-53) = -16$$

Let's try another one. We'll add $-74 + (-27)$. Imagine 74 red counters and 27 more red counters, so we have 101 red counters all together. This means the sum is -101 .

Equation:

$$-74 + (-27) = -101$$

Look again at the results of [\[link\]](#) - [\[link\]](#).

$5 + 3$	$-5 + (-3)$
both positive, sum positive	both negative, sum negative
When the signs are the same, the counters would be all the same color, so add them.	

$-5 + 3$	$5 + (-3)$
different signs, more negatives	different signs, more positives
Sum negative	sum positive
When the signs are different, some counters would make neutral pairs; subtract to see how many are left.	

Addition of Positive and Negative Integers

Example:

Exercise:

Problem: Simplify:

Ⓐ $19 + (-47)$

Ⓑ $-32 + 40$

Solution:

Solution

Ⓐ Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

Equation:

$$\begin{array}{r} 19 + (-47) \\ -28 \end{array}$$

Ⓑ The signs are different so we subtract 32 from 40. The answer will be positive because there are more positives than negatives

Equation:

$$\frac{-32 + 40}{8}$$

Note:

Exercise:

Problem: Simplify each expression:

Ⓐ $15 + (-32)$

Ⓑ $-19 + 76$

Solution:

Ⓐ -17

Ⓑ 57

Note:

Exercise:

Problem: Simplify each expression:

Ⓐ $-55 + 9$

Ⓑ $43 + (-17)$

Solution:

Ⓐ -46

Ⓑ 26

Example:

Exercise:

Problem: Simplify: $-14 + (-36)$.

Solution:

Solution

Since the signs are the same, we add. The answer will be negative because there are only negatives.

Equation:

$$\begin{array}{r} -14 + (-36) \\ -50 \end{array}$$

Note:

Exercise:

Problem: Simplify the expression:

$$-31 + (-19)$$

Solution:

$$-50$$

Note:

Exercise:

Problem: Simplify the expression:

$$-42 + (-28)$$

Solution:

$$-70$$

The techniques we have used up to now extend to more complicated expressions. Remember to follow the order of operations.

Example:

Exercise:

Problem: Simplify: $-5 + 3(-2 + 7)$.

Solution:

Solution

	$-5 + 3(-2 + 7)$
Simplify inside the parentheses.	$-5 + 3(5)$
Multiply.	$-5 + 15$
Add left to right.	10

Note:

Exercise:

Problem: Simplify the expression:

$$-2 + 5(-4 + 7)$$

Solution:

13

Note:

Exercise:

Problem: Simplify the expression:

$$-4 + 2(-3 + 5)$$

Solution:

0

Evaluate Variable Expressions with Integers

Remember that to evaluate an expression means to substitute a number for the variable in the expression. Now we can use negative numbers as well as positive numbers when evaluating expressions.

Example:

Exercise:

Problem: Evaluate $x + 7$ when

Ⓐ $x = -2$

Ⓑ $x = -11$.

Solution:

Solution

Ⓐ Evaluate $x + 7$ when $x = -2$

$$-2 + 7$$

Substitute -2 for x .

$$-2 + 7$$

Simplify.

5

Ⓑ Evaluate $x + 7$ when $x = -11$

	$x + 7$
Substitute -11 for x .	$-11 + 7$
Simplify.	-4

Note:

Exercise:

Problem: Evaluate each expression for the given values:

$x + 5$ when

- Ⓐ $x = -3$ and
- Ⓑ $x = -17$

Solution:

- Ⓐ 2
- Ⓑ -12

Note:

Exercise:

Problem: Evaluate each expression for the given values: $y + 7$ when

Ⓐ $y = -5$

Ⓑ $y = -8$

Solution:

Ⓐ 2

Ⓑ -1

Example:

Exercise:

Problem: When $n = -5$, evaluate

Ⓐ $n + 1$

Ⓑ $-n + 1$.

Solution:

Solution

Ⓐ Evaluate $n + 1$ when $n = -5$	
	$n + 1$

Substitute -5 for n .

$$-5 + 1$$

Simplify.

$$-4$$

⑥ Evaluate $-n + 1$ when $n = -5$

$$-n + 1$$

Substitute -5 for n .

$$-(-5) + 1$$

Simplify.

$$5 + 1$$

Add.

$$6$$

Note:

Exercise:

Problem: When $n = -8$, evaluate

- Ⓐ $n + 2$
- Ⓑ $-n + 2$

Solution:

- Ⓐ -6
- Ⓑ 10

Note:

Exercise:

Problem: When $y = -9$, evaluate

- Ⓐ $y + 8$
- Ⓑ $-y + 8$.

Solution:

- Ⓐ -1
- Ⓑ 17

Next we'll evaluate an expression with two variables.

Example:

Exercise:

Problem: Evaluate $3a + b$ when $a = 12$ and $b = -30$.

Solution:**Solution**

	$3a + b$
Substitute 12 for a and -30 for b .	$3(12) + (-30)$
Multiply.	$36 + (-30)$
Add.	6

Note:**Exercise:**

Problem: Evaluate the expression:

$a + 2b$ when $a = -19$ and $b = 14$.

Solution:

9

Note:

Exercise:

Problem: Evaluate the expression:

$5p + q$ when $p = 4$ and $q = -7$.

Solution:

13

Example:

Exercise:

Problem: Evaluate $(x + y)^2$ when $x = -18$ and $y = 24$.

Solution:

Solution

This expression has two variables. Substitute -18 for x and 24 for y .

	$(x + y)^2$

Substitute -18 for x and 24 for y .	$(-18 + 24)^2$
Add inside the parentheses.	$(6)^2$
Simplify	36

Note:

Exercise:

Problem: Evaluate:

$(x + y)^2$ when $x = -15$ and $y = 29$.

Solution:

196

Note:

Exercise:

Problem: Evaluate:

$(x + y)^3$ when $x = -8$ and $y = 10$.

Solution:

8

Translate Word Phrases to Algebraic Expressions

All our earlier work translating word phrases to algebra also applies to expressions that include both positive and negative numbers. Remember that the phrase *the sum* indicates addition.

Example:

Exercise:

Problem: Translate and simplify: the sum of -9 and 5 .

Solution:

Solution

The sum of -9 and 5 indicates addition.	the sum of -9 and 5
Translate.	$-9 + 5$
Simplify.	-4

Note:

Exercise:

Problem: Translate and simplify the expression:

the sum of -7 and 4

Solution:

$$-7 + 4 = -3$$

Note:

Exercise:

Problem: Translate and simplify the expression:

the sum of -8 and -6

Solution:

$$-8 + (-6) = -14$$

Example:

Exercise:

Problem:

Translate and simplify: the sum of 8 and -12 , increased by 3 .

Solution:

Solution

The phrase *increased by* indicates addition.

	The sum of 8 and -12 , increased by 3
Translate.	$[8 + (-12)] + 3$
Simplify.	$-4 + 3$
Add.	-1

Note:

Exercise:

Problem: Translate and simplify:

the sum of 9 and -16 , increased by 4.

Solution:

$$[9 + (-16)] + 4 = -3$$

Note:

Exercise:

Problem: Translate and simplify:

the sum of -8 and -12 , increased by 7.

Solution:

$$[-8 + (-12)] + 7 = -13$$

Add Integers in Applications

Recall that we were introduced to some situations in everyday life that use positive and negative numbers, such as temperatures, banking, and sports. For example, a debt of \$5 could be represented as $-\$5$. Let's practice translating and solving a few applications.

Solving applications is easy if we have a plan. First, we determine what we are looking for. Then we write a phrase that gives the information to find it. We translate the phrase into math notation and then simplify to get the answer. Finally, we write a sentence to answer the question.

Example:

Exercise:

Problem:

The temperature in Buffalo, NY, one morning started at 7 degrees below zero Fahrenheit. By noon, it had warmed up 12 degrees. What was the temperature at noon?

Solution:

Solution

We are asked to find the temperature at noon.

Write a phrase for the temperature.	The temperature warmed up 12 degrees from 7 degrees below zero.
Translate to math notation.	$-7 + 12$

Simplify.	5
Write a sentence to answer the question.	The temperature at noon was 5 degrees Fahrenheit.

Note:

Exercise:

Problem:

The temperature in Chicago at 5 A.M. was 10 degrees below zero Celsius. Six hours later, it had warmed up 14 degrees Celsius. What is the temperature at 11 A.M.?

Solution:

4 degrees Celsius

Note:

Exercise:

Problem:

A scuba diver was swimming 16 feet below the surface and then dove down another 17 feet. What is her new depth?

Solution:

−33 feet

Example:**Exercise:****Problem:**

A football team took possession of the football on their 42-yard line. In the next three plays, they lost 6 yards, gained 4 yards, and then lost 8 yards. On what yard line was the ball at the end of those three plays?

Solution:**Solution**

We are asked to find the yard line the ball was on at the end of three plays.

Write a word phrase for the position of the ball.	Start at 42, then lose 6, gain 4, lose 8.
Translate to math notation.	$42 - 6 + 4 - 8$
Simplify.	32
Write a sentence to answer the question.	At the end of the three plays, the ball is on the 32-yard line.

Note:**Exercise:**

Problem:

The Bears took possession of the football on their 20-yard line. In the next three plays, they lost 9 yards, gained 7 yards, then lost 4 yards. On what yard line was the ball at the end of those three plays?

Solution:

14-yard line

Note:**Exercise:****Problem:**

The Chargers began with the football on their 25-yard line. They gained 5 yards, lost 8 yards and then gained 15 yards on the next three plays. Where was the ball at the end of these plays?

Solution:

37-yard line

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Adding Integers with Same Sign Using Color Counters](#)
- [Adding Integers with Different Signs Using Counters](#)
- [Ex1: Adding Integers](#)
- [Ex2: Adding Integers](#)

Key Concepts

- **Addition of Positive and Negative Integers**

$5 + 3$	$-5 + (-3)$
both positive, sum positive	both negative, sum negative
When the signs are the same, the counters would be all the same color, so add them.	
$-5 + 3$	$5 + (-3)$
different signs, more negatives	different signs, more positives
Sum negative	sum positive
When the signs are different, some counters would make neutral pairs; subtract to see how many are left.	

Practice Makes Perfect

Model Addition of Integers

In the following exercises, model the expression to simplify.

Exercise:

Problem: $7 + 4$

Solution:



11

Exercise:

Problem: $8 + 5$

Exercise:

Problem: $-6 + (-3)$

Solution:



-9

Exercise:

Problem: $-5 + (-5)$

Exercise:

Problem: $-7 + 5$

Solution:



-2

Exercise:

Problem: $-9 + 6$

Exercise:

Problem: $8 + (-7)$

Solution:



1

Exercise:

Problem: $9 + (-4)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $-21 + (-59)$

Solution:

-80

Exercise:

Problem: $-35 + (-47)$

Exercise:

Problem: $48 + (-16)$

Solution:

32

Exercise:

Problem: $34 + (-19)$

Exercise:

Problem: $-200 + 65$

Solution:

-135

Exercise:

Problem: $-150 + 45$

Exercise:

Problem: $2 + (-8) + 6$

Solution:

0

Exercise:

Problem: $4 + (-9) + 7$

Exercise:

Problem: $-14 + (-12) + 4$

Solution:

-22

Exercise:

Problem: $-17 + (-18) + 6$

Exercise:

Problem: $135 + (-110) + 83$

Solution:

108

Exercise:

Problem: $140 + (-75) + 67$

Exercise:

Problem: $-32 + 24 + (-6) + 10$

Solution:

-4

Exercise:

Problem: $-38 + 27 + (-8) + 12$

Exercise:

Problem: $19 + 2(-3 + 8)$

Solution:

29

Exercise:

Problem: $24 + 3(-5 + 9)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $x + 8$ when

Ⓐ $x = -26$

Ⓑ $x = -95$

Solution:

Ⓐ -18

Ⓑ -87

Exercise:

Problem: $y + 9$ when

Ⓐ $y = -29$

Ⓑ $y = -84$

Exercise:

Problem: $y + (-14)$ when

Ⓐ $y = -33$

Ⓑ $y = 30$

Solution:

Ⓐ -47

Ⓑ 16

Exercise:

Problem: $x + (-21)$ when

Ⓐ $x = -27$

Ⓑ $x = 44$

Exercise:

Problem: When $a = -7$, evaluate:

Ⓐ $a + 3$

Ⓑ $-a + 3$

Solution:

Ⓐ -4

Ⓑ 10

Exercise:

Problem: When $b = -11$, evaluate:

- Ⓐ $b + 6$
- Ⓑ $-b + 6$

Exercise:

Problem: When $c = -9$, evaluate:

- Ⓐ $c + (-4)$
- Ⓑ $-c + (-4)$

Solution:

- Ⓐ -13
- Ⓑ 5

Exercise:

Problem: When $d = -8$, evaluate:

- Ⓐ $d + (-9)$
- Ⓑ $-d + (-9)$

Exercise:

Problem: $m + n$ when, $m = -15$, $n = 7$

Solution:

-8

Exercise:

Problem: $p + q$ when, $p = -9$, $q = 17$

Exercise:

Problem: $r - 3s$ when, $r = 16, s = 2$

Solution:

10

Exercise:

Problem: $2t + u$ when, $t = -6, u = -5$

Exercise:

Problem: $(a + b)^2$ when, $a = -7, b = 15$

Solution:

64

Exercise:

Problem: $(c + d)^2$ when, $c = -5, d = 14$

Exercise:

Problem: $(x + y)^2$ when, $x = -3, y = 14$

Solution:

121

Exercise:

Problem: $(y + z)^2$ when, $y = -3, z = 15$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate each phrase into an algebraic expression and then simplify.

Exercise:

Problem: The sum of -14 and 5

Solution:

$$-14 + 5 = -9$$

Exercise:

Problem: The sum of -22 and 9

Exercise:

Problem: 8 more than -2

Solution:

$$-2 + 8 = 6$$

Exercise:

Problem: 5 more than -1

Exercise:

Problem: -10 added to -15

Solution:

$$-15 + (-10) = -25$$

Exercise:

Problem: -6 added to -20

Exercise:

Problem: 6 more than the sum of -1 and -12

Solution:

$$[-1 + (-12)] + 6 = -7$$

Exercise:

Problem: 3 more than the sum of -2 and -8

Exercise:

Problem: the sum of 10 and -19 , increased by 4

Solution:

$$[10 + (-19)] + 4 = -5$$

Exercise:

Problem: the sum of 12 and -15 , increased by 1

Add Integers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The temperature in St. Paul, Minnesota was -19°F at sunrise. By noon the temperature had risen 26°F . What was the temperature at noon?

Solution:

$$7^{\circ}\text{F}$$

Exercise:

Problem:

Temperature The temperature in Chicago was -15°F at 6 am. By afternoon the temperature had risen 28°F . What was the afternoon temperature?

Exercise:**Problem:**

Credit Cards Lupe owes \$73 on her credit card. Then she charges \$45 more. What is the new balance?

Solution:

-\$118

Exercise:**Problem:**

Credit Cards Frank owes \$212 on his credit card. Then he charges \$105 more. What is the new balance?

Exercise:**Problem:**

Weight Loss Angie lost 3 pounds the first week of her diet. Over the next three weeks, she lost 2 pounds, gained 1 pound, and then lost 4 pounds. What was the change in her weight over the four weeks?

Solution:

-8 pounds

Exercise:

Problem:

Weight Loss April lost 5 pounds the first week of her diet. Over the next three weeks, she lost 3 pounds, gained 2 pounds, and then lost 1 pound. What was the change in her weight over the four weeks?

Exercise:

Problem:

Football The Rams took possession of the football on their own 35-yard line. In the next three plays, they lost 12 yards, gained 8 yards, then lost 6 yards. On what yard line was the ball at the end of those three plays?

Solution:

25-yard line

Exercise:

Problem:

Football The Cowboys began with the ball on their own 20-yard line. They gained 15 yards, lost 3 yards and then gained 6 yards on the next three plays. Where was the ball at the end of these plays?

Exercise:

Problem:

Calories Lisbeth walked from her house to get a frozen yogurt, and then she walked home. By walking for a total of 20 minutes, she burned 90 calories. The frozen yogurt she ate was 110 calories. What was her total calorie gain or loss?

Solution:

20 calories

Exercise:

Problem:

Calories Ozzie rode his bike for 30 minutes, burning 168 calories. Then he had a 140-calorie iced blended mocha. Represent the change in calories as an integer.

Everyday Math**Exercise:****Problem:**

Stock Market The week of September 15, 2008, was one of the most volatile weeks ever for the U.S. stock market. The change in the Dow Jones Industrial Average each day was:

Monday	−504	Tuesday	+142	Wednesday	−449
Thursday	+410	Friday	+369		

What was the overall change for the week?

Solution:

−32

Exercise:**Problem:**

Stock Market During the week of June 22, 2009, the change in the Dow Jones Industrial Average each day was:

Monday	−201	Tuesday	−16	Wednesday	−23
Thursday	+172	Friday	−34		

What was the overall change for the week?

Writing Exercises

Exercise:

Problem:

Explain why the sum of -8 and 2 is negative, but the sum of 8 and -2 and is positive.

Solution:

Sample answer: In the first case, there are more negatives so the sum is negative. In the second case, there are more positives so the sum is positive.

Exercise:

Problem:

Give an example from your life experience of adding two negative numbers.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model addition of integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			
add integers in applications.			

⑥ After reviewing this checklist, what will you do to become confident for all objectives?

Subtract Integers

By the end of this section, you will be able to:

- Model subtraction of integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate words phrases to algebraic expressions
- Subtract integers in applications

Note:

Before you get started, take this readiness quiz.

1. Simplify: $12 - (8 - 1)$.

If you missed this problem, review [\[link\]](#).

2. Translate *the difference of 20 and -15* into an algebraic expression.

If you missed this problem, review [\[link\]](#).

3. Add: $-18 + 7$.

If you missed this problem, review [\[link\]](#).

Model Subtraction of Integers

Remember the story in the last section about the toddler and the cookies? Children learn how to subtract numbers through their everyday experiences. Real-life experiences serve as models for subtracting positive numbers, and in some cases, such as temperature, for adding negative as well as positive numbers. But it is difficult to relate subtracting negative numbers to common life experiences. Most people do not have an intuitive understanding of subtraction when negative numbers are involved. Math teachers use several different models to explain subtracting negative numbers.

We will continue to use counters to model subtraction. Remember, the blue counters represent positive numbers and the red counters represent negative

numbers.

Perhaps when you were younger, you read $5 - 3$ as *five take away three*. When we use counters, we can think of subtraction the same way.

Note:Doing the Manipulative Mathematics activity "Subtraction of Signed Numbers" will help you develop a better understanding of subtracting integers.

We will model four subtraction facts using the numbers 5 and 3.

Equation:

$$5 - 3 \quad -5 - (-3) \quad -5 - 3 \quad 5 - (-3)$$

Example:
Exercise:

Problem: Model: $5 - 3$.

Solution:
Solution

Interpret the expression.	$5 - 3$ means 5 take away 3.
Model the first number. Start with 5	

positives.



Take away the second number. So take away 3 positives.



Find the counters that are left.



$5 - 3 = 2.$
The difference
between 5 and 3 is 2.

Note:

Exercise:

Problem: Model the expression:

$$6 - 4$$

Solution:



Note:

Exercise:

Problem: Model the expression:

$$7 - 4$$

Solution:



3

Example:

Exercise:

Problem: Model: $-5 - (-3)$.

Solution:

Solution

Interpret the expression.

$-5 - (-3)$ means -5

	take away -3 .
Model the first number. Start with 5 negatives.	
Take away the second number. So take away 3 negatives.	
Find the number of counters that are left.	
	$-5 - (-3) = -2.$ The difference between -5 and -3 is -2 .

Note:

Exercise:

Problem: Model the expression:

$$-6 - (-4)$$

Solution:



-2

Note:

Exercise:

Problem: Model the expression:

$$-7 - (-4)$$

Solution:



-3

Notice that [\[link\]](#) and [\[link\]](#) are very much alike.

- First, we subtracted 3 positives from 5 positives to get 2 positives.
- Then we subtracted 3 negatives from 5 negatives to get 2 negatives.

Each example used counters of only one color, and the “take away” model of subtraction was easy to apply.

$$5 - 3 = 2$$



$$-5 - (-3) = -2$$



Now let's see what happens when we subtract one positive and one negative number. We will need to use both positive and negative counters and sometimes some neutral pairs, too. Adding a neutral pair does not change the value.

Example:

Exercise:

Problem: Model: $-5 - 3$.

Solution:

Solution

Interpret the expression.	$-5 - 3$ means -5 take away 3.
Model the first number. Start with 5 negatives.	 -5
Take away the second number. So we need to take away 3 positives.	
But there are no positives to take away. Add neutral pairs until you have 3 positives.	

Now take away 3 positives.



Count the number of counters that are left.



$$-5 - 3 = -8.$$

The difference of -5 and 3 is -8 .

Note:

Exercise:

Problem: Model the expression:

$$-6 - 4$$

Solution:



$$-10$$

Note:

Exercise:

Problem: Model the expression:

$$-7 - 4$$

Solution:



$$-11$$

Example:

Exercise:

Problem: Model: $5 - (-3)$.

Solution:

Solution

Interpret the expression.	$5 - (-3)$ means 5 take away -3 .

Model the first number. Start with 5 positives.	
Take away the second number, so take away 3 negatives.	
But there are no negatives to take away. Add neutral pairs until you have 3 negatives.	
Then take away 3 negatives.	
Count the number of counters that are left.	 8 positives
	The difference of 5 and -3 is 8. $5 - (-3) = 8$

Note:

Exercise:

Problem: Model the expression:

$$6 - (-4)$$

Solution:



10

Note:

Exercise:

Problem: Model the expression:

$$7 - (-4)$$

Solution:



11

Example:

Exercise:

Problem: Model each subtraction.

- Ⓐ $8 - 2$
- Ⓑ $-5 - 4$
- Ⓒ $6 - (-6)$
- Ⓓ $-8 - (-3)$

Solution:

Ⓐ	
	$8 - 2$ This means 8 take away 2.
Start with 8 positives.	
Take away 2 positives.	
How many are left?	6
	$8 - 2 = 6$

⑥

$$-5 - 4$$

This means -5 take away 4 .

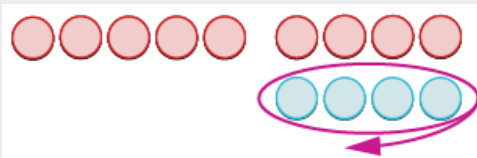
Start with 5 negatives.



You need to take away 4 positives.
Add 4 neutral pairs to get 4 positives.



Take away 4 positives.



How many are left?



$$-9$$

$$-5 - 4 = -9$$

Ⓒ

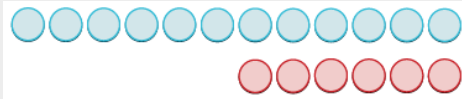
$$6 - (-6)$$

This means 6 take away -6 .

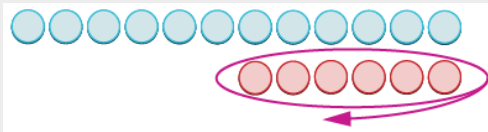
Start with 6 positives.



Add 6 neutrals to get 6 negatives to take away.



Remove 6 negatives.



How many are left?



12

$$6 - (-6) = 12$$

Ⓓ

$$-8 - (-3)$$

This means -8 take away -3 .

Start with 8 negatives.	
Take away 3 negatives.	
How many are left?	
	-5
	$-8 - (-3) = -5$

Note:

Exercise:

Problem: Model each subtraction.

- Ⓐ $7 - (-8)$
- Ⓑ $-2 - (-2)$
- Ⓒ $4 - 1$
- Ⓓ $-6 - 8$

Solution:

Ⓐ



(b)



(c)



(d)



Note:

Exercise:

Problem: Model each subtraction.

- (a) $4 - (-6)$
- (b) $-8 - (-1)$
- (c) $7 - 3$
- (d) $-4 - 2$

Solution:

(a)



(b)



(c)



(d)



Example:

Exercise:

Problem: Model each subtraction expression:

(a) $2 - 8$

(b) $-3 - (-8)$

Solution:

Solution

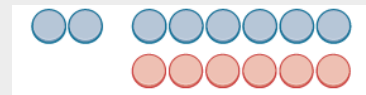
Ⓐ

We start with 2 positives.

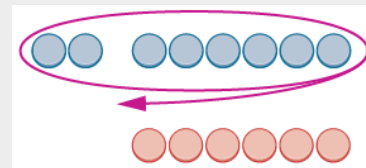


We need to take away 8 positives, but we have only 2.

Add neutral pairs until there are 8 positives to take away.



Then take away eight positives.

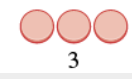
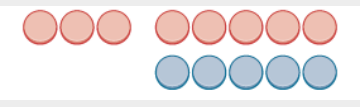
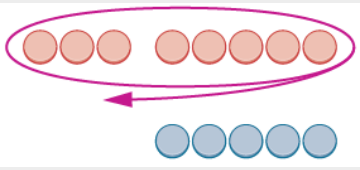


Find the number of counters that are left.

There are 6 negatives.



$$2 - 8 = -6$$

⑥ We start with 3 negatives.	
We need to take away 8 negatives, but we have only 3.	
Add neutral pairs until there are 8 negatives to take away.	
Then take away the 8 negatives.	
Find the number of counters that are left. There are 5 positives.	
	$-3 - (-8) = 5$

Note:

Exercise:

Problem: Model each subtraction expression.

① $7 - 9$

② $-5 - (-9)$

Solution:

Ⓐ



-2

Ⓑ



4

Note:

Exercise:

Problem: Model each subtraction expression.

Ⓐ $4 - 7$

Ⓑ $-7 - (-10)$

Solution:

Ⓐ



-3

(b)



3

Simplify Expressions with Integers

Do you see a pattern? Are you ready to subtract integers without counters? Let's do two more subtractions. We'll think about how we would model these with counters, but we won't actually use the counters.

- Subtract $-23 - 7$.

Think: We start with 23 negative counters.

We have to subtract 7 positives, but there are no positives to take away.

So we add 7 neutral pairs to get the 7 positives. Now we take away the 7 positives.

So what's left? We have the original 23 negatives plus 7 more negatives from the neutral pair. The result is 30 negatives.

Equation:

$$-23 - 7 = -30$$

Notice, that to subtract 7, we added 7 negatives.

- Subtract $30 - (-12)$.

Think: We start with 30 positives.

We have to subtract 12 negatives, but there are no negatives to take away.

So we add 12 neutral pairs to the 30 positives. Now we take away the 12 negatives.

What's left? We have the original 30 positives plus 12 more positives from the neutral pairs. The result is 42 positives.

Equation:

$$30 - (-12) = 42$$

Notice that to subtract -12 , we added 12.

While we may not always use the counters, especially when we work with large numbers, practicing with them first gave us a concrete way to apply the concept, so that we can visualize and remember how to do the subtraction without the counters.

Have you noticed that subtraction of signed numbers can be done by adding the opposite? You will often see the idea, the Subtraction Property, written as follows:

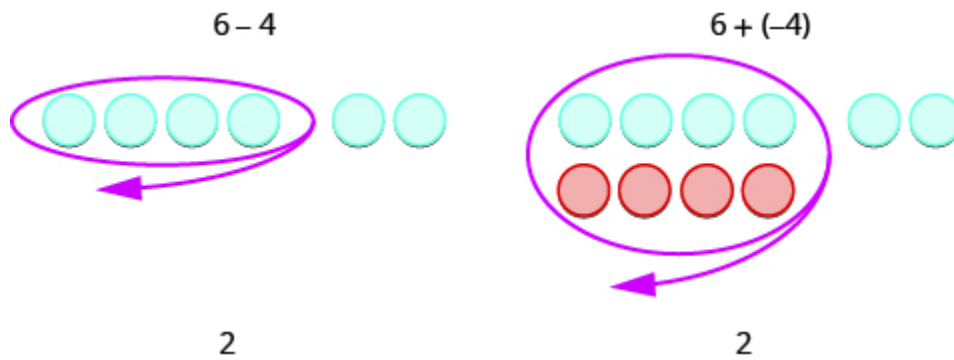
Note:

Subtraction Property

Equation:

$$a - b = a + (-b)$$

Look at these two examples.



We see that $6 - 4$ gives the same answer as $6 + (-4)$.

Of course, when we have a subtraction problem that has only positive numbers, like the first example, we just do the subtraction. We already knew how to subtract $6 - 4$ long ago. But knowing that $6 - 4$ gives the same answer as $6 + (-4)$ helps when we are subtracting negative numbers.

Example:

Exercise:

Problem: Simplify:

- Ⓐ $13 - 8$ and $13 + (-8)$
- Ⓑ $-17 - 9$ and $-17 + (-9)$

Solution:

Solution

Ⓐ	
	$13 - 8$ and

	$13 + (-8)$
Subtract to simplify.	$13 - 8 = 5$
Add to simplify.	$13 + (-8) = 5$
Subtracting 8 from 13 is the same as adding -8 to 13.	

ⓑ	
	$-17 - 9$ and $-17 + (-9)$
Subtract to simplify.	$-17 - 9 = -26$
Add to simplify.	$-17 + (-9) = -26$
Subtracting 9 from -17 is the same as adding -9 to -17 .	

Note:

Exercise:

Problem: Simplify each expression:

ⓐ $21 - 13$ and $21 + (-13)$

ⓑ $-11 - 7$ and $-11 + (-7)$

Solution:

- Ⓐ 8, 8
- Ⓑ -18, -18

Note:

Exercise:

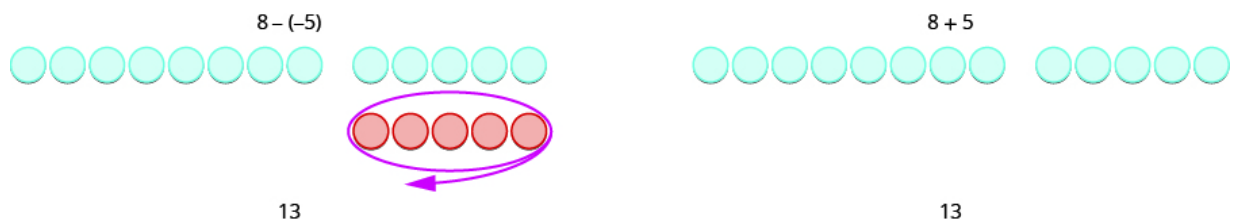
Problem: Simplify each expression:

- Ⓐ $15 - 7$ and $15 + (-7)$
- Ⓑ $-14 - 8$ and $-14 + (-8)$

Solution:

- Ⓐ 8, 8
- Ⓑ -22, -22

Now look what happens when we subtract a negative.



We see that $8 - (-5)$ gives the same result as $8 + 5$. Subtracting a negative number is like adding a positive.

Example:

Exercise:

Problem: Simplify:

Ⓐ $9 - (-15)$ and $9 + 15$

Ⓑ $-7 - (-4)$ and $-7 + 4$

Solution:

Solution

Ⓐ	
	$9 - (-15)$ and $9 + 15$
Subtract to simplify.	$9 - (-15) = -24$
Add to simplify.	$9 + 15 = 24$
Subtracting -15 from 9 is the same as adding 15 to 9.	

Ⓑ	
	$-7 - (-4)$ and

	$-7 + 4$
Subtract to simplify.	$-7 - (-4) = -3$
Add to simplify.	$-7 + 4 = -3$
Subtracting -4 from -7 is the same as adding 4 to -7	

Note:

Exercise:

Problem: Simplify each expression:

- Ⓐ $6 - (-13)$ and $6 + 13$
- Ⓑ $-5 - (-1)$ and $-5 + 1$

Solution:

- Ⓐ 19, 19
- Ⓑ $-4, -4$

Note:

Exercise:

Problem: Simplify each expression:

- Ⓐ $4 - (-19)$ and $4 + 19$
- Ⓑ $-4 - (-7)$ and $-4 + 7$

Solution:

Ⓐ 23, 23

Ⓑ 3, 3

Look again at the results of [\[link\]](#) - [\[link\]](#).

$5 - 3$	$-5 - (-3)$
2	-2
2 positives	2 negatives
When there would be enough counters of the color to take away, subtract.	
$-5 - 3$	$5 - (-3)$
-8	8
5 negatives, want to subtract 3 positives	5 positives, want to subtract 3 negatives
need neutral pairs	need neutral pairs
When there would not be enough of the counters to take away, add neutral pairs.	

Subtraction of Integers

Example:

Exercise:

Problem: Simplify: $-74 - (-58)$.

Solution:

Solution

We are taking 58 negatives away from 74 negatives.

$$-74 - (-58)$$

Subtract.

$$-16$$

Note:

Exercise:

Problem: Simplify the expression:

$$-67 - (-38)$$

Solution:

$$-29$$

Note:

Exercise:

Problem: Simplify the expression:

$$-83 - (-57)$$

Solution:

$$-26$$

Example:

Exercise:

Problem: Simplify: $7 - (-4 - 3) - 9$.

Solution:

Solution

We use the order of operations to simplify this expression, performing operations inside the parentheses first. Then we subtract from left to right.

Simplify inside the parentheses first.

$$7 - (-4 - 3) - 9$$

Subtract from left to right.

$$7 - (-7) - 9$$

Subtract.

$$14 - 9$$

$$5$$

Note:

Exercise:

Problem: Simplify the expression:

$$8 - (-3 - 1) - 9$$

Solution:

$$3$$

Note:

Exercise:

Problem: Simplify the expression:

$$12 - (-9 - 6) - 14$$

Solution:

$$13$$

Example:

Exercise:

Problem: Simplify: $3 \cdot 7 - 4 \cdot 7 - 5 \cdot 8$.

Solution:

Solution

We use the order of operations to simplify this expression. First we multiply, and then subtract from left to right.

Multiply first.	$3 \cdot 7 - 4 \cdot 7 - 5 \cdot 8$
Subtract from left to right.	$21 - 28 - 40$
Subtract.	$-7 - 40$
	-47

Note:

Exercise:

Problem: Simplify the expression:

$$6 \cdot 2 - 9 \cdot 1 - 8 \cdot 9.$$

Solution:

$$-69$$

Note:

Exercise:

Problem: Simplify the expression:

$$2 \cdot 5 - 3 \cdot 7 - 4 \cdot 9$$

Solution:

$$-47$$

Evaluate Variable Expressions with Integers

Now we'll practice evaluating expressions that involve subtracting negative numbers as well as positive numbers.

Example:

Exercise:

Problem: Evaluate $x - 4$ when

Ⓐ $x = 3$

ⓑ $x = -6$.

Solution:
Solution

ⓐ To evaluate $x - 4$ when $x = 3$, substitute 3 for x in the expression.

	$x - 4$
Substitute 3 for x .	$3 - 4$
Subtract.	-1

ⓑ To evaluate $x - 4$ when $x = -6$, substitute -6 for x in the expression.

	$x - 4$

Substitute -6 for x .

$$-6 - 4$$

Subtract.

$$-10$$

Note:

Exercise:

Problem: Evaluate each expression:

$y - 7$ when

Ⓐ $y = 5$

Ⓑ $y = -8$

Solution:

Ⓐ -2

Ⓑ -15

Note:

Exercise:

Problem: Evaluate each expression:

$m - 3$ when

Ⓐ $m = 1$

ⓑ $m = -4$

Solution:

ⓐ -2

ⓑ -7

Example:

Exercise:

Problem: Evaluate $20 - z$ when

ⓐ $z = 12$

ⓑ $z = -12$

Solution:

Solution

ⓐ To evaluate $20 - z$ when $z = 12$, substitute 12 for z in the expression.

	$20 - z$
Substitute 12 for z .	$20 - 12$

Subtract.

8

⑥ To evaluate $20 - z$ when $z = -12$, substitute -12 for z in the expression.

$20 - z$

Substitute -12 for z .

$20 - (-12)$

Subtract.

32

Note:

Exercise:

Problem: Evaluate each expression:

$17 - k$ when

① $k = 19$

② $k = -19$

Solution:

Ⓐ -2

Ⓑ 36

Note:

Exercise:

Problem: Evaluate each expression:

$-5 - b$ when

Ⓐ $b = 14$

Ⓑ $b = -14$

Solution:

Ⓐ -19

Ⓑ 9

Translate Word Phrases to Algebraic Expressions

When we first introduced the operation symbols, we saw that the expression $a - b$ may be read in several ways as shown below.

$a - b$
a minus b
the difference of a and b
subtract b from a
b subtracted from a
b less than a

Be careful to get a and b in the right order!

Example:

Exercise:

Translate and then simplify:

Problem: (a) the difference of 13 and -21

(b) subtract 24 from -19

Solution:

Solution

(a) A *difference* means subtraction. Subtract the numbers in the order they are given.

	the difference of 13 and -21
Translate.	$13 - (-21)$

Simplify.

34

ⓑ *Subtract* means to take 24 away from -19 .

subtract 24 from -19

Translate.

$-19 - 24$

Simplify.

-43

Note:

Exercise:

Problem: Translate and simplify:

- ⓐ the difference of 14 and -23
- ⓑ subtract 21 from -17

Solution:

Ⓐ $-14 - (-23) = 37$

Ⓑ $-17 - 21 = -38$

Note:

Exercise:

Problem: Translate and simplify:

Ⓐ the difference of 11 and -19

Ⓑ subtract 18 from -11

Solution:

Ⓐ $11 - (-19) = 30$

Ⓑ $-11 - 18 = -29$

Subtract Integers in Applications

It's hard to find something if we don't know what we're looking for or what to call it. So when we solve an application problem, we first need to determine what we are asked to find. Then we can write a phrase that gives the information to find it. We'll translate the phrase into an expression and then simplify the expression to get the answer. Finally, we summarize the answer in a sentence to make sure it makes sense.

Note:

Solve Application Problems.

Identify what you are asked to find.
Write a phrase that gives the information to find it.
Translate the phrase to an expression.
Simplify the expression.
Answer the question with a complete sentence.

Example:

Exercise:

Problem:

The temperature in Urbana, Illinois one morning was 11 degrees Fahrenheit. By mid-afternoon, the temperature had dropped to -9 degrees Fahrenheit. What was the difference between the morning and afternoon temperatures?

Solution:

Solution

Step 1. Identify what we are asked to find.

the difference between the morning and afternoon temperatures

Step 2. Write a phrase that gives the information to find it.

the difference of 11 and -9

Step 3. Translate the phrase to an expression.

$11 - (-9)$

The word <i>difference</i> indicates subtraction.	
Step 4. Simplify the expression.	20
Step 5. Write a complete sentence that answers the question.	The difference in temperature was 20 degrees Fahrenheit.

Note:

Exercise:

Problem:

The temperature in Anchorage, Alaska one morning was 15 degrees Fahrenheit. By mid-afternoon the temperature had dropped to 30 degrees below zero. What was the difference between the morning and afternoon temperatures?

Solution:

45 degrees Fahrenheit

Note:

Exercise:

Problem:

The temperature in Denver was -6 degrees Fahrenheit at lunchtime. By sunset the temperature had dropped to -15 degree Fahrenheit. What was the difference between the lunchtime and sunset temperatures?

Solution:

9 degrees Fahrenheit

Geography provides another application of negative numbers with the elevations of places below sea level.

Example:**Exercise:****Problem:**

Dinesh hiked from Mt. Whitney, the highest point in California, to Death Valley, the lowest point. The elevation of Mt. Whitney is 14,497 feet above sea level and the elevation of Death Valley is 282 feet below sea level. What is the difference in elevation between Mt. Whitney and Death Valley?

Solution:**Solution**

Step 1. What are we asked to find?

The difference in elevation between Mt. Whitney and Death Valley

Step 2. Write a phrase.

elevation of Mt. Whitney – elevation of Death Valley

Step 3. Translate.	$14,497 - (-282)$
Step 4. Simplify.	14,779
Step 5. Write a complete sentence that answers the question.	The difference in elevation is 14,779 feet.

Note:

Exercise:

Problem:

One day, John hiked to the 10,023 foot summit of Haleakala volcano in Hawaii. The next day, while scuba diving, he dove to a cave 80 feet below sea level. What is the difference between the elevation of the summit of Haleakala and the depth of the cave?

Solution:

10,103 feet

Note:

Exercise:

Problem:

The submarine Nautilus is at 340 feet below the surface of the water and the submarine Explorer is 573 feet below the surface of the water. What is the difference in the position of the Nautilus and the Explorer?

Solution:

233 feet

Managing your money can involve both positive and negative numbers. You might have overdraft protection on your checking account. This means the bank lets you write checks for more money than you have in your account (as long as they know they can get it back from you!)

Example:**Exercise:****Problem:**

Leslie has \$25 in her checking account and she writes a check for \$8.

- Ⓐ What is the balance after she writes the check?
- Ⓑ She writes a second check for \$20. What is the new balance after this check?
- Ⓒ Leslie's friend told her that she had lost a check for \$10 that Leslie had given her with her birthday card. What is the balance in Leslie's checking account now?

Solution:**Solution**

Ⓐ	

What are we asked to find?	The balance of the account
Write a phrase.	\$25 minus \$8
Translate	$\$25 - \$8$
Simplify.	$\$17$
Write a sentence answer.	The balance is \$17.

ⓑ	
What are we asked to find?	The new balance
Write a phrase.	\$17 minus \$20
Translate	$\$17 - \$20$
Simplify.	$-\$3$
Write a sentence answer.	She is overdrawn by \$3.

Ⓒ	
What are we asked to find?	The new balance
Write a phrase.	\$10 more than $-\$3$
Translate	$-\$3 + \$10$
Simplify.	$\$7$
Write a sentence answer.	The balance is now \$7.

Note:

Exercise:

Problem:

Araceli has \$75 in her checking account and writes a check for \$27.

- Ⓐ What is the balance after she writes the check?
- Ⓑ She writes a second check for \$50. What is the new balance?
- Ⓒ The check for \$20 that she sent a charity was never cashed. What is the balance in Araceli's checking account now?

Solution:

- Ⓐ \$48
- Ⓑ $-\$2$
- Ⓒ \$18

Note:**Exercise:****Problem:**

Genevieve's bank account was overdrawn and the balance is $-\$78$.

- Ⓐ She deposits a check for $\$24$ that she earned babysitting. What is the new balance?
- Ⓑ She deposits another check for $\$49$. Is she out of debt yet? What is her new balance?

Solution:

- Ⓐ $-\$54$
- Ⓑ No, $-\$5$

Note: The Links to Literacy activity "Elevator Magic" will provide you with another view of the topics covered in this section.

Note:**ACCESS ADDITIONAL ONLINE RESOURCES**

- [Adding and Subtracting Integers](#)
- [Subtracting Integers with Color Counters](#)
- [Subtracting Integers Basics](#)
- [Subtracting Integers](#)
- [Integer Application](#)

Key Concepts

- **Subtraction of Integers**

$5 - 3$	$-5 - (-3)$
2	-2
2 positives	2 negatives
When there would be enough counters of the color to take away, subtract.	
$-5 - 3$	$5 - (-3)$
-8	8
5 negatives, want to subtract 3 positives	5 positives, want to subtract 3 negatives
need neutral pairs	need neutral pairs
When there would not be enough of the counters to take away, add neutral pairs.	

- **Subtraction Property**

- $a - b = a + (-b)$
- $a - (-b) = a + b$

- **Solve Application Problems**

- Step 1. Identify what you are asked to find.
- Step 2. Write a phrase that gives the information to find it.
- Step 3. Translate the phrase to an expression.
- Step 4. Simplify the expression.
- Step 5. Answer the question with a complete sentence.

Practice Makes Perfect

Model Subtraction of Integers

In the following exercises, model each expression and simplify.

Exercise:

Problem: $8 - 2$

Solution:



6

Exercise:

Problem: $9 - 3$

Exercise:

Problem: $-5 - (-1)$

Solution:



-4

Exercise:

Problem: $-6 - (-4)$

Exercise:

Problem: $-5 - 4$

Solution:



-9

Exercise:

Problem: $-7 - 2$

Exercise:

Problem: $8 - (-4)$

Solution:



12

Exercise:

Problem: $7 - (-3)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem:

Ⓐ $15 - 6$

Ⓑ $15 + (-6)$

Solution:

Ⓐ 9

Ⓑ 9

Exercise:

Problem:

Ⓐ $12 - 9$

Ⓑ $12 + (-9)$

Exercise:

Problem:

- Ⓐ $44 - 28$
- Ⓑ $44 + (-28)$

Solution:

- Ⓐ 16
- Ⓑ 16

Exercise:

Problem:

- Ⓐ $35 - 16$
- Ⓑ $35 + (-16)$

Exercise:

Problem:

- Ⓐ $8 - (-9)$
- Ⓑ $8 + 9$

Solution:

- Ⓐ 17
- Ⓑ 17

Exercise:

Problem:

- Ⓐ $4 - (-4)$
- Ⓑ $4 + 4$

Exercise:

Problem:

Ⓐ $27 - (-18)$

Ⓑ $27 + 18$

Solution:

Ⓐ 45

Ⓑ 45

Exercise:

Problem:

Ⓐ $46 - (-37)$

Ⓑ $46 + 37$

In the following exercises, simplify each expression.

Exercise:

Problem: $15 - (-12)$

Solution:

27

Exercise:

Problem: $14 - (-11)$

Exercise:

Problem: $10 - (-19)$

Solution:

29

Exercise:

Problem: $11 - (-18)$

Exercise:

Problem: $48 - 87$

Solution:

-39

Exercise:

Problem: $45 - 69$

Exercise:

Problem: $31 - 79$

Solution:

-48

Exercise:

Problem: $39 - 81$

Exercise:

Problem: $-31 - 11$

Solution:

-42

Exercise:

Problem: $-32 - 18$

Exercise:

Problem: $-17 - 42$

Solution:

-59

Exercise:

Problem: $-19 - 46$

Exercise:

Problem: $-103 - (-52)$

Solution:

-51

Exercise:

Problem: $-105 - (-68)$

Exercise:

Problem: $-45 - (-54)$

Solution:

9

Exercise:

Problem: $-58 - (-67)$

Exercise:

Problem: $8 - 3 - 7$

Solution:

-2

Exercise:

Problem: $9 - 6 - 5$

Exercise:

Problem: $-5 - 4 + 7$

Solution:

-2

Exercise:

Problem: $-3 - 8 + 4$

Exercise:

Problem: $-14 - (-27) + 9$

Solution:

22

Exercise:

Problem: $-15 - (-28) + 5$

Exercise:

Problem: $71 + (-10) - 8$

Solution:

53

Exercise:

Problem: $64 + (-17) - 9$

Exercise:

Problem: $-16 - (-4 + 1) - 7$

Solution:

-20

Exercise:

Problem: $-15 - (-6 + 4) - 3$

Exercise:

Problem: $(2 - 7) - (3 - 8)$

Solution:

0

Exercise:

Problem: $(1 - 8) - (2 - 9)$

Exercise:

Problem: $-(6 - 8) - (2 - 4)$

Solution:

4

Exercise:

Problem: $-(4 - 5) - (7 - 8)$

Exercise:

Problem: $25 - [10 - (3 - 12)]$

Solution:

6

Exercise:

Problem: $32 - [5 - (15 - 20)]$

Exercise:

Problem: $6 \cdot 3 - 4 \cdot 3 - 7 \cdot 2$

Solution:

-8

Exercise:

Problem: $5 \cdot 7 - 8 \cdot 2 - 4 \cdot 9$

Exercise:

Problem: $5^2 - 6^2$

Solution:

-11

Exercise:

Problem: $6^2 - 7^2$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression for the given values.

Exercise:

Problem: $x - 6$ when

Ⓐ $x = 3$

Ⓑ $x = -3$

Solution:

Ⓐ -3

Ⓑ -9

Exercise:

Problem: $x - 4$ when

Ⓐ $x = 5$

Ⓑ $x = -5$

Exercise:

Problem: $5 - y$ when

Ⓐ $y = 2$

Ⓑ $y = -2$

Solution:

- Ⓐ 3
- Ⓑ 7

Exercise:

Problem: $8 - y$ when

- Ⓐ $y = 3$
- Ⓑ $y = -3$

Exercise:

Problem: $4x^2 - 15x + 1$ when $x = 3$

Solution:

-8

Exercise:

Problem: $5x^2 - 14x + 7$ when $x = 2$

Exercise:

Problem: $-12 - 5x^2$ when $x = 6$

Solution:

-192

Exercise:

Problem: $-19 - 4x^2$ when $x = 5$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate each phrase into an algebraic expression and then simplify.

Exercise:

Problem:

- Ⓐ The difference of 3 and -10
- Ⓑ Subtract -20 from 45

Solution:

- Ⓐ $-3 - (-10) = 13$
- Ⓑ $45 - (-20) = 65$

Exercise:

Problem:

- Ⓐ The difference of 8 and -12
- Ⓑ Subtract -13 from 50

Exercise:

Problem:

- Ⓐ The difference of -6 and 9
- Ⓑ Subtract -12 from -16

Solution:

- Ⓐ $-6 - 9 = -15$
- Ⓑ $-16 - (-12) = -4$

Exercise:

Problem:

- Ⓐ The difference of -8 and 9
- Ⓑ Subtract -15 from -19

Exercise:

Problem:

- Ⓐ 8 less than -17
- Ⓑ -24 minus 37

Solution:

- Ⓐ $-17 - 8 = -25$
- Ⓑ $-24 - 37 = -61$

Exercise:

Problem:

- Ⓐ 5 less than -14
- Ⓑ -13 minus 42

Exercise:

Problem:

- Ⓐ 21 less than 6
- Ⓑ 31 subtracted from -19

Solution:

- Ⓐ $6 - 21 = -15$
- Ⓑ $-19 - 31 = -50$

Exercise:

Problem:

- Ⓐ 34 less than 7
- Ⓑ 29 subtracted from -50

Subtract Integers in Applications

In the following exercises, solve the following applications.

Exercise:

Problem:

Temperature One morning, the temperature in Urbana, Illinois, was 28° Fahrenheit. By evening, the temperature had dropped 38° Fahrenheit. What was the temperature that evening?

Solution:

-10°

Exercise:

Problem:

Temperature On Thursday, the temperature in Spincich Lake, Michigan, was 22° Fahrenheit. By Friday, the temperature had dropped 35° Fahrenheit. What was the temperature on Friday?

Exercise:

Problem:

Temperature On January 15, the high temperature in Anaheim, California, was 84° Fahrenheit. That same day, the high temperature in Embarrass, Minnesota was -12° Fahrenheit. What was the difference between the temperature in Anaheim and the temperature in Embarrass?

Solution:

96°

Exercise:**Problem:**

Temperature On January 21, the high temperature in Palm Springs, California, was 89° , and the high temperature in Whitefield, New Hampshire was -31° . What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

Exercise:**Problem:**

Football At the first down, the Warriors football team had the ball on their 30-yard line. On the next three downs, they gained 2 yards, lost 7 yards, and lost 4 yards. What was the yard line at the end of the third down?

Solution:

21-yard line

Exercise:**Problem:**

Football At the first down, the Barons football team had the ball on their 20-yard line. On the next three downs, they lost 8 yards, gained 5 yards, and lost 6 yards. What was the yard line at the end of the third down?

Exercise:**Problem:**

Checking Account John has \$148 in his checking account. He writes a check for \$83. What is the new balance in his checking account?

Solution:

\$65

Exercise:

Problem:

Checking Account Ellie has \$426 in her checking account. She writes a check for \$152. What is the new balance in her checking account?

Exercise:

Problem:

Checking Account Gina has \$210 in her checking account. She writes a check for \$250. What is the new balance in her checking account?

Solution:

−\$40

Exercise:

Problem:

Checking Account Frank has \$94 in his checking account. He writes a check for \$110. What is the new balance in his checking account?

Exercise:

Problem:

Checking Account Bill has a balance of −\$14 in his checking account. He deposits \$40 to the account. What is the new balance?

Solution:

\$26

Exercise:

Problem:

Checking Account Patty has a balance of −\$23 in her checking account. She deposits \$80 to the account. What is the new balance?

Everyday Math

Exercise:

Problem:

Camping Rene is on an Alpine hike. The temperature is -7° . Rene's sleeping bag is rated "comfortable to -20° ". How much can the temperature change before it is too cold for Rene's sleeping bag?

Solution:

13°

Exercise:

Problem:

Scuba Diving Shelly's scuba watch is guaranteed to be watertight to -100 feet. She is diving at -45 feet on the face of an underwater canyon. By how many feet can she change her depth before her watch is no longer guaranteed?

Writing Exercises

Exercise:

Problem: Explain why the difference of 9 and -6 is 15.

Solution:

Sample answer: On a number line, 9 is 15 units away from -6 .

Exercise:

Problem:

Why is the result of subtracting $3 - (-4)$ the same as the result of adding $3 + 4$?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
model subtraction of integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			
subtract integers in applications.			

Ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Multiply and Divide Integers

By the end of this section, you will be able to:

- Multiply integers
- Divide integers
- Simplify expressions with integers
- Evaluate variable expressions with integers
- Translate word phrases to algebraic expressions

Note:

Before you get started, take this readiness quiz.

1. Translate the quotient of 20 and 13 into an algebraic expression.
If you missed this problem, review [\[link\]](#).
2. Add: $-5 + (-5) + (-5)$.
If you missed this problem, review [\[link\]](#).
3. Evaluate $n + 4$ when $n = -7$.
If you missed this problem, review [\[link\]](#).

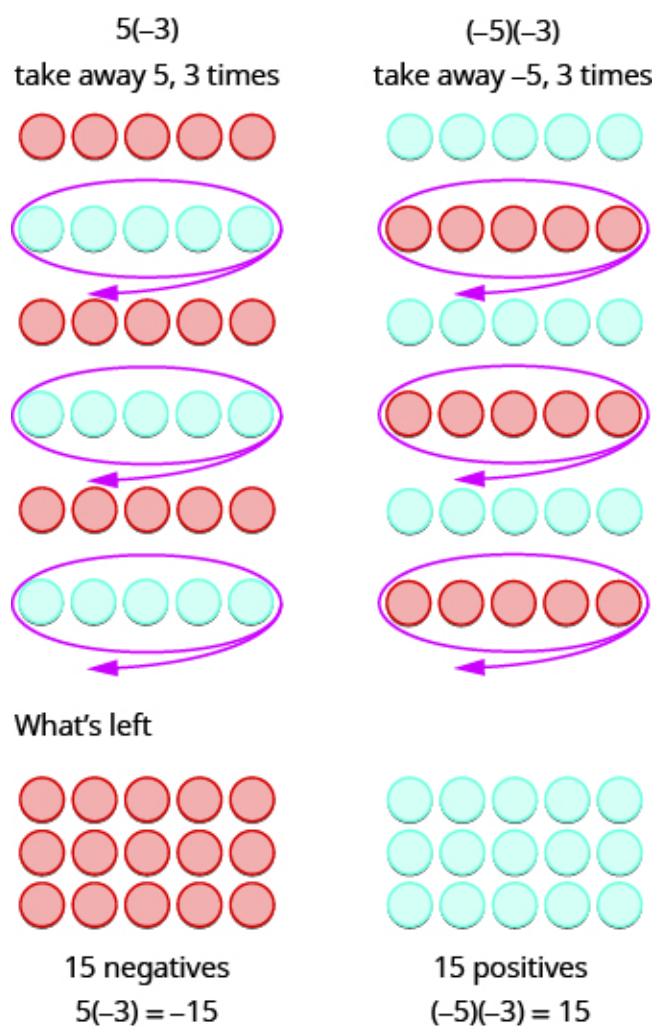
Multiply Integers

Since multiplication is mathematical shorthand for repeated addition, our counter model can easily be applied to show multiplication of integers. Let's look at this concrete model to see what patterns we notice. We will use the same examples that we used for addition and subtraction.

We remember that $a \cdot b$ means add a , b times. Here, we are using the model shown in [\[link\]](#) just to help us discover the pattern.



Now consider what it means to multiply 5 by -3 . It means subtract 5, 3 times. Looking at subtraction as *taking away*, it means to take away 5, 3 times. But there is nothing to take away, so we start by adding neutral pairs as shown in [\[link\]](#).



In both cases, we started with **15** neutral pairs. In the case on the left, we took away **5**, **3** times and the result was **−15**. To multiply $(-5)(-3)$, we took away **−5**, **3** times and the result was **15**. So we found that

Equation:

$$\begin{array}{ll} 5 \cdot 3 = 15 & -5(3) = -15 \\ 5(-3) = -15 & (-5)(-3) = 15 \end{array}$$

Notice that for multiplication of two signed numbers, when the signs are the same, the product is positive, and when the signs are different, the product is negative.

Note:

Multiplication of Signed Numbers

The sign of the product of two numbers depends on their signs.

Same signs	Product
•Two positives •Two negatives	Positive Positive
Different signs	Product
•Positive • negative •Negative • positive	Negative Negative

Example:

Exercise:

Problem: Multiply each of the following:

- Ⓐ $-9 \cdot 3$
- Ⓑ $-2(-5)$
- Ⓒ $4(-8)$
- Ⓓ $7 \cdot 6$

Solution:

Solution

Ⓐ	
	$-9 \cdot 3$
Multiply, noting that the signs are different and so the product is negative.	-27

Ⓑ	
	$-2(-5)$

Multiply, noting that the signs are the same and so the product is positive.

10

Ⓒ

$4(-8)$

Multiply, noting that the signs are different and so the product is negative.

-32

Ⓓ

$7 \cdot 6$

The signs are the same, so the product is positive.

42

Note:

Exercise:

Problem: Multiply:

Ⓐ $-6 \cdot 8$

Ⓑ $-4(-7)$

Ⓒ $9(-7)$

Ⓓ $5 \cdot 12$

Solution:

- Ⓐ -48
- Ⓑ 28
- Ⓒ -63
- Ⓓ 60

Note:

Exercise:

Problem: Multiply:

- Ⓐ $-8 \cdot 7$
- Ⓑ $-6(-9)$
- Ⓒ $7(-4)$
- Ⓓ $3 \cdot 13$

Solution:

- Ⓐ -56
- Ⓑ 54
- Ⓒ -28
- Ⓓ 39

When we multiply a number by 1, the result is the same number. What happens when we multiply a number by -1 ? Let's multiply a positive number and then a negative number by -1 to see what we get.

Equation:

$$-1 \cdot 4$$

$$-4$$

-4 is the opposite of **4**

$$-1(-3)$$

$$3$$

3 is the opposite of -3

Each time we multiply a number by -1 , we get its opposite.

Note:

Multiplication by -1

Multiplying a number by -1 gives its opposite.

Equation:

$$-1a = -a$$

Example:

Exercise:

Problem: Multiply each of the following:

Ⓐ $-1 \cdot 7$

Ⓑ $-1(-11)$

Solution:

Solution

Ⓐ

The signs are different, so the product will be negative.

$$-1 \cdot 7$$

Notice that -7 is the opposite of 7 .

-7

ⓑ

The signs are the same, so the product will be positive.

$-1(-11)$

Notice that 11 is the opposite of -11 .

11

Note:

Exercise:

Problem: Multiply.

ⓐ $-1 \cdot 9$

ⓑ $-1 \cdot (-17)$

Solution:

ⓐ -9

ⓑ 17

Note:

Exercise:

Problem: Multiply.

Ⓐ $-1 \cdot 8$

Ⓑ $-1 \cdot (-16)$

Solution:

Ⓐ -8

Ⓑ 16

Divide Integers

Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \cdot 3 = 15$. In words, this expression says that **15** can be divided into **3** groups of **5** each because adding five three times gives **15**. If we look at some examples of multiplying integers, we might figure out the rules for dividing integers.

Equation:

$$5 \cdot 3 = 15 \text{ so } 15 \div 3 = 5$$

$$(-5)(-3) = 15 \text{ so } 15 \div (-3) = -5$$

$$-5(3) = -15 \text{ so } -15 \div 3 = -5$$

$$5(-3) = -15 \text{ so } -15 \div -3 = 5$$

Division of signed numbers follows the same rules as multiplication. When the signs are the same, the quotient is positive, and when the signs are different, the quotient is negative.

Note:

Division of Signed Numbers

The sign of the quotient of two numbers depends on their signs.

Same signs	Quotient
•Two positives •Two negatives	Positive Positive
Different signs	Quotient
•Positive & negative •Negative & positive	Negative Negative

Remember, you can always check the answer to a division problem by multiplying.

Example:

Exercise:

Problem: Divide each of the following:

Ⓐ $-27 \div 3$

Ⓑ $-100 \div (-4)$

Solution:

Solution

Ⓐ	
	$-27 \div 3$
Divide, noting that the signs are different and so the quotient is negative.	-9

Ⓑ	
	$-100 \div (-4)$
Divide, noting that the signs are the same and so the quotient is positive.	25

Note:

Exercise:

Problem: Divide:

Ⓐ $-42 \div 6$

Ⓑ $-117 \div (-3)$

Solution:

Ⓐ -7

Ⓑ 39

Note:

Exercise:

Problem: Divide:

Ⓐ $-63 \div 7$

Ⓑ $-115 \div (-5)$

Solution:

Ⓐ -9

Ⓑ 23

Just as we saw with multiplication, when we divide a number by 1, the result is the same number. What happens when we divide a number by -1 ? Let's divide a positive number and then a negative number by -1 to see what we get.

Equation:

$$8 \div (-1)$$

$$-8$$

-8 is the opposite of 8

$$-9 \div (-1)$$

$$9$$

9 is the opposite of -9

When we divide a number by, -1 we get its opposite.

Note:

Division by -1

Dividing a number by -1 gives its opposite.

Equation:

$$a \div (-1) = -a$$

Example:

Exercise:

Problem: Divide each of the following:

Ⓐ $16 \div (-1)$

Ⓑ $-20 \div (-1)$

Solution:

Solution

Ⓐ	
	$16 \div (-1)$
The dividend, 16, is being divided by -1 .	-16
Dividing a number by -1 gives its opposite.	
Notice that the signs were different, so the result was negative.	

Ⓑ	
	$-20 \div (-1)$
The dividend, -20 , is being divided by -1 .	20

Dividing a number by -1 gives its opposite.

Notice that the signs were the same, so the quotient was positive.

Note:

Exercise:

Problem: Divide:

- Ⓐ $6 \div (-1)$
- Ⓑ $-36 \div (-1)$

Solution:

- Ⓐ -6
- Ⓑ 36

Note:

Exercise:

Problem: Divide:

- Ⓐ $28 \div (-1)$
- Ⓑ $-52 \div (-1)$

Solution:

- Ⓐ -28
- Ⓑ 52

Simplify Expressions with Integers

Now we'll simplify expressions that use all four operations—addition, subtraction, multiplication, and division—with integers. Remember to follow the order of operations.

Example:

Exercise:

Problem: Simplify: $7(-2) + 4(-7) - 6$.

Solution:

Solution

We use the order of operations. Multiply first and then add and subtract from left to right.

	$7(-2) + 4(-7) - 6$
Multiply first.	$-14 + (-28) - 6$
Add.	$-42 - 6$
Subtract.	-48

Note:

Exercise:

Problem: Simplify:

$$8(-3) + 5(-7) - 4$$

Solution:

$$-63$$

Note:

Exercise:

Problem: Simplify:

$$9(-3) + 7(-8) - 1$$

Solution:

$$-84$$

Example:

Exercise:

Problem: Simplify:

Ⓐ $(-2)^4$
Ⓑ -2^4

Solution:

Solution

The exponent tells how many times to multiply the base.

Ⓐ The exponent is 4 and the base is -2 . We raise -2 to the fourth power.

	$(-2)^4$
Write in expanded form.	$(-2)(-2)(-2)(-2)$
Multiply.	$4(-2)(-2)$
Multiply.	$-8(-2)$
Multiply.	16

Ⓑ The exponent is 4 and the base is 2. We raise 2 to the fourth power and then take the opposite.

	-2^4
Write in expanded form.	$-(2 \cdot 2 \cdot 2 \cdot 2)$
Multiply.	$-(4 \cdot 2 \cdot 2)$
Multiply.	$-(8 \cdot 2)$
Multiply.	-16

Note:

Exercise:

Problem: Simplify:

- Ⓐ $(-3)^4$
- Ⓑ -3^4

Solution:

- Ⓐ 81
- Ⓑ -81

Note:

Exercise:

Problem: Simplify:

- Ⓐ $(-7)^2$
- Ⓑ -7^2

Solution:

- Ⓐ 49
- Ⓑ -49

Example:

Exercise:

Problem: Simplify: $12 - 3(9 - 12)$.

Solution:
Solution

According to the order of operations, we simplify inside parentheses first. Then we will multiply and finally we will subtract.

	$12 - 3(9 - 12)$
Subtract the parentheses first.	$12 - 3(-3)$
Multiply.	$12 - (-9)$
Subtract.	21

Note:
Exercise:

Problem: Simplify:

$$17 - 4(8 - 11)$$

Solution:

29

Note:
Exercise:

Problem: Simplify:

$$16 - 6(7 - 13)$$

Solution:

52

Example:

Exercise:

Problem: Simplify: $8(-9) \div (-2)^3$.

Solution:

Solution

We simplify the exponent first, then multiply and divide.

	$8(-9) \div (-2)^3$
Simplify the exponent.	$8(-9) \div (-8)$
Multiply.	$-72 \div (-8)$
Divide.	9

Note:

Exercise:**Problem:** Simplify:

$$12(-9) \div (-3)^3$$

Solution:

4

Note:**Exercise:****Problem:** Simplify:

$$18(-4) \div (-2)^3$$

Solution:

9

Example:**Exercise:****Problem:** Simplify: $-30 \div 2 + (-3)(-7)$.**Solution:**
Solution

First we will multiply and divide from left to right. Then we will add.

	$-30 \div 2 + (-3)(-7)$
Divide.	$-15 + (-3)(-7)$
Multiply.	$-15 + 21$
Add.	6

Note:

Exercise:

Problem: Simplify:

$$-27 \div 3 + (-5)(-6)$$

Solution:

21

Note:

Exercise:

Problem: Simplify:

$$-32 \div 4 + (-2)(-7)$$

Solution:

6

Evaluate Variable Expressions with Integers

Now we can evaluate expressions that include multiplication and division with integers. Remember that to evaluate an expression, substitute the numbers in place of the variables, and then simplify.

Example:

Exercise:

Problem: Evaluate $2x^2 - 3x + 8$ when $x = -4$.

Solution:

Solution

	$2x^2 - 3x + 8$
Substitute -4 for x .	$2(-4)^2 - 3(-4) + 8$
Simplify exponents.	$2(16) - 3(-4) + 8$
Multiply.	$32 - (-12) + 8$
Subtract.	$44 + 8$

Add.

52

Keep in mind that when we substitute -4 for x , we use parentheses to show the multiplication. Without parentheses, it would look like $2 \cdot -4^2 - 3 \cdot -4 + 8$.

Note:

Exercise:

Problem: Evaluate:

$$3x^2 - 2x + 6 \text{ when } x = -3$$

Solution:

39

Note:

Exercise:

Problem: Evaluate:

$$4x^2 - x - 5 \text{ when } x = -2$$

Solution:

13

Example:

Exercise:

Problem: Evaluate $3x + 4y - 6$ when $x = -1$ and $y = 2$.

Solution:
Solution

	$3x + 4y - 6$
Substitute $x = -1$ and $y = 2$.	$3(-1) + 4(2) - 6$
Multiply.	$-3 + 8 - 6$
Simplify.	-1

Note:
Exercise:

Problem: Evaluate:

$7x + 6y - 12$ when $x = -2$ and $y = 3$

Solution:

−8

Note:

Exercise:

Problem: Evaluate:

$8x - 6y + 13$ when $x = -3$ and $y = -5$

Solution:

19

Translate Word Phrases to Algebraic Expressions

Once again, all our prior work translating words to algebra transfers to phrases that include both multiplying and dividing integers. Remember that the key word for multiplication is *product* and for division is *quotient*.

Example:

Exercise:

Problem:

Translate to an algebraic expression and simplify if possible: the product of -2 and 14 .

Solution:

Solution

The word *product* tells us to multiply.

	the product of -2 and 14
Translate.	$(-2)(14)$
Simplify.	-28

Note:

Exercise:

Problem: Translate to an algebraic expression and simplify if possible:
the product of -5 and 12

Solution:

$$-5 (12) = -60$$

Note:

Exercise:

Problem: Translate to an algebraic expression and simplify if possible:
the product of 8 and -13

Solution:

$$8 (-13) = -104$$

Example:

Exercise:

Problem:

Translate to an algebraic expression and simplify if possible: the quotient of -56 and -7 .

Solution:
Solution

The word *quotient* tells us to divide.

	the quotient of -56 and -7
Translate.	$-56 \div (-7)$
Simplify.	8

Note:**Exercise:**

Problem: Translate to an algebraic expression and simplify if possible:
the quotient of -63 and -9

Solution:

$$-63 \div -9 = 7$$

Note:

Exercise:

Problem: Translate to an algebraic expression and simplify if possible:
the quotient of -72 and -9

Solution:

$$-72 \div -9 = 8$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Multiplying Integers Using Color Counters](#)
- [Multiplying Integers Using Color Counters With Neutral Pairs](#)
- [Multiplying Integers Basics](#)
- [Dividing Integers Basics](#)
- [Ex. Dividing Integers](#)
- [Multiplying and Dividing Signed Numbers](#)

Key Concepts

- **Multiplication of Signed Numbers**
 - To determine the sign of the product of two signed numbers:

Same Signs	Product
------------	---------

Same Signs	Product
Two positives Two negatives	Positive Positive

Different Signs	Product
Positive • negative Negative • positive	Negative Negative

- **Division of Signed Numbers**

- To determine the sign of the quotient of two signed numbers:

Same Signs	Quotient
Two positives Two negatives	Positive Positive

Different Signs	Quotient
Positive • negative Negative • Positive	Negative Negative

- **Multiplication by -1**

- Multiplying a number by -1 gives its opposite: $-1a = -a$

- **Division by -1**

- Dividing a number by -1 gives its opposite: $a \div (-1) = -a$

Practice Makes Perfect

Multiply Integers

In the following exercises, multiply each pair of integers.

Exercise:

Problem: $-4 \cdot 8$

Solution:

-32

Exercise:

Problem: $-3 \cdot 9$

Exercise:

Problem: $-5(7)$

Solution:

-35

Exercise:

Problem: $-8(6)$

Exercise:

Problem: $-18(-2)$

Solution:

36

Exercise:

Problem: $-10(-6)$

Exercise:

Problem: $9(-7)$

Solution:

-63

Exercise:

Problem: $13(-5)$

Exercise:

Problem: $-1 \cdot 6$

Solution:

-6

Exercise:

Problem: $-1 \cdot 3$

Exercise:

Problem: $-1(-14)$

Solution:

14

Exercise:

Problem: $-1(-19)$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $-24 \div 6$

Solution:

-4

Exercise:

Problem: $-28 \div 7$

Exercise:

Problem: $56 \div (-7)$

Solution:

-8

Exercise:

Problem: $35 \div (-7)$

Exercise:

Problem: $-52 \div (-4)$

Solution:

13

Exercise:

Problem: $-84 \div (-6)$

Exercise:

Problem: $-180 \div 15$

Solution:

-12

Exercise:

Problem: $-192 \div 12$

Exercise:

Problem: $49 \div (-1)$

Solution:

-49

Exercise:

Problem: $62 \div (-1)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $5(-6) + 7(-2) - 3$

Solution:

-47

Exercise:

Problem: $8(-4) + 5(-4) - 6$

Exercise:

Problem: $-8(-2) - 3(-9)$

Solution:

43

Exercise:

Problem: $-7(-4) - 5(-3)$

Exercise:

Problem: $(-5)^3$

Solution:

-125

Exercise:

Problem: $(-4)^3$

Exercise:

Problem: $(-2)^6$

Solution:

64

Exercise:

Problem: $(-3)^5$

Exercise:

Problem: -4^2

Solution:

-16

Exercise:

Problem: -6^2

Exercise:

Problem: $-3(-5)(6)$

Solution:

90

Exercise:

Problem: $-4(-6)(3)$

Exercise:

Problem: $-4 \cdot 2 \cdot 11$

Solution:

-88

Exercise:

Problem: $-5 \cdot 3 \cdot 10$

Exercise:

Problem: $(8 - 11)(9 - 12)$

Solution:

9

Exercise:

Problem: $(6 - 11)(8 - 13)$

Exercise:

Problem: $26 - 3(2 - 7)$

Solution:

41

Exercise:

Problem: $23 - 2(4 - 6)$

Exercise:

Problem: $-10(-4) \div (-8)$

Solution:

-5

Exercise:

Problem: $-8(-6) \div (-4)$

Exercise:

Problem: $65 \div (-5) + (-28) \div (-7)$

Solution:

-9

Exercise:

Problem: $52 \div (-4) + (-32) \div (-8)$

Exercise:

Problem: $9 - 2[3 - 8(-2)]$

Solution:

-29

Exercise:

Problem: $11 - 3[7 - 4(-2)]$

Exercise:

Problem: $(-3)^2 - 24 \div (8 - 2)$

Solution:

5

Exercise:

Problem: $(-4)^2 - 32 \div (12 - 4)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $-2x + 17$ when

- Ⓐ $x = 8$
- Ⓑ $x = -8$

Solution:

- Ⓐ 1
- Ⓑ 33

Exercise:

Problem: $-5y + 14$ when

- Ⓐ $y = 9$
- Ⓑ $y = -9$

Exercise:

Problem: $10 - 3m$ when

- Ⓐ $m = 5$
- Ⓑ $m = -5$

Solution:

- Ⓐ -5
- Ⓑ 25

Exercise:

Problem: $18 - 4n$ when

- Ⓐ $n = 3$
- Ⓑ $n = -3$

Exercise:

Problem: $p^2 - 5p + 5$ when $p = -1$

Solution:

8

Exercise:

Problem: $q^2 - 2q + 9$ when $q = -2$

Exercise:

Problem: $2w^2 - 3w + 7$ when $w = -2$

Solution:

21

Exercise:

Problem: $3u^2 - 4u + 5$ when $u = -3$

Exercise:

Problem: $6x - 5y + 15$ when $x = 3$ and $y = -1$

Solution:

38

Exercise:

Problem: $3p - 2q + 9$ when $p = 8$ and $q = -2$

Exercise:

Problem: $9a - 2b - 8$ when $a = -6$ and $b = -3$

Solution:

−56

Exercise:

Problem: $7m - 4n - 2$ when $m = -4$ and $n = -9$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise:

Problem: The product of -3 and 15

Solution:

$$-3 \cdot 15 = -45$$

Exercise:

Problem: The product of -4 and 16

Exercise:

Problem: The quotient of -60 and -20

Solution:

$$-60 \div (-20) = 3$$

Exercise:

Problem: The quotient of -40 and -20

Exercise:

Problem: The quotient of -6 and the sum of a and b

Solution:

$$\frac{-6}{a+b}$$

Exercise:

Problem: The quotient of -7 and the sum of m and n

Exercise:

Problem: The product of -10 and the difference of p and q

Solution:

$$-10(p - q)$$

Exercise:

Problem: The product of -13 and the difference of c and d

Everyday Math

Exercise:

Problem:

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price dropped \$12 per share. What was the total effect on Javier's portfolio?

Solution:

$$-\$3,600$$

Exercise:

Problem:

Weight loss In the first week of a diet program, eight women lost an average of 3 pounds each. What was the total weight change for the eight women?

Writing Exercises**Exercise:**

Problem: In your own words, state the rules for multiplying two integers.

Solution:

Sample answer: Multiplying two integers with the same sign results in a positive product. Multiplying two integers with different signs results in a negative product.

Exercise:

Problem: In your own words, state the rules for dividing two integers.

Exercise:

Problem: Why is $-2^4 \neq (-2)^4$?

Solution:

Sample answer: In one expression the base is positive and then we take the opposite, but in the other the base is negative.

Exercise:

Problem: Why is $-4^2 \neq (-4)^2$?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			

Ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Solve Equations Using Integers; The Division Property of Equality
By the end of this section, you will be able to:

- Determine whether an integer is a solution of an equation
- Solve equations with integers using the Addition and Subtraction Properties of Equality
- Model the Division Property of Equality
- Solve equations using the Division Property of Equality
- Translate to an equation and solve

Note:

Before you get started, take this readiness quiz.

1. Evaluate $x + 4$ when $x = -4$.
If you missed this problem, review [\[link\]](#).
2. Solve: $y - 6 = 10$.
If you missed this problem, review [\[link\]](#).
3. Translate into an algebraic expression *5 less than x* .
If you missed this problem, review [\[link\]](#).

Determine Whether a Number is a Solution of an Equation

In [Solve Equations with the Subtraction and Addition Properties of Equality](#), we saw that a solution of an equation is a value of a variable that makes a true statement when substituted into that equation. In that section, we found solutions that were whole numbers. Now that we've worked with integers, we'll find integer solutions to equations.

The steps we take to determine whether a number is a solution to an equation are the same whether the solution is a whole number or an integer.

Note:

How to determine whether a number is a solution to an equation.

Substitute the number for the variable in the equation.

Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true.

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example:**Exercise:****Problem:**

Determine whether each of the following is a solution of $2x - 5 = -13$:

- Ⓐ $x = 4$
- Ⓑ $x = -4$
- Ⓒ $x = -9$.

Solution:**Solution**

- | | |
|--|--|
| Ⓐ Substitute 4 for x in the equation to determine if it is true. | |
| | |

	$2x - 5 = -13$
Substitute 4 for x.	$2(4) - 5 \stackrel{?}{=} -13$
Multiply.	$8 - 5 \stackrel{?}{=} -13$
Subtract.	$3 \neq -13$

Since $x = 4$ does not result in a true equation, 4 is not a solution to the equation.

⑥ Substitute -4 for x in the equation to determine if it is true.	$2x - 5 = -13$
Substitute -4 for x.	$2(-4) - 5 \stackrel{?}{=} -13$
Multiply.	$-8 - 5 \stackrel{?}{=} -13$
Subtract.	$-13 = -13 \checkmark$

Since $x = -4$ results in a true equation, -4 is a solution to the equation.

© Substitute -9 for x in the equation to determine if it is true.	
	$2x - 5 = -13$
Substitute -9 for x .	$2(-9) - 5 \stackrel{?}{=} -13$
Multiply.	$-18 - 5 \stackrel{?}{=} -13$
Subtract.	$-23 \neq -13$

Since $x = -9$ does not result in a true equation, -9 is not a solution to the equation.

Note:

Exercise:

Problem:

Determine whether each of the following is a solution of $2x - 8 = -14$:

- Ⓐ $x = -11$
- Ⓑ $x = 11$
- Ⓒ $x = -3$

Solution:

- Ⓐ no
- Ⓑ no
- Ⓒ yes

Note:

Exercise:

Problem:

Determine whether each of the following is a solution of $2y + 3 = -11$:

- Ⓐ $y = 4$
- Ⓑ $y = -4$
- Ⓒ $y = -7$

Solution:

- Ⓐ no
- Ⓑ no
- Ⓒ yes

Solve Equations with Integers Using the Addition and Subtraction Properties of Equality

In [Solve Equations with the Subtraction and Addition Properties of Equality](#), we solved equations similar to the two shown here using the Subtraction and Addition Properties of Equality. Now we can use them again with integers.

$$\begin{array}{rcl} x + 4 & = & 12 \\ x + 4 - 4 & = & 12 - 4 \\ x & = & 8 \end{array} \qquad \begin{array}{rcl} y - 5 & = & 9 \\ y - 5 + 5 & = & 9 + 5 \\ y & = & 14 \end{array}$$

When you add or subtract the same quantity from both sides of an equation, you still have equality.

Note:

Properties of Equalities

Subtraction Property of Equality	Addition Property of Equality
For any numbers a, b, c , if $a = b$ then $a - c = b - c$.	For any numbers a, b, c , if $a = b$ then $a + c = b + c$.

Example:

Exercise:

Problem: Solve: $y + 9 = 5$.

Solution:
Solution

	$y + 9 = 5$
Subtract 9 from each side to undo the addition.	$y + 9 - 9 = 5 - 9$
Simplify.	$y = -4$

Check the result by substituting -4 into the original equation.

	$y + 9 = 5$
Substitute -4 for y	$-4 + 9 \stackrel{?}{=} 5$
	$5 = 5 \checkmark$

Since $y = -4$ makes $y + 9 = 5$ a true statement, we found the solution to this equation.

Note:

Exercise:

Problem: Solve:

$$y + 11 = 7$$

Solution:

$$-4$$

Note:

Exercise:

Problem: Solve:

$$y + 15 = -4$$

Solution:

$$-19$$

Example:

Exercise:

Problem: Solve: $a - 6 = -8$

Solution:
Solution

	$a - 6 = -8$
Add 6 to each side to undo the subtraction.	$a - 6 + 6 = -8 + 6$
Simplify.	$a = -2$
Check the result by substituting -2 into the original equation:	$a - 6 = -8$
Substitute -2 for a	$-2 - 6 \stackrel{?}{=} -8$
	$-8 = -8 \checkmark$

The solution to $a - 6 = -8$ is -2 .

Since $a = -2$ makes $a - 6 = -8$ a true statement, we found the solution to this equation.

Note:

Exercise:

Problem: Solve:

$$a - 2 = -8$$

Solution:

-6

Note:

Exercise:

Problem: Solve:

$$n - 4 = -8$$

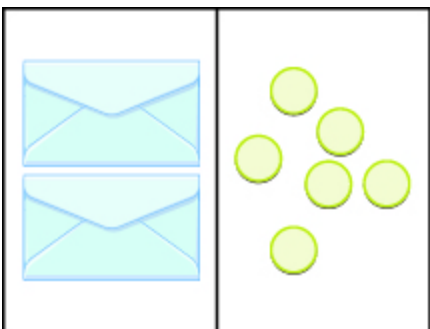
Solution:

-4

Model the Division Property of Equality

All of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term. Now we'll see how to solve equations that involve division.

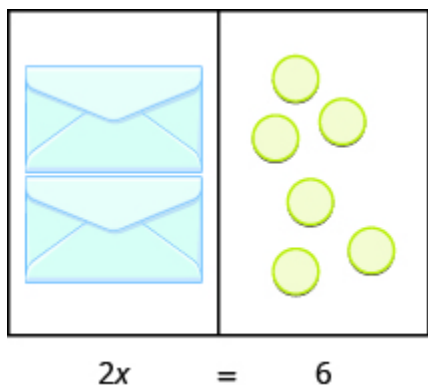
We will model an equation with envelopes and counters in [\[link\]](#).



Here, there are two identical envelopes that contain the same number of counters. Remember, the left side of the workspace must equal the right side, but the counters on the left side are “hidden” in the envelopes. So how many counters are in each envelope?

To determine the number, separate the counters on the right side into 2 groups of the same size. So 6 counters divided into 2 groups means there must be 3 counters in each group (since $6 \div 2 = 3$).

What equation models the situation shown in [\[link\]](#)? There are two envelopes, and each contains x counters. Together, the two envelopes must contain a total of 6 counters. So the equation that models the situation is $2x = 6$.

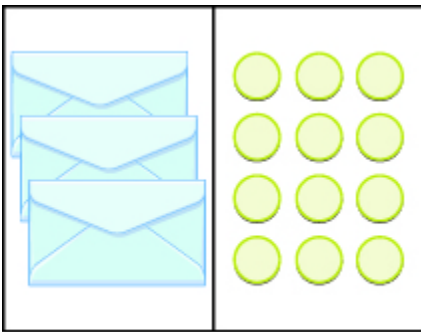


We can divide both sides of the equation by 2 as we did with the envelopes and counters.

$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

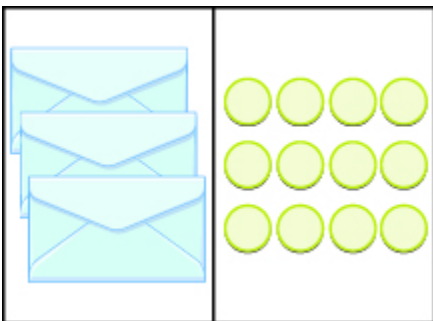
We found that each envelope contains 3 counters. Does this check? We know $2 \cdot 3 = 6$, so it works. Three counters in each of two envelopes does equal six.

[\[link\]](#) shows another example.



$$3x = 12$$

Now we have 3 identical envelopes and 12 counters. How many counters are in each envelope? We have to separate the 12 counters into 3 groups. Since $12 \div 3 = 4$, there must be 4 counters in each envelope. See [\[link\]](#).



The equation that models the situation is $3x = 12$. We can divide both sides of the equation by 3.

$$\frac{3x}{3} = \frac{12}{3}$$

$$x = 4$$

Does this check? It does because $3 \cdot 4 = 12$.

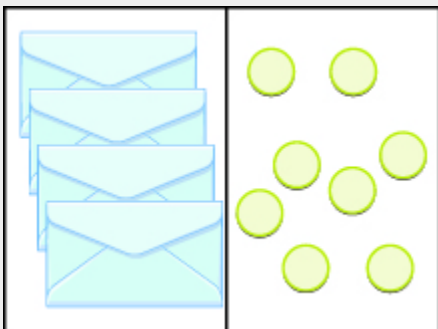
Note: Doing the Manipulative Mathematics activity “Division Property of Equality” will help you develop a better understanding of how to solve equations using the Division Property of Equality.

Example:

Exercise:

Problem:

Write an equation modeled by the envelopes and counters, and then solve it.



Solution:

Solution

There are 4 envelopes, or 4 unknown values, on the left that match the 8 counters on the right. Let's call the unknown quantity in the envelopes x .

Write the equation.

$$4x = 8$$

Divide both sides by 4.

$$\frac{4x}{4} = \frac{8}{4}$$

Simplify.

$$x = 2$$

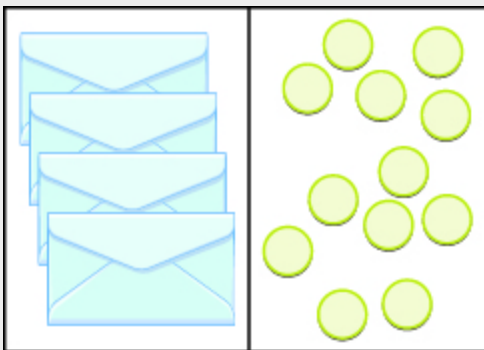
There are 2 counters in each envelope.

Note:

Exercise:

Problem:

Write the equation modeled by the envelopes and counters. Then solve it.



Solution:

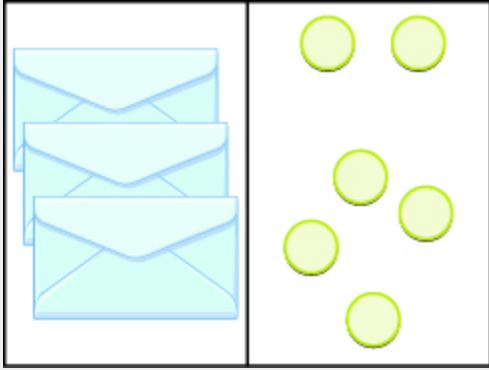
$$4x = 12; x = 3$$

Note:

Exercise:

Problem:

Write the equation modeled by the envelopes and counters. Then solve it.



Solution:

$$3x = 6; x = 2$$

Solve Equations Using the Division Property of Equality

The previous examples lead to the Division Property of Equality. When you divide both sides of an equation by any nonzero number, you still have equality.

Note:

Division Property of Equality

Equation:

For any numbers a, b, c , and $c \neq 0$,
If $a = b$ then $\frac{a}{c} = \frac{b}{c}$.

Example:

Exercise:

Problem: Solve: $7x = -49$.

Solution:
Solution

To isolate x , we need to undo multiplication.

	$7x = -49$
Divide each side by 7.	$\frac{7x}{7} = \frac{-49}{7}$
Simplify.	$x = -7$

Check the solution.

	$7x = -49$
Substitute -7 for x .	$7(-7) \stackrel{?}{=} -49$
	$-49 = -49 \checkmark$

Therefore, -7 is the solution to the equation.

Note:

Exercise:

Problem: Solve:

$$8a = 56$$

Solution:

7

Note:

Exercise:

Problem: Solve:

$$11n = 121$$

Solution:

11

Example:

Exercise:

Problem: Solve: $-3y = 63$.

Solution:
Solution

To isolate y , we need to undo the multiplication.

	$-3y = 63$
Divide each side by -3 .	$\frac{-3y}{-3} = \frac{63}{-3}$
Simplify	$y = -21$

Check the solution.

	$-3y = 63$
Substitute -21 for y .	$-3(-21) \stackrel{?}{=} 63$
	$63 = 63 \checkmark$

Since this is a true statement, $y = -21$ is the solution to the equation.

Note:

Exercise:

Problem: Solve:

$$-8p = 96$$

Solution:

$$-12$$

Note:

Exercise:

Problem: Solve:

$$-12m = 108$$

Solution:

$$-9$$

Translate to an Equation and Solve

In the past several examples, we were given an equation containing a variable. In the next few examples, we'll have to first translate word sentences into equations with variables and then we will solve the equations.

Example:

Exercise:

Problem: Translate and solve: five more than x is equal to -3 .

Solution:

Solution

	five more than x is equal to -3
Translate	$x + 5 = -3$
Subtract 5 from both sides.	$x + 5 - 5 = -3 - 5$
Simplify.	$x = -8$

Check the answer by substituting it into the original equation.

$$x + 5 = -3$$

$$-8 + 5 \stackrel{?}{=} -3$$

$$-3 = -3 \checkmark$$

Note:

Exercise:

Problem: Translate and solve:

Seven more than x is equal to -2 .

Solution:

$$x + 7 = -2; x = -9$$

Note:

Exercise:

Problem: Translate and solve:

Eleven more than y is equal to 2.

Solution:

$$y + 11 = 2; y = -9$$

Example:

Exercise:

Problem: Translate and solve: the difference of n and 6 is -10 .

Solution:

Solution

	the difference of n and 6 is -10

Translate.	$n - 6 = -10$
Add 6 to each side.	$n - 6 + 6 = -10 + 6$
Simplify.	$n = -4$

Check the answer by substituting it into the original equation.

$$n - 6 = -10$$

$$-4 - 6 \stackrel{?}{=} -10$$

$$-10 = -10 \checkmark$$

Note:

Exercise:

Problem: Translate and solve:

The difference of p and 2 is -4 .

Solution:

$$p - 2 = -4; p = -2$$

Note:

Exercise:

Problem: Translate and solve:

The difference of q and 7 is -3 .

Solution:

$$q - 7 = -3; q = 4$$

Example:**Exercise:****Problem:**

Translate and solve: the number 108 is the product of -9 and y .

Solution:**Solution**

	the number of 108 is the product of -9 and y
Translate.	$108 = -9y$
Divide by -9 .	$\frac{108}{-9} = \frac{-9y}{-9}$
Simplify.	$-12 = y$

Check the answer by substituting it into the original equation.

$$108 = -9y$$

$$108 \stackrel{?}{=} -9(-12)$$

$$108 = 108 \checkmark$$

Note:

Exercise:

Problem: Translate and solve:

The number 132 is the product of -12 and y .

Solution:

$$132 = -12y; y = -11$$

Note:

Exercise:

Problem: Translate and solve:

The number 117 is the product of -13 and z .

Solution:

$$117 = -13z; z = -9$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [One-Step Equations With Adding Or Subtracting](#)
- [One-Step Equations With Multiplying Or Dividing](#)

Key Concepts

- **How to determine whether a number is a solution to an equation.**

- Step 1. Substitute the number for the variable in the equation.
- Step 2. Simplify the expressions on both sides of the equation.
- Step 3. Determine whether the resulting equation is true.

If it is true, the number is a solution.

If it is not true, the number is not a solution.

- **Properties of Equalities**

Subtraction Property of Equality	Addition Property of Equality
For any numbers a, b, c , if $a = b$ then $a - c = b - c$.	For any numbers a, b, c , if $a = b$ then $a + c = b + c$.

- **Division Property of Equality**

- For any numbers a, b, c , and $c \neq 0$
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Section Exercises

Practice Makes Perfect

Determine Whether a Number is a Solution of an Equation

In the following exercises, determine whether each number is a solution of the given equation.

Exercise:

Problem: $4x - 2 = 6$

- Ⓐ $x = -2$
- Ⓑ $x = -1$
- Ⓒ $x = 2$

Solution:

- Ⓐ no
- Ⓑ no
- Ⓒ yes

Exercise:

Problem: $4y - 10 = -14$

- Ⓐ $y = -6$
- Ⓑ $y = -1$
- Ⓒ $y = 1$

Exercise:

Problem: $9a + 27 = -63$

- Ⓐ $a = 6$
- Ⓑ $a = -6$
- Ⓒ $a = -10$

Solution:

- Ⓐ no
- Ⓑ no
- Ⓒ yes

Exercise:

Problem: $7c + 42 = -56$

- Ⓐ $c = 2$
- Ⓑ $c = -2$
- Ⓒ $c = -14$

Solve Equations Using the Addition and Subtraction Properties of Equality

In the following exercises, solve for the unknown.

Exercise:

Problem: $n + 12 = 5$

Solution:

$$-7$$

Exercise:

Problem: $m + 16 = 2$

Exercise:

Problem: $p + 9 = -8$

Solution:

$$-17$$

Exercise:

Problem: $q + 5 = -6$

Exercise:

Problem: $u - 3 = -7$

Solution:

-4

Exercise:

Problem: $v - 7 = -8$

Exercise:

Problem: $h - 10 = -4$

Solution:

6

Exercise:

Problem: $k - 9 = -5$

Exercise:

Problem: $x + (-2) = -18$

Solution:

-16

Exercise:

Problem: $y + (-3) = -10$

Exercise:

Problem: $r - (-5) = -9$

Solution:

-14

Exercise:

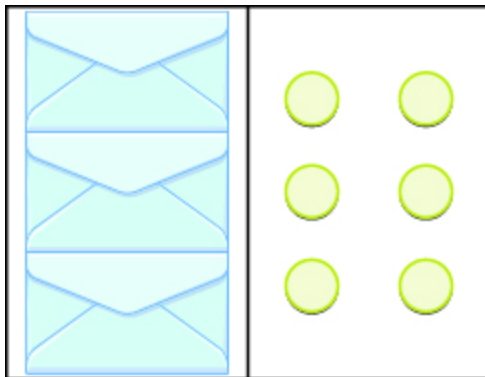
Problem: $s - (-2) = -11$

Model the Division Property of Equality

In the following exercises, write the equation modeled by the envelopes and counters and then solve it.

Exercise:

Problem:

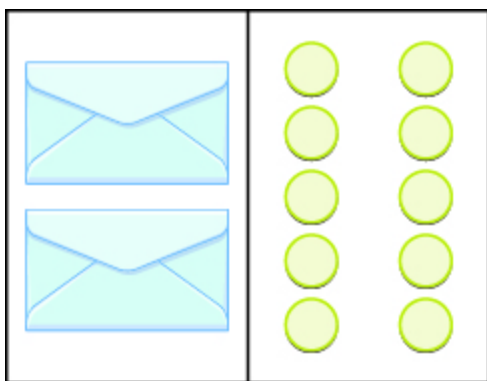


Solution:

$3x = 6; x = 2$

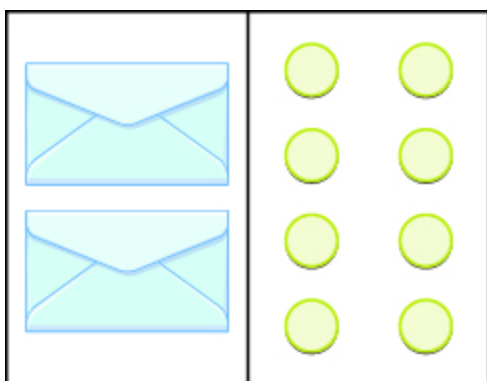
Exercise:

Problem:



Exercise:

Problem:

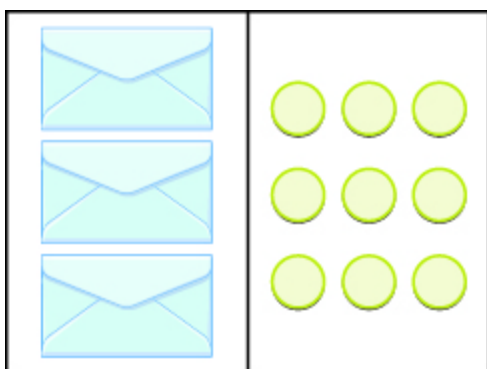


Solution:

$$2x = 8; x = 4$$

Exercise:

Problem:



Solve Equations Using the Division Property of Equality

In the following exercises, solve each equation using the division property of equality and check the solution.

Exercise:

Problem: $5x = 45$

Solution:

9

Exercise:

Problem: $4p = 64$

Exercise:

Problem: $-7c = 56$

Solution:

-8

Exercise:

Problem: $-9x = 54$

Exercise:

Problem: $-14p = -42$

Solution:

3

Exercise:

Problem: $-8m = -40$

Exercise:

Problem: $-120 = 10q$

Solution:

-12

Exercise:

Problem: $-75 = 15y$

Exercise:

Problem: $24x = 480$

Solution:

20

Exercise:

Problem: $18n = 540$

Exercise:

Problem: $-3z = 0$

Solution:

0

Exercise:

Problem: $4u = 0$

Translate to an Equation and Solve

In the following exercises, translate and solve.

Exercise:

Problem: Four more than n is equal to 1.

Solution:

$$n + 4 = 1; n = -3$$

Exercise:

Problem: Nine more than m is equal to 5.

Exercise:

Problem: The sum of eight and p is -3 .

Solution:

$$8 + p = -3; p = -11$$

Exercise:

Problem: The sum of two and q is -7 .

Exercise:

Problem: The difference of a and three is -14 .

Solution:

$$a - 3 = -14; a = -11$$

Exercise:

Problem: The difference of b and 5 is -2 .

Exercise:

Problem: The number -42 is the product of -7 and x .

Solution:

$$-42 = -7x; x = 6$$

Exercise:

Problem: The number -54 is the product of -9 and y .

Exercise:

Problem: The product of f and -15 is 75 .

Solution:

$$f(-15) = 75; f = 5$$

Exercise:

Problem: The product of g and -18 is 36 .

Exercise:

Problem: -6 plus c is equal to 4 .

Solution:

$$-6 + c = 4; c = 10$$

Exercise:

Problem: -2 plus d is equal to 1 .

Exercise:

Problem: Nine less than n is -4 .

Solution:

$$m - 9 = -4; m = 5$$

Exercise:

Problem: Thirteen less than n is -10 .

Mixed Practice

In the following exercises, solve.

Exercise:

Problem:

Ⓐ $x + 2 = 10$

Ⓑ $2x = 10$

Solution:

Ⓐ 8

Ⓑ 5

Exercise:

Problem:

Ⓐ $y + 6 = 12$

Ⓑ $6y = 12$

Exercise:

Problem:

Ⓐ $-3p = 27$

Ⓑ $p - 3 = 27$

Solution:

Ⓐ -9

Ⓑ 30

Exercise:

Problem:

Ⓐ $-2q = 34$

Ⓑ $q - 2 = 34$

Exercise:

Problem: $a - 4 = 16$

Solution:

20

Exercise:

Problem: $b - 1 = 11$

Exercise:

Problem: $-8m = -56$

Solution:

7

Exercise:

Problem: $-6n = -48$

Exercise:

Problem: $-39 = u + 13$

Solution:

$$-52$$

Exercise:

Problem: $-100 = v + 25$

Exercise:

Problem: $11r = -99$

Solution:

$$-9$$

Exercise:

Problem: $15s = -300$

Exercise:

Problem: $100 = 20d$

Solution:

$$5$$

Exercise:

Problem: $250 = 25n$

Exercise:

Problem: $-49 = x - 7$

Solution:

-42

Exercise:

Problem: $64 = y - 4$

Everyday Math

Exercise:

Problem:

Cookie packaging A package of 51 cookies has 3 equal rows of cookies. Find the number of cookies in each row, c , by solving the equation $3c = 51$.

Solution:

17 cookies

Exercise:

Problem:

Kindergarten class Connie's kindergarten class has 24 children. She wants them to get into 4 equal groups. Find the number of children in each group, g , by solving the equation $4g = 24$.

Writing Exercises

Exercise:

Problem:

Is modeling the Division Property of Equality with envelopes and counters helpful to understanding how to solve the equation $3x = 15$? Explain why or why not.

Solution:

Sample answer: It is helpful because it shows how the counters can be divided among the envelopes.

Exercise:**Problem:**

Suppose you are using envelopes and counters to model solving the equations $x + 4 = 12$ and $4x = 12$. Explain how you would solve each equation.

Exercise:**Problem:**

Frida started to solve the equation $-3x = 36$ by adding 3 to both sides. Explain why Frida's method will not solve the equation.

Solution:

Sample answer: The operation used in the equation is multiplication. The inverse of multiplication is division, not addition.

Exercise:**Problem:**

Raoul started to solve the equation $4y = 40$ by subtracting 4 from both sides. Explain why Raoul's method will not solve the equation.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether an integer is a solution of an equation.			
solve equations with integers using the addition and subtraction properties of equality.			
model division property of equality.			
solve equations using the division property of equality.			
translate to an equation and solve.			

Ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Introduction to Integers

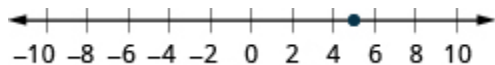
Locate Positive and Negative Numbers on the Number Line

In the following exercises, locate and label the integer on the number line.

Exercise:

Problem: 5

Solution:



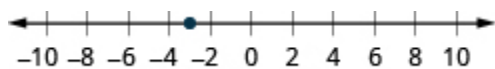
Exercise:

Problem: -5

Exercise:

Problem: -3

Solution:



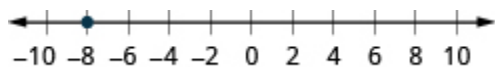
Exercise:

Problem: 3

Exercise:

Problem: -8

Solution:



Exercise:

Problem: -7

Order Positive and Negative Numbers

In the following exercises, order each of the following pairs of numbers, using $<$ or $>$.

Exercise:

Problem: 4_8

Solution:

$<$

Exercise:

Problem: -6_3

Exercise:

Problem: $-5\_ -10$

Solution:

$>$

Exercise:

Problem: $-9\_ -4$

Exercise:

Problem: $2\_ -7$

Solution:

$>$

Exercise:

Problem: -3_1

Find Opposites

In the following exercises, find the opposite of each number.

Exercise:

Problem: 6

Solution:

-6

Exercise:

Problem: -2

Exercise:

Problem: -4

Solution:

4

Exercise:

Problem: 3

In the following exercises, simplify.

Exercise:

Problem:

Ⓐ $-(8)$

Ⓑ $-(-8)$

Solution:

Ⓐ -8

Ⓑ 8

Exercise:

Problem:

Ⓐ $-(9)$

Ⓑ $-(-9)$

In the following exercises, evaluate.

Exercise:

Problem: $-x$, when

Ⓐ $x = 32$

Ⓑ $x = -32$

Solution:

Ⓐ -32

Ⓑ 32

Exercise:

Problem: $-n$, when

Ⓐ $n = 20$

Ⓑ $n = -20$

Simplify Absolute Values

In the following exercises, simplify.

Exercise:

Problem: $|-21|$

Solution:

21

Exercise:

Problem: $|-42|$

Exercise:

Problem: $|36|$

Solution:

36

Exercise:

Problem: $-|15|$

Exercise:

Problem: $|0|$

Solution:

0

Exercise:

Problem: $-|-75|$

In the following exercises, evaluate.

Exercise:

Problem: $|x|$ when $x = -14$

Solution:

14

Exercise:

Problem: $-|r|$ when $r = 27$

Exercise:

Problem: $-|-y|$ when $y = 33$

Solution:

-33

Exercise:

Problem: $|-n|$ when $n = -4$

In the following exercises, fill in $<$, $>$, or $=$ for each of the following pairs of numbers.

Exercise:

Problem: $-|-4|$ ___ 4

Solution:

$<$

Exercise:

Problem: -2 ___ $|-2|$

Exercise:

Problem: $-|-6|$ ___ -6

Solution:

=

Exercise:

Problem: $-|-9|$ $\underline{\hspace{1cm}}$ $|-9|$

In the following exercises, simplify.

Exercise:

Problem: $-(-55)$ and $-|-55|$

Solution:

-55 ; -55

Exercise:

Problem: $-(-48)$ and $-|-48|$

Exercise:

Problem: $|12 - 5|$

Solution:

7

Exercise:

Problem: $|9 + 7|$

Exercise:

Problem: $6|-9|$

Solution:

Exercise:

Problem: $|14 - 8| - |-2|$

Exercise:

Problem: $|9 - 3| - |5 - 12|$

Solution:

-1

Exercise:

Problem: $5 + 4|15 - 3|$

Translate Phrases to Expressions with Integers

In the following exercises, translate each of the following phrases into expressions with positive or negative numbers.

Exercise:**Problem:** the opposite of 16

Solution:

-16

Exercise:**Problem:** the opposite of -8 **Exercise:****Problem:** negative 3

Solution:

-3

Exercise:

Problem: 19 minus negative 12

Exercise:

Problem: a temperature of 10 below zero

Solution:

-10°

Exercise:

Problem: an elevation of 85 feet below sea level

Add Integers

Model Addition of Integers

In the following exercises, model the following to find the sum.

Exercise:

Problem: $3 + 7$

Solution:

10

Exercise:

Problem: $-2 + 6$

Exercise:

Problem: $5 + (-4)$

Solution:

1

Exercise:

Problem: $-3 + (-6)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $14 + 82$

Solution:

96

Exercise:

Problem: $-33 + (-67)$

Exercise:

Problem: $-75 + 25$

Solution:

-50

Exercise:

Problem: $54 + (-28)$

Exercise:

Problem: $11 + (-15) + 3$

Solution:

-1

Exercise:

Problem: $-19 + (-42) + 12$

Exercise:

Problem: $-3 + 6(-1 + 5)$

Solution:

21

Exercise:

Problem: $10 + 4(-3 + 7)$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $n + 4$ when

Ⓐ $n = -1$

Ⓑ $n = -20$

Solution:

Ⓐ 3

Ⓑ -16

Exercise:

Problem: $x + (-9)$ when

Ⓐ $x = 3$

Ⓑ $x = -3$

Exercise:

Problem: $(x + y)^3$ when $x = -4, y = 1$

Solution:

-27

Exercise:

Problem: $(u + v)^2$ when $u = -4, v = 11$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate each phrase into an algebraic expression and then simplify.

Exercise:

Problem: the sum of -8 and 2

Solution:

$-8 + 2 = -6$

Exercise:

Problem: 4 more than -12

Exercise:

Problem: 10 more than the sum of -5 and -6

Solution:

$$10 + [-5 + (-6)] = -1$$

Exercise:

Problem: the sum of 3 and -5 , increased by 18

Add Integers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature On Monday, the high temperature in Denver was -4 degrees. Tuesday's high temperature was 20 degrees more. What was the high temperature on Tuesday?

Solution:

16 degrees

Exercise:

Problem:

Credit Frida owed \$75 on her credit card. Then she charged \$21 more. What was her new balance?

Subtract Integers

Model Subtraction of Integers

In the following exercises, model the following.

Exercise:

Problem: $6 - 1$

Solution:



5

Exercise:

Problem: $-4 - (-3)$

Exercise:

Problem: $2 - (-5)$

Solution:



7

Exercise:

Problem: $-1 - 4$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $24 - 16$

Solution:

8

Exercise:

Problem: $19 - (-9)$

Exercise:

Problem: $-31 - 7$

Solution:

-38

Exercise:

Problem: $-40 - (-11)$

Exercise:

Problem: $-52 - (-17) - 23$

Solution:

-58

Exercise:

Problem: $25 - (-3 - 9)$

Exercise:

Problem: $(1 - 7) - (3 - 8)$

Solution:

-1

Exercise:

Problem: $3^2 - 7^2$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $x - 7$ when

Ⓐ $x = 5$

Ⓑ $x = -4$

Solution:

Ⓐ -2

Ⓑ -11

Exercise:

Problem: $10 - y$ when

Ⓐ $y = 15$

ⓑ $y = -16$

Exercise:

Problem: $2n^2 - n + 5$ when $n = -4$

Solution:

41

Exercise:

Problem: $-15 - 3u^2$ when $u = -5$

Translate Phrases to Algebraic Expressions

In the following exercises, translate each phrase into an algebraic expression and then simplify.

Exercise:

Problem: the difference of -12 and 5

Solution:

$$-12 - 5 = -17$$

Exercise:

Problem: subtract 23 from -50

Subtract Integers in Applications

In the following exercises, solve the given applications.

Exercise:

Problem:

Temperature One morning the temperature in Bangor, Maine was 18 degrees. By afternoon, it had dropped 20 degrees. What was the afternoon temperature?

Solution:

−2 degrees

Exercise:**Problem:**

Temperature On January 4, the high temperature in Laredo, Texas was 78 degrees, and the high in Houlton, Maine was −28 degrees. What was the difference in temperature of Laredo and Houlton?

Multiply and Divide Integers

Multiply Integers

In the following exercises, multiply.

Exercise:

Problem: $-9 \cdot 4$

Solution:

−36

Exercise:

Problem: $5(-7)$

Exercise:

Problem: $(-11)(-11)$

Solution:

121

Exercise:

Problem: $-1 \cdot 6$

Divide Integers

In the following exercises, divide.

Exercise:

Problem: $56 \div (-8)$

Solution:

-7

Exercise:

Problem: $-120 \div (-6)$

Exercise:

Problem: $-96 \div 12$

Solution:

-8

Exercise:

Problem: $96 \div (-16)$

Exercise:

Problem: $45 \div (-1)$

Solution:

-45

Exercise:

Problem: $-162 \div (-1)$

Simplify Expressions with Integers

In the following exercises, simplify each expression.

Exercise:

Problem: $5(-9) - 3(-12)$

Solution:

-9

Exercise:

Problem: $(-2)^5$

Exercise:

Problem: -3^4

Solution:

-81

Exercise:

Problem: $(-3)(4)(-5)(-6)$

Exercise:

Problem: $42 - 4(6 - 9)$

Solution:

54

Exercise:

Problem: $(8 - 15)(9 - 3)$

Exercise:

Problem: $-2(-18) \div 9$

Solution:

4

Exercise:

Problem: $45 \div (-3) - 12$

Evaluate Variable Expressions with Integers

In the following exercises, evaluate each expression.

Exercise:

Problem: $7x - 3$ when $x = -9$

Solution:

-66

Exercise:

Problem: $16 - 2n$ when $n = -8$

Exercise:

Problem: $5a + 8b$ when $a = -2, b = -6$

Solution:

-58

Exercise:

Problem: $x^2 + 5x + 4$ when $x = -3$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate to an algebraic expression and simplify if possible.

Exercise:

Problem: the product of -12 and 6

Solution:

$-12(6) = -72$

Exercise:

Problem: the quotient of 3 and the sum of -7 and s

Solve Equations using Integers; The Division Property of Equality

Determine Whether a Number is a Solution of an Equation

In the following exercises, determine whether each number is a solution of the given equation.

Exercise:

Problem: $5x - 10 = -35$

- Ⓐ $x = -9$
- Ⓑ $x = -5$
- Ⓒ $x = 5$

Solution:

- Ⓐ no
- Ⓑ yes
- Ⓒ no

Exercise:

Problem: $8u + 24 = -32$

- Ⓐ $u = -7$
- Ⓑ $u = -1$
- Ⓒ $u = 7$

Using the Addition and Subtraction Properties of Equality

In the following exercises, solve.

Exercise:

Problem: $a + 14 = 2$

Solution:

-12

Exercise:

Problem: $b - 9 = -15$

Solution:

-6

Exercise:

Problem: $c + (-10) = -17$

Solution:

-7

Exercise:

Problem: $d - (-6) = -26$

Solution:

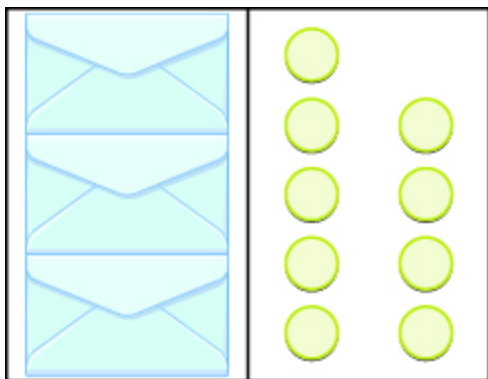
-32

Model the Division Property of Equality

In the following exercises, write the equation modeled by the envelopes and counters. Then solve it.

Exercise:

Problem:

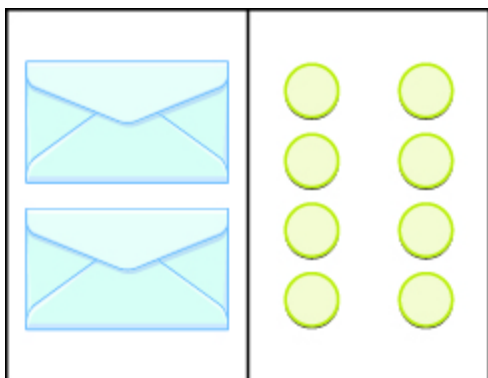


Solution:

$$3x = 9; x = 3 >$$

Exercise:

Problem:



Solve Equations Using the Division Property of Equality

In the following exercises, solve each equation using the division property of equality and check the solution.

Exercise:

Problem: $8p = 72$

Solution:

Exercise:

Problem: $-12q = 48$

Exercise:

Problem: $-16r = -64$

Solution:

4

Exercise:

Problem: $-5s = -100$

Translate to an Equation and Solve.

In the following exercises, translate and solve.

Exercise:

Problem: The product of -6 and y is -42

Solution:

$$-6y = -42; y = 7$$

Exercise:

Problem: The difference of z and -13 is -18 .

Exercise:

Problem: Four more than m is -48 .

Solution:

$$m + 4 = -48; m = -52$$

Exercise:

Problem: The product of -21 and n is 63 .

Everyday Math

Exercise:

Problem:

Describe how you have used two topics from this chapter in your life outside of your math class during the past month.

Chapter Practice Test

Exercise:

Problem: Locate and label 0 , 2 , -4 , and -1 on a number line.

In the following exercises, compare the numbers, using $<$ or $>$ or $=$.

Exercise:

Problem:

Ⓐ $-6 \underline{\hspace{1cm}} 3$

Ⓑ $-1 \underline{\hspace{1cm}} -4$

Solution:

Ⓐ $<$

Ⓑ $>$

Exercise:

Problem:

Ⓐ $-5 \underline{\hspace{1cm}} |-5|$

Ⓑ $- |-2| \underline{\hspace{1cm}} -2$

In the following exercises, find the opposite of each number.

Exercise:

Problem:

Ⓐ -7

Ⓑ 8

Solution:

Ⓐ 7

Ⓑ -8

In the following exercises, simplify.

Exercise:

Problem: $-(-22)$

Exercise:

Problem: $|4 - 9|$

Solution:

5

Exercise:

Problem: $-8 + 6$

Exercise:

Problem: $-15 + (-12)$

Solution:

$$-27$$

Exercise:

Problem: $-7 - (-3)$

Exercise:

Problem: $10 - (5 - 6)$

Solution:

$$11$$

Exercise:

Problem: $-3 \cdot 8$

Exercise:

Problem: $-6(-9)$

Solution:

$$54$$

Exercise:

Problem: $70 \div (-7)$

Exercise:

Problem: $(-2)^3$

Solution:

-8

Exercise:

Problem: -4^2

Exercise:

Problem: $16 - 3(5 - 7)$

Solution:

22

Exercise:

Problem: $|21 - 6| - |-8|$

In the following exercises, evaluate.

Exercise:

Problem: $35 - a$ when $a = -4$

Solution:

39

Exercise:

Problem: $(-2r)^2$ when $r = 3$

Exercise:

Problem: $3m - 2n$ when $m = 6$, $n = -8$

Solution:

34

Exercise:

Problem: $-|-y|$ when $y = 17$

In the following exercises, translate each phrase into an algebraic expression and then simplify, if possible.

Exercise:

Problem: the difference of -7 and -4

Solution:

$$-7 - (-4) = -3$$

Exercise:

Problem: the quotient of 25 and the sum of m and n .

In the following exercises, solve.

Exercise:

Problem:

Early one morning, the temperature in Syracuse was -8°F . By noon, it had risen 12° . What was the temperature at noon?

Solution:

4°F

Exercise:

Problem:

Collette owed \$128 on her credit card. Then she charged \$65. What was her new balance?

In the following exercises, solve.

Exercise:

Problem: $n + 6 = 5$

Solution:

$$n = -1$$

Exercise:

Problem: $p - 11 = -4$

Exercise:

Problem: $-9r = -54$

Solution:

$$r = 6$$

In the following exercises, translate and solve.

Exercise:

Problem: The product of 15 and x is 75.

Exercise:

Problem: Eight less than y is -32 .

Solution:

$$y - 8 = -32; y = -24$$

Introduction to the Properties of Real Numbers

class="introduction"

Quiltmakers know that
by
rearranging
the same
basic
blocks the
resulting
quilts can
look very
different.

What
happens
when we
rearrange
the
numbers in
an
expression?

Does the
resulting
value
change? We
will answer
these
questions in
this chapter
as we will
learn about
the
properties
of numbers.

(credit:
Hans,

Public
Domain)



A quilt is formed by sewing many different pieces of fabric together. The pieces can vary in color, size, and shape. The combinations of different kinds of pieces provide for an endless possibility of patterns. Much like the pieces of fabric, mathematicians distinguish among different types of numbers. The kinds of numbers in an expression provide for an endless possibility of outcomes. We have already described counting numbers, whole numbers, and integers. In this chapter, we will learn about other types of numbers and their properties.

Rational and Irrational Numbers

By the end of this section, you will be able to:

- Identify rational numbers and irrational numbers
- Classify different types of real numbers

Note:

Before you get started, take this readiness quiz.

1. Write 3.19 as an improper fraction.
If you missed this problem, review [\[link\]](#).
2. Write $\frac{5}{11}$ as a decimal.
If you missed this problem, review [\[link\]](#).
3. Simplify: $\sqrt{144}$.
If you missed this problem, review [\[link\]](#).

Identify Rational Numbers and Irrational Numbers

Congratulations! You have completed the first six chapters of this book! It's time to take stock of what you have done so far in this course and think about what is ahead. You have learned how to add, subtract, multiply, and divide whole numbers, fractions, integers, and decimals. You have become familiar with the language and symbols of algebra, and have simplified and evaluated algebraic expressions. You have solved many different types of applications. You have established a good solid foundation that you need so you can be successful in algebra.

In this chapter, we'll make sure your skills are firmly set. We'll take another look at the kinds of numbers we have worked with in all previous chapters. We'll work with properties of numbers that will help you improve your number sense. And we'll practice using them in ways that we'll use when we solve equations and complete other procedures in algebra.

We have already described numbers as counting numbers, whole numbers, and integers. Do you remember what the difference is among these types of numbers?

counting numbers	1, 2, 3, 4. . .
whole numbers	0, 1, 2, 3, 4. . .
integers	. . . -3, -2, -1, 0, 1, 2, 3, 4. . .

Rational Numbers

What type of numbers would you get if you started with all the integers and then included all the fractions? The numbers you would have form the set of rational numbers. A **rational number** is a number that can be written as a ratio of two integers.

Note:

Rational Numbers

A rational number is a number that can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

All fractions, both positive and negative, are rational numbers. A few examples are

Equation:

$$\frac{4}{5}, -\frac{7}{8}, \frac{13}{4}, \text{ and } -\frac{20}{3}$$

Each numerator and each denominator is an integer.

We need to look at all the numbers we have used so far and verify that they are rational. The definition of rational numbers tells us that all fractions are rational. We will now look at the counting numbers, whole numbers, integers, and decimals to make sure they are rational.

Are integers rational numbers? To decide if an integer is a rational number, we try to write it as a ratio of two integers. An easy way to do this is to write it as a fraction with denominator one.

Equation:

$$3 = \frac{3}{1} \quad -8 = \frac{-8}{1} \quad 0 = \frac{0}{1}$$

Since any integer can be written as the ratio of two integers, all integers are rational numbers. Remember that all the counting numbers and all the whole numbers are also integers, and so they, too, are rational.

What about decimals? Are they rational? Let's look at a few to see if we can write each of them as the ratio of two integers. We've already seen that integers are rational numbers. The integer -8 could be written as the decimal -8.0 . So, clearly, some decimals are rational.

Think about the decimal 7.3 . Can we write it as a ratio of two integers? Because 7.3 means $7\frac{3}{10}$, we can write it as an improper fraction, $\frac{73}{10}$. So 7.3 is the ratio of the integers 73 and 10 . It is a rational number.

In general, any decimal that ends after a number of digits (such as 7.3 or -1.2684) is a rational number. We can use the place value of the last digit as the denominator when writing the decimal as a fraction.

Example:

Exercise:

Problem: Write each as the ratio of two integers: Ⓐ -15 Ⓑ 6.81 Ⓒ $-3\frac{6}{7}$.

Solution:
Solution

Ⓐ	
	-15
Write the integer as a fraction with denominator 1.	$\frac{-15}{1}$

Ⓑ	
	6.81
Write the decimal as a mixed number.	$6\frac{81}{100}$
Then convert it to an improper fraction.	$\frac{681}{100}$

Ⓒ	
	$-3\frac{6}{7}$
Convert the mixed number to an improper fraction.	$-\frac{27}{7}$

Note:
Exercise:

Problem: Write each as the ratio of two integers: Ⓐ -24 Ⓑ 3.57 .

Solution:

a) $\frac{-24}{1}$

b) $\frac{357}{100}$

(a) $\frac{-24}{1}$
(b) $\frac{357}{100}$

Exercise:

Problem: Write each as the ratio of two integers: Ⓐ -19 Ⓑ 8.41 .

Solution:

(a) $\frac{-19}{1}$

(b) $\frac{841}{100}$

(a) $\frac{-19}{1}$

(b) $\frac{841}{100}$

Integer $-2, -1, 0, 1, 2, 3$

We have also seen that every fraction is a rational number. Look at the decimal form of the fractions we just considered.

Decimal Forms 0.8, -0.875, 3.25, -6.666... These decimals either stop or repeat.

Rational Numbers		
	Fractions	Integers

Rational Numbers		
Number	$\frac{4}{5}, -\frac{7}{8}, \frac{13}{4}, \frac{-20}{3}$	$-2, -1, 0, 1, 2, 3$
Ratio of Integer	$\frac{4}{5}, \frac{-7}{8}, \frac{13}{4}, \frac{-20}{3}$	$\frac{-2}{1}, \frac{-1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}$
Decimal number	$0.8, -0.875, 3.25, -6.\bar{6},$	$-2.0, -1.0, 0.0, 1.0, 2.0, 3.0$

Irrational Numbers

Are there any decimals that do not stop or repeat? Yes. The number π (the Greek letter pi, pronounced 'pie'), which is very important in describing circles, has a decimal form that does not stop or repeat.

Equation:

$$\pi = 3.141592654.....$$

Similarly, the decimal representations of square roots of numbers that are not perfect squares never stop and never repeat. For example,

Equation:

$$\sqrt{5} = 2.236067978.....$$

A decimal that does not stop and does not repeat cannot be written as the ratio of integers. We call this kind of number an **irrational number**.

Note:

Irrational Number

An irrational number is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

Let's summarize a method we can use to determine whether a number is rational or irrational.

If the decimal form of a number

- stops or repeats, the number is rational.
- does not stop and does not repeat, the number is irrational.

Example:

Exercise:

Problem: Identify each of the following as rational or irrational:

- Ⓐ $0.58\bar{3}$
- Ⓑ 0.475
- Ⓒ $3.605551275\dots$

Solution:
Solution

- Ⓐ $0.58\bar{3}$

The bar above the 3 indicates that it repeats. Therefore, $0.58\bar{3}$ is a repeating decimal, and is therefore a rational number.

- Ⓑ 0.475

This decimal stops after the 5, so it is a rational number.

- Ⓒ $3.605551275\dots$

The ellipsis ($\dots$) means that this number does not stop. There is no repeating pattern of digits. Since the number doesn't stop and doesn't repeat, it is irrational.

Note:
Exercise:

Problem: Identify each of the following as rational or irrational:

- Ⓐ 0.29 Ⓑ $0.81\bar{6}$ Ⓒ $2.515115111\dots$

Solution:

- Ⓐ rational
- Ⓑ rational
- Ⓒ irrational

Note:
Exercise:

Problem: Identify each of the following as rational or irrational:

- Ⓐ $0.2\bar{3}$ Ⓑ 0.125 Ⓒ $0.418302\dots$

Solution:

- Ⓐ rational
- Ⓑ rational
- Ⓒ irrational

Let's think about square roots now. Square roots of perfect squares are always whole numbers, so they are rational. But the decimal forms of square roots of numbers that are not perfect squares never stop and never repeat, so these square roots are irrational.

Example:

Exercise:

Problem: Identify each of the following as rational or irrational:

Ⓐ $\sqrt{36}$

Ⓑ $\sqrt{44}$

Solution:

Solution

Ⓐ The number 36 is a perfect square, since $6^2 = 36$. So $\sqrt{36} = 6$. Therefore $\sqrt{36}$ is rational.

Ⓑ Remember that $6^2 = 36$ and $7^2 = 49$, so 44 is not a perfect square.

This means $\sqrt{44}$ is irrational.

Note:

Exercise:

Problem: Identify each of the following as rational or irrational:

Ⓐ $\sqrt{81}$

Ⓑ $\sqrt{17}$

Solution:

Ⓐ rational

Ⓑ irrational

Note:

Exercise:

Problem: Identify each of the following as rational or irrational:

Ⓐ $\sqrt{116}$

Ⓑ $\sqrt{121}$

Solution:

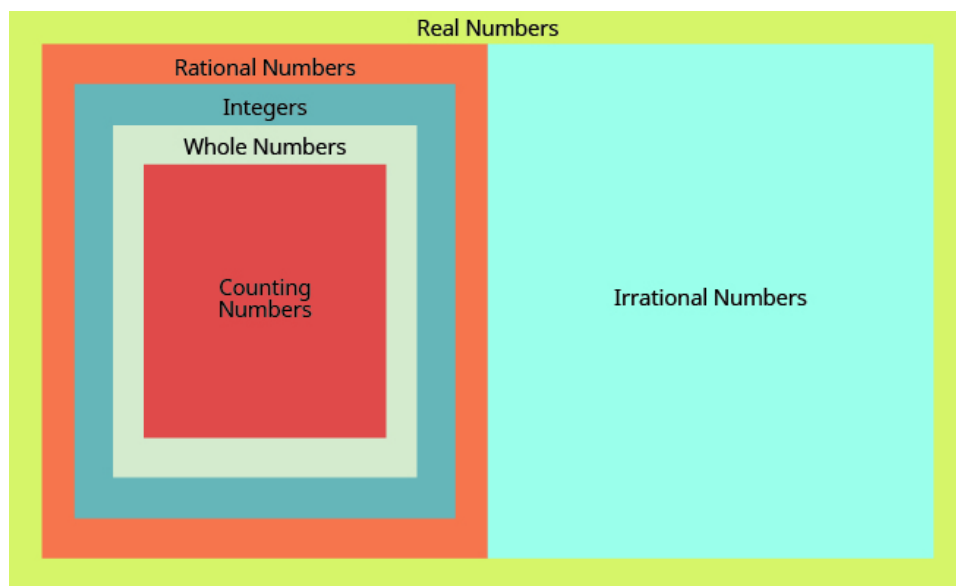
Ⓐ irrational

Ⓑ rational

Classify Real Numbers

We have seen that all counting numbers are whole numbers, all whole numbers are integers, and all integers are rational numbers. Irrational numbers are a separate category of their own. When we put together the rational numbers and the irrational numbers, we get the set of **real numbers**.

[\[link\]](#) illustrates how the number sets are related.



This diagram illustrates the relationships between the different types of real numbers.

Note:

Real Numbers

Real numbers are numbers that are either rational or irrational.

Does the term “real numbers” seem strange to you? Are there any numbers that are not “real”, and, if so, what could they be? For centuries, the only numbers people knew about were what we now call the real numbers. Then mathematicians discovered the set of *imaginary numbers*. You won't encounter imaginary numbers in this course, but you will later on in your studies of algebra.

Example:

Exercise:

Problem:

Determine whether each of the numbers in the following list is a ① whole number, ② integer, ③ rational number, ④ irrational number, and ⑤ real number.

Equation:

$$-7, \frac{14}{5}, 8, \sqrt{5}, 5.9, -\sqrt{64}$$

Solution:

Solution

- ① The whole numbers are 0, 1, 2, 3, . . . The number 8 is the only whole number given.
- ② The integers are the whole numbers, their opposites, and 0. From the given numbers, -7 and 8 are integers. Also, notice that 64 is the square of 8 so $-\sqrt{64} = -8$. So the integers are $-7, 8, -\sqrt{64}$.
- ③ Since all integers are rational, the numbers $-7, 8$, and $-\sqrt{64}$ are also rational. Rational numbers also include fractions and decimals that terminate or repeat, so $\frac{14}{5}$ and 5.9 are rational.
- ④ The number 5 is not a perfect square, so $\sqrt{5}$ is irrational.
- ⑤ All of the numbers listed are real.

We'll summarize the results in a table.

Number	Whole	Integer	Rational	Irrational	Real
-7					

Number	Whole	Integer	Rational	Irrational	Real
$\frac{14}{5}$					
8					
$\sqrt{5}$					
5.9					
$-\sqrt{64}$					

Note:

Exercise:

Problem:

Determine whether each number is a ① whole number, ② integer, ③ rational number, ④ irrational number, and ⑤ real number: -3 , $-\sqrt{2}$, $0.\bar{3}$, $\frac{9}{5}$, 4 , $\sqrt{49}$.

Solution:

Number	Whole	Integer	Rational
-3		✓	✓
$-\sqrt{2}$			
$0.\bar{3}$			✓
$\sqrt{-49}$			
4	✓	✓	✓
$\frac{9}{5}$			✓
Number	Irrational	Real	
-3		✓	
$-\sqrt{2}$	✓	✓	
$0.\bar{3}$		✓	
$\sqrt{-49}$			
4		✓	
$\frac{9}{5}$		✓	

Note:

Exercise:

Problem:

Determine whether each number is a (a) whole number, (b) integer, (c) rational number, (d) irrational number, and (e) real number: $-\sqrt{25}$, $-\frac{3}{8}$, -1 , 6 , $\sqrt{121}$, $2.041975\ldots$

Solution:

Number	Whole	Integer	Rational
$-\sqrt{25}$		✓	✓
$-\frac{3}{8}$			✓
-1		✓	✓
6	✓	✓	✓
$\sqrt{121}$	✓	✓	✓
$2.041975\ldots$			
Number	Irrational		Real
$-\sqrt{25}$			✓
$-\frac{3}{8}$			✓
-1			✓
6			✓
$\sqrt{121}$			✓
$2.041975\ldots$	✓		✓

Note:

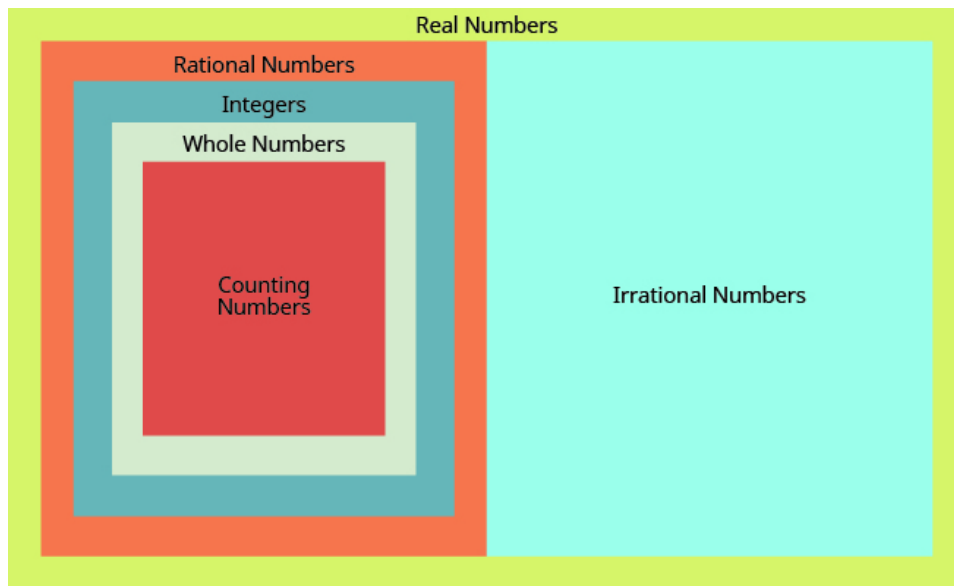
ACCESS ADDITIONAL ONLINE RESOURCES

- [Sets of Real Numbers](#)
- [Real Numbers](#)

Key Concepts

- Real numbers

○



Practice Makes Perfect

Rational Numbers

In the following exercises, write as the ratio of two integers.

Exercise:

Problem:

- Ⓐ 5
- Ⓑ 3.19

Solution:

- Ⓐ $\frac{5}{1}$
- Ⓑ $\frac{319}{100}$

Exercise:

Problem:

- Ⓐ 8
- Ⓑ -1.61

Exercise:

Problem:

- Ⓐ -12
- Ⓑ 9.279

Solution:

- Ⓐ $\frac{-12}{1}$
- Ⓑ $\frac{9279}{1000}$

Exercise:

Problem:

- Ⓐ -16
- Ⓑ 4.399

In the following exercises, determine which of the given numbers are rational and which are irrational.

Exercise:

Problem: 0.75 , $0.22\bar{3}$, $1.39174\dots$

Solution:

Rational: 0.75 , $0.22\bar{3}$. Irrational: $1.39174\dots$

Exercise:

Problem: 0.36 , $0.94729\dots$, $2.52\bar{8}$

Exercise:

Problem: $0.\bar{45}$, $1.919293\dots$, 3.59

Solution:

Rational: $0.\bar{45}$, 3.59 . Irrational: $1.919293\dots$

Exercise:

Problem: $0.1\bar{3}$, $0.42982\dots$, 1.875

In the following exercises, identify whether each number is rational or irrational.

Exercise:

Problem:

- Ⓐ $\sqrt{25}$
- Ⓑ $\sqrt{30}$

Solution:

- Ⓐ rational

- ⓑ irrational

Exercise:

Problem:

- ⓐ $\sqrt{44}$
ⓑ $\sqrt{49}$

Exercise:

Problem:

- ⓐ $\sqrt{164}$
ⓑ $\sqrt{169}$

Solution:

- ⓐ irrational
ⓑ rational

Exercise:

Problem:

- ⓐ $\sqrt{225}$
ⓑ $\sqrt{216}$

Classifying Real Numbers

In the following exercises, determine whether each number is whole, integer, rational, irrational, and real.

Exercise:

Problem: -8 , 0 , $1.95286\dots$, $\frac{12}{5}$, $\sqrt{36}$, 9

Solution:

Number	Whole	Integer	Rational
-8		✓	✓
0	✓	✓	✓
1.95286...			
$\frac{12}{5}$			✓
$\sqrt{36}$	✓	✓	✓
9	✓	✓	✓
Number	Irrational	Real	
-8		✓	
0		✓	
1.95286...	✓	✓	
$\frac{12}{5}$		✓	
$\sqrt{36}$		✓	
9		✓	

Exercise:

Problem: -9 , $-3\frac{4}{9}$, $-\sqrt{9}$, $0.\overline{409}$, $\frac{11}{6}$, 7

Exercise:

Problem: $-\sqrt{100}$, -7 , $-\frac{8}{3}$, -1 , 0.77 , $3\frac{1}{4}$

Solution:

Number	Whole	Integer	Rational
$-\sqrt{100}$		✓	✓
-7		✓	✓
$-\frac{8}{5}$			✓
-1		✓	✓
0.77			✓
$3\frac{1}{4}$			✓
Number	Irrational	Real	
$-\sqrt{100}$		✓	
-7		✓	
$-\frac{8}{5}$		✓	
-1		✓	
0.77		✓	
$3\frac{1}{4}$		✓	

Everyday Math

Exercise:

Problem:

Field trip All the 5th graders at Lincoln Elementary School will go on a field trip to the science museum. Counting all the children, teachers, and chaperones, there will be 147 people. Each bus holds 44 people.

- Ⓐ How many buses will be needed?
- Ⓑ Why must the answer be a whole number?
- Ⓒ Why shouldn't you round the answer the usual way?

Exercise:**Problem:**

Child care Serena wants to open a licensed child care center. Her state requires that there be no more than 12 children for each teacher. She would like her child care center to serve 40 children.

- Ⓐ How many teachers will be needed?
- Ⓑ Why must the answer be a whole number?
- Ⓒ Why shouldn't you round the answer the usual way?

Solution:

- Ⓐ 4
- Ⓑ Teachers cannot be divided
- Ⓒ It would result in a lower number.

Writing Exercises**Exercise:****Problem:**

In your own words, explain the difference between a rational number and an irrational number.

Exercise:**Problem:**

Explain how the sets of numbers (counting, whole, integer, rational, irrationals, reals) are related to each other.

Solution:

Answers will vary.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify rational and irrational numbers.			
classify different types of real numbers.			

Ⓑ If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

...no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

Irrational number

An **irrational number** is a number that cannot be written as the ratio of two integers. Its decimal form does not stop and does not repeat.

Rational number

A **rational number** is a number that can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Its decimal form stops or repeats.

Real number

a **real number** is a number that is either rational or irrational.

Commutative and Associative Properties

By the end of this section, you will be able to:

- Use the commutative and associative properties
- Evaluate expressions using the commutative and associative properties
- Simplify expressions using the commutative and associative properties

Note:

Before you get started, take this readiness quiz.

1. Simplify: $7y + 2 + y + 13$.

If you missed this problem, review [\[link\]](#).

2. Multiply: $\frac{2}{3} \cdot 18$.

If you missed this problem, review [\[link\]](#).

3. Find the opposite of 15.

If you missed this problem, review [\[link\]](#).

In the next few sections, we will take a look at the properties of real numbers. Many of these properties will describe things you already know, but it will help to give names to the properties and define them formally. This way we'll be able to refer to them and use them as we solve equations in the next chapter.

Use the Commutative and Associative Properties

Think about adding two numbers, such as 5 and 3.

Equation:

$$\begin{array}{cc} 5 + 3 & 3 + 5 \\ 8 & 8 \end{array}$$

The results are the same. $5 + 3 = 3 + 5$

Notice, the order in which we add does not matter. The same is true when multiplying 5 and 3.

Equation:

$$\begin{array}{cc} 5 \cdot 3 & 3 \cdot 5 \\ 15 & 15 \end{array}$$

Again, the results are the same! $5 \cdot 3 = 3 \cdot 5$. The order in which we multiply does not matter.

These examples illustrate the commutative properties of addition and multiplication.

Note:

Commutative Properties

Commutative Property of Addition: if a and b are real numbers, then

Equation:

$$a + b = b + a$$

Commutative Property of Multiplication: if a and b are real numbers, then

Equation:

$$a \cdot b = b \cdot a$$

The commutative properties have to do with order. If you change the order of the numbers when adding or multiplying, the result is the same.

Example:

Exercise:

Problem:

Use the commutative properties to rewrite the following expressions:

Ⓐ $-1 + 3 = \underline{\hspace{2cm}}$

Ⓑ $4 \cdot 9 = \underline{\hspace{2cm}}$

Solution:**Solution**

Ⓐ	
	$-1 + 3 = \underline{\hspace{2cm}}$
Use the commutative property of addition to change the order.	$-1 + 3 = 3 + (-1)$

Ⓑ	
	$4 \cdot 9 = \underline{\hspace{2cm}}$
Use the commutative property of multiplication to change the order.	$4 \cdot 9 = 9 \cdot 4$

Note:

Exercise:

Problem: Use the commutative properties to rewrite the following:

Ⓐ $-4 + 7 = \underline{\hspace{2cm}}$

Ⓑ $6 \cdot 12 = \underline{\hspace{2cm}}$

Solution:

Ⓐ $-4 + 7 = 7 + (-4)$

Ⓑ $6 \cdot 12 = 12 \cdot 6$

Note:

Exercise:

Problem: Use the commutative properties to rewrite the following:

Ⓐ $14 + (-2) = \underline{\hspace{2cm}}$

Ⓑ $3(-5) = \underline{\hspace{2cm}}$

Solution:

Ⓐ $14 + (-2) = -2 + 14$

Ⓑ $3(-5) = (-5)3$

What about subtraction? Does order matter when we subtract numbers?

Does $7 - 3$ give the same result as $3 - 7$?

Equation:

$$\begin{array}{r} 7 - 3 \\ 4 \end{array} \qquad \begin{array}{r} 3 - 7 \\ -4 \end{array}$$

$$4 \neq -4$$

Equation:

The results are not the same. $7 - 3 \neq 3 - 7$

Since changing the order of the subtraction did not give the same result, we can say that subtraction is not commutative.

Let's see what happens when we divide two numbers. Is division commutative?

Equation:

$$\begin{array}{r} 12 \div 4 \\ \frac{12}{4} \\ 3 \end{array} \qquad \begin{array}{r} 4 \div 12 \\ \frac{4}{12} \\ \frac{1}{3} \end{array}$$

$$3 \neq \frac{1}{3}$$

Equation:

The results are not the same. So $12 \div 4 \neq 4 \div 12$

Since changing the order of the division did not give the same result, division is not commutative.

Addition and multiplication are commutative. Subtraction and division are not commutative.

Suppose you were asked to simplify this expression.

Equation:

$$7 + 8 + 2$$

How would you do it and what would your answer be?

Some people would think $7 + 8$ is 15 and then $15 + 2$ is 17. Others might start with $8 + 2$ makes 10 and then $7 + 10$ makes 17.

Both ways give the same result, as shown in [\[link\]](#). (Remember that parentheses are grouping symbols that indicate which operations should be done first.)

$$\begin{array}{|c|c|} \hline (7 + 8) + 2 = 15 + 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline 7 + (8 + 2) = 7 + 10 \\ \hline \end{array}$$

17

When adding three numbers, changing the grouping of the numbers does not change the result. This is known as the Associative Property of Addition.

The same principle holds true for multiplication as well. Suppose we want to find the value of the following expression:

Equation:

$$5 \cdot \frac{1}{3} \cdot 3$$

Changing the grouping of the numbers gives the same result, as shown in [\[link\]](#).

$$\begin{array}{l} (5 \cdot \frac{1}{3}) \cdot 3 = 5 \cdot (\frac{1}{3} \cdot 3) \\ (\frac{5}{3}) \cdot 3 = 5 \cdot 1 \end{array}$$

When multiplying three numbers, changing the grouping of the numbers does not change the result. This is known as the Associative Property of Multiplication.

If we multiply three numbers, changing the grouping does not affect the product.

You probably know this, but the terminology may be new to you. These examples illustrate the *Associative Properties*.

Note:

Associative Properties

Associative Property of Addition: if a , b , and c are real numbers, then

Equation:

$$(a + b) + c = a + (b + c)$$

Associative Property of Multiplication: if a , b , and c are real numbers, then

Equation:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Example:

Exercise:

Problem: Use the associative properties to rewrite the following:

Ⓐ $(3 + 0.6) + 0.4 = \underline{\hspace{2cm}}$

Ⓑ $(-4 \cdot \frac{2}{5}) \cdot 15 = \underline{\hspace{2cm}}$

Solution:
Solution

Ⓐ	
	$(3 + 0.6) + 0.4 = \underline{\hspace{2cm}}$
Change the grouping.	$(3 + 0.6) + 0.4 = 3 + (0.6 + 0.4)$

Notice that $0.6 + 0.4$ is 1, so the addition will be easier if we group as shown on the right.

Ⓑ	
	$(-4 \cdot \frac{2}{5}) \cdot 15 = \underline{\hspace{2cm}}$
Change the grouping.	$(-4 \cdot \frac{2}{5}) \cdot 15 = -4 \cdot (\frac{2}{5} \cdot 15)$

Notice that $\frac{2}{5} \cdot 15$ is 6. The multiplication will be easier if we group as shown on the right.

Note:

Exercise:

Problem:

Use the associative properties to rewrite the following:

Ⓐ $(1 + 0.7) + 0.3 = \underline{\hspace{2cm}}$ Ⓑ $(-9 \cdot 8) \cdot \frac{3}{4} = \underline{\hspace{2cm}}$

Solution:

Ⓐ $(1 + 0.7) + 0.3 = 1 + (0.7 + 0.3)$
Ⓑ $(-9 \cdot 8) \cdot \frac{3}{4} = -9(8 \cdot \frac{3}{4})$

Note:

Exercise:

Problem:

Use the associative properties to rewrite the following:

Ⓐ $(4 + 0.6) + 0.4 = \underline{\hspace{2cm}}$ Ⓑ $(-2 \cdot 12) \cdot \frac{5}{6} = \underline{\hspace{2cm}}$

Solution:

Ⓐ $(4 + 0.6) + 0.4 = 4 + (0.6 + 0.4)$
Ⓑ $(-2 \cdot 12) \cdot \frac{5}{6} = -2(12 \cdot \frac{5}{6})$

Besides using the associative properties to make calculations easier, we will often use it to simplify expressions with variables.

Example:

Exercise:

Problem:

Use the Associative Property of Multiplication to simplify: $6(3x)$.

Solution:

Solution

	$6(3x)$
Change the grouping.	$(6 \cdot 3)x$
Multiply in the parentheses.	$18x$

Notice that we can multiply $6 \cdot 3$, but we could not multiply $3 \cdot x$ without having a value for x .

Note:

Exercise:

Problem:

Use the Associative Property of Multiplication to simplify the given expression: $8(4x)$.

Solution:

$$32x$$

Note:

Exercise:

Problem:

Use the Associative Property of Multiplication to simplify the given expression: $-9(7y)$.

Solution:

$$-63y$$

Evaluate Expressions using the Commutative and Associative Properties

The commutative and associative properties can make it easier to evaluate some algebraic expressions. Since order does not matter when adding or multiplying three or more terms, we can rearrange and re-group terms to make our work easier, as the next several examples illustrate.

Example:

Exercise:

Problem: Evaluate each expression when $x = \frac{7}{8}$.

Ⓐ $x + 0.37 + (-x)$

Ⓑ $x + (-x) + 0.37$

Solution:
Solution

Ⓐ	
	$x + 0.37 + (-x)$
Substitute $\frac{7}{8}$ for x .	$\frac{7}{8} + 0.37 + \left(-\frac{7}{8}\right)$
Convert fractions to decimals.	$0.875 + 0.37 + (-0.875)$
Add left to right.	$1.245 - 0.875$
Subtract.	0.37

Ⓑ	
---	--

	$x + (-x) + 0.37$
Substitute $\frac{7}{8}$ for x.	$\frac{7}{8} + \left(-\frac{7}{8}\right) + 0.37$
Add opposites first.	0.37

What was the difference between part ① and part ②? Only the order changed. By the Commutative Property of Addition, $x + 0.37 + (-x) = x + (-x) + 0.37$. But wasn't part ② much easier?

Note:

Exercise:

Problem:

Evaluate each expression when $y = \frac{3}{8}$: ① $y + 0.84 + (-y)$ ② $y + (-y) + 0.84$.

Solution:

① 0.84

② 0.84

Note:

Exercise:

Problem:

Evaluate each expression when $f = \frac{17}{20}$: ① $f + 0.975 + (-f)$ ② $f + (-f) + 0.975$.

Solution:

① 0.975

② 0.975

Let's do one more, this time with multiplication.

Example:

Exercise:

Problem: Evaluate each expression when $n = 17$.

① $\frac{4}{3} \left(\frac{3}{4} n \right)$

② $\left(\frac{4}{3} \cdot \frac{3}{4} \right) n$

Solution:

Solution

①	

	$\frac{4}{3}\left(\frac{3}{4}n\right)$
Substitute 17 for n.	$\frac{4}{3}\left(\frac{3}{4} \cdot 17\right)$
Multiply in the parentheses first.	$\frac{4}{3}\left(\frac{51}{4}\right)$
Multiply again.	17

ⓑ	
	$\left(\frac{4}{3} \cdot \frac{3}{4}\right)n$
Substitute 17 for n.	$\left(\frac{4}{3} \cdot \frac{3}{4}\right) \cdot 17$
Multiply. The product of reciprocals is 1.	$(1) \cdot 17$
Multiply again.	

What was the difference between part ① and part ② here? Only the grouping changed. By the Associative Property of Multiplication, $\frac{4}{3} \left(\frac{3}{4} n \right) = \left(\frac{4}{3} \cdot \frac{3}{4} \right) n$. By carefully choosing how to group the factors, we can make the work easier.

Note:

Exercise:

Problem:

Evaluate each expression when $p = 24$: ① $\frac{5}{9} \left(\frac{9}{5} p \right)$ ② $\left(\frac{5}{9} \cdot \frac{9}{5} \right) p$.

Solution:

① 24

② 24

Note:

Exercise:

Problem:

Evaluate each expression when $q = 15$: ① $\frac{7}{11} \left(\frac{11}{7} q \right)$ ② $\left(\frac{7}{11} \cdot \frac{11}{7} \right) q$

Solution:

① 15

Simplify Expressions Using the Commutative and Associative Properties

When we have to simplify algebraic expressions, we can often make the work easier by applying the Commutative or Associative Property first instead of automatically following the order of operations. Notice that in [\[link\]](#) part ⓑ was easier to simplify than part ⓐ because the opposites were next to each other and their sum is 0. Likewise, part ⓑ in [\[link\]](#) was easier, with the reciprocals grouped together, because their product is 1. In the next few examples, we'll use our number sense to look for ways to apply these properties to make our work easier.

Example:

Exercise:

Problem: Simplify: $-84n + (-73n) + 84n$.

Solution:

Solution

Notice the first and third terms are opposites, so we can use the commutative property of addition to reorder the terms.

	$-84n + (-73n) + 84n$

Re-order the terms.	$-84n + 84n + (-73n)$
Add left to right.	$0 + (-73n)$
Add.	$-73n$

Note:

Exercise:

Problem: Simplify: $-27a + (-48a) + 27a$.

Solution:

$$-48a$$

Note:

Exercise:

Problem: Simplify: $39x + (-92x) + (-39x)$.

Solution:

$$-92x$$

Now we will see how recognizing reciprocals is helpful. Before multiplying left to right, look for reciprocals—their product is 1.

Example:

Exercise:

Problem: Simplify: $\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$.

Solution:

Solution

Notice the first and third terms are reciprocals, so we can use the Commutative Property of Multiplication to reorder the factors.

	$\frac{7}{15} \cdot \frac{8}{23} \cdot \frac{15}{7}$
Re-order the terms.	$\frac{7}{15} \cdot \frac{15}{7} \cdot \frac{8}{23}$
Multiply left to right.	$1 \cdot \frac{8}{23}$
Multiply.	$\frac{8}{23}$

Note:

Exercise:

Problem: Simplify: $\frac{9}{16} \cdot \frac{5}{49} \cdot \frac{16}{9}$.

Solution:

$$\frac{5}{49}$$

Note:

Exercise:

Problem: Simplify: $\frac{6}{17} \cdot \frac{11}{25} \cdot \frac{17}{6}$.

Solution:

$$\frac{11}{25}$$

In expressions where we need to add or subtract three or more fractions, combine those with a common denominator first.

Example:

Exercise:

Problem: Simplify: $\left(\frac{5}{13} + \frac{3}{4}\right) + \frac{1}{4}$.

Solution:

Solution

Notice that the second and third terms have a common denominator, so this work will be easier if we change the grouping.

$$\left(\frac{5}{13} + \frac{3}{4}\right) + \frac{1}{4}$$

Group the terms with a common denominator.	$\frac{5}{13} + \left(\frac{3}{4} + \frac{1}{4}\right)$
Add in the parentheses first.	$\frac{5}{13} + \left(\frac{4}{4}\right)$
Simplify the fraction.	$\frac{5}{13} + 1$
Add.	$1\frac{5}{13}$
Convert to an improper fraction.	$\frac{18}{13}$

Note:

Exercise:

Problem: Simplify: $\left(\frac{7}{15} + \frac{5}{8}\right) + \frac{3}{8}$.

Solution:

$$\frac{22}{15}$$

Note:

Exercise:

Problem: Simplify: $\left(\frac{2}{9} + \frac{7}{12}\right) + \frac{5}{12}$.

Solution:

$$\frac{11}{9}$$

When adding and subtracting three or more terms involving decimals, look for terms that combine to give whole numbers.

Example:

Exercise:

Problem: Simplify: $(6.47q + 9.99q) + 1.01q$.

Solution:

Solution

Notice that the sum of the second and third coefficients is a whole number.

	$(6.47q + 9.99q) + 1.01q$
Change the grouping.	$6.47q + (9.99q + 1.01q)$
Add in the parentheses first.	$6.47q + (11.00q)$
Add.	$17.47q$

Many people have good number sense when they deal with money. Think about adding 99 cents and 1 cent. Do you see how this applies to adding $9.99 + 1.01$?

Note:

Exercise:

Problem: Simplify: $(5.58c + 8.75c) + 1.25c$.

Solution:

$15.58c$

Note:**Exercise:**

Problem: Simplify: $(8.79d + 3.55d) + 5.45d$.

Solution:

$17.79d$

No matter what you are doing, it is always a good idea to think ahead. When simplifying an expression, think about what your steps will be. The next example will show you how using the Associative Property of Multiplication can make your work easier if you plan ahead.

Example:**Exercise:**

Problem: Simplify the expression: $[1.67(8)](0.25)$.

Solution:

Solution

Notice that multiplying $(8)(0.25)$ is easier than multiplying $1.67(8)$ because it gives a whole number. (Think about having 8 quarters—that makes \$2.)

	$[1.67(8)](0.25)$
Regroup.	$1.67 [(8)(0.25)]$
Multiply in the brackets first.	$1.67 [2]$
Multiply.	3.34

Note:

Exercise:

Problem: Simplify: $[1.17(4)](2.25)$.

Solution:

10.53

Note:

Exercise:

Problem: Simplify: $[3.52(8)](2.5)$.

Solution:

70.4

When simplifying expressions that contain variables, we can use the commutative and associative properties to re-order or regroup terms, as shown in the next pair of examples.

Example:

Exercise:

Problem: Simplify: $6(9x)$.

Solution:

Solution

	$6(9x)$
Use the associative property of multiplication to regroup.	$(6 \cdot 9)x$
Multiply in the parentheses.	$54x$

Note:

Exercise:**Problem:** Simplify: $8(3y)$.**Solution:** $24y$ **Note:****Exercise:****Problem:** Simplify: $12(5z)$.**Solution:** $60z$

In [The Language of Algebra](#), we learned to combine like terms by rearranging an expression so the like terms were together. We simplified the expression $3x + 7 + 4x + 5$ by rewriting it as $3x + 4x + 7 + 5$ and then simplified it to $7x + 12$. We were using the Commutative Property of Addition.

Example:**Exercise:****Problem:** Simplify: $18p + 6q + (-15p) + 5q$.**Solution:****Solution**

Use the Commutative Property of Addition to re-order so that like terms are together.

	$18p + 6q + (-15p) + 5q$
Re-order terms.	$18p + (-15p) + 6q + 5q$
Combine like terms.	$3p + 11q$

Note:

Exercise:

Problem: Simplify: $23r + 14s + 9r + (-15s)$.

Solution:

$$32r - s$$

Note:

Exercise:

Problem: Simplify: $37m + 21n + 4m + (-15n)$.

Solution:

$$41m + 6n$$

Note: The *Links to Literacy* activity, "Each Orange Had 8 Slices" will provide you with another view of the topics covered in this section.

Key Concepts

- **Commutative Properties**

- **Commutative Property of Addition:**

- If a, b are real numbers, then $a + b = b + a$

- **Commutative Property of Multiplication:**

- If a, b are real numbers, then $a \cdot b = b \cdot a$

- **Associative Properties**

- **Associative Property of Addition:**

- If a, b, c are real numbers then $(a + b) + c = a + (b + c)$

- **Associative Property of Multiplication:**

- If a, b, c are real numbers then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Practice Makes Perfect

Use the Commutative and Associative Properties

In the following exercises, use the commutative properties to rewrite the given expression.

Exercise:

Problem: $8 + 9 = \underline{\hspace{1cm}}$

Exercise:

Problem: $7 + 6 = \underline{\hspace{1cm}}$

Solution:

$$7 + 6 = 6 + 7$$

Exercise:

Problem: $8(-12) = \underline{\hspace{1cm}}$

Exercise:

Problem: $7(-13) = \underline{\hspace{1cm}}$

Solution:

$$7(-13) = (-13)7$$

Exercise:

Problem: $(-19)(-14) = \underline{\hspace{1cm}}$

Exercise:

Problem: $(-12)(-18) = \underline{\hspace{1cm}}$

Solution:

$$(-12)(-18) = (-18)(-12)$$

Exercise:

Problem: $-11 + 8 = \underline{\hspace{1cm}}$

Exercise:

Problem: $-15 + 7 = \underline{\hspace{2cm}}$

Solution:

$$-15 + 7 = 7 + (-15)$$

Exercise:

Problem: $x + 4 = \underline{\hspace{2cm}}$

Exercise:

Problem: $y + 1 = \underline{\hspace{2cm}}$

Solution:

$$y + 1 = 1 + y$$

Exercise:

Problem: $-2a = \underline{\hspace{2cm}}$

Exercise:

Problem: $-3m = \underline{\hspace{2cm}}$

Solution:

$$-3m = m(-3)$$

In the following exercises, use the associative properties to rewrite the given expression.

Exercise:

Problem: $(11 + 9) + 14 = \underline{\hspace{2cm}}$

Exercise:

Problem: $(21 + 14) + 9 = \underline{\hspace{2cm}}$

Solution:

$$(21 + 14) + 9 = 21 + (14 + 9)$$

Exercise:

Problem: $(12 \cdot 5) \cdot 7 = \underline{\hspace{2cm}}$

Exercise:

Problem: $(14 \cdot 6) \cdot 9 = \underline{\hspace{2cm}}$

Solution:

$$(14 \cdot 6) \cdot 9 = 14(6 \cdot 9)$$

Exercise:

Problem: $(-7 + 9) + 8 = \underline{\hspace{2cm}}$

Exercise:

Problem: $(-2 + 6) + 7 = \underline{\hspace{2cm}}$

Solution:

$$(-2 + 6) + 7 = -2 + (6 + 7)$$

Exercise:

Problem: $(16 \cdot \frac{4}{5}) \cdot 15 = \underline{\hspace{2cm}}$

Exercise:

Problem: $(13 \cdot \frac{2}{3}) \cdot 18 = \underline{\hspace{2cm}}$

Solution:

$$(13 \cdot \frac{2}{3}) \cdot 18 = 13 (\frac{2}{3} \cdot 18)$$

Exercise:

Problem: $3(4x) = \underline{\hspace{2cm}}$

Exercise:

Problem: $4(7x) = \underline{\hspace{2cm}}$

Solution:

$$4(7x) = (4 \cdot 7)x$$

Exercise:

Problem: $(12 + x) + 28 = \underline{\hspace{2cm}}$

Exercise:

Problem: $(17 + y) + 33 = \underline{\hspace{2cm}}$

Solution:

$$(17 + y) + 33 = 17 + (y + 33)$$

Evaluate Expressions using the Commutative and Associative Properties

In the following exercises, evaluate each expression for the given value.

Exercise:

Problem: If $y = \frac{5}{8}$, evaluate:

Ⓐ $y + 0.49 + (-y)$

Ⓑ $y + (-y) + 0.49$

Exercise:

Problem: If $z = \frac{7}{8}$, evaluate:

Ⓐ $z + 0.97 + (-z)$

Ⓑ $z + (-z) + 0.97$

Solution:

Ⓐ 0.97

Ⓑ 0.97

Exercise:

Problem: If $c = -\frac{11}{4}$, evaluate:

Ⓐ $c + 3.125 + (-c)$

Ⓑ $c + (-c) + 3.125$

Exercise:

Problem: If $d = -\frac{9}{4}$, evaluate:

Ⓐ $d + 2.375 + (-d)$

Ⓑ $d + (-d) + 2.375$

Solution:

- Ⓐ 2.375
- Ⓑ 2.375

Exercise:

Problem: If $j = 11$, evaluate:

- Ⓐ $\frac{5}{6} \left(\frac{6}{5} j \right)$
- Ⓑ $\left(\frac{5}{6} \cdot \frac{6}{5} \right) j$

Exercise:

Problem: If $k = 21$, evaluate:

- Ⓐ $\frac{4}{13} \left(\frac{13}{4} k \right)$
- Ⓑ $\left(\frac{4}{13} \cdot \frac{13}{4} \right) k$

Solution:

- Ⓐ 21
- Ⓑ 21

Exercise:

Problem: If $m = -25$, evaluate:

- Ⓐ $-\frac{3}{7} \left(\frac{7}{3} m \right)$
- Ⓑ $\left(-\frac{3}{7} \cdot \frac{7}{3} \right) m$

Exercise:

Problem: If $n = -8$, evaluate:

- Ⓐ $-\frac{5}{21} \left(\frac{21}{5} n \right)$
Ⓑ $\left(-\frac{5}{21} \cdot \frac{21}{5} \right) n$

Solution:

- Ⓐ -8
Ⓑ -8

Simplify Expressions Using the Commutative and Associative Properties

In the following exercises, simplify.

Exercise:

Problem: $-45a + 15 + 45a$

Exercise:

Problem: $9y + 23 + (-9y)$

Solution:

23

Exercise:

Problem: $\frac{1}{2} + \frac{7}{8} + \left(-\frac{1}{2} \right)$

Exercise:

Problem: $\frac{2}{5} + \frac{5}{12} + \left(-\frac{2}{5} \right)$

Solution:

$\frac{5}{12}$

Exercise:

Problem: $\frac{3}{20} \cdot \frac{49}{11} \cdot \frac{20}{3}$

Exercise:

Problem: $\frac{13}{18} \cdot \frac{25}{7} \cdot \frac{18}{13}$

Solution:

$$\frac{25}{7}$$

Exercise:

Problem: $\frac{7}{12} \cdot \frac{9}{17} \cdot \frac{24}{7}$

Exercise:

Problem: $\frac{3}{10} \cdot \frac{13}{23} \cdot \frac{50}{3}$

Solution:

$$\frac{65}{23}$$

Exercise:

Problem: $-24 \cdot 7 \cdot \frac{3}{8}$

Exercise:

Problem: $-36 \cdot 11 \cdot \frac{4}{9}$

Solution:

$$-176$$

Exercise:

Problem: $\left(\frac{5}{6} + \frac{8}{15}\right) + \frac{7}{15}$

Exercise:

Problem: $\left(\frac{1}{12} + \frac{4}{9}\right) + \frac{5}{9}$

Solution:

$$\frac{13}{12}$$

Exercise:

Problem: $\frac{5}{13} + \frac{3}{4} + \frac{1}{4}$

Exercise:

Problem: $\frac{8}{15} + \frac{5}{7} + \frac{2}{7}$

Solution:

$$\frac{23}{15}$$

Exercise:

Problem: $(4.33p + 1.09p) + 3.91p$

Exercise:

Problem: $(5.89d + 2.75d) + 1.25d$

Solution:

$$9.89d$$

Exercise:

Problem: $17(0.25)(4)$

Exercise:

Problem: $36(0.2)(5)$

Solution:

36

Exercise:

Problem: $[2.48(12)](0.5)$

Exercise:

Problem: $[9.731(4)](0.75)$

Solution:

29.193

Exercise:

Problem: $7(4a)$

Exercise:

Problem: $9(8w)$

Solution:

$72w$

Exercise:

Problem: $-15(5m)$

Exercise:

Problem: $-23(2n)$

Solution:

$$-46n$$

Exercise:

Problem: $12\left(\frac{5}{6}p\right)$

Exercise:

Problem: $20\left(\frac{3}{5}q\right)$

Solution:

$$12q$$

Exercise:

Problem: $14x + 19y + 25x + 3y$

Exercise:

Problem: $15u + 11v + 27u + 19v$

Solution:

$$42u + 30v$$

Exercise:

Problem: $43m + (-12n) + (-16m) + (-9n)$

Exercise:

Problem: $-22p + 17q + (-35p) + (-27q)$

Solution:

$$-57p + (-10q)$$

Exercise:

Problem: $\frac{3}{8}g + \frac{1}{12}h + \frac{7}{8}g + \frac{5}{12}h$

Exercise:

Problem: $\frac{5}{6}a + \frac{3}{10}b + \frac{1}{6}a + \frac{9}{10}b$

Solution:

$$a + \frac{6}{5}b$$

Exercise:

Problem: $6.8p + 9.14q + (-4.37p) + (-0.88q)$

Exercise:

Problem: $9.6m + 7.22n + (-2.19m) + (-0.65n)$

Solution:

$$7.41m + 6.57n$$

Everyday Math

Exercise:

Problem:

Stamps Allie and Loren need to buy stamps. Allie needs four \$0.49 stamps and nine \$0.02 stamps. Loren needs eight \$0.49 stamps and three \$0.02 stamps.

- Ⓐ How much will Allie's stamps cost?
- Ⓑ How much will Loren's stamps cost?
- Ⓒ What is the total cost of the girls' stamps?
- Ⓓ How many \$0.49 stamps do the girls need altogether? How much will they cost?
- Ⓔ How many \$0.02 stamps do the girls need altogether? How much will they cost?

Exercise:**Problem:**

Counting Cash Grant is totaling up the cash from a fundraising dinner. In one envelope, he has twenty-three \$5 bills, eighteen \$10 bills, and thirty-four \$20 bills. In another envelope, he has fourteen \$5 bills, nine \$10 bills, and twenty-seven \$20 bills.

- Ⓐ How much money is in the first envelope?
- Ⓑ How much money is in the second envelope?
- Ⓒ What is the total value of all the cash?
- Ⓓ What is the value of all the \$5 bills?
- Ⓔ What is the value of all \$10 bills?
- Ⓕ What is the value of all \$20 bills?

Solution:

- Ⓐ \$975
- Ⓑ \$700
- Ⓒ \$1675
- Ⓓ \$185
- Ⓔ \$270
- Ⓕ \$1220

Writing Exercises**Exercise:****Problem:**

In your own words, state the Commutative Property of Addition and explain why it is useful.

Exercise:**Problem:**

In your own words, state the Associative Property of Multiplication and explain why it is useful.

Solution:

Answers will vary.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use the Commutative and Associative Properties.			
evaluate expressions using the Commutative and Associative Properties.			
simplify expressions using the Commutative and Associative Properties.			

⑥ After reviewing this checklist, what will you do to become confident for all objectives?

Distributive Property

By the end of this section, you will be able to:

- Simplify expressions using the distributive property
- Evaluate expressions using the distributive property

Note:

Before you get started, take this readiness quiz.

1. Multiply: $3(0.25)$.

If you missed this problem, review [\[link\]](#)

2. Simplify: $10 - (-2)(3)$.

If you missed this problem, review [\[link\]](#)

3. Combine like terms: $9y + 17 + 3y - 2$.

If you missed this problem, review [\[link\]](#).

Simplify Expressions Using the Distributive Property

Suppose three friends are going to the movies. They each need \$9.25; that is, 9 dollars and 1 quarter. How much money do they need all together? You can think about the dollars separately from the quarters.

$$3 \times 9 = 27$$



$$3 \times \$0.25 = \$0.75$$



They need 3 times \$9, so \$27, and 3 times 1 quarter, so 75 cents. In total, they need \$27.75.

If you think about doing the math in this way, you are using the Distributive Property.

Note:

Distributive Property

If a, b, c are real numbers, then

Equation:

$$a(b + c) = ab + ac$$

Back to our friends at the movies, we could show the math steps we take to find the total amount of money they need like this:

Equation:

$$\begin{array}{rcl} & 3(9.25) & \\ 3(9) & + & 0.25) \\ 3(9) & + & 3(0.25) \\ 27 & + & 0.75 \\ & 27.75 & \end{array}$$

In algebra, we use the Distributive Property to remove parentheses as we simplify expressions. For example, if we are asked to simplify the expression $3(x + 4)$, the order of operations says to work in the parentheses first. But we cannot add x and 4, since they are not like terms. So we use the Distributive Property, as shown in [\[link\]](#).

Example:**Exercise:**

Problem: Simplify: $3(x + 4)$.

Solution:**Solution**

	$3(x + 4)$
Distribute.	$3 \cdot x + 3 \cdot 4$
Multiply.	$3x + 12$

Note:

Exercise:

Problem: Simplify: $4(x + 2)$.

Solution:

$$4x + 8$$

Note:

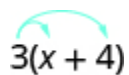
Exercise:

Problem: Simplify: $6(x + 7)$.

Solution:

$$6x + 42$$

Some students find it helpful to draw in arrows to remind them how to use the Distributive Property. Then the first step in [\[link\]](#) would look like this:


$$3(x + 4)$$

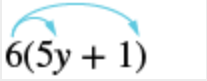
$$3 \cdot x + 3 \cdot 4$$

Example:

Exercise:

Problem: Simplify: $6(5y + 1)$.

Solution:
Solution

	 $6(5y + 1)$
Distribute.	$6 \cdot 5y + 6 \cdot 1$
Multiply.	$30y + 6$

Note:
Exercise:

Problem: Simplify: $9(3y + 8)$.

Solution:

$$27y + 72$$

Note:

Exercise:

Problem: Simplify: $5(5w + 9)$.

Solution:

$$25w + 45$$

The distributive property can be used to simplify expressions that look slightly different from $a(b + c)$. Here are two other forms.

Note:

Distributive Property

If a, b, c are real numbers, then

Equation:

$$a(b + c) = ab + ac$$

Other forms

Equation:

$$a(b - c) = ab - ac$$

Equation:

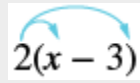
$$(b + c)a = ba + ca$$

Example:

Exercise:

Problem: Simplify: $2(x - 3)$.

Solution:
Solution


$$2(x - 3)$$

Distribute.

$$2 \cdot x - 2 \cdot 3$$

Multiply.

$$2x - 6$$

Note:

Exercise:

Problem: Simplify: $7(x - 6)$.

Solution:

$$7x - 42$$

Note:

Exercise:

Problem: Simplify: $8(x - 5)$.

Solution:

$$8x - 40$$

Do you remember how to multiply a fraction by a whole number? We'll need to do that in the next two examples.

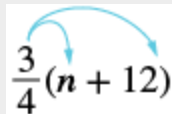
Example:

Exercise:

Problem: Simplify: $\frac{3}{4}(n + 12)$.

Solution:

Solution


$$\frac{3}{4}(n + 12)$$

Distribute.

$$\frac{3}{4} \cdot n + \frac{3}{4} \cdot 12$$

Simplify.

$$\frac{3}{4}n + 9$$

Note:

Exercise:

Problem: Simplify: $\frac{2}{5}(p + 10)$.

Solution:

$$\frac{2}{5}p + 4$$

Note:

Exercise:

Problem: Simplify: $\frac{3}{7}(u + 21)$.

Solution:

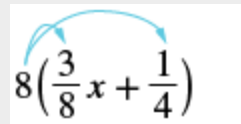
$$\frac{3}{7}u + 9$$

Example:

Exercise:

Problem: Simplify: $8\left(\frac{3}{8}x + \frac{1}{4}\right)$.

Solution:
Solution


$$8\left(\frac{3}{8}x + \frac{1}{4}\right)$$

Distribute.

$$8 \cdot \frac{3}{8}x + 8 \cdot \frac{1}{4}$$

Multiply.

$$3x + 2$$

Note:
Exercise:

Problem: Simplify: $6\left(\frac{5}{6}y + \frac{1}{2}\right)$.

Solution:

$$5y + 3$$

Note:

Exercise:

Problem: Simplify: $12\left(\frac{1}{3}n + \frac{3}{4}\right)$.

Solution:

$$4n + 9$$

Using the Distributive Property as shown in the next example will be very useful when we solve money applications later.

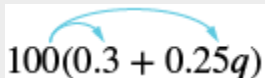
Example:

Exercise:

Problem: Simplify: $100(0.3 + 0.25q)$.

Solution:

Solution


$$100(0.3 + 0.25q)$$

Distribute.

$$100(0.3) + 100(0.25q)$$

Multiply.

$$30 + 25q$$

Note:

Exercise:

Problem: Simplify: $100(0.7 + 0.15p)$.

Solution:

$$70 + 15p$$

Note:

Exercise:

Problem: Simplify: $100(0.04 + 0.35d)$.

Solution:

$$4 + 35d$$

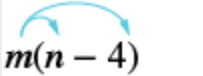
In the next example we'll multiply by a variable. We'll need to do this in a later chapter.

Example:

Exercise:

Problem: Simplify: $m(n - 4)$.

Solution:
Solution

	 $m(n - 4)$
Distribute.	$m \cdot n - m \cdot 4$
Multiply.	$mn - 4m$

Notice that we wrote $m \cdot 4$ as $4m$. We can do this because of the Commutative Property of Multiplication. When a term is the product of a number and a variable, we write the number first.

Note:
Exercise:

Problem: Simplify: $r(s - 2)$.

Solution:

$$rs - 2r$$

Note:

Exercise:

Problem: Simplify: $y(z - 8)$.

Solution:

$$yz - 8y$$

The next example will use the ‘backwards’ form of the Distributive Property, $(b + c)a = ba + ca$.

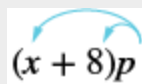
Example:

Exercise:

Problem: Simplify: $(x + 8)p$.

Solution:

Solution


$$(x + 8)p$$

Distribute.

$$px + 8p$$

Note:

Exercise:

Problem: Simplify: $(x + 2)p$.

Solution:

$$xp + 2p$$

Note:

Exercise:

Problem: Simplify: $(y + 4)q$.

Solution:

$$yq + 4q$$

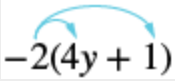
When you distribute a negative number, you need to be extra careful to get the signs correct.

Example:

Exercise:

Problem: Simplify: $-2(4y + 1)$.

Solution:
Solution

	 $-2(4y + 1)$
Distribute.	$-2 \cdot 4y + (-2) \cdot 1$
Simplify.	$-8y - 2$

Note:
Exercise:

Problem: Simplify: $-3(6m + 5)$.

Solution:

$$-18m - 15$$

Note:

Exercise:

Problem: Simplify: $-6(8n + 11)$.

Solution:

$$-48n - 66$$

Example:

Exercise:

Problem: Simplify: $-11(4 - 3a)$.

Solution:

Solution

	$-11(4 - 3a)$
Distribute.	$-11 \cdot 4 - (-11) \cdot 3a$
Multiply.	$-44 + (-33a)$
Simplify.	$-44 + 33a$

You could also write the result as $33a - 44$. Do you know why?

Note:

Exercise:

Problem: Simplify: $-5(2 - 3a)$.

Solution:

$$-10 + 15a$$

Note:

Exercise:

Problem: Simplify: $-7(8 - 15y)$.

Solution:

$$-56 + 105y$$

In the next example, we will show how to use the Distributive Property to find the opposite of an expression. Remember, $-a = -1 \cdot a$.

Example:

Exercise:

Problem: Simplify: $-(y + 5)$.

Solution:
Solution

	$-(y + 5)$
Multiplying by -1 results in the opposite.	$-1(y + 5)$
Distribute.	$-1 \cdot y + (-1) \cdot 5$
Simplify.	$-y + (-5)$
Simplify.	$-y - 5$

Note:
Exercise:

Problem: Simplify: $-(z - 11)$.

Solution:

$$-z + 11$$

Note:

Exercise:

Problem: Simplify: $-(x - 4)$.

Solution:

$$-x + 4$$

Sometimes we need to use the Distributive Property as part of the order of operations. Start by looking at the parentheses. If the expression inside the parentheses cannot be simplified, the next step would be multiply using the distributive property, which removes the parentheses. The next two examples will illustrate this.

Example:

Exercise:

Problem: Simplify: $8 - 2(x + 3)$.

Solution:
Solution

	$8 - 2(x + 3)$
Distribute.	$8 - 2 \cdot x - 2 \cdot 3$
Multiply.	$8 - 2x - 6$
Combine like terms.	$-2x + 2$

Note:

Exercise:

Problem: Simplify: $9 - 3(x + 2)$.

Solution:

$$-3x + 3$$

Note:

Exercise:

Problem: Simplify: $7x - 5(x + 4)$.

Solution:

$$2x - 20$$

Example:

Exercise:

Problem: Simplify: $4(x - 8) - (x + 3)$.

Solution:

Solution

	$4(x - 8) - (x + 3)$
Distribute.	$4x - 32 - x - 3$
Combine like terms.	$3x - 35$

Note:

Exercise:

Problem: Simplify: $6(x - 9) - (x + 12)$.

Solution:

$$5x - 66$$

Note:

Exercise:

Problem: Simplify: $8(x - 1) - (x + 5)$.

Solution:

$$7x - 13$$

Evaluate Expressions Using the Distributive Property

Some students need to be convinced that the Distributive Property always works.

In the examples below, we will practice evaluating some of the expressions from previous examples; in part (a), we will evaluate the form with parentheses, and in part (b) we will evaluate the form we got after distributing. If we evaluate both expressions correctly, this will show that they are indeed equal.

Example:

Exercise:

Problem: When $y = 10$ evaluate: (a) $6(5y + 1)$ (b) $6 \cdot 5y + 6 \cdot 1$.

Solution:

Solution

Ⓐ

$$6(5y + 1)$$

Substitute 10 for y .

$$6(5 \cdot 10 + 1)$$

Simplify in the parentheses.

$$6(51)$$

Multiply.

$$306$$

Ⓑ

$$6 \cdot 5y + 6 \cdot 1$$

Substitute 10 for y .

$$6 \cdot 5 \cdot 10 + 6 \cdot 1$$

Simplify.

$$300 + 6$$

Add.

306

Notice, the answers are the same. When $y = 10$,

Equation:

$$6(5y + 1) = 6 \cdot 5y + 6 \cdot 1.$$

Try it yourself for a different value of y .

Note:

Exercise:

Problem: Evaluate when $w = 3$: Ⓐ $5(5w + 9)$ Ⓑ $5 \cdot 5w + 5 \cdot 9$.

Solution:

Ⓐ 120

Ⓑ 120

Note:

Exercise:

Problem: Evaluate when $y = 2$: Ⓐ $9(3y + 8)$ Ⓑ $9 \cdot 3y + 9 \cdot 8$.

Solution:

Ⓐ 126

Ⓑ 126

Example:

Exercise:

Problem:

When $y = 3$, evaluate ① $-2(4y + 1)$ ② $-2 \cdot 4y + (-2) \cdot 1$.

Solution:

Solution

①	
	$-2(4y + 1)$
Substitute 3 for y .	$-2(4 \cdot 3 + 1)$
Simplify in the parentheses.	$-2(13)$
Multiply.	-26

②	
	$-2 \cdot 4y + (-2) \cdot 1$
Substitute 3 for y .	$-2 \cdot 4 \cdot 3 + (-2) \cdot 1$

Multiply.	$-24 - 2$
Subtract.	-26
The answers are the same. When $y = 3$,	$-2(4y + 1) = -8y - 2$

Note:

Exercise:

Problem:

Evaluate when $n = -2$: Ⓐ $-6(8n + 11)$ Ⓑ $-6 \cdot 8n + (-6) \cdot 11$.

Solution:

Ⓐ 30

Ⓑ 30

Note:

Exercise:

Problem:

Evaluate when $m = -1$: Ⓐ $-3(6m + 5)$ Ⓑ $-3 \cdot 6m + (-3) \cdot 5$.

Solution:

Ⓐ 3

Ⓑ 3

Example:

Exercise:

Problem:

When $y = 35$ evaluate ① $-(y + 5)$ and ② $-y - 5$ to show that $-(y + 5) = -y - 5$.

Solution:

Solution

①	
	$-(y + 5)$
Substitute 35 for y .	$-(35 + 5)$
Add in the parentheses.	$-(40)$
Simplify.	-40

②	

	$-y - 5$
Substitute 35 for y .	$-35 - 5$
Simplify.	-40
The answers are the same when $y = 35$, demonstrating that	$-(y + 5) = -y - 5$

Note:

Exercise:

Problem:

Evaluate when $x = 36$: (a) $-(x - 4)$ (b) $-x + 4$ to show that $-(x - 4) = -x + 4$.

Solution:

(a) -32

(b) -32

Note:

Exercise:

Problem:

Evaluate when $z = 55$: (a) $-(z - 10)$ (b) $-z + 10$ to show that $-(z - 10) = -z + 10$.

Solution:

Ⓐ -45

Ⓑ -45

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Model Distribution](#)
- [The Distributive Property](#)

Key Concepts

- **Distributive Property:**
 - If a, b, c are real numbers then
 - $a(b + c) = ab + ac$
 - $(b + c)a = ba + ca$
 - $a(b \cdot c) = ab \cdot ac$

Practice Makes Perfect

Simplify Expressions Using the Distributive Property

In the following exercises, simplify using the distributive property.

Exercise:

Problem: $4(x + 8)$

Exercise:

Problem: $3(a + 9)$

Solution:

$$3a + 27$$

Exercise:

Problem: $8(4y + 9)$

Exercise:

Problem: $9(3w + 7)$

Solution:

$$27w + 63$$

Exercise:

Problem: $6(c - 13)$

Exercise:

Problem: $7(y - 13)$

Solution:

$$7y - 91$$

Exercise:

Problem: $7(3p - 8)$

Exercise:

Problem: $5(7u - 4)$

Solution:

$$35u - 20$$

Exercise:

Problem: $\frac{1}{2}(n + 8)$

Exercise:

Problem: $\frac{1}{3}(u + 9)$

Solution:

$$\frac{1}{3}u + 3$$

Exercise:

Problem: $\frac{1}{4}(3q + 12)$

Exercise:

Problem: $\frac{1}{5}(4m + 20)$

Solution:

$$\frac{4}{5}m + 4$$

Exercise:

Problem: $9\left(\frac{5}{9}y - \frac{1}{3}\right)$

Exercise:

Problem: $10 \left(\frac{3}{10}x - \frac{2}{5} \right)$

Solution:

$$3x - 4$$

Exercise:

Problem: $12 \left(\frac{1}{4} + \frac{2}{3}r \right)$

Exercise:

Problem: $12 \left(\frac{1}{6} + \frac{3}{4}s \right)$

Solution:

$$2 + 9s$$

Exercise:

Problem: $r(s - 18)$

Exercise:

Problem: $u(v - 10)$

Solution:

$$uv - 10u$$

Exercise:

Problem: $(y + 4)p$

Exercise:

Problem: $(a + 7)x$

Solution:

$$ax + 7x$$

Exercise:

Problem: $-2(y + 13)$

Exercise:

Problem: $-3(a + 11)$

Solution:

$$-3a - 33$$

Exercise:

Problem: $-7(4p + 1)$

Exercise:

Problem: $-9(9a + 4)$

Solution:

$$-81a - 36$$

Exercise:

Problem: $-3(x - 6)$

Exercise:

Problem: $-4(q - 7)$

Solution:

$$-4q + 28$$

Exercise:

Problem: $-9(3a - 7)$

Exercise:

Problem: $-6(7x - 8)$

Solution:

$$-42x + 48$$

Exercise:

Problem: $-(r + 7)$

Exercise:

Problem: $-(q + 11)$

Solution:

$$-q - 11$$

Exercise:

Problem: $-(3x - 7)$

Exercise:

Problem: $-(5p - 4)$

Solution:

$$-5p + 4$$

Exercise:

Problem: $5 + 9(n - 6)$

Exercise:

Problem: $12 + 8(u - 1)$

Solution:

$$8u + 4$$

Exercise:

Problem: $16 - 3(y + 8)$

Exercise:

Problem: $18 - 4(x + 2)$

Solution:

$$-4x + 10$$

Exercise:

Problem: $4 - 11(3c - 2)$

Exercise:

Problem: $9 - 6(7n - 5)$

Solution:

$$-42n + 39$$

Exercise:

Problem: $22 - (a + 3)$

Exercise:

Problem: $8 - (r - 7)$

Solution:

$$-r + 15$$

Exercise:

Problem: $-12 - (u + 10)$

Exercise:

Problem: $-4 - (c - 10)$

Solution:

$$-c + 6$$

Exercise:

Problem: $(5m - 3) - (m + 7)$

Exercise:

Problem: $(4y - 1) - (y - 2)$

Solution:

$$3y + 1$$

Exercise:

Problem: $5(2n + 9) + 12(n - 3)$

Exercise:

Problem: $9(5u + 8) + 2(u - 6)$

Solution:

$$47u + 60$$

Exercise:

Problem: $9(8x - 3) - (-2)$

Exercise:

Problem: $4(6x - 1) - (-8)$

Solution:

$$24x + 4$$

Exercise:

Problem: $14(c - 1) - 8(c - 6)$

Exercise:

Problem: $11(n - 7) - 5(n - 1)$

Solution:

$$6n - 72$$

Exercise:

Problem: $6(7y + 8) - (30y - 15)$

Exercise:

Problem: $7(3n + 9) - (4n - 13)$

Solution:

$$17n + 76$$

Evaluate Expressions Using the Distributive Property

In the following exercises, evaluate both expressions for the given value.

Exercise:

Problem: If $v = -2$, evaluate

Ⓐ $6(4v + 7)$

Ⓑ $6 \cdot 4v + 6 \cdot 7$

Exercise:

Problem: If $u = -1$, evaluate

Ⓐ $8(5u + 12)$

Ⓑ $8 \cdot 5u + 8 \cdot 12$

Solution:

Ⓐ 56

Ⓑ 56

Exercise:

Problem: If $n = \frac{2}{3}$, evaluate

Ⓐ $3\left(n + \frac{5}{6}\right)$

Ⓑ $3 \cdot n + 3 \cdot \frac{5}{6}$

Exercise:

Problem: If $y = \frac{3}{4}$, evaluate

Ⓐ $4\left(y + \frac{3}{8}\right)$

Ⓑ $4 \cdot y + 4 \cdot \frac{3}{8}$

Solution:

Ⓐ $\frac{9}{2}$

Ⓑ $\frac{9}{2}$

Exercise:

Problem: If $y = \frac{7}{12}$, evaluate

Ⓐ $-3(4y + 15)$

Ⓑ $3 \cdot 4y + (-3) \cdot 15$

Exercise:

Problem: If $p = \frac{23}{30}$, evaluate

Ⓐ $-6(5p + 11)$

Ⓑ $-6 \cdot 5p + (-6) \cdot 11$

Solution:

Ⓐ -89

Ⓑ -89

Exercise:

Problem: If $m = 0.4$, evaluate

Ⓐ $-10(3m - 0.9)$

Ⓑ $-10 \cdot 3m - (-10)(0.9)$

Exercise:

Problem: If $n = 0.75$, evaluate

Ⓐ $-100(5n + 1.5)$

Ⓑ $-100 \cdot 5n + (-100)(1.5)$

Solution:

Ⓐ -525

Ⓑ -525

Exercise:

Problem: If $y = -25$, evaluate

Ⓐ $-(y - 25)$

Ⓑ $-y + 25$

Exercise:

Problem: If $w = -80$, evaluate

Ⓐ $-(w - 80)$

Ⓑ $-w + 80$

Solution:

- Ⓐ 160
- Ⓑ 160

Exercise:

Problem: If $p = 0.19$, evaluate

- Ⓐ $-(p + 0.72)$
- Ⓑ $-p - 0.72$

Exercise:

Problem: If $q = 0.55$, evaluate

- Ⓐ $-(q + 0.48)$
- Ⓑ $-q - 0.48$

Solution:

- Ⓐ -1.03
- Ⓑ -1.03

Everyday Math

Exercise:

Problem:

Buying by the case Joe can buy his favorite ice tea at a convenience store for \$1.99 per bottle. At the grocery store, he can buy a case of 12 bottles for \$23.88.

Ⓐ Use the distributive property to find the cost of 12 bottles bought individually at the convenience store. (Hint: notice that \$1.99 is $\$2 - \$0.01$.)

Ⓑ Is it a bargain to buy the iced tea at the grocery store by the case?

Exercise:

Problem:

Multi-pack purchase Adele's shampoo sells for \$3.97 per bottle at the drug store. At the warehouse store, the same shampoo is sold as a 3-pack for \$10.49.

Ⓐ Show how you can use the distributive property to find the cost of 3 bottles bought individually at the drug store.

Ⓑ How much would Adele save by buying the 3-pack at the warehouse store?

Solution:

Ⓐ $3(4 - 0.03) = 11.91$

Ⓑ \$1.42

Writing Exercises

Exercise:

Problem:

Simplify $8\left(x - \frac{1}{4}\right)$ using the distributive property and explain each step.

Exercise:

Problem:

Explain how you can multiply $4(\$5.97)$ without paper or a calculator by thinking of $\$5.97$ as $6 - 0.03$ and then using the distributive property.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
simplify expressions using the Distributive Property.			
evaluate expressions using the Distributive Property.			

Ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Properties of Identity, Inverses, and Zero

By the end of this section, you will be able to:

- Recognize the identity properties of addition and multiplication
- Use the inverse properties of addition and multiplication
- Use the properties of zero
- Simplify expressions using the properties of identities, inverses, and zero

Note:

Before you get started, take this readiness quiz.

1. Find the opposite of -4 .

If you missed this problem, review [\[link\]](#).

2. Find the reciprocal of $\frac{5}{2}$.

If you missed this problem, review [\[link\]](#).

3. Multiply: $\frac{3a}{5} \cdot \frac{9}{2a}$.

If you missed this problem, review [\[link\]](#).

Recognize the Identity Properties of Addition and Multiplication

What happens when we add zero to any number? Adding zero doesn't change the value. For this reason, we call 0 the **additive identity**.

For example,

Equation:

$$\begin{array}{ccc} 13 + 0 & -14 + 0 & 0 + (-3x) \\ 13 & -14 & -3x \end{array}$$

What happens when you multiply any number by one? Multiplying by one doesn't change the value. So we call 1 the **multiplicative identity**.

For example,

Equation:

$$\begin{array}{ccc} 43 \cdot 1 & -27 \cdot 1 & 1 \cdot \frac{6y}{5} \\ 43 & -27 & \frac{6y}{5} \end{array}$$

Note:

Identity Properties

The **identity property of addition**: for any real number a ,
Equation:

$$a + 0 = a \qquad 0 + a = a$$

0 is called the **additive identity**

The **identity property of multiplication**: for any real number a
Equation:

$$a \cdot 1 = a \qquad 1 \cdot a = a$$

1 is called the **multiplicative identity**

Example:
Exercise:

Problem:

Identify whether each equation demonstrates the identity property of addition or multiplication.

- Ⓐ $7 + 0 = 7$
- Ⓑ $-16(1) = -16$

Solution:
Solution

Ⓐ	
	$7 + 0 = 7$
We are adding 0.	We are using the identity property of addition.

Ⓑ	
	$-16(1) = -16$

We are multiplying by 1.

We are using the identity property of multiplication.

Note:

Exercise:

Problem:

Identify whether each equation demonstrates the identity property of addition or multiplication:

Ⓐ $23 + 0 = 23$ Ⓑ $-37(1) = -37$.

Solution:

- Ⓐ identity property of addition
- Ⓑ identity property of multiplication

Note:

Exercise:

Problem:

Identify whether each equation demonstrates the identity property of addition or multiplication:

Ⓐ $1 \cdot 29 = 29$ Ⓑ $14 + 0 = 14$.

Solution:

- Ⓐ identity property of multiplication
- Ⓑ additive identity

Use the Inverse Properties of Addition and Multiplication

What number added to 5 gives the additive identity, 0?

$5 + \underline{\hspace{1cm}} = 0$	We know $5 + (-5) = 0$
What number added to -6 gives the additive identity, 0 ?	
$-6 + \underline{\hspace{1cm}} = 0$	We know $-6 + 6 = 0$

Notice that in each case, the missing number was the opposite of the number.

We call $-a$ the **additive inverse** of a . The opposite of a number is its additive inverse. A number and its opposite add to 0 , which is the additive identity.

What number multiplied by $\frac{2}{3}$ gives the multiplicative identity, 1 ? In other words, two-thirds times what results in 1 ?

$\frac{2}{3} \cdot \underline{\hspace{1cm}} = 1$	We know $\frac{2}{3} \cdot \frac{3}{2} = 1$
--	---

What number multiplied by 2 gives the multiplicative identity, 1 ? In other words two times what results in 1 ?

$2 \cdot \underline{\hspace{1cm}} = 1$	We know $2 \cdot \frac{1}{2} = 1$
--	-----------------------------------

Notice that in each case, the missing number was the reciprocal of the number.

We call $\frac{1}{a}$ the **multiplicative inverse** of a ($a \neq 0$). The reciprocal of a number is its multiplicative inverse. A number and its reciprocal multiply to 1 , which is the multiplicative identity.

We'll formally state the Inverse Properties here:

Note:

Inverse Properties

Inverse Property of Addition for any real number a ,**Equation:**

$$a + (-a) = 0$$

$-a$ is the **additive inverse** of a .

Inverse Property of Multiplication for any real number $a \neq 0$,**Equation:**

$$a \cdot \frac{1}{a} = 1$$

$\frac{1}{a}$ is the **multiplicative inverse** of a .

Example:**Exercise:****Problem:** Find the additive inverse of each expression: (a) 13 (b) $-\frac{5}{8}$ (c) 0.6.**Solution:****Solution**

To find the additive inverse, we find the opposite.

- (a) The additive inverse of 13 is its opposite, -13 .
- (b) The additive inverse of $-\frac{5}{8}$ is its opposite, $\frac{5}{8}$.
- (c) The additive inverse of 0.6 is its opposite, -0.6 .

Note:**Exercise:****Problem:** Find the additive inverse: (a) 18 (b) $\frac{7}{9}$ (c) 1.2.**Solution:**

- (a) -18
- (b) $-\frac{7}{9}$
- (c) -1.2

Note:

Exercise:

Problem: Find the additive inverse: Ⓐ 47 Ⓑ $\frac{7}{13}$ Ⓒ 8.4.

Solution:

- Ⓐ -47
- Ⓑ $-\frac{7}{13}$
- Ⓒ -8.4

Example:

Exercise:

Problem: Find the multiplicative inverse: Ⓐ 9 Ⓑ $-\frac{1}{9}$ Ⓒ 0.9.

Solution:

Solution

To find the multiplicative inverse, we find the reciprocal.

- Ⓐ The multiplicative inverse of 9 is its reciprocal, $\frac{1}{9}$.
- Ⓑ The multiplicative inverse of $-\frac{1}{9}$ is its reciprocal, -9 .
- Ⓒ To find the multiplicative inverse of 0.9, we first convert 0.9 to a fraction, $\frac{9}{10}$. Then we find the reciprocal, $\frac{10}{9}$.

Note:

Exercise:

Problem: Find the multiplicative inverse: Ⓐ 5 Ⓑ $-\frac{1}{7}$ Ⓒ 0.3.

Solution:

- Ⓐ $\frac{1}{5}$
- Ⓑ -7
- Ⓒ $\frac{10}{3}$

Note:

Exercise:

Problem: Find the multiplicative inverse: (a) 18 (b) $-\frac{4}{5}$ (c) 0.6.

Solution:

- (a) $\frac{1}{18}$
- (b) $-\frac{5}{4}$
- (c) $\frac{5}{3}$

Use the Properties of Zero

We have already learned that zero is the additive identity, since it can be added to any number without changing the number's identity. But zero also has some special properties when it comes to multiplication and division.

Multiplication by Zero

What happens when you multiply a number by 0? Multiplying by 0 makes the product equal zero. The product of any real number and 0 is 0.

Note:

Multiplication by Zero

For any real number a ,

Equation:

$$a \cdot 0 = 0 \quad 0 \cdot a = 0$$

Example:

Exercise:

Problem: Simplify: (a) $-8 \cdot 0$ (b) $\frac{5}{12} \cdot 0$ (c) $0(2.94)$.

Solution:
Solution

Ⓐ	
	$-8 \cdot 0$
The product of any real number and 0 is 0.	0

Ⓑ	
	$\frac{5}{12} \cdot 0$
The product of any real number and 0 is 0.	0

Ⓒ	
	$0(2.94)$
The product of any real number and 0 is 0.	0

Note:
Exercise:

Problem: Simplify: Ⓐ $-14 \cdot 0$ Ⓑ $0 \cdot \frac{2}{3}$ Ⓒ $(16.5) \cdot 0$.

Solution:

- Ⓐ 0
- Ⓑ 0
- Ⓒ 0

Note:

Exercise:

Problem: Simplify: Ⓐ $(1.95) \cdot 0$ Ⓑ $0(-17)$ Ⓒ $0 \cdot \frac{5}{4}$.

Solution:

- Ⓐ 0
- Ⓑ 0
- Ⓒ 0

Dividing with Zero

What about dividing with 0? Think about a real example: if there are no cookies in the cookie jar and three people want to share them, how many cookies would each person get? There are 0 cookies to share, so each person gets 0 cookies.

Equation:

$$0 \div 3 = 0$$

Remember that we can always check division with the related multiplication fact. So, we know that

Equation:

$$0 \div 3 = 0 \text{ because } 0 \cdot 3 = 0.$$

Note:

Division of Zero

For any real number a , except 0, $\frac{0}{a} = 0$ and $0 \div a = 0$.

Zero divided by any real number except zero is zero.

Example:

Exercise:

Problem: Simplify: (a) $0 \div 5$ (b) $\frac{0}{-2}$ (c) $0 \div \frac{7}{8}$.

Solution:
Solution

(a)	
	$0 \div 5$
Zero divided by any real number, except 0, is zero.	0

(b)	
	$\frac{0}{-2}$
Zero divided by any real number, except 0, is zero.	0

(c)	
	$0 \div \frac{7}{8}$
Zero divided by any real number, except 0, is zero.	0

Note:
Exercise:

Problem: Simplify: (a) $0 \div 11$ (b) $\frac{0}{-6}$ (c) $0 \div \frac{3}{10}$.

Solution:

- (a) 0
- (b) 0
- (c) 0

Note:

Exercise:

Problem: Simplify: (a) $0 \div \frac{8}{3}$ (b) $0 \div (-10)$ (c) $0 \div 12.75$.

Solution:

- (a) 0
- (b) 0
- (c) 0

Now let's think about dividing a number *by* zero. What is the result of dividing 4 by 0? Think about the related multiplication fact. Is there a number that multiplied by 0 gives 4?

Equation:

$$4 \div 0 = \underline{\hspace{1cm}} \text{ means } \underline{\hspace{1cm}} \cdot 0 = 4$$

Since any real number multiplied by 0 equals 0, there is no real number that can be multiplied by 0 to obtain 4. We can conclude that there is no answer to $4 \div 0$, and so we say that division by zero is undefined.

Note:

Division by Zero

For any real number a , $\frac{a}{0}$, and $a \div 0$ are undefined.

Division *by* zero is undefined.

Example:
Exercise:

Problem: Simplify: (a) $7.5 \div 0$ (b) $\frac{-32}{0}$ (c) $\frac{4}{9} \div 0$.

Solution:
Solution

(a)	
	$7.5 \div 0$
Division by zero is undefined.	undefined

(b)	
	$\frac{-32}{0}$
Division by zero is undefined.	undefined

(c)	
	$\frac{4}{9} \div 0$
Division by zero is undefined.	undefined

Note:
Exercise:

Problem: Simplify: (a) $16.4 \div 0$ (b) $\frac{-2}{0}$ (c) $\frac{1}{5} \div 0$.

Solution:

- (a) undefined
- (b) undefined
- (c) undefined

Note:

Exercise:

Problem: Simplify: (a) $\frac{-5}{0}$ (b) $96.9 \div 0$ (c) $\frac{4}{15} \div 0$

Solution:

- (a) undefined
- (b) undefined
- (c) undefined

We summarize the properties of zero.

Note:

Properties of Zero

Multiplication by Zero: For any real number a ,

$$a \cdot 0 = 0 \quad 0 \cdot a = 0 \quad \text{The product of any number and 0 is 0.}$$

Division by Zero: For any real number a , $a \neq 0$

$$\frac{0}{a} = 0 \quad \text{Zero divided by any real number, except itself, is zero.}$$

$\frac{a}{0}$ is undefined. Division by zero is undefined.

Simplify Expressions using the Properties of Identities, Inverses, and Zero

We will now practice using the properties of identities, inverses, and zero to simplify expressions.

Example:

Exercise:

Problem: Simplify: $3x + 15 - 3x$.

Solution:
Solution

	$3x + 15 - 3x$
Notice the additive inverses, $3x$ and $-3x$.	$0 + 15$
Add.	15

Note:
Exercise:

Problem: Simplify: $-12z + 9 + 12z$.

Solution:

9

Note:
Exercise:

Problem: Simplify: $-25u - 18 + 25u$.

Solution:

-18

Example:
Exercise:

Problem: Simplify: $4(0.25q)$.

Solution:
Solution

	$4(0.25q)$
Regroup, using the associative property.	$[4(0.25)]q$
Multiply.	$1.00q$
Simplify; 1 is the multiplicative identity.	q

Note:
Exercise:

Problem: Simplify: $2(0.5p)$.

Solution:

p

Note:
Exercise:

Problem: Simplify: $25(0.04r)$.

Solution:

r

Example:
Exercise:

Problem: Simplify: $\frac{0}{n+5}$, where $n \neq -5$.

Solution:
Solution

	$\frac{0}{n+5}$
Zero divided by any real number except itself is zero.	0

Note:
Exercise:

Problem: Simplify: $\frac{0}{m+7}$, where $m \neq -7$.

Solution:

0

Note:
Exercise:

Problem: Simplify: $\frac{0}{d-4}$, where $d \neq 4$.

Solution:

0

Example:
Exercise:

Problem: Simplify: $\frac{10-3p}{0}$.

Solution:
Solution

	$\frac{10-3p}{0}$
Division by zero is undefined.	undefined

Note:

Exercise:

Problem: Simplify: $\frac{18-6c}{0}$.

Solution:

undefined

Note:

Exercise:

Problem: Simplify: $\frac{15-4q}{0}$.

Solution:

undefined

Example:

Exercise:

Problem: Simplify: $\frac{3}{4} \cdot \frac{4}{3}(6x + 12)$.

Solution:

Solution

We cannot combine the terms in parentheses, so we multiply the two fractions first.

	$\frac{3}{4} \cdot \frac{4}{3}(6x + 12)$
--	--

Multiply; the product of reciprocals is 1.	$1(6x + 12)$
Simplify by recognizing the multiplicative identity.	$6x + 12$

Note:

Exercise:

Problem: Simplify: $\frac{2}{5} \cdot \frac{5}{2}(20y + 50)$.

Solution:

$$20y + 50$$

Note:

Exercise:

Problem: Simplify: $\frac{3}{8} \cdot \frac{8}{3}(12z + 16)$.

Solution:

$$12z + 16$$

All the properties of real numbers we have used in this chapter are summarized in [\[link\]](#).

Property	Of Addition	Of Multiplication
Commutative Property		
If a and b are real numbers then...	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative Property		

Property	Of Addition	Of Multiplication
If a , b , and c are real numbers then...	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity Property	0 is the additive identity	1 is the multiplicative identity
For any real number a ,	$a + 0 = a$ $0 + a = a$	$a \cdot 1 = a$ $1 \cdot a = a$
Inverse Property	$-a$ is the additive inverse of a	$a, a \neq 0$ $1a$ is the multiplicative inverse of a
For any real number a ,	$a + (-a) = 0$	$a \cdot 1a = 1$
Distributive Property If a, b, c are real numbers, then $a(b + c) = ab + ac$		
Properties of Zero		
For any real number a ,	$a \cdot 0 = 0$ $0 \cdot a = 0$	
For any real number $a, a \neq 0$	$\frac{0}{a} = 0$ $\frac{a}{0}$ is undefined	

Properties of Real Numbers

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Multiplying and Dividing Involving Zero](#)

Key Concepts

- **Identity Properties**

- **Identity Property of Addition:** For any real number a : $a + 0 = a$ $0 + a = a$ **0** is the **additive identity**
- **Identity Property of Multiplication:** For any real number a : $a \cdot 1 = a$ $1 \cdot a = a$ **1** is the **multiplicative identity**

- **Inverse Properties**

- **Inverse Property of Addition:** For any real number a : $a + (-a) = 0$ — $-a$ is the **additive inverse** of a
- **Inverse Property of Multiplication:** For any real number a :
($a \neq 0$) $a \cdot \frac{1}{a} = 1$ $\frac{1}{a}$ is the **multiplicative inverse** of a

- **Properties of Zero**

- **Multiplication by Zero:** For any real number a ,
 $a \cdot 0 = 0$ $0 \cdot a = 0$ The product of any number and 0 is 0.
- **Division of Zero:** For any real number a ,
 $\frac{0}{a} = 0$ $0 + a = 0$ Zero divided by any real number, except itself, is zero.
- **Division by Zero:** For any real number a , $\frac{0}{a}$ is undefined and $a \div 0$ is undefined.
Division by zero is undefined.

Practice Makes Perfect

Recognize the Identity Properties of Addition and Multiplication

In the following exercises, identify whether each example is using the identity property of addition or multiplication.

Exercise:

Problem: $101 + 0 = 101$

Exercise:

Problem: $\frac{3}{5}(1) = \frac{3}{5}$

Solution:

identity property of multiplication

Exercise:

Problem: $-9 \cdot 1 = -9$

Exercise:

Problem: $0 + 64 = 64$

Solution:

identity property of addition

Use the Inverse Properties of Addition and Multiplication

In the following exercises, find the multiplicative inverse.

Exercise:

Problem: 8

Exercise:

Problem: 14

Solution:

$$\frac{1}{14}$$

Exercise:

Problem: -17

Exercise:

Problem: -19

Solution:

$$-\frac{1}{19}$$

Exercise:

Problem: $\frac{7}{12}$

Exercise:

Problem: $\frac{8}{13}$

Solution:

$$\frac{13}{8}$$

Exercise:

Problem: $-\frac{3}{10}$

Exercise:

Problem: $-\frac{5}{12}$

Solution:

$$-\frac{12}{5}$$

Exercise:

Problem: 0.8

Exercise:

Problem: 0.4

Solution:

$$\frac{5}{2}$$

Exercise:

Problem: -0.2

Exercise:

Problem: -0.5

Solution:

$$-2$$

Use the Properties of Zero

In the following exercises, simplify using the properties of zero.

Exercise:

Problem: $48 \cdot 0$

Exercise:

Problem: $\frac{0}{6}$

Solution:

$$0$$

Exercise:

Problem: $\frac{3}{0}$

Exercise:

Problem: $22 \cdot 0$

Solution:

0

Exercise:

Problem: $0 \div \frac{11}{12}$

Exercise:

Problem: $\frac{6}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{0}{3}$

Exercise:

Problem: $0 \div \frac{7}{15}$

Solution:

0

Exercise:

Problem: $0 \cdot \frac{8}{15}$

Exercise:

Problem: $(-3.14)(0)$

Solution:

0

Exercise:

Problem: $5.72 \div 0$

Exercise:

Problem: $\frac{\frac{1}{10}}{0}$

Solution:

undefined

Simplify Expressions using the Properties of Identities, Inverses, and Zero

In the following exercises, simplify using the properties of identities, inverses, and zero.

Exercise:

Problem: $19a + 44 - 19a$

Exercise:

Problem: $27c + 16 - 27c$

Solution:

$$16$$

Exercise:

Problem: $38 + 11r - 38$

Exercise:

Problem: $92 + 31s - 92$

Solution:

$$31s$$

Exercise:

Problem: $10(0.1d)$

Exercise:

Problem: $100(0.01p)$

Solution:

$$p$$

Exercise:

Problem: $5(0.6q)$

Exercise:

Problem: $40(0.05n)$

Solution:

$$2n$$

Exercise:

Problem: $\frac{0}{r+20}$, where $r \neq -20$

Exercise:

Problem: $\frac{0}{s+13}$, where $s \neq -13$

Solution:

0

Exercise:

Problem: $\frac{0}{u-4.99}$, where $u \neq 4.99$

Exercise:

Problem: $\frac{0}{v-65.1}$, where $v \neq 65.1$

Solution:

0

Exercise:

Problem: $0 \div (x - \frac{1}{2})$, where $x \neq \frac{1}{2}$

Exercise:

Problem: $0 \div (y - \frac{1}{6})$, where $y \neq \frac{1}{6}$

Solution:

0

Exercise:

Problem: $\frac{32-5a}{0}$, where $32 - 5a \neq 0$

Exercise:

Problem: $\frac{28-9b}{0}$, where $28 - 9b \neq 0$

Solution:

undefined

Exercise:

Problem: $\frac{2.1+0.4c}{0}$, where $2.1 + 0.4c \neq 0$

Exercise:

Problem: $\frac{1.75+9f}{0}$, where $1.75 + 9f \neq 0$

Solution:

undefined

Exercise:

Problem: $\left(\frac{3}{4} + \frac{9}{10}m\right) \div 0$, where $\frac{3}{4} + \frac{9}{10}m \neq 0$

Exercise:

Problem: $\left(\frac{5}{16}n - \frac{3}{7}\right) \div 0$, where $\frac{5}{16}n - \frac{3}{7} \neq 0$

Solution:

undefined

Exercise:

Problem: $\frac{9}{10} \cdot \frac{10}{9}(18p - 21)$

Exercise:

Problem: $\frac{5}{7} \cdot \frac{7}{5}(20q - 35)$

Solution:

$20q - 35$

Exercise:

Problem: $15 \cdot \frac{3}{5}(4d + 10)$

Exercise:

Problem: $18 \cdot \frac{5}{6}(15h + 24)$

Solution:

$225h + 360$

Everyday Math

Exercise:

Problem:

Insurance copayment Carrie had to have 5 fillings done. Each filling cost \$80. Her dental insurance required her to pay 20% of the cost. Calculate Carrie's cost

- Ⓐ by finding her copay for each filling, then finding her total cost for 5 fillings, and
- Ⓑ by multiplying $5(0.20)(80)$.
- Ⓒ Which of the Properties of Real Numbers did you use for part (b)?

Exercise:

Problem:

Cooking time Helen bought a 24-pound turkey for her family's Thanksgiving dinner and wants to know what time to put the turkey in the oven. She wants to allow 20 minutes per pound cooking time.

- Ⓐ Calculate the length of time needed to roast the turkey by multiplying $24 \cdot 20$ to find the number of minutes and then multiplying the product by $\frac{1}{60}$ to convert minutes into hours.
- Ⓑ Multiply $24 \left(20 \cdot \frac{1}{60} \right)$.
- Ⓒ Which of the Properties of Real Numbers allows you to multiply $24 \left(20 \cdot \frac{1}{60} \right)$ instead of $(24 \cdot 20) \frac{1}{60}$?

Solution:

- Ⓐ 8 hours
- Ⓑ 8
- Ⓒ associative property of multiplication

Writing Exercises

Exercise:

Problem:

In your own words, describe the difference between the additive inverse and the multiplicative inverse of a number.

Exercise:

Problem:

How can the use of the properties of real numbers make it easier to simplify expressions?

Solution:

Answers will vary.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the Identity Properties of Addition and Multiplication.			
use the Inverse Properties of Addition and Multiplication.			
use the Properties of Zero.			
simplify expressions using the properties of identities, inverses, and zero.			

Ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary**Additive Identity**

The **additive identity** is 0. When zero is added to any number, it does not change the value.

Additive Inverse

The opposite of a number is its **additive inverse**. The **additive inverse** of a is $-a$.

Multiplicative Identity

The **multiplicative identity** is 1. When one multiplies any number, it does not change the value.

Multiplicative Inverse

The reciprocal of a number is its **multiplicative inverse**. The **multiplicative inverse** of a is $\frac{1}{a}$.

Introduction to Solving Linear Equations

class="introduction"

A
Calder
mobile
is
balance
d and
has
several
element
s on
each
side.
(credit:
paurian,
Flickr)



Teetering high above the floor, this amazing mobile remains aloft thanks to its carefully balanced mass. Any shift in either direction could cause the mobile to become lopsided, or even crash downward. In this chapter, we

will solve equations by keeping quantities on both sides of an equal sign in perfect balance.

Solve Equations Using the Subtraction and Addition Properties of Equality
By the end of this section, you will be able to:

- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that need to be simplified
- Translate an equation and solve
- Translate and solve applications

Note:

Before you get started, take this readiness quiz.

1. Solve: $n - 12 = 16$.
If you missed this problem, review [\[link\]](#).
2. Translate into algebra ‘five less than x .’
If you missed this problem, review [\[link\]](#).
3. Is $x = 2$ a solution to $5x - 3 = 7$?
If you missed this problem, review [\[link\]](#).

We are now ready to “get to the good stuff.” You have the basics down and are ready to begin one of the most important topics in algebra: solving equations. The applications are limitless and extend to all careers and fields. Also, the skills and techniques you learn here will help improve your critical thinking and problem-solving skills. This is a great benefit of studying mathematics and will be useful in your life in ways you may not see right now.

Solve Equations Using the Subtraction and Addition Properties of Equality

We began our work solving equations in previous chapters. It has been a while since we have seen an equation, so we will review some of the key

concepts before we go any further.

We said that solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle.

Note:

Solution of an Equation

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

In the earlier sections, we listed the steps to determine if a value is a solution. We restate them here.

Note:

Determine whether a number is a solution to an equation.

Substitute the number for the variable in the equation.

Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true.

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example:

Exercise:

Problem: Determine whether $y = \frac{3}{4}$ is a solution for $4y + 3 = 8y$.

Solution:
Solution

	$4y + 3 = 8y$
Substitute $\frac{3}{4}$ for y .	$4\left(\frac{3}{4}\right) + 3 \stackrel{?}{=} 8\left(\frac{3}{4}\right)$
Multiply.	$3 + 3 \stackrel{?}{=} 6$
Add.	$6 = 6 \checkmark$

Since $y = \frac{3}{4}$ results in a true equation, $\frac{3}{4}$ is a solution to the equation $4y + 3 = 8y$.

Note:
Exercise:

Problem: Is $y = \frac{2}{3}$ a solution for $9y + 2 = 6y$?

Solution:

no

Note:

Exercise:

Problem: Is $y = \frac{2}{5}$ a solution for $5y - 3 = 10y$?

Solution:

no

We introduced the Subtraction and Addition Properties of Equality in [Solving Equations Using the Subtraction and Addition Properties of Equality](#). In that section, we modeled how these properties work and then applied them to solving equations with whole numbers. We used these properties again each time we introduced a new system of numbers. Let's review those properties here.

Note:

Subtraction and Addition Properties of Equality

Subtraction Property of Equality

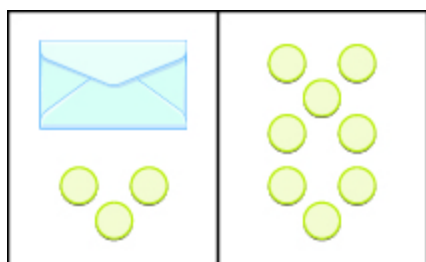
For all real numbers a , b , and c , if $a = b$, then $a - c = b - c$.

Addition Property of Equality

For all real numbers a , b , and c , if $a = b$, then $a + c = b + c$.

When you add or subtract the same quantity from both sides of an equation, you still have equality.

We introduced the Subtraction Property of Equality earlier by modeling equations with envelopes and counters. [\[link\]](#) models the equation $x + 3 = 8$.



The goal is to isolate the variable on one side of the equation. So we ‘took away’ 3 from both sides of the equation and found the solution $x = 5$.

Some people picture a balance scale, as in [\[link\]](#), when they solve equations.



1 mass on each side = balanced



2 masses on each side = balanced



1 mass on one side and 2 masses
on the other = unbalanced

The quantities on both sides of the equal sign in an equation are equal, or balanced. Just as with the balance scale, whatever you do to one side of the equation you must also do to the other to keep it balanced.

Let's review how to use Subtraction and Addition Properties of Equality to solve equations. We need to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Example:

Exercise:

Problem: Solve: $x + 11 = -3$.

Solution:
Solution

To isolate x , we undo the addition of 11 by using the Subtraction Property of Equality.

		$x - 11 = -3$
Subtract 11 from each side to "undo" the addition.		$x + 11 - 11 = -3 - 11$
Simplify.		$x = -14$
Check:	$x - 11 = -3$	
Substitute $x = -14$.	$-14 + 11 \stackrel{?}{=} -3$	
	$-3 = -3 \checkmark$	

Since $x = -14$ makes $x + 11 = -3$ a true statement, we know that it is a solution to the equation.

Note:

Exercise:

Problem: Solve: $x + 9 = -7$.

Solution:

$$x = -16$$

Note:

Exercise:

Problem: Solve: $x + 16 = -4$.

Solution:

$$x = -20$$

In the original equation in the previous example, 11 was added to the x , so we subtracted 11 to ‘undo’ the addition. In the next example, we will need to ‘undo’ subtraction by using the Addition Property of Equality.

Example:

Exercise:

Problem: Solve: $m - 4 = -5$.

Solution:
Solution

		$m + 4 = -5$
Add 4 to each side to "undo" the subtraction.		$m + 4 - 4 = -5 - 4$
Simplify.		$m = -1$
Check:	$m + 4 = -5$	
Substitute $m = -1$.	$-1 + 4 \stackrel{?}{=} -5$	
	$-5 = -5 \checkmark$	
		The solution to $m - 4 = -5$ is $m = -1$.

Note:

Exercise:

Problem: Solve: $n - 6 = -7$.

Solution:

-1

Note:**Exercise:**

Problem: Solve: $x - 5 = -9$.

Solution:

-4

Now let's review solving equations with fractions.

Example:**Exercise:**

Problem: Solve: $n - \frac{3}{8} = \frac{1}{2}$.

Solution:

Solution

		$n - \frac{3}{8} = \frac{1}{2}$
Use the Addition Property of Equality.		$n - \frac{3}{8} + \frac{3}{8} = \frac{1}{2} + \frac{3}{8}$
Find the LCD to add the fractions on the right.		$n - \frac{3}{8} + \frac{3}{8} = \frac{4}{8} + \frac{3}{8}$
Simplify		$n = \frac{7}{8}$
Check:	$n - \frac{3}{8} = \frac{1}{2}$	
Substitute $n = \frac{7}{8}$.	$\frac{7}{8} - \frac{3}{8} \stackrel{?}{=} \frac{1}{2}$	
Subtract.	$\frac{4}{8} \stackrel{?}{=} \frac{1}{2}$	
Simplify.	$\frac{1}{2} = \frac{1}{2} \checkmark$	
The solution checks.		

Note:

Exercise:

Problem: Solve: $p - \frac{1}{3} = \frac{5}{6}$.

Solution:

$$p = \frac{7}{6}$$

Note:

Exercise:

Problem: Solve: $q - \frac{1}{2} = \frac{1}{6}$.

Solution:

$$q = \frac{2}{3}$$

In [Solve Equations with Decimals](#), we solved equations that contained decimals. We'll review this next.

Example:

Exercise:

Problem: Solve $a - 3.7 = 4.3$.

Solution:
Solution

		$a - 3.7 = 4.3$
Use the Addition Property of Equality.		$a - 3.7 + 3.7 = 4.3 + 3.7$
Add.		$a = 8$
Check:	$a - 3.7 = 4.3$	
Substitute $a = 8$.	$8 - 3.7 \stackrel{?}{=} 4.3$	
Simplify.	$4.3 = 4.3 \checkmark$	
The solution checks.		

Note:

Exercise:

Problem: Solve: $b - 2.8 = 3.6$.

Solution:

$$b = 6.4$$

Note:**Exercise:**

Problem: Solve: $c - 6.9 = 7.1$.

Solution:

$$c = 14$$

Solve Equations That Need to Be Simplified

In the examples up to this point, we have been able to isolate the variable with just one operation. Many of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality. You should always simplify as much as possible before trying to isolate the variable.

Example:**Exercise:**

Problem: Solve: $3x - 7 - 2x - 4 = 1$.

Solution:
Solution

The left side of the equation has an expression that we should simplify before trying to isolate the variable.

	$3x - 7 - 2x - 4 = 1$
Rearrange the terms, using the Commutative Property of Addition.	$3x - 2x - 7 - 4 = 1$
Combine like terms.	$x - 11 = 1$
Add 11 to both sides to isolate x .	$x - 11 + 11 = 1 + 11$
Simplify.	$x = 12$
Check. Substitute $x = 12$ into the original equation. $\begin{aligned} 3x - 7 - 2x - 4 &= 1 \\ 3(12) - 7 - 2(12) - 4 &= 1 \\ 36 - 7 - 24 - 4 &= 1 \\ 29 - 24 - 4 &= 1 \\ 5 - 4 &= 1 \\ 1 &= 1 \checkmark \end{aligned}$	

The solution checks.

Note:

Exercise:

Problem: Solve: $8y - 4 - 7y - 7 = 4$.

Solution:

$$y = 15$$

Note:

Exercise:

Problem: Solve: $6z + 5 - 5z - 4 = 3$.

Solution:

$$z = 2$$

Example:

Exercise:

Problem: Solve: $3(n - 4) - 2n = -3$.

Solution:

Solution

The left side of the equation has an expression that we should simplify.

	$3(n - 4) - 2n = -3$
Distribute on the left.	$3n - 12 - 2n = -3$
Use the Commutative Property to rearrange terms.	$3n - 2n - 12 = -3$
Combine like terms.	$n - 12 = -3$
Isolate n using the Addition Property of Equality.	$n - 12 + 12 = -3 + 12$
Simplify.	$n = 9$
Check. Substitute $n = 9$ into the original equation. $\begin{aligned} 3(n - 4) - 2n &= -3 \\ 3(9 - 4) - 2 \cdot 9 &= -3 \\ 3(5) - 18 &= -3 \\ 15 - 18 &= -3 \\ -3 &= -3 \checkmark \end{aligned}$	

The solution checks.

Note:

Exercise:

Problem: Solve: $5(p - 3) - 4p = -10$.

Solution:

$$p = 5$$

Note:

Exercise:

Problem: Solve: $4(q + 2) - 3q = -8$.

Solution:

$$q = -16$$

Example:

Exercise:

Problem: Solve: $2(3k - 1) - 5k = -2 - 7$.

Solution:

Solution

Both sides of the equation have expressions that we should simplify before we isolate the variable.

	$2(3k - 1) - 5k = -2 - 7$
Distribute on the left, subtract on the right.	$6k - 2 - 5k = -9$
Use the Commutative Property of Addition.	$6k - 5k - 2 = -9$
Combine like terms.	$k - 2 = -9$
Undo subtraction by using the Addition Property of Equality.	$k - 2 + 2 = -9 + 2$
Simplify.	$k = -7$
Check. Let $k = -7$. $\begin{aligned} 2(3k - 1) - 5k &= -2 - 7 \\ 2(3(-7) - 1) - 5(-7) &= -2 - 7 \\ 2(-21 - 1) - 5(-7) &= -9 \\ 2(-22) + 35 &= -9 \\ -44 + 35 &= -9 \\ -9 &= -9 \checkmark \end{aligned}$	

The solution checks.

Note:

Exercise:

Problem: Solve: $4(2h - 3) - 7h = -6 - 7$.

Solution:

$$h = -1$$

Note:

Exercise:

Problem: Solve: $2(5x + 2) - 9x = -2 + 7$.

Solution:

$$x = 1$$

Translate an Equation and Solve

In previous chapters, we translated word sentences into equations. The first step is to look for the word (or words) that translate(s) to the equal sign.

[\[link\]](#) reminds us of some of the words that translate to the equal sign.

Equals (=)						
is	is equal to	is the same as	the result is	gives	was	will be

Let's review the steps we used to translate a sentence into an equation.

Note:

Translate a word sentence to an algebraic equation.

Locate the "equals" word(s). Translate to an equal sign.

Translate the words to the left of the "equals" word(s) into an algebraic expression.

Translate the words to the right of the "equals" word(s) into an algebraic expression.

Now we are ready to try an example.

Example:

Exercise:

Problem: Translate and solve: five more than x is equal to 26.

Solution:

Solution

Translate.	Five more than x is equal to 26 $x + 5 = 26$
Subtract 5 from both sides.	$x + 5 - 5 = 26 - 5$
Simplify.	$x = 21$
Check: Is 26 five more than 21? $21 + 5 \stackrel{?}{=} 26$ $26 = 26 \checkmark$	
The solution checks.	

Note:

Exercise:

Problem: Translate and solve: Eleven more than x is equal to 41.

Solution:

$$x + 11 = 41; x = 30$$

Note:

Exercise:

Problem: Translate and solve: Twelve less than y is equal to 51.

Solution:

$$y - 12 = 51; y = 63$$

Example:

Exercise:

Problem: Translate and solve: The difference of $5p$ and $4p$ is 23.

Solution:

Solution

Translate.

The difference of $5p$ and $4p$	is	23
$5p - 4p$	=	23

Simplify.

$$p = 23$$

Check:

$$5p - 4p = 23$$

$$5(23) - 4(23) \stackrel{?}{=} 23$$

$$115 - 22 \stackrel{?}{=} 23$$

$$23 = 23 \checkmark$$

The solution
checks.

Note:

Exercise:

Problem: Translate and solve: The difference of $4x$ and $3x$ is 14.

Solution:

$$4x - 3x = 14; x = 14$$

Note:

Exercise:

Problem: Translate and solve: The difference of $7a$ and $6a$ is -8 .

Solution:

$$7a - 6a = -8; a = -8$$

Translate and Solve Applications

In most of the application problems we solved earlier, we were able to find the quantity we were looking for by simplifying an algebraic expression. Now we will be using equations to solve application problems. We'll start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for.

Example:

Exercise:

Problem:

The Robles family has two dogs, Buster and Chandler. Together, they weigh 71 pounds.

Chandler weighs 28 pounds. How much does Buster weigh?

Solution:

Solution

Read the problem carefully.	
Identify what you are asked to find, and choose a variable to represent it.	How much does Buster weigh?

	Let b = Buster's weight
Write a sentence that gives the information to find it.	Buster's weight plus Chandler's weight equals 71 pounds.
We will restate the problem, and then include the given information.	Buster's weight plus 28 equals 71.
Translate the sentence into an equation, using the variable b .	$b + 28 = 71$
Solve the equation using good algebraic techniques.	$b + 28 - 28 = 71 - 28$ $b = 43$
Check the answer in the problem and make sure it makes sense.	
Is 43 pounds a reasonable weight for a dog? Yes. Does Buster's weight plus Chandler's weight equal 71 pounds?	
$43 + 28 \stackrel{?}{=} 71$	
$71 = 71 \checkmark$	
Write a complete sentence that answers the question, "How much does Buster weigh?"	Buster weighs 43 pounds

Note:

Exercise:

Problem:

Translate into an algebraic equation and solve: The Pappas family has two cats, Zeus and Athena. Together, they weigh 13 pounds. Zeus weighs 6 pounds. How much does Athena weigh?

Solution:

$a + 6 = 13$; Athena weighs 7 pounds.

Note:

Exercise:

Problem:

Translate into an algebraic equation and solve: Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Solution:

$26 + h = 68$; Henry has 42 books.

Note:

Devise a problem-solving strategy.

Read the problem. Make sure you understand all the words and ideas. Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

Answer the question with a complete sentence.

Example:

Exercise:

Problem:

Shayla paid \$24,575 for her new car. This was \$875 less than the sticker price. What was the sticker price of the car?

Solution:

Solution

What are you asked to find?	"What was the sticker price of the car?"
Assign a variable.	Let s = the sticker price of the car.
Write a sentence that gives the information to find it.	\$24,575 is \$875 less than the sticker price \$24,575 is \$875 less than s

Translate into an equation.	$24,575 = s - 875$
Solve.	$24,575 + 875 = s - 875 + 875$ $24,575 = s$
Check:	
Is \$875 less than \$25,450 equal to \$24,575?	
$25,450 - 875 \stackrel{?}{=} 24,575$	
$24,575 = 24,575 \checkmark$	
Write a sentence that answers the question.	The sticker price was \$25,450.

Note:

Exercise:

Problem:

Translate into an algebraic equation and solve: Eddie paid \$19,875 for his new car. This was \$1,025 less than the sticker price. What was the sticker price of the car?

Solution:

$19,875 = s - 1025$; the sticker price is \$20,900.

Note:

Exercise:

Problem:

Translate into an algebraic equation and solve: The admission price for the movies during the day is \$7.75. This is \$3.25 less than the price at night. How much does the movie cost at night?

Solution:

$7.75 = n - 3.25$; the price at night is \$11.00.

Note: The *Links to Literacy* activity, "The 100-pound Problem", will provide you with another view of the topics covered in this section.

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solving One Step Equations By Addition and Subtraction](#)
- [Solve One Step Equations By Add and Subtract Whole Numbers \(Variable on Left\)](#)
- [Solve One Step Equations By Add and Subtract Whole Numbers \(Variable on Right\)](#)

Key Concepts

- **Determine whether a number is a solution to an equation.**

Substitute the number for the variable in the equation.
Simplify the expressions on both sides of the equation.
Determine whether the resulting equation is true.

If it is true, the number is a solution.

If it is not true, the number is not a solution.

- **Subtraction and Addition Properties of Equality**

- **Subtraction Property of Equality**

For all real numbers a , b , and c ,
if $a = b$ then $a - c = b - c$.

- **Addition Property of Equality**

For all real numbers a , b , and c ,
if $a = b$ then $a + c = b + c$.

- **Translate a word sentence to an algebraic equation.**

Locate the “equals” word(s). Translate to an equal sign.

Translate the words to the left of the “equals” word(s) into an algebraic expression.

Translate the words to the right of the “equals” word(s) into an algebraic expression.

- **Problem-solving strategy**

Read the problem. Make sure you understand all the words and ideas.

Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.

Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.

Solve the equation using good algebra techniques.

Check the answer in the problem and make sure it makes sense.

Answer the question with a complete sentence.

Practice Makes Perfect

Solve Equations Using the Subtraction and Addition Properties of Equality

In the following exercises, determine whether the given value is a solution to the equation.

Exercise:

Problem: Is $y = \frac{1}{3}$ a solution of $4y + 2 = 10y$?

Solution:

yes

Exercise:

Problem: Is $x = \frac{3}{4}$ a solution of $5x + 3 = 9x$?

Exercise:

Problem: Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?

Solution:

no

Exercise:

Problem: Is $v = -\frac{1}{3}$ a solution of $9v - 2 = 3v$?

In the following exercises, solve each equation.

Exercise:

Problem: $x + 7 = 12$

Solution:

$$x = 5$$

Exercise:

Problem: $y + 5 = -6$

Exercise:

Problem: $b + \frac{1}{4} = \frac{3}{4}$

Solution:

$$b = \frac{1}{2}$$

Exercise:

Problem: $a + \frac{2}{5} = \frac{4}{5}$

Exercise:

Problem: $p + 2.4 = -9.3$

Solution:

$$p = -11.7$$

Exercise:

Problem: $m + 7.9 = 11.6$

Exercise:

Problem: $a - 3 = 7$

Solution:

$$a = 10$$

Exercise:

Problem: $m - 8 = -20$

Exercise:

Problem: $x - \frac{1}{3} = 2$

Solution:

$$x = \frac{7}{3}$$

Exercise:

Problem: $x - \frac{1}{5} = 4$

Exercise:

Problem: $y - 3.8 = 10$

Solution:

$$y = 13.8$$

Exercise:

Problem: $y - 7.2 = 5$

Exercise:

Problem: $x - 15 = -42$

Solution:

$$x = -27$$

Exercise:

Problem: $z + 5.2 = -8.5$

Exercise:

Problem: $q + \frac{3}{4} = \frac{1}{2}$

Solution:

$$q = -\frac{1}{4}$$

Exercise:

Problem: $p - \frac{2}{5} = \frac{2}{3}$

Exercise:

Problem: $y - \frac{3}{4} = \frac{3}{5}$

Solution:

$$y = \frac{27}{20}$$

Solve Equations that Need to be Simplified

In the following exercises, solve each equation.

Exercise:

Problem: $c + 3 - 10 = 18$

Exercise:

Problem: $m + 6 - 8 = 15$

Solution:

$$17$$

Exercise:

Problem: $9x + 5 - 8x + 14 = 20$

Exercise:

Problem: $6x + 8 - 5x + 16 = 32$

Solution:

8

Exercise:

Problem: $-6x - 11 + 7x - 5 = -16$

Exercise:

Problem: $-8n - 17 + 9n - 4 = -41$

Solution:

-20

Exercise:

Problem: $3(y - 5) - 2y = -7$

Exercise:

Problem: $4(y - 2) - 3y = -6$

Solution:

2

Exercise:

Problem: $8(u + 1.5) - 7u = 4.9$

Exercise:

Problem: $5(w + 2.2) - 4w = 9.3$

Solution:

$$-1.7$$

Exercise:

Problem: $-5(y - 2) + 6y = -7 + 4$

Exercise:

Problem: $-8(x - 1) + 9x = -3 + 9$

Solution:

$$-2$$

Exercise:

Problem: $3(5n - 1) - 14n + 9 = 1 - 2$

Exercise:

Problem: $2(8m + 3) - 15m - 4 = 3 - 5$

Solution:

$$-4$$

Exercise:

Problem: $-(j + 2) + 2j - 1 = 5$

Exercise:

Problem: $-(k + 7) + 2k + 8 = 7$

Solution:

6

Exercise:

Problem: $6a - 5(a - 2) + 9 = -11$

Exercise:

Problem: $8c - 7(c - 3) + 4 = -16$

Solution:

-41

Exercise:

Problem: $8(4x + 5) - 5(6x) - x = 53$

Exercise:

Problem: $6(9y - 1) - 10(5y) - 3y = 22$

Solution:

28

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise:

Problem: Five more than x is equal to 21.

Exercise:

Problem: The sum of x and -5 is 33.

Solution:

$$x + (-5) = 33; x = 38$$

Exercise:

Problem: Ten less than m is -14 .

Exercise:

Problem: Three less than y is -19 .

Solution:

$$y - 3 = -19; y = -16$$

Exercise:

Problem: The sum of y and -3 is 40.

Exercise:

Problem: Eight more than p is equal to 52.

Solution:

$$p + 8 = 52; p = 44$$

Exercise:

Problem: The difference of $9x$ and $8x$ is 17.

Exercise:

Problem: The difference of $5c$ and $4c$ is 60.

Solution:

$$5c - 4c = 60; 60$$

Exercise:

Problem: The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.

Exercise:

Problem: The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.

Solution:

$$f - \frac{1}{3} = \frac{1}{12}; \frac{5}{12}$$

Exercise:

Problem: The sum of $-4n$ and $5n$ is -32 .

Exercise:

Problem: The sum of $-9m$ and $10m$ is -25 .

Solution:

$$-9m + 10m = -25; m = -25$$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise:

Problem:

Pilar drove from home to school and then to her aunt's house, a total of 18 miles. The distance from Pilar's house to school is 7 miles. What is the distance from school to her aunt's house?

Exercise:**Problem:**

Jeff read a total of 54 pages in his English and Psychology textbooks. He read 41 pages in his English textbook. How many pages did he read in his Psychology textbook?

Solution:

Let p equal the number of pages read in the Psychology book $41 + p = 54$. Jeff read pages in his Psychology book.

Exercise:**Problem:**

Pablo's father is 3 years older than his mother. Pablo's mother is 42 years old. How old is his father?

Exercise:**Problem:**

Eva's daughter is 5 years younger than her son. Eva's son is 12 years old. How old is her daughter?

Solution:

Let d equal the daughter's age. $d = 12 - 5$. Eva's daughter's age is 7 years old.

Exercise:**Problem:**

Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?

Exercise:

Problem:

For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday dinner turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?

Solution:

21 pounds

Exercise:**Problem:**

The nurse reported that Tricia's daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia's daughter weigh at her last checkup?

Exercise:**Problem:**

Connor's temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?

Solution:

100.5 degrees

Exercise:**Problem:**

Melissa's math book cost \$22.85 less than her art book cost. Her math book cost \$93.75. How much did her art book cost?

Exercise:

Problem:

Ron's paycheck this week was \$17.43 less than his paycheck last week. His paycheck this week was \$103.76. How much was Ron's paycheck last week?

Solution:

\$121.19

Everyday Math**Exercise:****Problem:**

Baking Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{1}{4}$ cup of sugar and will borrow the rest from her neighbor. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{1}{4} + s = \frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Exercise:**Problem:**

Construction Miguel wants to drill a hole for a $\frac{5}{8}$ -inch screw. The screw should be $\frac{1}{12}$ inch larger than the hole. Let d equal the size of the hole he should drill. Solve the equation $d + \frac{1}{12} = \frac{5}{8}$ to see what size the hole should be.

Solution:

$$d = \frac{13}{24}$$

Writing Exercises

Exercise:

Problem:

Is -18 a solution to the equation $3x = 16 - 5x$? How do you know?

Exercise:

Problem:

Write a word sentence that translates the equation $y - 18 = 41$ and then make up an application that uses this equation in its solution.

Solution:

Answers will vary.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using the Subtraction and Addition Properties of Equality.			
solve equations that need to be simplified.			
translate an equation and solve.			
translate and solve applications.			

Ⓑ If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use

them. What did you do to become confident of your ability to do these things? Be specific.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

...no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

Solve Equations Using the Division and Multiplication Properties of Equality

By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that need to be simplified

Note:

Before you get started, take this readiness quiz.

1. Simplify: $-7\left(\frac{1}{-7}\right)$.

If you missed this problem, review [\[link\]](#).

2. What is the reciprocal of $-\frac{3}{8}$?

If you missed this problem, review [\[link\]](#).

3. Evaluate $9x + 2$ when $x = -3$.

If you missed this problem, review [\[link\]](#).

Solve Equations Using the Division and Multiplication Properties of Equality

We introduced the Multiplication and Division Properties of Equality in [Solve Equations Using Integers; The Division Property of Equality](#) and [Solve Equations with Fractions](#). We modeled how these properties worked using envelopes and counters and then applied them to solving equations (See [Solve Equations Using Integers; The Division Property of Equality](#)). We restate them again here as we prepare to use these properties again.

Note:

Division and Multiplication Properties of Equality

Division Property of Equality: For all real numbers a, b, c , and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.

Multiplication Property of Equality: For all real numbers a, b, c , if $a = b$, then $ac = bc$.

When you divide or multiply both sides of an equation by the same quantity, you still have equality.

Let's review how these properties of equality can be applied in order to solve equations. Remember, the goal is to 'undo' the operation on the variable. In the example below the variable is multiplied by 4, so we will divide both sides by 4 to 'undo' the multiplication.

Example:

Exercise:

Problem: Solve: $4x = -28$.

Solution:

Solution

We use the Division Property of Equality to divide both sides by 4.

$$4x = -28$$

Divide both sides by 4 to undo the multiplication.

$$\frac{4x}{4} = \frac{-28}{4}$$

Simplify.

$$x = -7$$

Check your answer. Let $x = -7$.

$$4x = -28$$

$$4(-7) \stackrel{?}{=} -28$$

$$-28 = -28 \checkmark$$

Since this is a true statement, $x = -7$ is a solution to $4x = -28$.

Note:

Exercise:

Problem: Solve: $3y = -48$.

Solution:

$$y = -16$$

Note:

Exercise:

Problem: Solve: $4z = -52$.

Solution:

$$z = -13$$

In the previous example, to ‘undo’ multiplication, we divided. How do you think we ‘undo’ division?

Example:

Exercise:

Problem: Solve: $\frac{a}{-7} = -42$.

Solution:

Solution

Here a is divided by -7 . We can multiply both sides by -7 to isolate a .

$$\frac{a}{-7} = -42$$

Multiply both sides by -7 .

$$-7\left(\frac{a}{-7}\right) = -7(-42)$$

	$\frac{-7a}{-7} = 294$
Simplify.	$a = 294$
Check your answer. Let $a = 294$.	
$\frac{a}{-7} = -42$	
$\frac{294}{-7} \stackrel{?}{=} -42$	
$-42 = -42 \quad \checkmark$	

Note:

Exercise:

Problem: Solve: $\frac{b}{-6} = -24$.

Solution:

$$b = 144$$

Note:

Exercise:

Problem: Solve: $\frac{c}{-8} = -16$.

Solution:

$$c = 128$$

Example:

Exercise:

Problem: Solve: $-r = 2$.

Solution:

Solution

Remember $-r$ is equivalent to $-1r$.

	$-r = 2$
Rewrite $-r$ as $-1r$.	$-1r = 2$
Divide both sides by -1 .	$\frac{-1r}{-1} = \frac{2}{-1}$

		$r = -2$
Check.	$-r = 2$	
Substitute $r = -2$	$-(-2) \stackrel{?}{=} 2$	
Simplify.	$2 = 2 \checkmark$	

In [Solve Equations with Fractions](#), we saw that there are two other ways to solve $-r = 2$.

We could multiply both sides by -1 .

We could take the opposite of both sides.

Note:

Exercise:

Problem: Solve: $-k = 8$.

Solution:

$$k = -8$$

Note:

Exercise:**Problem:** Solve: $-g = 3$.**Solution:**

$$g = -3$$

Example:**Exercise:****Problem:** Solve: $\frac{2}{3}x = 18$.**Solution:****Solution**

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{2}{3}$.

	$\frac{2}{3}x = 18$
Multiply by the reciprocal of $\frac{2}{3}$.	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 18$
Reciprocals multiply to one.	

	$1x = \frac{3}{2} \cdot \frac{18}{1}$
Multiply.	$x = 27$
Check your answer. Let $x = 27$	
$\frac{2}{3}x = 18$	
$\frac{2}{3} \cdot 27 \stackrel{?}{=} 18$	
$18 = 18 \checkmark$	

Notice that we could have divided both sides of the equation $\frac{2}{3}x = 18$ by $\frac{2}{3}$ to isolate x . While this would work, multiplying by the reciprocal requires fewer steps.

Note:

Exercise:

Problem: Solve: $\frac{2}{5}n = 14$.

Solution:

$$n = 35$$

Note:

Exercise:

Problem: Solve: $\frac{5}{6}y = 15$.

Solution:

$$y = 18$$

Solve Equations That Need to be Simplified

Many equations start out more complicated than the ones we've just solved. First, we need to simplify both sides of the equation as much as possible

Example:

Exercise:

Problem: Solve: $8x + 9x - 5x = -3 + 15$.

Solution:

Solution

Start by combining like terms to simplify each side.

--	--

	$8x + 9x - 5x = -3 + 15$
Combine like terms.	$12x = 12$
Divide both sides by 12 to isolate x.	$\frac{12x}{12} = \frac{12}{12}$
Simplify.	$x = 1$
Check your answer. Let $x = 1$	
$8x + 9x - 5x = -3 + 15$	
$8 \cdot 1 + 9 \cdot 1 - 5 \cdot 1 \stackrel{?}{=} -3 + 15$	
$8 + 9 - 5 \stackrel{?}{=} -3 + 15$	
$12 = 12 \checkmark$	

Note:

Exercise:

Problem: Solve: $7x + 6x - 4x = -8 + 26$.

Solution:

$$x = 2$$

Note:

Exercise:

Problem: Solve: $11n - 3n - 6n = 7 - 17$.

Solution:

$$n = -5$$

Example:

Exercise:

Problem: Solve: $11 - 20 = 17y - 8y - 6y$.

Solution:

Solution

Simplify each side by combining like terms.

$$11 - 20 = 17y - 8y - 6y$$

Simplify each side.

$$-9 = 3y$$

Divide both sides by 3 to isolate y.

$$\frac{-9}{3} = \frac{3y}{3}$$

Simplify.

$$-3 = y$$

Check your answer. Let
 $y = -3$

$$11 - 20 = 17y - 8y - 6y$$

$$11 - 20 \stackrel{?}{=} 17(-3) - 8(-3) - 6(-3)$$

$$11 - 20 \stackrel{?}{=} -51 + 24 + 18$$

$$-9 = -9 \checkmark$$

Notice that the variable ended up on the right side of the equal sign when we solved the equation. You may prefer to take one more step to write the solution with the variable on the left side of the equal sign.

Note:

Exercise:

Problem: Solve: $18 - 27 = 15c - 9c - 3c$.

Solution:

$$c = -3$$

Note:

Exercise:

Problem: Solve: $18 - 22 = 12x - x - 4x$.

Solution:

$$x = -\frac{4}{7}$$

Example:

Exercise:

Problem: Solve: $-3(n - 2) - 6 = 21$.

Solution:

Solution

Remember—always simplify each side first.

	$-3(n - 2) - 6 = 21$
Distribute.	$-3n + 6 - 6 = 21$
Simplify.	$-3n = 21$
Divide both sides by -3 to isolate n.	$\frac{-3n}{-3} = \frac{21}{-3}$ $n = -7$
Check your answer. Let $n = -7$.	
$-3(n - 2) - 6 = 21$	
$-3(-7 - 2) - 6 \stackrel{?}{=} 21$	
$-3(-9) - 6 \stackrel{?}{=} 21$	
$27 - 6 \stackrel{?}{=} 21$	

$$21 = 21 \checkmark$$

Note:

Exercise:

Problem: Solve: $-4(n - 2) - 8 = 24$.

Solution:

$$n = -6$$

Note:

Exercise:

Problem: Solve: $-6(n - 2) - 12 = 30$.

Solution:

$$n = -5$$

Note: The *Links to Literacy* activity, "Everybody Wins" will provide you with another view of the topics covered in this section.

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solving One Step Equation by Mult/Div. Integers \(Var on Left\)](#)
- [Solving One Step Equation by Mult/Div. Integers \(Var on Right\)](#)
- [Solving One Step Equation in the Form: \$-x = -a\$](#)

Key Concepts

- **Division and Multiplication Properties of Equality**
 - **Division Property of Equality:** For all real numbers a , b , c , and $c \neq 0$, if $a = b$, then $ac = bc$.
 - **Multiplication Property of Equality:** For all real numbers a , b , c , if $a = b$, then $ac = bc$.

Practice Makes Perfect

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation for the variable using the Division Property of Equality and check the solution.

Exercise:

Problem: $8x = 32$

Exercise:

Problem: $7p = 63$

Solution:

9

Exercise:

Problem: $-5c = 55$

Exercise:

Problem: $-9x = -27$

Solution:

3

Exercise:

Problem: $-90 = 6y$

Exercise:

Problem: $-72 = 12y$

Solution:

-6

Exercise:

Problem: $-16p = -64$

Exercise:

Problem: $-8m = -56$

Solution:

7

Exercise:

Problem: $0.25z = 3.25$

Exercise:

Problem: $0.75a = 11.25$

Solution:

15

Exercise:

Problem: $-3x = 0$

Exercise:

Problem: $4x = 0$

Solution:

0

In the following exercises, solve each equation for the variable using the Multiplication Property of Equality and check the solution.

Exercise:

Problem: $\frac{x}{4} = 15$

Exercise:

Problem: $\frac{z}{2} = 14$

Solution:

28

Exercise:

Problem: $-20 = \frac{q}{-5}$

Exercise:

Problem: $\frac{c}{-3} = -12$

Solution:

$$36$$

Exercise:

Problem: $\frac{y}{9} = -6$

Exercise:

Problem: $\frac{q}{6} = -8$

Solution:

$$-48$$

Exercise:

Problem: $\frac{m}{-12} = 5$

Exercise:

Problem: $-4 = \frac{p}{-20}$

Solution:

$$80$$

Exercise:

Problem: $\frac{2}{3}y = 18$

Exercise:

Problem: $\frac{3}{5}r = 15$

Solution:

25

Exercise:

Problem: $-\frac{5}{8}w = 40$

Exercise:

Problem: $24 = -\frac{3}{4}x$

Solution:

-32

Exercise:

Problem: $-\frac{2}{5} = \frac{1}{10}a$

Exercise:

Problem: $-\frac{1}{3}q = -\frac{5}{6}$

Solution:

5/2

Solve Equations That Need to be Simplified

In the following exercises, solve the equation.

Exercise:

Problem: $8a + 3a - 6a = -17 + 27$

Exercise:

Problem: $6y - 3y + 12y = -43 + 28$

Solution:

$$y = -1$$

Exercise:

Problem: $-9x - 9x + 2x = 50 - 2$

Exercise:

Problem: $-5m + 7m - 8m = -6 + 36$

Solution:

$$m = -5$$

Exercise:

Problem: $100 - 16 = 4p - 10p - p$

Exercise:

Problem: $-18 - 7 = 5t - 9t - 6t$

Solution:

$$t = \frac{5}{2}$$

Exercise:

Problem: $\frac{7}{8}n - \frac{3}{4}n = 9 + 2$

Exercise:

Problem: $\frac{5}{12}q + \frac{1}{2}q = 25 - 3$

Solution:

$$q = 24$$

Exercise:

Problem: $0.25d + 0.10d = 6 - 0.75$

Exercise:

Problem: $0.05p - 0.01p = 2 + 0.24$

Solution:

$$p = 56$$

Everyday Math

Exercise:

Problem:

Balloons Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. Find the number of balloons in each bunch, b , by solving the equation $3b = 18$.

Exercise:

Problem:

Teaching Connie's kindergarten class has 24 children. She wants them to get into 4 equal groups. Find the number of children in each group, g , by solving the equation $4g = 24$.

Solution:

6 children

Exercise:**Problem:**

Ticket price Daria paid \$36.25 for 5 children's tickets at the ice skating rink. Find the price of each ticket, p , by solving the equation $5p = 36.25$.

Exercise:**Problem:**

Unit price Nishant paid \$12.96 for a pack of 12 juice bottles. Find the price of each bottle, b , by solving the equation $12b = 12.96$.

Solution:

\$1.08

Exercise:**Problem:**

Fuel economy Tania's SUV gets half as many miles per gallon (mpg) as her husband's hybrid car. The SUV gets 18 mpg. Find the miles per gallons, m , of the hybrid car, by solving the equation $\frac{1}{2}m = 18$.

Exercise:**Problem:**

Fabric The drill team used 14 yards of fabric to make flags for one-third of the members. Find how much fabric, f , they would need to make flags for the whole team by solving the equation $\frac{1}{3}f = 14$.

Solution:

42 yards

Writing Exercises

Exercise:

Problem:

Frida started to solve the equation $-3x = 36$ by adding 3 to both sides. Explain why Frida's method will result in the correct solution.

Exercise:

Problem:

Emiliano thinks $x = 40$ is the solution to the equation $\frac{1}{2}x = 80$. Explain why he is wrong.

Solution:

Answer will vary.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using the Division and Multiplication Properties of Equality.			
solve equations that need to be simplified.			

Ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Solve Equations with Fractions

By the end of this section, you will be able to:

- Determine whether a fraction is a solution of an equation
- Solve equations with fractions using the Addition, Subtraction, and Division Properties of Equality
- Solve equations using the Multiplication Property of Equality
- Translate sentences to equations and solve

Note:

Before you get started, take this readiness quiz. If you miss a problem, go back to the section listed and review the material.

1. Evaluate $x + 4$ when $x = -3$

If you missed this problem, review [\[link\]](#).

2. Solve: $2y - 3 = 9$.

If you missed this problem, review [\[link\]](#).

3. Solve: $y - 3 = -9$

If you missed this problem, review [\[link\]](#).

Determine Whether a Fraction is a Solution of an Equation

As we saw in [Solve Equations with the Subtraction and Addition Properties of Equality](#) and [Solve Equations Using Integers; The Division Property of Equality](#), a solution of an equation is a value that makes a true statement when substituted for the variable in the equation. In those sections, we found whole number and integer solutions to equations. Now that we have worked with fractions, we are ready to find fraction solutions to equations.

The steps we take to determine whether a number is a solution to an equation are the same whether the solution is a whole number, an integer, or a fraction.

Note:

Determine whether a number is a solution to an equation.

Substitute the number for the variable in the equation.

Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true. If it is true, the number is a solution. If it is not true, the number is not a solution.

Example:**Exercise:****Problem:**

Determine whether each of the following is a solution of

$$x - \frac{3}{10} = \frac{1}{2}.$$

Ⓐ $x = 1$

Ⓑ $x = \frac{4}{5}$

Ⓒ $x = -\frac{4}{5}$

Solution:**Solution**

Ⓐ

$$x - \frac{3}{10} = \frac{1}{2}$$

Substitute 1 for x .

$$1 - \frac{3}{10} \stackrel{?}{=} \frac{1}{2}$$

Change to fractions with a LCD of 10.

$$\frac{10}{10} - \frac{3}{10} \stackrel{?}{=} \frac{5}{10}$$

Subtract.

$$\frac{7}{10} \neq \frac{5}{10}$$

Since $x = 1$ does not result in a true equation, 1 is not a solution to the equation.

ⓑ

$$x - \frac{3}{10} = \frac{1}{2}$$

Substitute $\frac{4}{5}$ for x .

$$\frac{4}{5} - \frac{3}{10} \stackrel{?}{=} \frac{1}{2}$$

$$\frac{8}{10} - \frac{3}{10} \stackrel{?}{=} \frac{5}{10}$$

Subtract.

$$\frac{5}{10} = \frac{5}{10} \checkmark$$

Since $x = \frac{4}{5}$ results in a true equation, $\frac{4}{5}$ is a solution to the equation $x - \frac{3}{10} = \frac{1}{2}$.

©

$$x - \frac{3}{10} = \frac{1}{2}$$

Substitute $-\frac{4}{5}$ for x .

$$-\frac{4}{5} - \frac{3}{10} \stackrel{?}{=} \frac{1}{2}$$

$$-\frac{8}{10} - \frac{3}{10} \stackrel{?}{=} \frac{5}{10}$$

Subtract.

$$\frac{11}{10} \neq \frac{5}{10}$$

Since $x = -\frac{4}{5}$ does not result in a true equation, $-\frac{4}{5}$ is not a solution to the equation.

Note:

Exercise:

Problem:

Determine whether each number is a solution of the given equation.

$$x - \frac{2}{3} = \frac{1}{6}:$$

Ⓐ $x = 1$

Ⓑ $x = \frac{5}{6}$

Ⓒ $x = -\frac{5}{6}$

Solution:

Ⓐ no

Ⓑ yes

Ⓒ no

Note:

Exercise:

Problem:

Determine whether each number is a solution of the given equation.

$$y - \frac{1}{4} = \frac{3}{8}:$$

Ⓐ $y = 1$

Ⓑ $y = -\frac{5}{8}$

Ⓒ $y = \frac{5}{8}$

Solution:

- Ⓐ no
- Ⓑ no
- Ⓒ yes

Solve Equations with Fractions using the Addition, Subtraction, and Division Properties of Equality

In [Solve Equations with the Subtraction and Addition Properties of Equality](#) and [Solve Equations Using Integers; The Division Property of Equality](#), we solved equations using the Addition, Subtraction, and Division Properties of Equality. We will use these same properties to solve equations with fractions.

Note:

Addition, Subtraction, and Division Properties of Equality

For any numbers a , b , and c ,

if $a = b$, then $a + c = b + c$.	Addition Property of Equality
if $a = b$, then $a - c = b - c$.	Subtraction Property of Equality
if $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$.	Division Property of Equality

In other words, when you add or subtract the same quantity from both sides of an equation, or divide both sides by the same quantity, you still have equality.

Example:
Exercise:

Problem: Solve: $y + \frac{9}{16} = \frac{5}{16}$.

Solution:
Solution

		$y + \frac{9}{16} = \frac{5}{16}$
Subtract $\frac{9}{16}$ from each side to undo the addition.		$y + \frac{9}{16} - \frac{9}{16} = \frac{5}{16} - \frac{9}{16}$
Simplify on each side of the equation.		$y + 0 = -\frac{4}{16}$
Simplify the fraction.		$y = -\frac{1}{4}$
Check:	$y + \frac{9}{16} = \frac{5}{16}$	
Substitute $y = -\frac{1}{4}$.	$-\frac{1}{4} + \frac{9}{16} \stackrel{?}{=} \frac{5}{16}$	

Rewrite as fractions
with the LCD.

$$-\frac{4}{16} + \frac{9}{16} \stackrel{?}{=} \frac{5}{16}$$

Add.

$$\frac{5}{16} = \frac{5}{16} \checkmark$$

Since $y = -\frac{1}{4}$ makes $y + \frac{9}{16} = \frac{5}{16}$ a true statement, we know we have found the solution to this equation.

Note:

Exercise:

Problem: Solve: $y + \frac{11}{12} = \frac{5}{12}$.

Solution:

$$-\frac{1}{2}$$

Note:

Exercise:

Problem: Solve: $y + \frac{8}{15} = \frac{4}{15}$.

Solution:

$$-\frac{4}{15}$$

We used the Subtraction Property of Equality in [\[link\]](#). Now we'll use the Addition Property of Equality.

Example:

Exercise:

Problem: Solve: $a - \frac{5}{9} = -\frac{8}{9}$.

Solution:
Solution

	$a - \frac{5}{9} = -\frac{8}{9}$
Add $\frac{5}{9}$ from each side to undo the addition.	$a - \frac{5}{9} + \frac{5}{9} = -\frac{8}{9} + \frac{5}{9}$
Simplify on each side of the equation.	$a + 0 = -\frac{3}{9}$
Simplify the fraction.	$a = -\frac{1}{3}$
Check:	$a - \frac{5}{9} = -\frac{8}{9}$

Substitute $a = -\frac{1}{3}$.

$$-\frac{1}{3} - \frac{5}{9} \stackrel{?}{=} -\frac{8}{9}$$

Change to common denominator.

$$-\frac{3}{9} - \frac{5}{9} \stackrel{?}{=} -\frac{8}{9}$$

Subtract.

$$-\frac{8}{9} = -\frac{8}{9} \checkmark$$

Since $a = -\frac{1}{3}$ makes the equation true, we know that $a = -\frac{1}{3}$ is the solution to the equation.

Note:

Exercise:

Problem: Solve: $a - \frac{3}{5} = -\frac{8}{5}$.

Solution:

-1

Note:

Exercise:

Problem: Solve: $n - \frac{3}{7} = -\frac{9}{7}$.

Solution:

$$-\frac{6}{7}$$

The next example may not seem to have a fraction, but let's see what happens when we solve it.

Example:

Exercise:

Problem: Solve: $10q = 44$.

Solution:

Solution

		$10q = 44$
Divide both sides by 10 to undo the multiplication.		$\frac{10q}{10} = \frac{44}{10}$
Simplify.		$q = \frac{22}{5}$
Check:		
Substitute $q = \frac{22}{5}$ into the original equation.	$10\left(\frac{22}{5}\right) \stackrel{?}{=} 44$	
Simplify.	$\cancel{10}^2 \left(\frac{22}{\cancel{5}}\right) \stackrel{?}{=} 44$	

Multiply.

$$44 = 44 \checkmark$$

The solution to the equation was the fraction $\frac{22}{5}$. We leave it as an improper fraction.

Note:

Exercise:

Problem: Solve: $12u = -76$.

Solution:

$$-\frac{19}{3}$$

Note:

Exercise:

Problem: Solve: $8m = 92$.

Solution:

$$\frac{23}{2}$$

Solve Equations with Fractions Using the Multiplication Property of Equality

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So to “undo” the division, we will need to multiply by 4. The

Multiplication Property of Equality will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

Note:

The Multiplication Property of Equality

For any numbers a , b , and c ,

Equation:

$$\text{if } a = b, \text{ then } ac = bc.$$

If you multiply both sides of an equation by the same quantity, you still have equality.

Let's use the Multiplication Property of Equality to solve the equation

$$\frac{x}{7} = -9.$$

Example:

Exercise:

Problem: Solve: $\frac{x}{7} = -9$.

Solution:

Solution

$$\frac{x}{7} = -9$$

Use the Multiplication Property of Equality to multiply both sides by 7. This will isolate the variable.		$7 \cdot \frac{x}{7} = 7(-9)$
Multiply.		$\frac{7x}{7} = -63$
Simplify.		$x = -63$
Check. Substitute -63 for x for in the original equation.	$\frac{-63}{7} \stackrel{?}{=} -9$	
The equation is true.	$-9 = -9 \checkmark$	

Note:

Exercise:

Problem: Solve: $\frac{f}{5} = -25$.

Solution:

-125

Note:

Exercise:

Problem: Solve: $\frac{h}{9} = -27$.

Solution:

-243

Example:

Exercise:

Problem: Solve: $\frac{p}{-8} = -40$.

Solution:

Solution

Here, p is divided by -8 . We must multiply by -8 to isolate p .

	$\frac{p}{-8} = -40$
Multiply both sides by -8	$-8\left(\frac{p}{-8}\right) = -8(-40)$
Multiply.	$\frac{-8p}{-8} = 320$

Simplify.

$$p = 320$$

Check:

Substitute $p = 320$
.

$$\frac{320}{-8} \stackrel{?}{=} -40$$

The equation is
true.

$$-40 = -40 \checkmark$$

Note:

Exercise:

Problem: Solve: $\frac{c}{-7} = -35$.

Solution:

245

Note:

Exercise:

Problem: Solve: $\frac{x}{-11} = -12$.

Solution:

Solve Equations with a Coefficient of -1

Look at the equation $-y = 15$. Does it look as if y is already isolated? But there is a negative sign in front of y , so it is not isolated.

There are three different ways to isolate the variable in this type of equation. We will show all three ways in [\[link\]](#).

Example:

Exercise:

Problem: Solve: $-y = 15$.

Solution:

Solution

One way to solve the equation is to rewrite $-y$ as $-1y$, and then use the Division Property of Equality to isolate y .

	$-y = 15$
Rewrite $-y$ as $-1y$.	$-1y = 15$

Divide both sides by -1 .

$$\frac{-1y}{-1} = \frac{15}{-1}$$

Simplify each side.

$$y = -15$$

Another way to solve this equation is to multiply both sides of the equation by -1 .

$$y = 15$$

Multiply both sides by -1 .

$$-1(-y) = -1(15)$$

Simplify each side.

$$y = -15$$

The third way to solve the equation is to read $-y$ as “the opposite of y .” What number has 15 as its opposite? The opposite of 15 is -15 . So $y = -15$.

For all three methods, we isolated y is isolated and solved the equation.

Check:

	$y = 15$
Substitute $y = -15$.	$-(-15) \stackrel{?}{=} (15)$
Simplify. The equation is true.	$15 = 15 \checkmark$

Note:

Exercise:

Problem: Solve: $-y = 48$.

Solution:

-48

Note:

Exercise:

Problem: Solve: $-c = -23$.

Solution:

23

Solve Equations with a Fraction Coefficient

When we have an equation with a fraction coefficient we can use the Multiplication Property of Equality to make the coefficient equal to 1.

For example, in the equation:

Equation:

$$\frac{3}{4}x = 24$$

The coefficient of x is $\frac{3}{4}$. To solve for x , we need its coefficient to be 1. Since the product of a number and its reciprocal is 1, our strategy here will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$. We will do this in [\[link\]](#).

Example:

Exercise:

Problem: Solve: $\frac{3}{4}x = 24$.

Solution:
Solution

$$\frac{3}{4}x = 24$$

Multiply both sides by the reciprocal of the

coefficient.		$\frac{4}{3} \cdot \frac{3}{4}x = \frac{4}{3} \cdot 24$
Simplify.		$1x = \frac{4}{3} \cdot \frac{24}{1}$
Multiply.		$x = 32$
Check:	$\frac{3}{4}x = 24$	
Substitute $x = 32$.	$\frac{3}{4} \cdot 32 \stackrel{?}{=} 24$	
Rewrite 32 as a fraction.	$\frac{3}{4} \cdot \frac{32}{1} \stackrel{?}{=} 24$	
Multiply. The equation is true.	$24 = 24 \quad \checkmark$	

Notice that in the equation $\frac{3}{4}x = 24$, we could have divided both sides by $\frac{3}{4}$ to get x by itself. Dividing is the same as multiplying by the reciprocal, so we would get the same result. But most people agree that multiplying by the reciprocal is easier.

Note:
Exercise:

Problem: Solve: $\frac{2}{5}n = 14$.

Solution:

35

Note:

Exercise:

Problem: Solve: $\frac{5}{6}y = 15$.

Solution:

18

Example:

Exercise:

Problem: Solve: $-\frac{3}{8}w = 72$.

Solution:

Solution

The coefficient is a negative fraction. Remember that a number and its reciprocal have the same sign, so the reciprocal of the coefficient must also be negative.

		$\frac{3}{8}w = 72$
Multiply both sides by the reciprocal of $-\frac{3}{8}$.		$-\frac{8}{3}\left(-\frac{3}{8}w\right) = \left(-\frac{8}{3}\right)72$
Simplify; reciprocals multiply to one.		$1w = -\frac{8}{3} \cdot \frac{72}{1}$
Multiply.		$w = -192$
Check:	$-\frac{3}{8}w = 72$	
Let $w = -192$.	$-\frac{3}{8}(-192) \stackrel{?}{=} 72$	
Multiply. It checks.	$72 = 72 \quad \checkmark$	

Note:

Exercise:

Problem: Solve: $-\frac{4}{7}a = 52$.

Solution:

-91

Note:

Exercise:

Problem: Solve: $-\frac{7}{9}w = 84$.

Solution:

-108

Translate Sentences to Equations and Solve

Now we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

Subtraction Property of Equality:

For any real numbers a , b , and c ,

if $a = b$, then $a - c = b - c$.

Addition Property of Equality:

For any real numbers a , b , and c ,

	if $a = b$, then $a + c = b + c$.
Division Property of Equality: For any numbers a , b , and c , where $c \neq 0$ if $a = b$, then $\frac{a}{c} = \frac{b}{c}$	Multiplication Property of Equality: For any real numbers a , b , and c if $a = b$, then $ac = bc$

When you add, subtract, multiply or divide the same quantity from both sides of an equation, you still have equality.

In the next few examples, we'll translate sentences into equations and then solve the equations. It might be helpful to review the translation table in [Evaluate, Simplify, and Translate Expressions](#).

Example:

Exercise:

Problem: Translate and solve: n divided by 6 is -24 .

Solution:

Solution

Translate.

$$\underbrace{n \text{ divided by } 6}_{\frac{n}{6}} \underbrace{\text{ is }}_{=} \underbrace{24}_{-24}$$

Multiply both sides by 6.		$6 \cdot \frac{n}{6} = 6(-24)$
Simplify.		$n = -144$
Check:	Is -144 divided by 6 equal to -24 ?	
Translate.	$\frac{-144}{6} \stackrel{?}{=} -24$	
Simplify. It checks.	$-24 = -24 \checkmark$	

Note:

Exercise:

Problem: Translate and solve: n divided by 7 is equal to -21 .

Solution:

$$\frac{n}{7} = -21; n = -147$$

Note:

Exercise:

Problem: Translate and solve: n divided by 8 is equal to -56 .

Solution:

$$\frac{n}{8} = -56; n = -448$$

Example:

Exercise:

Problem: Translate and solve: The quotient of q and -5 is 70 .

Solution:

Solution

Translate.		The quotient of q and -5 is 70 $\frac{q}{-5} = 70$
Multiply both sides by -5 .		$-5(\frac{q}{-5}) = -5(70)$
Simplify.		$q = -350$
Check:	Is the quotient of -350 and -5 equal to 70 ?	
Translate.	$\frac{-350}{-5} \stackrel{?}{=} 70$	

Simplify.
It checks.

$$70 = 70 \quad \checkmark$$

Note:

Exercise:

Problem: Translate and solve: The quotient of q and -8 is 72 .

Solution:

$$\frac{q}{-8} = 72; q = -576$$

Note:

Exercise:

Problem: Translate and solve: The quotient of p and -9 is 81 .

Solution:

$$\frac{p}{-9} = 81; p = -729$$

Example:

Exercise:

Problem: Translate and solve: Two-thirds of f is 18 .

Solution:
Solution

Translate.		$\underbrace{\text{Two-thirds of } f}_{\frac{2}{3}f} \quad \underbrace{\text{is}}_{=} \quad \underbrace{18}_{18}$
Multiply both sides by $\frac{3}{2}$.		$\frac{3}{2} \cdot \frac{2}{3}f = \frac{3}{2} \cdot 18$
Simplify.		$f = 27$
Check:	Is two-thirds of 27 equal to 18?	
Translate.	$\frac{2}{3}(27) \stackrel{?}{=} 18$	
Simplify. It checks.	$18 = 18 \quad \checkmark$	

Note:

Exercise:

Problem: Translate and solve: Two-fifths of f is 16.

Solution:

$$\frac{2}{5}f = 16; f = 40$$

Note:**Exercise:**

Problem: Translate and solve: Three-fourths of f is 21.

Solution:

$$\frac{3}{4}f = 21; f = 28$$

Example:**Exercise:**

Problem: Translate and solve: The quotient of m and $\frac{5}{6}$ is $\frac{3}{4}$.

Solution:

Solution

The quotient of m

		and $\frac{5}{6}$ is $\frac{3}{4}$.
Translate.		$\frac{m}{\frac{5}{6}} = \frac{3}{4}$
Multiply both sides by $\frac{5}{6}$ to isolate m .		$\frac{5}{6} \left(\frac{m}{\frac{5}{6}} \right) = \frac{5}{6} \left(\frac{3}{4} \right)$
Simplify.		$m = \frac{5 \cdot 3}{6 \cdot 4}$
Remove common factors and multiply.		$m = \frac{5}{8}$
Check:		
Is the quotient of $\frac{5}{8}$ and $\frac{5}{6}$ equal to $\frac{3}{4}$?	$\frac{\frac{5}{8}}{\frac{5}{6}} \stackrel{?}{=} \frac{3}{4}$	
Rewrite as division.	$\frac{5}{8} \div \frac{5}{6} \stackrel{?}{=} \frac{3}{4}$	
Multiply the first fraction by the reciprocal of the second.	$\frac{5}{8} \cdot \frac{6}{5} \stackrel{?}{=} \frac{3}{4}$	
Simplify.	$\frac{3}{4} = \frac{3}{4} \checkmark$	
Our solution checks.		

Note:

Exercise:

Problem: Translate and solve. The quotient of n and $\frac{2}{3}$ is $\frac{5}{12}$.

Solution:

$$\frac{\frac{n}{2}}{\frac{2}{3}} = \frac{5}{12}; n = \frac{5}{18}$$

Note:

Exercise:

Problem: Translate and solve The quotient of c and $\frac{3}{8}$ is $\frac{4}{9}$.

Solution:

$$\frac{\frac{c}{3}}{\frac{3}{8}} = \frac{4}{9}; c = \frac{1}{3}$$

Example:

Exercise:

Problem:

Translate and solve: The sum of three-eighths and x is three and one-half.

Solution:

Solution

Translate.

The sum of three-eighths and x	is	three and one-half
$\frac{3}{8} + x$	=	$3\frac{1}{2}$

Use the Subtraction Property of Equality to subtract $\frac{3}{8}$ from both sides.

$$\frac{3}{8} + x - \frac{3}{8} = 3\frac{1}{2} - \frac{3}{8}$$

Combine like terms on the left side.

$$x = 3\frac{1}{2} - \frac{3}{8}$$

Convert mixed number to improper fraction.

$$x = \frac{7}{2} - \frac{3}{8}$$

Convert to equivalent fractions with LCD of 8.

$$x = \frac{28}{8} - \frac{3}{8}$$

Subtract.

$$x = \frac{25}{8}$$

Write as a mixed number.

$$x = 3\frac{1}{8}$$

We write the answer as a mixed number because the original problem used a mixed number.

Check:

Is the sum of three-eighths and $3\frac{1}{8}$ equal to three and one-half?

$$\frac{3}{8} + 3\frac{1}{8} \stackrel{?}{=} 3\frac{1}{2}$$

Add.

$$3\frac{4}{8} \stackrel{?}{=} 3\frac{1}{2}$$

Simplify.

$$3\frac{1}{2} = 3\frac{1}{2} \checkmark$$

The solution checks.

Note:

Exercise:

Problem:

Translate and solve: The sum of five-eighths and x is one-fourth.

Solution:

$$\frac{5}{8} + x = \frac{1}{4}; x = -\frac{3}{8}$$

Note:

Exercise:

Problem:

Translate and solve: The difference of one-and-three-fourths and x is five-sixths.

Solution:

$$1\frac{3}{4} - x = \frac{5}{6}; x = \frac{11}{12}$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solve One Step Equations With Fractions](#)
- [Solve One Step Equations With Fractions by Adding or Subtracting](#)
- [Solve One Step Equations With Fraction by Multiplying](#)

Key Concepts

- **Determine whether a number is a solution to an equation.**

Substitute the number for the variable in the equation.

Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true. If it is true, the number is a solution. If it is not true, the number is not a solution.

- **Addition, Subtraction, and Division Properties of Equality**
 - For any numbers a , b , and c ,
if $a = b$, then $a + c = b + c$. Addition Property of Equality
 - if $a = b$, then $a - c = b - c$. Subtraction Property of Equality
 - if $a = b$, then $\frac{a}{c} = \frac{b}{c}$, $c \neq 0$. Division Property of Equality
- **The Multiplication Property of Equality**
 - For any numbers a , b , and c , $a = b$, then $ac = bc$.
 - If you multiply both sides of an equation by the same quantity, you still have equality.

Section Exercises

Practice Makes Perfect

Determine Whether a Fraction is a Solution of an Equation

In the following exercises, determine whether each number is a solution of the given equation.

Exercise:

Problem: $x - \frac{2}{5} = \frac{1}{10}:$

Ⓐ $x = 1$

Ⓑ $x = \frac{1}{2}$

Ⓒ $x = -\frac{1}{2}$

Exercise:

Problem: $y - \frac{1}{3} = \frac{5}{12}:$

Ⓐ $y = 1$

Ⓑ $y = \frac{3}{4}$

Ⓒ $y = -\frac{3}{4}$

Solution:

Ⓐ no

Ⓑ yes

Ⓒ no

Exercise:

Problem: $h + \frac{3}{4} = \frac{2}{5}:$

Ⓐ $h = 1$

Ⓑ $h = \frac{7}{20}$

Ⓒ $h = -\frac{7}{20}$

Exercise:

Problem: $k + \frac{2}{5} = \frac{5}{6}$:

Ⓐ $k = 1$

Ⓑ $k = \frac{13}{30}$

Ⓒ $k = -\frac{13}{30}$

Solution:

Ⓐ no

Ⓑ yes

Ⓒ no

Solve Equations with Fractions using the Addition, Subtraction, and Division Properties of Equality

In the following exercises, solve.

Exercise:

Problem: $y + \frac{1}{3} = \frac{4}{3}$

Exercise:

Problem: $m + \frac{3}{8} = \frac{7}{8}$

Solution:

$$m = \frac{1}{2}$$

Exercise:

Problem: $f + \frac{9}{10} = \frac{2}{5}$

Exercise:

Problem: $h + \frac{5}{6} = \frac{1}{6}$

Solution:

$$h = -\frac{2}{3}$$

Exercise:

Problem: $a - \frac{5}{8} = -\frac{7}{8}$

Exercise:

Problem: $c - \frac{1}{4} = -\frac{5}{4}$

Solution:

$$c = -1$$

Exercise:

Problem: $x - \left(-\frac{3}{20}\right) = -\frac{11}{20}$

Exercise:

Problem: $z - \left(-\frac{5}{12}\right) = -\frac{7}{12}$

Solution:

$$z = -1$$

Exercise:

Problem: $n - \frac{1}{6} = \frac{3}{4}$

Exercise:

Problem: $p - \frac{3}{10} = \frac{5}{8}$

Solution:

$$p = \frac{37}{40}$$

Exercise:

Problem: $s + \left(-\frac{1}{2}\right) = -\frac{8}{9}$

Exercise:

Problem: $k + \left(-\frac{1}{3}\right) = -\frac{4}{5}$

Solution:

$$k = -\frac{7}{15}$$

Exercise:

Problem: $5j = 17$

Exercise:

Problem: $7k = 18$

Solution:

$$k = \frac{18}{7}$$

Exercise:

Problem: $-4w = 26$

Exercise:

Problem: $-9v = 33$

Solution:

$$v = -\frac{11}{3}$$

Solve Equations with Fractions Using the Multiplication Property of Equality

In the following exercises, solve.

Exercise:

Problem: $\frac{f}{4} = -20$

Exercise:

Problem: $\frac{b}{3} = -9$

Solution:

$$b = -27$$

Exercise:

Problem: $\frac{y}{7} = -21$

Exercise:

Problem: $\frac{x}{8} = -32$

Solution:

$$x = -256$$

Exercise:

Problem: $\frac{p}{-5} = -40$

Exercise:

Problem: $\frac{q}{-4} = -40$

Solution:

$$q = 160$$

Exercise:

Problem: $\frac{r}{-12} = -6$

Exercise:

Problem: $\frac{s}{-15} = -3$

Solution:

$$s = 45$$

Exercise:

Problem: $-x = 23$

Exercise:

Problem: $-y = 42$

Solution:

$$y = -42$$

Exercise:

Problem: $-h = -\frac{5}{12}$

Exercise:

Problem: $-k = -\frac{17}{20}$

Solution:

$$k = \frac{17}{20}$$

Exercise:

Problem: $\frac{4}{5}n = 20$

Exercise:

Problem: $\frac{3}{10}p = 30$

Solution:

$$p = 100$$

Exercise:

Problem: $\frac{3}{8}q = -48$

Exercise:

Problem: $\frac{5}{2}m = -40$

Solution:

$$m = -16$$

Exercise:

Problem: $-\frac{2}{9}a = 16$

Exercise:

Problem: $-\frac{3}{7}b = 9$

Solution:

$$b = -21$$

Exercise:

Problem: $-\frac{6}{11}u = -24$

Exercise:

Problem: $-\frac{5}{12}v = -15$

Solution:

$$v = 36$$

Mixed Practice

In the following exercises, solve.

Exercise:

Problem: $3x = 0$

Exercise:

Problem: $8y = 0$

Solution:

$$y = 0$$

Exercise:

Problem: $4f = \frac{4}{5}$

Exercise:

Problem: $7g = \frac{7}{9}$

Solution:

$$g = \frac{1}{9}$$

Exercise:

Problem: $p + \frac{2}{3} = \frac{1}{12}$

Exercise:

Problem: $q + \frac{5}{6} = \frac{1}{12}$

Solution:

$$q = -\frac{3}{4}$$

Exercise:

Problem: $\frac{7}{8}m = \frac{1}{10}$

Exercise:

Problem: $\frac{1}{4}n = \frac{7}{10}$

Solution:

$$n = \frac{14}{5}$$

Exercise:

Problem: $-\frac{2}{5} = x + \frac{3}{4}$

Exercise:

Problem: $-\frac{2}{3} = y + \frac{3}{8}$

Solution:

$$y = -\frac{25}{24}$$

Exercise:

Problem: $\frac{11}{20} = -f$

Exercise:

Problem: $\frac{8}{15} = -d$

Solution:

$$d = -\frac{8}{15}$$

Translate Sentences to Equations and Solve

In the following exercises, translate to an algebraic equation and solve.

Exercise:

Problem: n divided by eight is -16 .

Exercise:

Problem: n divided by six is -24 .

Solution:

$$\frac{n}{6} = -24; n = -144$$

Exercise:

Problem: m divided by -9 is -7 .

Exercise:

Problem: m divided by -7 is -8 .

Solution:

$$\frac{m}{-7} = -8; m = 56$$

Exercise:

Problem: The quotient of f and -3 is -18 .

Exercise:

Problem: The quotient of f and -4 is -20 .

Solution:

$$\frac{f}{-4} = -20; f = 80$$

Exercise:

Problem: The quotient of g and twelve is 8 .

Exercise:

Problem: The quotient of g and nine is 14 .

Solution:

$$\frac{g}{9} = 14; g = 126$$

Exercise:

Problem: Three-fourths of q is 12 .

Exercise:

Problem: Two-fifths of q is 20.

Solution:

$$\frac{2}{5}q = 20; q = 50$$

Exercise:

Problem: Seven-tenths of p is -63 .

Exercise:

Problem: Four-ninths of p is -28 .

Solution:

$$\frac{4}{9}p = -28; p = -63$$

Exercise:

Problem: m divided by 4 equals negative 6.

Exercise:

Problem: The quotient of h and 2 is 43.

Solution:

$$\frac{h}{2} = 43$$

Exercise:

Problem: Three-fourths of z is the same as 15.

Exercise:

Problem: The quotient of a and $\frac{2}{3}$ is $\frac{3}{4}$.

Solution:

$$\frac{\frac{a}{2}}{\frac{2}{3}} = \frac{3}{4}$$

Exercise:

Problem: The sum of five-sixths and x is $\frac{1}{2}$.

Exercise:

Problem: The sum of three-fourths and x is $\frac{1}{8}$.

Solution:

$$\frac{3}{4} + x = \frac{1}{8}; x = -\frac{5}{8}$$

Exercise:

Problem: The difference of y and one-fourth is $-\frac{1}{8}$.

Exercise:

Problem: The difference of y and one-third is $-\frac{1}{6}$.

Solution:

$$y - \frac{1}{3} = -\frac{1}{6}; y = \frac{1}{6}$$

Everyday Math

Exercise:

Problem:

Shopping Teresa bought a pair of shoes on sale for \$48. The sale price was $\frac{2}{3}$ of the regular price. Find the regular price of the shoes by solving the equation $\frac{2}{3}p = 48$

Exercise:**Problem:**

Playhouse The table in a child's playhouse is $\frac{3}{5}$ of an adult-size table. The playhouse table is 18 inches high. Find the height of an adult-size table by solving the equation $\frac{3}{5}h = 18$.

Solution:

30 inches

Writing Exercises**Exercise:****Problem:**

[\[link\]](#) describes three methods to solve the equation $-y = 15$. Which method do you prefer? Why?

Exercise:**Problem:**

Richard thinks the solution to the equation $\frac{3}{4}x = 24$ is 16. Explain why Richard is wrong.

Solution:

Answers will vary.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether a fraction is a solution of an equation.			
solve equations with fractions using the addition, subtraction, and division properties of equality.			
solve equations using the multiplication property of equality.			
translate sentences to equations and solve.			

Ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Visualize Fractions

In the following exercises, name the fraction of each figure that is shaded.

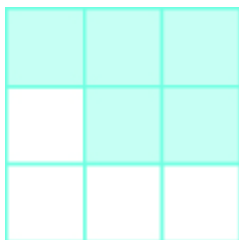
Exercise:

Problem:



Exercise:

Problem:



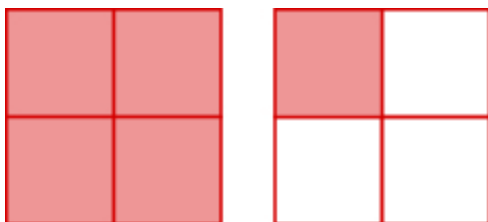
Solution:

$$\frac{5}{9}$$

In the following exercises, name the improper fractions. Then write each improper fraction as a mixed number.

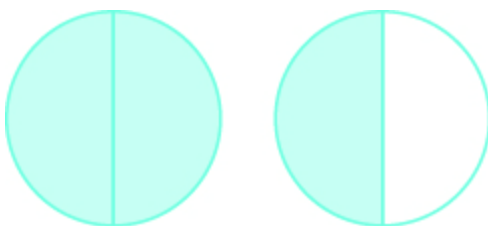
Exercise:

Problem:



Exercise:

Problem:



Solution:

$$\frac{3}{2}$$

In the following exercises, convert the improper fraction to a mixed number.

Exercise:

Problem: $\frac{58}{15}$

Exercise:

Problem: $\frac{63}{11}$

Solution:

$$5\frac{8}{11}$$

In the following exercises, convert the mixed number to an improper fraction.

Exercise:

Problem: $12\frac{1}{4}$

Exercise:

Problem: $9\frac{4}{5}$

Solution:

$$\frac{49}{5}$$

Exercise:

Problem:

Find three fractions equivalent to $\frac{2}{5}$. Show your work, using figures or algebra.

Exercise:

Problem:

Find three fractions equivalent to $-\frac{4}{3}$. Show your work, using figures or algebra.

Solution:

Answers may vary.

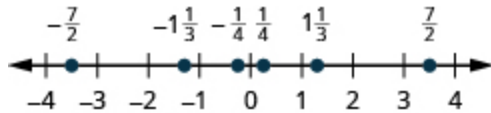
In the following exercises, locate the numbers on a number line.

Exercise:

Problem: $\frac{5}{8}, \frac{4}{3}, 3\frac{3}{4}, 4$

Exercise:

Problem: $\frac{1}{4}, -\frac{1}{4}, 1\frac{1}{3}, -1\frac{1}{3}, \frac{7}{2}, -\frac{7}{2}$

Solution:

In the following exercises, order each pair of numbers, using $<$ or $>$.

Exercise:

Problem: -1 ____ $-\frac{2}{5}$

Exercise:

Problem: $-2\frac{1}{2}$ ____ -3

Solution:

>

Multiply and Divide Fractions

In the following exercises, simplify.

Exercise:

Problem: $-\frac{63}{84}$

Exercise:

Problem: $-\frac{90}{120}$

Solution:

$$-\frac{3}{4}$$

Exercise:

Problem: $-\frac{14a}{14b}$

Exercise:

Problem: $-\frac{8x}{8y}$

Solution:

$$-\frac{x}{y}$$

In the following exercises, multiply.

Exercise:

Problem: $\frac{2}{5} \cdot \frac{8}{13}$

Exercise:

Problem: $-\frac{1}{3} \cdot \frac{12}{7}$

Solution:

$$-\frac{4}{7}$$

Exercise:

Problem: $\frac{2}{9} \cdot \left(-\frac{45}{32}\right)$

Exercise:

Problem: $6m \cdot \frac{4}{11}$

Solution:

$$\frac{24}{11}m$$

Exercise:

Problem: $-\frac{1}{4}(-32)$

Exercise:

Problem: $3\frac{1}{5} \cdot 1\frac{7}{8}$

Solution:

$$6$$

In the following exercises, find the reciprocal.

Exercise:

Problem: $\frac{2}{9}$

Exercise:

Problem: $\frac{15}{4}$

Solution:

$$\frac{4}{15}$$

Exercise:

Problem: 3

Exercise:

Problem: $-\frac{1}{4}$

Solution:

$$-4$$

Exercise:

Problem: Fill in the chart.

	Opposite	Absolute Value	Reciprocal
$-\frac{5}{13}$			

	Opposite	Absolute Value	Reciprocal
$\frac{3}{10}$			
$\frac{9}{4}$			
-12			

In the following exercises, divide.

Exercise:

Problem: $\frac{2}{3} \div \frac{1}{6}$

Solution:

4

Exercise:

Problem: $\left(-\frac{3x}{5}\right) \div \left(-\frac{2y}{3}\right)$

Exercise:

Problem: $\frac{4}{5} \div 3$

Solution:

$\frac{4}{15}$

Exercise:

Problem: $8 \div 2\frac{2}{3}$

Exercise:

Problem: $8\frac{2}{3} \div 1\frac{1}{12}$

Solution:

8

Multiply and Divide Mixed Numbers and Complex Fractions

In the following exercises, perform the indicated operation.

Exercise:

Problem: $3\frac{1}{5} \cdot 1\frac{7}{8}$

Exercise:

Problem: $-5\frac{7}{12} \cdot 4\frac{4}{11}$

Solution:

$$-\frac{268}{11}$$

Exercise:

Problem: $8 \div 2\frac{2}{3}$

Exercise:

Problem: $8\frac{2}{3} \div 1\frac{1}{12}$

Solution:

8

In the following exercises, translate the English phrase into an algebraic expression.

Exercise:

Problem: the quotient of 8 and y

Exercise:

Problem: the quotient of V and the difference of h and 6

Solution:

$$\frac{V}{h-6}$$

In the following exercises, simplify the complex fraction

Exercise:

Problem: $\frac{\frac{5}{8}}{\frac{4}{5}}$

Exercise:

Problem: $\frac{\frac{8}{9}}{-4}$

Solution:

$$-\frac{2}{9}$$

Exercise:

Problem: $\frac{\frac{n}{4}}{\frac{3}{8}}$

Exercise:

Problem: $\frac{-1\frac{5}{6}}{-\frac{1}{12}}$

Solution:

22

In the following exercises, simplify.

Exercise:

Problem: $\frac{5+16}{5}$

Exercise:

Problem: $\frac{8 \cdot 4 - 5^2}{3 \cdot 12}$

Solution:

$$\frac{7}{36}$$

Exercise:

Problem: $\frac{8 \cdot 7 + 5(8-10)}{9 \cdot 3 - 6 \cdot 4}$

Add and Subtract Fractions with Common Denominators

In the following exercises, add.

Exercise:

Problem: $\frac{3}{8} + \frac{2}{8}$

Solution:

$$\frac{5}{8}$$

Exercise:

Problem: $\frac{4}{5} + \frac{1}{5}$

Exercise:

Problem: $\frac{2}{5} + \frac{1}{5}$

Solution:

$$\frac{3}{5}$$

Exercise:

Problem: $\frac{15}{32} + \frac{9}{32}$

Exercise:

Problem: $\frac{x}{10} + \frac{7}{10}$

Solution:

$$\frac{x+7}{10}$$

In the following exercises, subtract.

Exercise:

Problem: $\frac{8}{11} - \frac{6}{11}$

Exercise:

Problem: $\frac{11}{12} - \frac{5}{12}$

Solution:

$$\frac{1}{2}$$

Exercise:

Problem: $\frac{4}{5} - \frac{y}{5}$

Exercise:

Problem: $-\frac{31}{30} - \frac{7}{30}$

Solution:

$$-\frac{19}{15}$$

Exercise:

Problem: $\frac{3}{2} - \left(\frac{3}{2}\right)$

Exercise:

Problem: $\frac{11}{15} - \frac{5}{15} - \left(-\frac{2}{15}\right)$

Solution:

$$\frac{8}{15}$$

Add and Subtract Fractions with Different Denominators

In the following exercises, find the least common denominator.

Exercise:

Problem: $\frac{1}{3}$ and $\frac{1}{12}$

Exercise:

Problem: $\frac{1}{3}$ and $\frac{4}{5}$

Solution:

15

Exercise:

Problem: $\frac{8}{15}$ and $\frac{11}{20}$

Exercise:

Problem: $\frac{3}{4}$, $\frac{1}{6}$, and $\frac{5}{10}$

Solution:

60

In the following exercises, change to equivalent fractions using the given LCD.

Exercise:

Problem: $\frac{1}{3}$ and $\frac{1}{5}$, LCD = 15

Exercise:

Problem: $\frac{3}{8}$ and $\frac{5}{6}$, LCD = 24

Solution:

$\frac{9}{24}$ and $\frac{20}{24}$

Exercise:

Problem: $-\frac{9}{16}$ and $\frac{5}{12}$, LCD = 48

Exercise:

Problem: $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$, LCD = 60

Solution:

$$\frac{20}{60}, \frac{15}{60} \text{ and } \frac{48}{60}$$

In the following exercises, perform the indicated operations and simplify.

Exercise:

Problem: $\frac{1}{5} + \frac{2}{3}$

Exercise:

Problem: $\frac{11}{12} - \frac{2}{3}$

Solution:

$$\frac{1}{4}$$

Exercise:

Problem: $-\frac{9}{10} - \frac{3}{4}$

Exercise:

Problem: $-\frac{11}{36} - \frac{11}{20}$

Solution:

$$-\frac{77}{90}$$

Exercise:

Problem: $-\frac{22}{25} + \frac{9}{40}$

Exercise:

Problem: $\frac{y}{10} - \frac{1}{3}$

Solution:

$$\frac{3y-10}{30}$$

Exercise:

Problem: $\frac{2}{5} + \left(-\frac{5}{9}\right)$

Exercise:

Problem: $\frac{4}{11} \div \frac{2}{7d}$

Solution:

$$\frac{14d}{11}$$

Exercise:

Problem: $\frac{2}{5} + \left(-\frac{3n}{8}\right) \left(-\frac{2}{9n}\right)$

Exercise:

Problem: $\frac{\left(\frac{2}{3}\right)^2}{\left(\frac{5}{8}\right)^2}$

Solution:

$$\frac{256}{225}$$

Exercise:

Problem: $\left(\frac{11}{12} + \frac{3}{8}\right) \div \left(\frac{5}{6} - \frac{1}{10}\right)$

In the following exercises, evaluate.

Exercise:

Problem: $y - \frac{4}{5}$ when

Ⓐ $y = -\frac{4}{5}$

Ⓑ $y = \frac{1}{4}$

Solution:

Ⓐ $-\frac{8}{5}$

Ⓑ $-\frac{11}{20}$

Exercise:

Problem: $6mn^2$ when $m = \frac{3}{4}$ and $n = -\frac{1}{3}$

Add and Subtract Mixed Numbers

In the following exercises, perform the indicated operation.

Exercise:

Problem: $4\frac{1}{3} + 9\frac{1}{3}$

Solution:

$13\frac{2}{3}$

Exercise:

Problem: $6\frac{2}{5} + 7\frac{3}{5}$

Exercise:

Problem: $5\frac{8}{11} + 2\frac{4}{11}$

Solution:

$$8\frac{1}{11}$$

Exercise:

Problem: $3\frac{5}{8} + 3\frac{7}{8}$

Exercise:

Problem: $9\frac{13}{20} - 4\frac{11}{20}$

Solution:

$$5\frac{1}{10}$$

Exercise:

Problem: $2\frac{3}{10} - 1\frac{9}{10}$

Exercise:

Problem: $2\frac{11}{12} - 1\frac{7}{12}$

Solution:

$$\frac{10}{3}$$

Exercise:

Problem: $8\frac{6}{11} - 2\frac{9}{11}$

Solve Equations with Fractions

In the following exercises, determine whether the each number is a solution of the given equation.

Exercise:

Problem: $x - \frac{1}{2} = \frac{1}{6}:$

Ⓐ $x = 1$

Ⓑ $x = \frac{2}{3}$

Ⓒ $x = -\frac{1}{3}$

Solution:

Ⓐ no

Ⓑ yes

Ⓒ no

Exercise:

Problem: $y + \frac{3}{5} = \frac{5}{9}:$

Ⓐ $y = \frac{1}{2}$

Ⓑ $y = \frac{52}{45}$

Ⓒ $y = -\frac{2}{45}$

In the following exercises, solve the equation.

Exercise:

Problem: $n + \frac{9}{11} = \frac{4}{11}$

Solution:

$$n = -\frac{5}{11}$$

Exercise:

Problem: $x - \frac{1}{6} = \frac{7}{6}$

Exercise:

Problem: $h - \left(-\frac{7}{8}\right) = -\frac{2}{5}$

Solution:

$$h = -\frac{51}{40}$$

Exercise:

Problem: $\frac{x}{5} = -10$

Exercise:

Problem: $-z = 23$

Solution:

$$z = -23$$

In the following exercises, translate and solve.

Exercise:

Problem: The sum of two-thirds and n is $-\frac{3}{5}$.

Exercise:

Problem: The difference of q and one-tenth is $\frac{1}{2}$.

Solution:

$$q - \frac{1}{10} = \frac{1}{2}; q = \frac{3}{5}$$

Exercise:

Problem: The quotient of p and -4 is -8 .

Exercise:

Problem: Three-eighths of y is 24.

Solution:

$$\frac{3}{8}y = 24; y = 64$$

Chapter Practice Test

Convert the improper fraction to a mixed number.

Exercise:

Problem: $\frac{19}{5}$

Convert the mixed number to an improper fraction.

Exercise:

Problem: $3\frac{2}{7}$

Solution:

$$\frac{23}{7}$$

Locate the numbers on a number line.

Exercise:

Problem: $\frac{1}{2}$, $1\frac{2}{3}$, $-2\frac{3}{4}$, and $\frac{9}{4}$

In the following exercises, simplify.

Exercise:

Problem: $\frac{5}{20}$

Solution:

$$\frac{1}{4}$$

Exercise:

Problem: $\frac{18r}{27s}$

Exercise:

Problem: $\frac{1}{3} \cdot \frac{3}{4}$

Solution:

$$\frac{1}{4}$$

Exercise:

Problem: $\frac{3}{5} \cdot 15$

Exercise:

Problem: $-36u \left(-\frac{4}{9}\right)$

Solution:

$$16u$$

Exercise:

Problem: $-5\frac{7}{12} \cdot 4\frac{4}{11}$

Exercise:

Problem: $-\frac{5}{6} \div \frac{5}{12}$

Solution:

$$-2$$

Exercise:

Problem: $\frac{7}{11} \div \left(-\frac{7}{11}\right)$

Exercise:

Problem: $\frac{9a}{10} \div \frac{15a}{8}$

Solution:

$$\frac{12}{25}$$

Exercise:

Problem: $-6\frac{2}{5} \div 4$

Exercise:

Problem: $\left(-15\frac{5}{6}\right) \div \left(-3\frac{1}{6}\right)$

Solution:

$$5$$

Exercise:

Problem: $\frac{-6}{\frac{6}{11}}$

Exercise:

Problem: $\frac{\frac{p}{2}}{\frac{q}{5}}$

Solution:

$$\frac{5p}{2q}$$

Exercise:

Problem: $\frac{-\frac{4}{15}}{-2\frac{2}{3}}$

Exercise:

Problem: $\frac{9^2-4^2}{9-4}$

Solution:

$$13$$

Exercise:

Problem: $\frac{2}{d} + \frac{9}{d}$

Exercise:

Problem: $-\frac{3}{13} + \left(-\frac{4}{13}\right)$

Solution:

$$-\frac{7}{13}$$

Exercise:

Problem: $-\frac{22}{25} + \frac{9}{40}$

Exercise:

Problem: $\frac{2}{5} + \left(-\frac{7}{5}\right)$

Solution:

$$-1$$

Exercise:

Problem: $-\frac{3}{10} + \left(-\frac{5}{8}\right)$

Exercise:

Problem: $-\frac{3}{4} \div \frac{x}{3}$

Solution:

$$-\frac{9}{4x}$$

Exercise:

Problem: $\frac{2^3 - 2^2}{\left(\frac{3}{4}\right)^2}$

Exercise:

Problem: $\frac{\frac{5}{14} + \frac{1}{8}}{\frac{9}{56}}$

Solution:

$$3$$

Evaluate.

Exercise:

Problem: $x + \frac{1}{3}$ when

$$\textcircled{a} x = \frac{2}{3}$$

$$\textcircled{b} x = -\frac{5}{6}$$

In the following exercises, solve the equation.

Exercise:

Problem: $y + \frac{3}{5} = \frac{7}{5}$

Solution:

$$y = \frac{4}{5}$$

Exercise:

Problem: $a - \frac{3}{10} = -\frac{9}{10}$

Exercise:

Problem: $f + \left(-\frac{2}{3}\right) = \frac{5}{12}$

Solution:

$$f = \frac{13}{12}$$

Exercise:

Problem: $\frac{m}{-2} = -16$

Exercise:

Problem: $-\frac{2}{3}c = 18$

Solution:

$$c = -27$$

Exercise:

Problem:

Translate and solve: The quotient of p and -4 is -8 . Solve for p .

Solve Equations with Decimals

By the end of this section, you will be able to:

- Determine whether a decimal is a solution of an equation
- Solve equations with decimals
- Translate to an equation and solve

Note:

Before you get started, take this readiness quiz.

1. Evaluate $x + \frac{2}{3}$ when $x = -\frac{1}{4}$.

If you missed this problem, review [\[link\]](#).

2. Evaluate $15 - y$ when $y = -5$.

If you missed this problem, review [\[link\]](#).

3. Solve $\frac{n}{-7} = 42$.

If you missed this problem, review [\[link\]](#).

Determine Whether a Decimal is a Solution of an Equation

Solving equations with decimals is important in our everyday lives because money is usually written with decimals. When applications involve money, such as shopping for yourself, making your family's budget, or planning for the future of your business, you'll be solving equations with decimals.

Now that we've worked with decimals, we are ready to find solutions to equations involving decimals. The steps we take to determine whether a number is a solution to an equation are the same whether the solution is a whole number, an integer, a fraction, or a decimal. We'll list these steps here again for easy reference.

Note:

Determine whether a number is a solution to an equation.

Substitute the number for the variable in the equation.

Simplify the expressions on both sides of the equation.

Determine whether the resulting equation is true.

- If so, the number is a solution.
- If not, the number is not a solution.

Example:

Exercise:

Problem:

Determine whether each of the following is a solution of $x - 0.7 = 1.5$:

Ⓐ $x = 1$ Ⓑ $x = -0.8$ Ⓒ $x = 2.2$

Solution:

Solution

Ⓐ

$$x - 0.7 = 1.5$$

Substitute 1 for x .

	$1 - 0.7 \stackrel{?}{=} 1.5$
Subtract.	$0.3 \neq 1.5$

Since $x = 1$ does not result in a true equation, 1 is not a solution to the equation.

ⓑ	
	$x - 0.7 = 1.5$
Substitute -0.8 for x .	$-0.8 - 0.7 \stackrel{?}{=} 1.5$
Subtract.	$-1.5 \neq 1.5$

Since $x = -0.8$ does not result in a true equation, -0.8 is not a solution to the equation.

ⓒ	
---	--

	$x - 0.7 = 1.5$
Substitute 2.2 for x .	$2.2 - 0.7 \stackrel{?}{=} 1.5$
Subtract.	$1.5 = 1.5 \checkmark$

Since $x = 2.2$ results in a true equation, 2.2 is a solution to the equation.

Note:

Exercise:

Problem:

Determine whether each value is a solution of the given equation.

$$x - 0.6 = 1.3 : \textcircled{a} x = 0.7 \textcircled{b} x = 1.9 \textcircled{c} x = -0.7$$

Solution:

- $\textcircled{a}$ no
- $\textcircled{b}$ yes
- $\textcircled{c}$ no

Note:

Exercise:

Problem:

Determine whether each value is a solution of the given equation.

$$y - 0.4 = 1.7 : \textcircled{a} y = 2.1 \textcircled{b} y = 1.3 \textcircled{c} -1.3$$

Solution:

- ☐ (a) yes
- ☐ (b) no
- ☐ (c) no

Solve Equations with Decimals

In previous chapters, we solved equations using the Properties of Equality. We will use these same properties to solve equations with decimals.

Note:

Properties of Equality

Subtraction Property of Equality

For any numbers a , b , and c ,
If $a = b$, then $a - c = b - c$.

Addition Property of Equality

For any numbers a , b , and c ,
If $a = b$, then $a + c = b + c$.

The Division Property of Equality**The Multiplication Property of Equality**

For any numbers
 a , b , and c , and $c \neq 0$
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$

For any numbers a , b , and c ,
If $a = b$, then $ac = bc$

When you add, subtract, multiply or divide the same quantity from both sides of an equation, you still have equality.

Example:

Exercise:

Problem: Solve: $y + 2.3 = -4.7$.

Solution:

Solution

We will use the Subtraction Property of Equality to isolate the variable.

	$y + 2.3 = -4.7$
Subtract 2.3 from each side, to undo the addition.	$y + 2.3 - 2.3 = -4.7 - 2.3$
Simplify.	$y = -7$

Check:	$y + 2.3 = -4.7$	
Substitute $y = -7$.	$-7 + 2.3 \stackrel{?}{=} -4.7$	
Simplify.	$-4.7 = -4.7 \checkmark$	

Since $y = -7$ makes $y + 2.3 = -4.7$ a true statement, we know we have found a solution to this equation.

Note:

Exercise:

Problem: Solve: $y + 2.7 = -5.3$.

Solution:

$$y = -8$$

Note:

Exercise:

Problem: Solve: $y + 3.6 = -4.8$.

Solution:

$$y = -8.4$$

Example:

Exercise:

Problem: Solve: $a - 4.75 = -1.39$.

Solution:

Solution

We will use the Addition Property of Equality.

		$a - 4.75 = -1.39$
Add 4.75 to each side, to undo the subtraction.		$a - 4.75 + 4.75 = -1.39 + 4.75$
Simplify.		$a = 3.36$
Check:	$a - 4.75 = -1.39$	
Substitute $a = 3.36$.	$3.36 - 4.75 \stackrel{?}{=} -1.39$	
	$-1.39 = -1.39 \checkmark$	

Since the result is a true statement, $a = 3.36$ is a solution to the equation.

Note:

Exercise:

Problem: Solve: $a - 3.93 = -2.86$.

Solution:

$$a = 1.07$$

Note:

Exercise:

Problem: Solve: $n - 3.47 = -2.64$.

Solution:

$$n = 0.83$$

Example:

Exercise:

Problem: Solve: $-4.8 = 0.8n$.

Solution:
Solution

We will use the Division Property of Equality.

Use the Properties of Equality to find a value for n .

		$-4.8 = 0.8n$
We must divide both sides by 0.8 to isolate n .		$\frac{-4.8}{0.8} = \frac{0.8n}{0.8}$
Simplify.		$-6 = n$
Check:	$-4.8 = 0.8n$	
Substitute $n = -6$.	$-4.8 \stackrel{?}{=} 0.8(-6)$	
	$-4.8 = -4.8 \checkmark$	

Since $n = -6$ makes $-4.8 = 0.8n$ a true statement, we know we have a solution.

Note:

Exercise:

Problem: Solve: $-8.4 = 0.7b$.

Solution:

$$b = -12$$

Note:

Exercise:

Problem: Solve: $-5.6 = 0.7c$.

Solution:

$$c = -8$$

Example:

Exercise:

Problem: Solve: $\frac{p}{-1.8} = -6.5$.

Solution:

Solution

We will use the Multiplication Property of Equality.

		$\frac{p}{-1.8} = -6.5$
Here, p is divided by -1.8 . We must multiply by -1.8 to isolate p		$-1.8\left(\frac{p}{-1.8}\right) = -1.8(-6.5)$
Multiply.		$p = 11.7$
Check:	$\frac{p}{-1.8} = -6.5$	
Substitute $p = 11.7$.	$\frac{11.7}{-1.8} \stackrel{?}{=} -6.5$	
	$-6.5 = -6.5 \checkmark$	
A solution to $\frac{p}{-1.8} = -6.5$ is $p = 11.7$.		

Note:

Exercise:

Problem: Solve: $\frac{c}{-2.6} = -4.5$.

Solution:

$$c = 11.7$$

Note:

Exercise:

Problem: Solve: $\frac{b}{-1.2} = -5.4$.

Solution:

$$b = 6.48$$

Translate to an Equation and Solve

Now that we have solved equations with decimals, we are ready to translate word sentences to equations and solve. Remember to look for words and phrases that indicate the operations to use.

Example:

Exercise:

Problem: Translate and solve: The difference of n and 4.3 is 2.1.

Solution:

Solution

Translate.

The difference of n and 4.3		is	2.1.
$n - 4.3$		=	2.1

Add 4.3 to both sides of the equation.		$n - 4.3 + 4.3 = 2.1 + 4.3$
Simplify.		$n = 6.4$
Check:	Is the difference of n and 4.3 equal to 2.1?	
Let $n = 6.4$:	Is the difference of 6.4 and 4.3 equal to 2.1?	
Translate.	$6.4 - 4.3 \stackrel{?}{=} 2.1$	
Simplify.	$2.1 = 2.1 \checkmark$	

Note:
Exercise:
Problem: Translate and solve: The difference of y and 4.9 is 2.8.
Solution:
$y - 4.9 = 2.8; y = 7.7$

Note:
Exercise:

Problem: Translate and solve: The difference of z and 5.7 is 3.4.

Solution:

$$z - 5.7 = 3.4; z = 9.1$$

Example:

Exercise:

Problem: Translate and solve: The product of -3.1 and x is 5.27.

Solution:

Solution

Translate.		The product of 3.1 and x is 5.27. $-3.1x = 5.27$
Divide both sides by -3.1 .		$\frac{-3.1x}{-3.1} = \frac{5.27}{-3.1}$
Simplify.		$x = -1.7$
Check:	Is the product of -3.1 and x equal to 5.27?	
Let	Is the product of -3.1	

$x = -1.7$:	and -1.7 equal to 5.27 ?	
Translate.	$-3.1(-1.7) \stackrel{?}{=} 5.27$	
Simplify.	$5.27 = 5.27 \checkmark$	

Note:

Exercise:

Problem: Translate and solve: The product of -4.3 and x is 12.04 .

Solution:

$$-4.3x = 12.04; x = -2.8$$

Note:

Exercise:

Problem: Translate and solve: The product of -3.1 and m is 26.66 .

Solution:

$$-3.1m = 26.66; m = -8.6$$

Example:

Exercise:

Problem: Translate and solve: The quotient of p and -2.4 is 6.5 .

Solution:
Solution

Translate.		The quotient of p and -2.4 is 6.5. $\frac{p}{-2.4} = 6.5$
Multiply both sides by -2.4 .		$-2.4(\frac{p}{-2.4}) = -2.4(6.5)$
Simplify.		$p = -15.6$
Check:	Is the quotient of p and -2.4 equal to 6.5 ?	
Let $p = -15.6$:	Is the quotient of -15.6 and -2.4 equal to 6.5 ?	
Translate.	$\frac{-15.6}{-2.4} \stackrel{?}{=} 6.5$	
Simplify.	$6.5 = 6.5 \quad \checkmark$	

Note:

Exercise:

Problem: Translate and solve: The quotient of q and -3.4 is 4.5 .

Solution:

$$\frac{q}{-3.4} = 4.5; q = -15.3$$

Note:

Exercise:

Problem: Translate and solve: The quotient of r and -2.6 is 2.5 .

Solution:

$$\frac{r}{-2.6} = 2.5; r = -6.5$$

Example:

Exercise:

Problem: Translate and solve: The sum of n and 2.9 is 1.7 .

Solution:

Solution

Translate.		The sum of n and 2.9 is 1.7. $n + 2.9 = 1.7$
Subtract 2.9 from each side.		$n + 2.9 - 2.9 = 1.7 - 2.9$
Simplify.		$n = -1.2$
Check:	Is the sum n and 2.9 equal to 1.7?	
Let $n = -1.2$:	Is the sum -1.2 and 2.9 equal to 1.7?	
Translate.	$-1.2 + 2.9 \stackrel{?}{=} 1.7$	
Simplify.	$1.7 = 1.7 \quad \checkmark$	

Note:

Exercise:

Problem: Translate and solve: The sum of j and 3.8 is 2.6.

Solution:

$$j + 3.8 = 2.6; j = -1.2$$

Note:

Exercise:

Problem: Translate and solve: The sum of k and 4.7 is 0.3.

Solution:

$$k + 4.7 = 0.3; k = -4.4$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solving One Step Equations Involving Decimals](#)
- [Solve a One Step Equation With Decimals by Adding and Subtracting](#)
- [Solve a One Step Equation With Decimals by Multiplying](#)
- [Solve a One Step Equation With Decimals by Dividing](#)

Key Concepts

- **Determine whether a number is a solution to an equation.**
 - Substitute the number for the variable in the equation.
 - Simplify the expressions on both sides of the equation.
 - Determine whether the resulting equation is true.
 - If so, the number is a solution.
 - If not, the number is not a solution.
- **Properties of Equality**

Subtraction Property of Equality	Addition Property of Equality
For any numbers a , b , and c , If $a = b$ then $a - c = b - c$	For any numbers a , b , and c , If $a = b$ then $a + c = b + c$
Division Property of Equality	Multiplication Property of Equality
For any numbers a , b , and $c \neq 0$, If $a = b$ then $\frac{a}{c} = \frac{b}{c}$	For any numbers a , b , and c , If $a = b$ then $a \cdot c = b \cdot c$

Practice Makes Perfect

Determine Whether a Decimal is a Solution of an Equation

In the following exercises, determine whether each number is a solution of the given equation.

Exercise:

$$x - 0.8 = 2.3$$

Problem: (a) $x = 2$ (b) $x = -1.5$ (c) $x = 3.1$

Solution:

- (a) no
- (b) no
- (c) yes

Exercise:

$$y + 0.6 = -3.4$$

Problem: Ⓐ $y = -4$ Ⓑ $y = -2.8$ Ⓒ $y = 2.6$

Exercise:

$$\frac{h}{1.5} = -4.3$$

Problem: Ⓐ $h = 6.45$ Ⓑ $h = -6.45$ Ⓒ $h = -2.1$

Solution:

- Ⓐ no
- Ⓑ yes
- Ⓒ no

Exercise:

$$0.75k = -3.6$$

Problem: Ⓐ $k = -0.48$ Ⓑ $k = -4.8$ Ⓒ $k = -2.7$

Solve Equations with Decimals

In the following exercises, solve the equation.

Exercise:

Problem: $y + 2.9 = 5.7$

Solution:

$$y = 2.8$$

Exercise:

Problem: $m + 4.6 = 6.5$

Exercise:

Problem: $f + 3.45 = 2.6$

Solution:

$$f = -0.85$$

Exercise:

Problem: $h + 4.37 = 3.5$

Exercise:

Problem: $a + 6.2 = -1.7$

Solution:

$$a = -7.9$$

Exercise:

Problem: $b + 5.8 = -2.3$

Exercise:

Problem: $c + 1.15 = -3.5$

Solution:

$$c = -4.65$$

Exercise:

Problem: $d + 2.35 = -4.8$

Exercise:

Problem: $n - 2.6 = 1.8$

Solution:

$$n = 4.4$$

Exercise:

Problem: $p - 3.6 = 1.7$

Exercise:

Problem: $x - 0.4 = -3.9$

Solution:

$$x = -3.5$$

Exercise:

Problem: $y - 0.6 = -4.5$

Exercise:

Problem: $j - 1.82 = -6.5$

Solution:

$$j = -4.68$$

Exercise:

Problem: $k - 3.19 = -4.6$

Exercise:

Problem: $m - 0.25 = -1.67$

Solution:

$$m = -1.42$$

Exercise:

Problem: $q - 0.47 = -1.53$

Exercise:

Problem: $0.5x = 3.5$

Solution:

$$x = 7$$

Exercise:

Problem: $0.4p = 9.2$

Exercise:

Problem: $-1.7c = 8.5$

Solution:

$$c = -5$$

Exercise:

Problem: $-2.9x = 5.8$

Exercise:

Problem: $-1.4p = -4.2$

Solution:

$$p = 3$$

Exercise:

Problem: $-2.8m = -8.4$

Exercise:

Problem: $-120 = 1.5q$

Solution:

$$q = -80$$

Exercise:

Problem: $-75 = 1.5y$

Exercise:

Problem: $0.24x = 4.8$

Solution:

$$x = 20$$

Exercise:

Problem: $0.18n = 5.4$

Exercise:

Problem: $-3.4z = -9.18$

Solution:

$$z = 2.7$$

Exercise:

Problem: $-2.7u = -9.72$

Exercise:

Problem: $\frac{a}{0.4} = -20$

Solution:

$$a = -8$$

Exercise:

Problem: $\frac{b}{0.3} = -9$

Exercise:

Problem: $\frac{x}{0.7} = -0.4$

Solution:

$$x = -0.28$$

Exercise:

Problem: $\frac{y}{0.8} = -0.7$

Exercise:

Problem: $\frac{p}{-5} = -1.65$

Solution:

$$p = 8.25$$

Exercise:

Problem: $\frac{q}{-4} = -5.92$

Exercise:

Problem: $\frac{r}{-1.2} = -6$

Solution:

$$r = 7.2$$

Exercise:

Problem: $\frac{s}{-1.5} = -3$

Mixed Practice

In the following exercises, solve the equation. Then check your solution.

Exercise:

Problem: $x - 5 = -11$

Solution:

$$x = -6$$

Exercise:

Problem: $-\frac{2}{5} = x + \frac{3}{4}$

Exercise:

Problem: $p + 8 = -2$

Solution:

$$p = -10$$

Exercise:

Problem: $p + \frac{2}{3} = \frac{1}{12}$

Exercise:

Problem: $-4.2m = -33.6$

Solution:

$$m = 8$$

Exercise:

Problem: $q + 9.5 = -14$

Exercise:

Problem: $q + \frac{5}{6} = \frac{1}{12}$

Solution:

$$q = -\frac{3}{4}$$

Exercise:

Problem: $\frac{8.6}{15} = -d$

Exercise:

Problem: $\frac{7}{8}m = \frac{1}{10}$

Solution:

$$m = \frac{4}{35}$$

Exercise:

Problem: $\frac{j}{-6.2} = -3$

Exercise:

Problem: $-\frac{2}{3} = y + \frac{3}{8}$

Solution:

$$y = -\frac{25}{24}$$

Exercise:

Problem: $s - 1.75 = -3.2$

Exercise:

Problem: $\frac{11}{20} = -f$

Solution:

$$f = -\frac{11}{20}$$

Exercise:

Problem: $-3.6b = 2.52$

Exercise:

Problem: $-4.2a = 3.36$

Solution:

$$a = -0.8$$

Exercise:

Problem: $-9.1n = -63.7$

Exercise:

Problem: $r - 1.25 = -2.7$

Solution:

$$r = -1.45$$

Exercise:

Problem: $\frac{1}{4}n = \frac{7}{10}$

Exercise:

Problem: $\frac{h}{-3} = -8$

Solution:

$$h = 24$$

Exercise:

Problem: $y - 7.82 = -16$

Translate to an Equation and Solve

In the following exercises, translate and solve.

Exercise:

Problem: The difference of n and 1.9 is 3.4.

Solution:

$$n - 1.9 = 3.4; 5.3$$

Exercise:

Problem: The difference n and 1.5 is 0.8.

Exercise:

Problem: The product of -6.2 and x is -4.96 .

Solution:

$$-6.2x = -4.96; 0.8$$

Exercise:

Problem: The product of -4.6 and x is -3.22 .

Exercise:

Problem: The quotient of y and -1.7 is -5 .

Solution:

$$\frac{y}{-1.7} = -5; 8.5$$

Exercise:

Problem: The quotient of z and -3.6 is 3 .

Exercise:

Problem: The sum of n and -7.3 is 2.4 .

Solution:

$$n + (-7.3) = 2.4; 9.7$$

Exercise:

Problem: The sum of n and -5.1 is 3.8 .

Exercise:**Problem:**

Shawn bought a pair of shoes on sale for \$78. Solve the equation $0.75p = 78$ to find the original price of the shoes, p .

Solution:

\$104

Exercise:**Problem:**

Mary bought a new refrigerator. The total price including sales tax was \$1,350. Find the retail price, r , of the refrigerator before tax by solving the equation $1.08r = 1,350$.

Writing Exercises**Exercise:****Problem:**

Think about solving the equation $1.2y = 60$, but do not actually solve it. Do you think the solution should be greater than 60 or less than 60? Explain your reasoning. Then solve the equation to see if your thinking was correct.

Solution:

Answers will vary.

Exercise:

Problem:

Think about solving the equation $0.8x = 200$, but do not actually solve it. Do you think the solution should be greater than 200 or less than 200? Explain your reasoning. Then solve the equation to see if your thinking was correct.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
determine whether a decimal is a solution of an equation.			
solve equations with decimals.			
translate to an equation and solve.			

Ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Solve Proportions and their Applications

By the end of this section, you will be able to:

- Use the definition of proportion
- Solve proportions
- Solve applications using proportions
- Write percent equations as proportions
- Translate and solve percent proportions

Note:

Before you get started, take this readiness quiz.

1. Simplify: $\frac{\frac{1}{3}}{4}$.

If you missed this problem, review [\[link\]](#).

2. Solve: $\frac{x}{4} = 20$.

If you missed this problem, review [\[link\]](#).

3. Write as a rate: Sale rode his bike 24 miles in 2 hours.

If you missed this problem, review [\[link\]](#).

Use the Definition of Proportion

In the section on Ratios and Rates we saw some ways they are used in our daily lives. When two ratios or rates are equal, the equation relating them is called a **proportion**.

Note:

Proportion

A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$.

The proportion states two ratios or rates are equal. The proportion is read “ a is to b , as c is to d ”.

The equation $\frac{1}{2} = \frac{4}{8}$ is a proportion because the two fractions are equal. The proportion $\frac{1}{2} = \frac{4}{8}$ is read “1 is to 2 as 4 is to 8”.

If we compare quantities with units, we have to be sure we are comparing them in the right order. For example, in the proportion $\frac{20 \text{ students}}{1 \text{ teacher}} = \frac{60 \text{ students}}{3 \text{ teachers}}$ we compare the number of students to the number of teachers. We put students in the numerators and teachers in the denominators.

Example:

Exercise:

Problem: Write each sentence as a proportion:

- (a) 3 is to 7 as 15 is to 35.
 (b) 5 hits in 8 at bats is the same as 30 hits in 48 at-bats.
 (c) \$1.50 for 6 ounces is equivalent to \$2.25 for 9 ounces.

Solution:

Solution

(a)	
	3 is to 7 as 15 is to 35.
Write as a proportion.	$\frac{3}{7} = \frac{15}{35}$

	5 hits in 8 at-bats is the same as 30 hits in 48 at-bats.
Write each fraction to compare hits to at-bats.	$\frac{\text{hits}}{\text{at-bats}} = \frac{\text{hits}}{\text{at-bats}}$
Write as a proportion.	$\frac{5}{8} = \frac{30}{48}$
Ⓒ	
	\$1.50 for 6 ounces is equivalent to \$2.25 for 9 ounces.
Write each fraction to compare dollars to ounces.	$\frac{\$}{\text{ounces}} = \frac{\$}{\text{ounces}}$
Write as a proportion.	$\frac{1.50}{6} = \frac{2.25}{9}$

Note:

Exercise:

Problem: Write each sentence as a proportion:

- Ⓐ 5 is to 9 as 20 is to 36.
- Ⓑ 7 hits in 11 at-bats is the same as 28 hits in 44 at-bats.
- Ⓒ \$2.50 for 8 ounces is equivalent to \$3.75 for 12 ounces.

Solution:

Ⓐ $\frac{5}{9} = \frac{20}{36}$

$$\begin{array}{l} \textcircled{b} \frac{7}{11} = \frac{28}{44} \\ \textcircled{c} \frac{2.50}{8} = \frac{3.75}{12} \end{array}$$

Note:

Exercise:

Problem: Write each sentence as a proportion:

- Ⓐ 6 is to 7 as 36 is to 42.
- Ⓑ 8 adults for 36 children is the same as 12 adults for 54 children.
- Ⓒ \$3.75 for 6 ounces is equivalent to \$2.50 for 4 ounces.

Solution:

$$\begin{array}{l} \textcircled{a} \frac{6}{7} = \frac{36}{42} \\ \textcircled{b} \frac{8}{36} = \frac{12}{54} \\ \textcircled{c} \frac{3.75}{6} = \frac{2.50}{4} \end{array}$$

Look at the proportions $\frac{1}{2} = \frac{4}{8}$ and $\frac{2}{3} = \frac{6}{9}$. From our work with equivalent fractions we know these equations are true. But how do we know if an equation is a proportion with equivalent fractions if it contains fractions with larger numbers?

To determine if a proportion is true, we find the **cross products** of each proportion. To find the cross products, we multiply each denominator with the opposite numerator (diagonally across the equal sign). The results are called a cross products because of the cross formed. The cross products of a proportion are equal.

$$\begin{array}{cc} 8 \cdot 1 = 8 & 2 \cdot 4 = 8 \\ \frac{1}{2} & \frac{4}{8} \end{array} \qquad \begin{array}{cc} 9 \cdot 2 = 18 & 3 \cdot 6 = 18 \\ \frac{2}{3} & \frac{6}{9} \end{array}$$

Note:**Cross Products of a Proportion**

For any proportion of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0, d \neq 0$, its cross products are equal.

$$a \cdot d = b \cdot c$$


$$\frac{a}{b} = \frac{c}{d}$$

Cross products can be used to test whether a proportion is true. To test whether an equation makes a proportion, we find the cross products. If they are the equal, we have a proportion.

Example:**Exercise:**

Problem: Determine whether each equation is a proportion:

Ⓐ $\frac{4}{9} = \frac{12}{28}$

Ⓑ $\frac{17.5}{37.5} = \frac{7}{15}$

Solution:**Solution**

To determine if the equation is a proportion, we find the cross products. If they are equal, the equation is a proportion.

Ⓐ	

$$\frac{4}{9} = \frac{12}{28}$$

Find the cross products.

$$28 \cdot 4 = 112 \quad 9 \cdot 12 = 108$$

$$\frac{4}{9} \neq \frac{12}{28}$$

Since the cross products are not equal, $28 \cdot 4 \neq 9 \cdot 12$, the equation is not a proportion.

ⓑ

$$\frac{17.5}{37.5} = \frac{7}{15}$$

Find the cross products.

$$15 \cdot 17.5 = 262.5 \quad 37.5 \cdot 7 = 262.5$$

$$\frac{17.5}{37.5} = \frac{7}{15}$$

Since the cross products are equal, $15 \cdot 17.5 = 37.5 \cdot 7$, the equation is a proportion.

Note:

Exercise:

Problem: Determine whether each equation is a proportion:

ⓐ $\frac{7}{9} = \frac{54}{72}$

Ⓑ $\frac{24.5}{45.5} = \frac{7}{13}$

Solution:

- Ⓐ no
Ⓑ yes

Note:

Exercise:

Problem: Determine whether each equation is a proportion:

- Ⓐ $\frac{8}{9} = \frac{56}{73}$
Ⓑ $\frac{28.5}{52.5} = \frac{8}{15}$

Solution:

- Ⓐ no
Ⓑ no

Solve Proportions

To solve a proportion containing a variable, we remember that the proportion is an equation. All of the techniques we have used so far to solve equations still apply. In the next example, we will solve a proportion by multiplying by the Least Common Denominator (LCD) using the Multiplication Property of Equality.

Example:

Exercise:

Problem: Solve: $\frac{x}{63} = \frac{4}{7}$.

Solution:
Solution

		$\frac{x}{63} = \frac{4}{7}$
To isolate x , multiply both sides by the LCD, 63.		$63\left(\frac{x}{63}\right) = 63\left(\frac{4}{7}\right)$
Simplify.		$x = \frac{9 \cdot 7 \cdot 4}{7}$
Divide the common factors.		$x = 36$
Check: To check our answer, we substitute into the original proportion.		
	$\frac{x}{63} = \frac{4}{7}$	
Substitute $x = 36$	$\frac{36}{63} \stackrel{?}{=} \frac{4}{7}$	
Show common factors.	$\frac{4 \cdot 9}{7 \cdot 9} \stackrel{?}{=} \frac{4}{7}$	
Simplify.	$\frac{4}{7} = \frac{4}{7} \checkmark$	

Note:

Exercise:

Problem: Solve the proportion: $\frac{n}{84} = \frac{11}{12}$.

Solution:

77

Note:

Exercise:

Problem: Solve the proportion: $\frac{y}{96} = \frac{13}{12}$.

Solution:

104

When the variable is in a denominator, we'll use the fact that the cross products of a proportion are equal to solve the proportions.

We can find the cross products of the proportion and then set them equal. Then we solve the resulting equation using our familiar techniques.

Example:

Exercise:

Problem: Solve: $\frac{144}{a} = \frac{9}{4}$.

Solution:
Solution

Notice that the variable is in the denominator, so we will solve by finding the cross products and setting them equal.

		$\frac{144}{a} \neq \frac{9}{4}$
Find the cross products and set them equal.		$4 \cdot 144 = a \cdot 9$
Simplify.		$576 = 9a$
Divide both sides by 9.		$\frac{576}{9} = \frac{9a}{9}$
Simplify.		$64 = a$
Check your answer.		
	$\frac{144}{a} = \frac{9}{4}$	
Substitute $a = 64$	$\frac{144}{64} \stackrel{?}{=} \frac{9}{4}$	

Show common factors..	$\frac{9 \cdot 16}{4 \cdot 16} = \frac{9}{4}$	
Simplify.	$\frac{9}{4} = \frac{9}{4} \checkmark$	

Another method to solve this would be to multiply both sides by the LCD, $4a$. Try it and verify that you get the same solution.

Note:

Exercise:

Problem: Solve the proportion: $\frac{91}{b} = \frac{7}{5}$.

Solution:

65

Note:

Exercise:

Problem: Solve the proportion: $\frac{39}{c} = \frac{13}{8}$.

Solution:

24

Example:

Exercise:

Problem: Solve: $\frac{52}{91} = \frac{-4}{y}$.

Solution:
Solution

Find the cross products and set them equal.

$$\frac{52}{91} \neq \frac{-4}{y}$$

$$y \cdot 52 = 91(-4)$$

Simplify.

$$52y = -364$$

Divide both sides by 52.

$$\frac{52y}{52} = \frac{-364}{52}$$

Simplify.

$$y = -7$$

Check:

$$\frac{52}{91} = \frac{-4}{y}$$

Substitute $y = -7$

$$\frac{52}{91} \stackrel{?}{=} \frac{-4}{-7}$$

Show common factors.	$\frac{13 \cdot 4}{13 \cdot 4} = \frac{?}{-7}$	
Simplify.	$\frac{4}{7} = \frac{4}{7} \checkmark$	

Note:

Exercise:

Problem: Solve the proportion: $\frac{84}{98} = \frac{-6}{x}$.

Solution:

-7

Note:

Exercise:

Problem: Solve the proportion: $\frac{-7}{y} = \frac{105}{135}$.

Solution:

-9

Solve Applications Using Proportions

The strategy for solving applications that we have used earlier in this chapter, also works for proportions, since proportions are equations. When we set up the

proportion, we must make sure the units are correct—the units in the numerators match and the units in the denominators match.

Example:

Exercise:

Problem:

When pediatricians prescribe acetaminophen to children, they prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of the child’s weight. If Zoe weighs 80 pounds, how many milliliters of acetaminophen will her doctor prescribe?

Solution:

Solution

Identify what you are asked to find.	How many ml of acetaminophen the doctor will prescribe
Choose a variable to represent it.	Let a = ml of acetaminophen.
Write a sentence that gives the information to find it.	If 5 ml is prescribed for every 25 pounds, how much will be prescribed for 80 pounds?
Translate into a proportion.	$\frac{\text{ml}}{\text{pounds}} = \frac{\text{ml}}{\text{pounds}}$
Substitute given values—be careful of the units.	$\frac{5}{25} = \frac{a}{80}$

Multiply both sides by 80.	$80 \cdot \frac{5}{25} = 80 \cdot \frac{a}{80}$
Multiply and show common factors.	$\frac{16 \cdot 5 \cdot 5}{5 \cdot 5} = \frac{80a}{80}$
Simplify.	$16 = a$
Check if the answer is reasonable.	
Yes. Since 80 is about 3 times 25, the medicine should be about 3 times 5.	
Write a complete sentence.	The pediatrician would prescribe 16 ml of acetaminophen to Zoe.

You could also solve this proportion by setting the cross products equal.

Note:

Exercise:

Problem:

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Emilia, who weighs 60 pounds?

Solution:

12 ml

Note:

Exercise:

Problem:

For every 1 kilogram (kg) of a child's weight, pediatricians prescribe 15 milligrams (mg) of a fever reducer. If Isabella weighs 12 kg, how many milligrams of the fever reducer will the pediatrician prescribe?

Solution:

180 mg

Example:

Exercise:

Problem:

One brand of microwave popcorn has 120 calories per serving. A whole bag of this popcorn has 3.5 servings. How many calories are in a whole bag of this microwave popcorn?

Solution:

Solution

Identify what you are asked to find.	How many calories are in a whole bag of microwave popcorn?
Choose a variable to represent it.	Let c = number of calories.
Write a sentence that gives the information to find it.	If there are 120 calories per serving, how many calories are in a whole bag with 3.5 servings?

Translate into a proportion.	$\frac{\text{calories}}{\text{serving}} = \frac{\text{calories}}{\text{serving}}$
Substitute given values.	$\frac{120}{1} = \frac{c}{3.5}$
Multiply both sides by 3.5.	$(3.5)\left(\frac{120}{1}\right) = (3.5)\left(\frac{c}{3.5}\right)$
Multiply.	$420 = c$
Check if the answer is reasonable.	
Yes. Since 3.5 is between 3 and 4, the total calories should be between 360 ($3 \cdot 120$) and 480 ($4 \cdot 120$).	
Write a complete sentence.	The whole bag of microwave popcorn has 420 calories.

Note:

Exercise:

Problem:

Marissa loves the Caramel Macchiato at the coffee shop. The 16 oz. medium size has 240 calories. How many calories will she get if she drinks the large 20 oz. size?

Solution:

Note:**Exercise:****Problem:**

Yaneli loves Starburst candies, but wants to keep her snacks to 100 calories. If the candies have 160 calories for 8 pieces, how many pieces can she have in her snack?

Solution:

5

Example:**Exercise:****Problem:**

Josiah went to Mexico for spring break and changed \$325 dollars into Mexican pesos. At that time, the exchange rate had \$1 U.S. is equal to 12.54 Mexican pesos. How many Mexican pesos did he get for his trip?

Solution:**Solution**

Identify what you are asked to find.	How many Mexican pesos did Josiah get?
Choose a variable to represent it.	Let p = number of pesos.
Write a sentence that gives the	If \$1 U.S. is equal to 12.54

information to find it.	Mexican pesos, then \$325 is how many pesos?
Translate into a proportion.	$\frac{\$}{\text{pesos}} = \frac{\$}{\text{pesos}}$
Substitute given values.	$\frac{1}{12.54} = \frac{325}{p}$
The variable is in the denominator, so find the cross products and set them equal.	$p \cdot 1 = 12.54(325)$
Simplify.	$c = 4,075.5$
Check if the answer is reasonable.	
Yes, \$100 would be \$1,254 pesos. \$325 is a little more than 3 times this amount.	
Write a complete sentence.	Josiah has 4075.5 pesos for his spring break trip.

Note:

Exercise:

Problem:

Yurianna is going to Europe and wants to change \$800 dollars into Euros. At the current exchange rate, \$1 US is equal to 0.738 Euro. How many Euros will she have for her trip?

Solution:

590 Euros

Note:**Exercise:****Problem:**

Corey and Nicole are traveling to Japan and need to exchange \$600 into Japanese yen. If each dollar is 94.1 yen, how many yen will they get?

Solution:

56,460 yen

Write Percent Equations As Proportions

Previously, we solved percent equations by applying the properties of equality we have used to solve equations throughout this text. Some people prefer to solve percent equations by using the proportion method. The proportion method for solving percent problems involves a percent proportion. A **percent proportion** is an equation where a percent is equal to an equivalent ratio.

For example, $60\% = \frac{60}{100}$ and we can simplify $\frac{60}{100} = \frac{3}{5}$. Since the equation $\frac{60}{100} = \frac{3}{5}$ shows a percent equal to an equivalent ratio, we call it a **percent proportion**. Using the vocabulary we used earlier:

Equation:

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

Equation:

$$\frac{3}{5} = \frac{60}{100}$$

Note:**Percent Proportion**

The amount is to the base as the percent is to 100.

Equation:

$$\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$$

If we restate the problem in the words of a proportion, it may be easier to set up the proportion:

Equation:

The amount is to the base as the percent is to one hundred.

We could also say:

Equation:

The amount out of the base is the same as the percent out of one hundred.

First we will practice translating into a percent proportion. Later, we'll solve the proportion.

Example:**Exercise:**

Problem: Translate to a proportion. What number is 75% of 90?

Solution:**Solution**

If you look for the word "of", it may help you identify the base.

Identify the parts of the percent proportion.

What number is 75% of 90?
amount percent base

Restate as a proportion.

What number out of 90 is the same as 75 out of 100?

Set up the proportion. Let n = number.

$$\frac{n}{90} = \frac{75}{100}$$

Note:

Exercise:

Problem: Translate to a proportion: What number is 60% of 105?

Solution:

$$\frac{n}{105} = \frac{60}{100}$$

Note:

Exercise:

Problem: Translate to a proportion: What number is 40% of 85?

Solution:

$$\frac{n}{85} = \frac{40}{100}$$

Example:

Exercise:

Problem: Translate to a proportion. 19 is 25% of what number?

Solution:
Solution

Identify the parts of the percent proportion.

19 is 25% of what number?
amount percent base

Restate as a proportion.

19 out of what number is the same as 25 out of 100?

Set up the proportion. Let $n = \text{number}$.

$$\frac{19}{n} = \frac{25}{100}$$

Note:

Exercise:

Problem: Translate to a proportion: 36 is 25% of what number?

Solution:

$$\frac{36}{n} = \frac{25}{100}$$

Note:

Exercise:

Problem: Translate to a proportion: 27 is 36% of what number?

Solution:

$$\frac{27}{n} = \frac{36}{100}$$

Example:

Exercise:

Problem: Translate to a proportion. What percent of 27 is 9?

Solution:

Solution

Identify the parts of the percent proportion.

What percent of 27 is 9?
percent base amount

Restate as a proportion.

9 out of 27 is the same as what number out of 100?

Set up the proportion. Let p = percent.

$$\frac{9}{27} = \frac{p}{100}$$

Note:

Exercise:

Problem: Translate to a proportion: What percent of 52 is 39?

Solution:

$$\frac{n}{100} = \frac{39}{52}$$

Note:

Exercise:

Problem: Translate to a proportion: What percent of 92 is 23?

Solution:

$$\frac{n}{100} = \frac{23}{92}$$

Translate and Solve Percent Proportions

Now that we have written percent equations as proportions, we are ready to solve the equations.

Example:

Exercise:

Problem: Translate and solve using proportions: What number is 45% of 80?

Solution:

Solution

Identify the parts of the percent proportion.	What number amount is 45% percent of 80? base
Restate as a proportion.	What number out of 80 is the same as 45 out of 100?
Set up the proportion. Let n = number.	$\frac{n}{80} = \frac{45}{100}$
Find the cross products and set them equal.	$100 \cdot n = 80 \cdot 45$
Simplify.	$100n = 3,600$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,600}{100}$
Simplify.	$n = 36$
Check if the answer is reasonable.	
Yes. 45 is a little less than half of 100 and 36 is a little less than half 80.	
Write a complete sentence that answers the question.	36 is 45% of 80.

Note:

Exercise:

Problem: Translate and solve using proportions: What number is 65% of 40?

Solution:

26

Note:

Exercise:

Problem: Translate and solve using proportions: What number is 85% of 40?

Solution:

34

In the next example, the percent is more than 100, which is more than one whole. So the unknown number will be more than the base.

Example:

Exercise:

Problem: Translate and solve using proportions: 125% of 25 is what number?

Solution:

Solution

Identify the parts of the percent proportion.

125%	is	25	of	what number ?
				
percent		base		amount

Restate as a proportion.	What number out of 25 is the same as 125 out of 100?
Set up the proportion. Let n = number.	$\frac{n}{25} = \frac{125}{100}$
Find the cross products and set them equal.	$100 \cdot n = 25 \cdot 125$
Simplify.	$100n = 3,125$
Divide both sides by 100.	$\frac{100n}{100} = \frac{3,125}{100}$
Simplify.	$n = 31.25$
Check if the answer is reasonable.	
Yes. 125 is more than 100 and 31.25 is more than 25.	
Write a complete sentence that answers the question.	125% of 25 is 31.25.

Note:

Exercise:

Problem: Translate and solve using proportions: 125% of 64 is what number?

Solution:

80

Note:

Exercise:

Problem: Translate and solve using proportions: 175% of 84 is what number?

Solution:

147

Percents with decimals and money are also used in proportions.

Example:

Exercise:

Problem: Translate and solve: 6.5% of what number is \$1.56?

Solution:

Solution

Identify the parts of the percent proportion.

6.5% of what number is \$1.56?
percent base amount

Restate as a proportion.

\$1.56 out of what number is the same as 6.5 out of 100?

Set up the proportion. Let n = number.	$\frac{1.56}{n} = \frac{6.5}{100}$
Find the cross products and set them equal.	$100(1.56) = n \cdot 6.5$
Simplify.	$156 = 6.5n$
Divide both sides by 6.5 to isolate the variable.	$\frac{156}{6.5} = \frac{6.5n}{6.5}$
Simplify.	$24 = n$
Check if the answer is reasonable.	
Yes. 6.5% is a small amount and \$1.56 is much less than \$24.	
Write a complete sentence that answers the question.	6.5% of \$24 is \$1.56.

Note:

Exercise:

Problem:

Translate and solve using proportions: 8.5% of what number is \$3.23?

Solution:

Note:

Exercise:

Problem:

Translate and solve using proportions: 7.25% of what number is \$4.64?

Solution:

64

Example:

Exercise:

Problem: Translate and solve using proportions: What percent of 72 is 9?

Solution:

Solution

Identify the parts of the percent proportion.

What percent of 72 is 9?
percent base amount

Restate as a proportion.

9 out of 72 is the same as what number out of 100?

Set up the proportion. Let $n =$ number.

$$\frac{9}{72} = \frac{n}{100}$$

Find the cross products and set them equal.	$72 \cdot n = 100 \cdot 9$
Simplify.	$72n = 900$
Divide both sides by 72.	$\frac{72n}{72} = \frac{900}{72}$
Simplify.	$n = 12.5$
Check if the answer is reasonable.	
Yes. 9 is $\frac{1}{8}$ of 72 and $\frac{1}{8}$ is 12.5%.	
Write a complete sentence that answers the question.	12.5% of 72 is 9.

Note:

Exercise:

Problem: Translate and solve using proportions: What percent of 72 is 27?

Solution:

37.5%

Note:

Exercise:

Problem: Translate and solve using proportions: What percent of 92 is 23?

Solution:

25%

Key Concepts

- **Proportion**

- A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$, $d \neq 0$. The proportion states two ratios or rates are equal. The proportion is read “ a is to b , as c is to d ”.

- **Cross Products of a Proportion**

- For any proportion of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$, its cross products are equal: $a \cdot d = b \cdot c$.

- **Percent Proportion**

- The amount is to the base as the percent is to 100. $\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$

Section Exercises

Practice Makes Perfect

Use the Definition of Proportion

In the following exercises, write each sentence as a proportion.

Exercise:

Problem: 4 is to 15 as 36 is to 135.

Solution:

$$\frac{4}{15} = \frac{36}{135}$$

Exercise:

Problem: 7 is to 9 as 35 is to 45.

Exercise:

Problem: 12 is to 5 as 96 is to 40.

Solution:

$$\frac{12}{5} = \frac{96}{40}$$

Exercise:

Problem: 15 is to 8 as 75 is to 40.

Exercise:

Problem: 5 wins in 7 games is the same as 115 wins in 161 games.

Solution:

$$\frac{5}{7} = \frac{115}{161}$$

Exercise:

Problem: 4 wins in 9 games is the same as 36 wins in 81 games.

Exercise:

Problem: 8 campers to 1 counselor is the same as 48 campers to 6 counselors.

Solution:

$$\frac{8}{1} = \frac{48}{6}$$

Exercise:

Problem: 6 campers to 1 counselor is the same as 48 campers to 8 counselors.

Exercise:

Problem: \$9.36 for 18 ounces is the same as \$2.60 for 5 ounces.

Solution:

$$\frac{9.36}{18} = \frac{2.60}{5}$$

Exercise:

Problem: \$3.92 for 8 ounces is the same as \$1.47 for 3 ounces.

Exercise:

Problem: \$18.04 for 11 pounds is the same as \$4.92 for 3 pounds.

Solution:

$$\frac{18.04}{11} = \frac{4.92}{3}$$

Exercise:

Problem: \$12.42 for 27 pounds is the same as \$5.52 for 12 pounds.

In the following exercises, determine whether each equation is a proportion.

Exercise:

Problem: $\frac{7}{15} = \frac{56}{120}$

Solution:

yes

Exercise:

Problem: $\frac{5}{12} = \frac{45}{108}$

Exercise:

Problem: $\frac{11}{6} = \frac{21}{16}$

Solution:

no

Exercise:

Problem: $\frac{9}{4} = \frac{39}{34}$

Exercise:

Problem: $\frac{12}{18} = \frac{4.99}{7.56}$

Solution:

no

Exercise:

Problem: $\frac{9}{16} = \frac{2.16}{3.89}$

Exercise:

Problem: $\frac{13.5}{8.5} = \frac{31.05}{19.55}$

Solution:

yes

Exercise:

Problem: $\frac{10.1}{8.4} = \frac{3.03}{2.52}$

Solve Proportions

In the following exercises, solve each proportion.

Exercise:

Problem: $\frac{x}{56} = \frac{7}{8}$

Solution:

49

Exercise:

Problem: $\frac{n}{91} = \frac{8}{13}$

Exercise:

Problem: $\frac{49}{63} = \frac{z}{9}$

Solution:

7

Exercise:

Problem: $\frac{56}{72} = \frac{y}{9}$

Exercise:

Problem: $\frac{5}{a} = \frac{65}{117}$

Solution:

9

Exercise:

Problem: $\frac{4}{b} = \frac{64}{144}$

Exercise:

Problem: $\frac{98}{154} = \frac{-7}{p}$

Solution:

-11

Exercise:

Problem: $\frac{72}{156} = \frac{-6}{q}$

Exercise:

Problem: $\frac{a}{-8} = \frac{-42}{48}$

Solution:

7

Exercise:

Problem: $\frac{b}{-7} = \frac{-30}{42}$

Exercise:

Problem: $\frac{2.6}{3.9} = \frac{c}{3}$

Solution:

2

Exercise:

Problem: $\frac{2.7}{3.6} = \frac{d}{4}$

Exercise:

Problem: $\frac{2.7}{j} = \frac{0.9}{0.2}$

Solution:

0.6

Exercise:

Problem: $\frac{2.8}{k} = \frac{2.1}{1.5}$

Exercise:

Problem: $\frac{\frac{1}{2}}{1} = \frac{m}{8}$

Solution:

Exercise:

Problem: $\frac{\frac{1}{3}}{3} = \frac{9}{n}$

Solve Applications Using Proportions

In the following exercises, solve the proportion problem.

Exercise:**Problem:**

Pediatricians prescribe 5 milliliters (ml) of acetaminophen for every 25 pounds of a child's weight. How many milliliters of acetaminophen will the doctor prescribe for Jocelyn, who weighs 45 pounds?

Solution:

9 ml

Exercise:**Problem:**

Brianna, who weighs 6 kg, just received her shots and needs a pain killer. The pain killer is prescribed for children at 15 milligrams (mg) for every 1 kilogram (kg) of the child's weight. How many milligrams will the doctor prescribe?

Exercise:**Problem:**

At the gym, Carol takes her pulse for 10 sec and counts 19 beats. How many beats per minute is this? Has Carol met her target heart rate of 140 beats per minute?

Solution:

114, no

Exercise:

Problem:

Kevin wants to keep his heart rate at 160 beats per minute while training. During his workout he counts 27 beats in 10 seconds. How many beats per minute is this? Has Kevin met his target heart rate?

Exercise:**Problem:**

A new energy drink advertises 106 calories for 8 ounces. How many calories are in 12 ounces of the drink?

Solution:

159 cal

Exercise:**Problem:**

One 12 ounce can of soda has 150 calories. If Josiah drinks the big 32 ounce size from the local mini-mart, how many calories does he get?

Exercise:**Problem:**

Karen eats $\frac{1}{2}$ cup of oatmeal that counts for 2 points on her weight loss program. Her husband, Joe, can have 3 points of oatmeal for breakfast. How much oatmeal can he have?

Solution:

$\frac{3}{4}$ cup

Exercise:**Problem:**

An oatmeal cookie recipe calls for $\frac{1}{2}$ cup of butter to make 4 dozen cookies. Hilda needs to make 10 dozen cookies for the bake sale. How many cups of butter will she need?

Exercise:

Problem:

Janice is traveling to Canada and will change \$250 US dollars into Canadian dollars. At the current exchange rate, \$1 US is equal to \$1.01 Canadian. How many Canadian dollars will she get for her trip?

Solution:

\$252.50

Exercise:**Problem:**

Todd is traveling to Mexico and needs to exchange \$450 into Mexican pesos. If each dollar is worth 12.29 pesos, how many pesos will he get for his trip?

Exercise:**Problem:**

Steve changed \$600 into 480 Euros. How many Euros did he receive per US dollar?

Solution:

1.25

Exercise:**Problem:**

Martha changed \$350 US into 385 Australian dollars. How many Australian dollars did she receive per US dollar?

Exercise:**Problem:**

At the laundromat, Lucy changed \$12.00 into quarters. How many quarters did she get?

Solution:

48 quarters

Exercise:

Problem:

When she arrived at a casino, Gerty changed \$20 into nickels. How many nickels did she get?

Exercise:**Problem:**

Jesse's car gets 30 miles per gallon of gas. If Las Vegas is 285 miles away, how many gallons of gas are needed to get there and then home? If gas is \$3.09 per gallon, what is the total cost of the gas for the trip?

Solution:

19, \$58.71

Exercise:**Problem:**

Danny wants to drive to Phoenix to see his grandfather. Phoenix is 370 miles from Danny's home and his car gets 18.5 miles per gallon. How many gallons of gas will Danny need to get to and from Phoenix? If gas is \$3.19 per gallon, what is the total cost for the gas to drive to see his grandfather?

Exercise:**Problem:**

Hugh leaves early one morning to drive from his home in Chicago to go to Mount Rushmore, 812 miles away. After 3 hours, he has gone 190 miles. At that rate, how long will the whole drive take?

Solution:

12.8 hours

Exercise:**Problem:**

Kelly leaves her home in Seattle to drive to Spokane, a distance of 280 miles. After 2 hours, she has gone 152 miles. At that rate, how long will the whole drive take?

Exercise:

Problem:

Phil wants to fertilize his lawn. Each bag of fertilizer covers about 4,000 square feet of lawn. Phil's lawn is approximately 13,500 square feet. How many bags of fertilizer will he have to buy?

Solution:

4 bags

Exercise:**Problem:**

April wants to paint the exterior of her house. One gallon of paint covers about 350 square feet, and the exterior of the house measures approximately 2000 square feet. How many gallons of paint will she have to buy?

Write Percent Equations as Proportions

In the following exercises, translate to a proportion.

Exercise:

Problem: What number is 35% of 250?

Solution:

$$\frac{n}{250} = \frac{35}{100}$$

Exercise:

Problem: What number is 75% of 920?

Exercise:

Problem: What number is 110% of 47?

Solution:

$$\frac{n}{47} = \frac{110}{100}$$

Exercise:

Problem: What number is 150% of 64?

Exercise:

Problem: 45 is 30% of what number?

Solution:

$$\frac{45}{n} = \frac{30}{100}$$

Exercise:

Problem: 25 is 80% of what number?

Exercise:

Problem: 90 is 150% of what number?

Solution:

$$\frac{90}{n} = \frac{150}{100}$$

Exercise:

Problem: 77 is 110% of what number?

Exercise:

Problem: What percent of 85 is 17?

Solution:

$$\frac{17}{85} = \frac{p}{100}$$

Exercise:

Problem: What percent of 92 is 46?

Exercise:

Problem: What percent of 260 is 340?

Solution:

$$\frac{340}{260} = \frac{p}{100}$$

Exercise:

Problem: What percent of 180 is 220?

Translate and Solve Percent Proportions

In the following exercises, translate and solve using proportions.

Exercise:

Problem: What number is 65% of 180?

Solution:

117

Exercise:

Problem: What number is 55% of 300?

Solution:

165

Exercise:

Problem: 18% of 92 is what number?

Solution:

16.56

Exercise:

Problem: 22% of 74 is what number?

Exercise:

Problem: 175% of 26 is what number?

Solution:

45.5

Exercise:

Problem: 250% of 61 is what number?

Exercise:

Problem: What is 300% of 488?

Solution:

1464

Exercise:

Problem: What is 500% of 315?

Exercise:

Problem: 17% of what number is \$7.65?

Solution:

\$45

Exercise:

Problem: 19% of what number is \$6.46?

Exercise:

Problem: \$13.53 is 8.25% of what number?

Solution:

\$164

Exercise:

Problem: \$18.12 is 7.55% of what number?

Exercise:

Problem: What percent of 56 is 14?

Solution:

25%

Exercise:

Problem: What percent of 80 is 28?

Exercise:

Problem: What percent of 96 is 12?

Solution:

12.5%

Exercise:

Problem: What percent of 120 is 27?

Everyday Math

Exercise:

Problem:

Mixing a concentrate Sam bought a large bottle of concentrated cleaning solution at the warehouse store. He must mix the concentrate with water to make a solution for washing his windows. The directions tell him to mix 3 ounces of concentrate with 5 ounces of water. If he puts 12 ounces of concentrate in a bucket, how many ounces of water should he add? How many ounces of the solution will he have altogether?

Solution:

20, 32

Exercise:**Problem:**

Mixing a concentrate Travis is going to wash his car. The directions on the bottle of car wash concentrate say to mix 2 ounces of concentrate with 15 ounces of water. If Travis puts 6 ounces of concentrate in a bucket, how much water must he mix with the concentrate?

Writing Exercises**Exercise:****Problem:**

To solve “what number is 45% of 350” do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.

Solution:

Answers will vary.

Exercise:**Problem:**

To solve “what percent of 125 is 25” do you prefer to use an equation like you did in the section on Decimal Operations or a proportion like you did in this section? Explain your reason.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use definition of proportion.			
solve proportions.			
solve applications using proportions.			
write percent equations as proportions.			
translate and solve percent proportions.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Understand Percent

In the following exercises, write each percent as a ratio.

Exercise:

Problem: 32% admission rate for the university

Solution:

$$\frac{32}{100}$$

Exercise:

Problem: 53.3% rate of college students with student loans

In the following exercises, write as a ratio and then as a percent.

Exercise:

Problem: 13 out of 100 architects are women.

Solution:

$$\frac{13}{100}, 13\%$$

Exercise:

Problem: 9 out of every 100 nurses are men.

In the following exercises, convert each percent to a fraction.

Exercise:

Problem: 48%

Solution:

$$\frac{12}{25}$$

Exercise:

Problem: 175%

Exercise:

Problem: 64.1%

Solution:

$$\frac{641}{1000}$$

Exercise:

Problem: $8\frac{1}{4}\%$

In the following exercises, convert each percent to a decimal.

Exercise:

Problem: 6%

Solution:

$$0.06$$

Exercise:

Problem: 23%

Exercise:

Problem: 128%

Solution:

1.28

Exercise:

Problem: 4.9%

In the following exercises, convert each percent to (a) a simplified fraction and (b) a decimal.

Exercise:

Problem:

In 2012, 13.5% of the United States population was age 65 or over. (Source: www.census.gov)

Solution:

- (a) $\frac{27}{200}$
- (b) 0.135

Exercise:

Problem:

In 2012, 6.5% of the United States population was under 5 years old. (Source: www.census.gov)

Exercise:

Problem:

When a die is tossed, the probability it will land with an even number of dots on the top side is 50%.

Solution:

- (a) $\frac{1}{2}$

ⓑ 0.5

Exercise:

Problem:

A couple plans to have three children. The probability they will all be girls is 12.5%.

In the following exercises, convert each decimal to a percent.

Exercise:

Problem: 0.04

Solution:

4%

Exercise:

Problem: 0.15

Exercise:

Problem: 2.82

Solution:

282%

Exercise:

Problem: 3

Exercise:

Problem: 0.003

Solution:

0.3%

Exercise:

Problem: 1.395

In the following exercises, convert each fraction to a percent.

Exercise:

Problem: $\frac{3}{4}$

Solution:

75%

Exercise:

Problem: $\frac{11}{5}$

Exercise:

Problem: $3\frac{5}{8}$

Solution:

362.5%

Exercise:

Problem: $\frac{2}{9}$

Exercise:

Problem:

According to the Centers for Disease Control, $\frac{2}{5}$ of adults do not take a vitamin or supplement.

Solution:

40%

Exercise:

Problem:

According to the Centers for Disease Control, among adults who do take a vitamin or supplement, $\frac{3}{4}$ take a multivitamin.

In the following exercises, translate and solve.

Exercise:

Problem: What number is 46% of 350?

Solution:

161

Exercise:

Problem: 120% of 55 is what number?

Exercise:

Problem: 84 is 35% of what number?

Solution:

240

Exercise:

Problem: 15 is 8% of what number?

Exercise:

Problem: 200% of what number is 50?

Solution:

25

Exercise:

Problem: 7.9% of what number is \$4.74?

Exercise:

Problem: What percent of 120 is 81.6?

Solution:

68%

Exercise:

Problem: What percent of 340 is 595?

Solve General Applications of Percents

In the following exercises, solve.

Exercise:

Problem:

When Aurelio and his family ate dinner at a restaurant, the bill was \$83.50. Aurelio wants to leave 20% of the total bill as a tip. How much should the tip be?

Solution:

\$16.70

Exercise:

Problem:

One granola bar has 2 grams of fiber, which is 8% of the recommended daily amount. What is the total recommended daily amount of fiber?

Exercise:

Problem:

The nutrition label on a package of granola bars says that each granola bar has 190 calories, and 54 calories are from fat. What percent of the total calories is from fat?

Solution:

28.4%

Exercise:

Problem:

Elsa gets paid \$4,600 per month. Her car payment is \$253. What percent of her monthly pay goes to her car payment?

In the following exercises, solve.

Exercise:

Problem:

Jorge got a raise in his hourly pay, from \$19.00 to \$19.76. Find the percent increase.

Solution:

4%

Exercise:

Problem:

Last year Bernard bought a new car for \$30,000. This year the car is worth \$24,000. Find the percent decrease.

Solve Sales Tax, Commission, and Discount Applications

In the following exercises, find (a) the sales tax (b) the total cost.

Exercise:

Problem:

The cost of a lawn mower was \$750. The sales tax rate is 6% of the purchase price.

Solution:

- (a) \$45
- (b) \$795

Exercise:

Problem:

The cost of a water heater is \$577. The sales tax rate is 8.75% of the purchase price.

In the following exercises, find the sales tax rate.

Exercise:

Problem:

Andy bought a piano for \$4,600. The sales tax on the purchase was \$333.50.

Solution:

7.25%

Exercise:

Problem:

Nahomi bought a purse for \$200. The sales tax on the purchase was \$16.75.

In the following exercises, find the commission.

Exercise:

Problem:

Ginny is a realtor. She receives 3% commission when she sells a house. How much commission will she receive for selling a house for \$380,000?

Solution:

\$11,400

Exercise:

Problem:

Jackson receives 16.5% commission when he sells a dinette set. How much commission will he receive for selling a dinette set for \$895?

In the following exercises, find the rate of commission.

Exercise:

Problem:

Ruben received \$675 commission when he sold a \$4,500 painting at the art gallery where he works. What was the rate of commission?

Solution:

15%

Exercise:**Problem:**

Tori received \$80.75 for selling a \$950 membership at her gym. What was her rate of commission?

In the following exercises, find the sale price.

Exercise:**Problem:**

Aya bought a pair of shoes that was on sale for \$30 off. The original price of the shoes was \$75.

Solution:

\$45

Exercise:**Problem:**

Takwanna saw a cookware set she liked on sale for \$145 off. The original price of the cookware was \$312.

In the following exercises, find (a) the amount of discount and (b) the sale price.

Exercise:**Problem:**

Nga bought a microwave for her office. The microwave was discounted 30% from an original price of \$84.90.

Solution:

- Ⓐ \$25.47
- Ⓑ \$59.43

Exercise:

Problem:

Jarrett bought a tie that was discounted 65% from an original price of \$45.

In the following exercises, find Ⓐ the amount of discount Ⓑ the discount rate.
(Round to the nearest tenth of a percent if needed.)

Exercise:

Problem:

Hilda bought a bedspread on sale for \$37. The original price of the bedspread was \$50.

Solution:

- Ⓐ \$13
- Ⓑ 26%

Exercise:

Problem:

Tyler bought a phone on sale for \$49.99. The original price of the phone was \$79.99.

In the following exercises, find

- Ⓐ the amount of the mark-up
- Ⓑ the list price

Exercise:

Problem:

Manny paid \$0.80 a pound for apples. He added 60% mark-up before selling them at his produce stand. What price did he charge for the apples?

Solution:

- Ⓐ \$0.48
- Ⓑ \$1.28

Exercise:

Problem:

It cost Noelle \$17.40 for the materials she used to make a purse. She added a 325% mark-up before selling it at her friend's store. What price did she ask for the purse?

Solve Simple Interest Applications

In the following exercises, solve the simple interest problem.

Exercise:

Problem:

Find the simple interest earned after 4 years on \$2,250 invested at an interest rate of 5%.

Solution:

\$450

Exercise:

Problem:

Find the simple interest earned after 7 years on \$12,000 invested at an interest rate of 8.5%.

Exercise:

Problem:

Find the principal invested if \$660 interest was earned in 5 years at an interest rate of 3%.

Solution:

\$4400

Exercise:

Problem:

Find the interest rate if \$2,898 interest was earned from a principal of \$23,000 invested for 3 years.

Exercise:**Problem:**

Kazuo deposited \$10,000 in a bank account with interest rate 4.5%. How much interest was earned in 2 years?

Solution:

\$900

Exercise:**Problem:**

Brent invested \$23,000 in a friend's business. In 5 years the friend paid him the \$23,000 plus \$9,200 interest. What was the rate of interest?

Exercise:**Problem:**

Fresia lent her son \$5,000 for college expenses. Three years later he repaid her the \$5,000 plus \$375 interest. What was the rate of interest?

Solution:

2.5%

Exercise:**Problem:**

In 6 years, a bond that paid 5.5% earned \$594 interest. What was the principal of the bond?

Solve Proportions and their Applications

In the following exercises, write each sentence as a proportion.

Exercise:

Problem: 3 is to 8 as 12 is to 32.

Solution:

$$\frac{3}{8} = \frac{12}{32}$$

Exercise:

Problem: 95 miles to 3 gallons is the same as 475 miles to 15 gallons.

Exercise:

Problem: 1 teacher to 18 students is the same as 23 teachers to 414 students.

Solution:

$$\frac{1}{18} = \frac{23}{414}$$

Exercise:

Problem: \$7.35 for 15 ounces is the same as \$2.94 for 6 ounces.

In the following exercises, determine whether each equation is a proportion.

Exercise:

Problem: $\frac{5}{13} = \frac{30}{78}$

Solution:

yes

Exercise:

Problem: $\frac{16}{7} = \frac{48}{23}$

Exercise:

Problem: $\frac{12}{18} = \frac{6.99}{10.99}$

Solution:

no

Exercise:

Problem: $\frac{11.6}{9.2} = \frac{37.12}{29.44}$

In the following exercises, solve each proportion.

Exercise:

Problem: $\frac{x}{36} = \frac{5}{9}$

Solution:

20

Exercise:

Problem: $\frac{7}{a} = \frac{-6}{84}$

Exercise:

Problem: $\frac{1.2}{1.8} = \frac{d}{6}$

Solution:

4

Exercise:

Problem: $\frac{\frac{1}{2}}{2} = \frac{m}{20}$

In the following exercises, solve the proportion problem.

Exercise:

Problem:

The children's dosage of acetaminophen is 5 milliliters (ml) for every 25 pounds of a child's weight. How many milliliters of acetaminophen will be prescribed for a 60 pound child?

Solution:

12

Exercise:**Problem:**

After a workout, Dennis takes his pulse for 10 sec and counts 21 beats. How many beats per minute is this?

Exercise:**Problem:**

An 8 ounce serving of ice cream has 272 calories. If Lavonne eats 10 ounces of ice cream, how many calories does she get?

Solution:

340

Exercise:**Problem:**

Alma is going to Europe and wants to exchange \$1,200 into Euros. If each dollar is 0.75 Euros, how many Euros will Alma get?

Exercise:**Problem:**

Zack wants to drive from Omaha to Denver, a distance of 494 miles. If his car gets 38 miles to the gallon, how many gallons of gas will Zack need to get to Denver?

Solution:

13 gallons

Exercise:

Problem:

Teresa is planning a party for 100 people. Each gallon of punch will serve 18 people. How many gallons of punch will she need?

In the following exercises, translate to a proportion.

Exercise:

Problem: What number is 62% of 395?

Solution:

$$\frac{n}{395} = \frac{62}{100}$$

Exercise:

Problem: 42 is 70% of what number?

Exercise:

Problem: What percent of 1,000 is 15?

Solution:

$$\frac{15}{1000} = \frac{p}{100}$$

Exercise:

Problem: What percent of 140 is 210?

In the following exercises, translate and solve using proportions.

Exercise:

Problem: What number is 85% of 900?

Solution:

765

Exercise:

Problem: 6% of what number is \$24?

Exercise:

Problem: \$3.51 is 4.5% of what number?

Solution:

\$78

Exercise:

Problem: What percent of 3,100 is 930?

In the following exercises, convert each percent to ① a decimal ② a simplified fraction.

Exercise:

Problem: 24%

Solution:

0.24, $\frac{6}{25}$

Exercise:

Problem: 5%

Exercise:

Problem: 350%

Solution:

3.5, $3\frac{1}{2}$

In the following exercises, convert each fraction to a percent. (Round to 3 decimal places if needed.)

Exercise:

Problem: $\frac{7}{8}$

Exercise:

Problem: $\frac{1}{3}$

Solution:

33.333%

Exercise:

Problem: $\frac{11}{12}$

In the following exercises, solve the percent problem.

Exercise:

Problem: 65 is what percent of 260?

Solution:

25%

Exercise:

Problem: What number is 27% of 3,000?

Exercise:

Problem: 150% of what number is 60?

Solution:

40

Exercise:

Problem:

Yuki's monthly paycheck is \$3,825. She pays \$918 for rent. What percent of her paycheck goes to rent?

Exercise:

Problem:

The total number of vehicles on one freeway dropped from 84,000 to 74,000. Find the percent decrease (round to the nearest tenth of a percent).

Solution:

11.9%

Exercise:

Problem:

Kyle bought a bicycle in Denver where the sales tax was 7.72% of the purchase price. The purchase price of the bicycle was \$600. What was the total cost?

Exercise:

Problem:

Mara received \$31.80 commission when she sold a \$795 suit. What was her rate of commission?

Solution:

4%

Exercise:

Problem:

Kiyoshi bought a television set on sale for \$899. The original price was \$1,200. Find:

- Ⓐ the amount of discount
- Ⓑ the discount rate (round to the nearest tenth of a percent)

Exercise:

Problem:

Oxana bought a dresser at a garage sale for \$20. She refinished it, then added a 250% markup before advertising it for sale. What price did she ask for the dresser?

Solution:

\$50

Exercise:**Problem:**

Find the simple interest earned after 5 years on \$3000 invested at an interest rate of 4.2%.

Exercise:**Problem:**

Brenda borrowed \$400 from her brother. Two years later, she repaid the \$400 plus \$50 interest. What was the rate of interest?

Solution:

6.2%

Exercise:**Problem:**

Write as a proportion: 4 gallons to 144 miles is the same as 10 gallons to 360 miles.

Exercise:

Problem: Solve for a: $\frac{12}{a} = \frac{-15}{65}$

Solution:

-52

Exercise:

Problem:

Vin read 10 pages of a book in 12 minutes. At that rate, how long will it take him to read 35 pages?

Glossary

proportion

A proportion is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$, $d \neq 0$. The proportion states two ratios or rates are equal. The proportion is read “ a is to b , as c is to d ”.

Solve Equations with Variables and Constants on Both Sides

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides
- Solve equations using a general strategy

Note:

Before you get started, take this readiness quiz.

1. Simplify: $4y - 9 + 9$.

If you missed this problem, review [\[link\]](#).

2. Solve: $y + 12 = 16$.

If you missed this problem, review [\[link\]](#).

3. Solve: $-3y = 63$.

If you missed this problem, review [\[link\]](#).

Solve an Equation with Constants on Both Sides

You may have noticed that in all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we'll see how to solve equations where the variable terms and/or constant terms are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the variable side, and the other side of the equation to be the constant side. Then, we will use the Subtraction and Addition Properties of Equality, step by step, to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that started with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example:
Exercise:

Problem: Solve: $4x + 6 = -14$.

Solution:
Solution

In this equation, the variable is only on the left side. It makes sense to call the left side the variable side. Therefore, the right side will be the constant side. We'll write the labels above the equation to help us remember what goes where.

	Variable
Since the left side is the variable side, the 6 is out of place. We must "undo" adding 6 by subtracting 6, and to keep the equality we must subtract 6 from both sides. Use the Subtraction Property of Equality.	Constant
Simplify.	
Now all the x s are on the left and the constant on the right.	

Use the Division Property of Equality.		
Simplify.		
Check:	$4x + 6 = -14$	
Let $x = -5$.	$4(-5) + 6 = -14$	
	$-20 + 6 = -14$	
	$-14 = -14 \checkmark$	

Note:

Exercise:

Problem: Solve: $3x + 4 = -8$.

Solution:

$$x = -4$$

Note:

Exercise:

Problem: Solve: $5a + 3 = -37$.

Solution:

$$a = -8$$

Example:**Exercise:**

Problem: Solve: $2y - 7 = 15$.

Solution:

Solution

Notice that the variable is only on the left side of the equation, so this will be the variable side and the right side will be the constant side. Since the left side is the variable side, the 7 is out of place. It is subtracted from the $2y$, so to ‘undo’ subtraction, add 7 to both sides.

	variable constant $2y - 7 = 15$
Add 7 to both sides.	$2y - 7 + 7 = 15 + 7$
Simplify.	$2y = 22$

The variables are now on one side and the constants on the other.		
Divide both sides by 2.		$\frac{2y}{2} = \frac{22}{2}$
Simplify.		$y = 11$
Check:	$2y - 7 = 15$	
Substitute: $y = 11$.	$2 \cdot 11 - 7 \stackrel{?}{=} 15$	
	$22 - 7 \stackrel{?}{=} 15$	
	$15 = 15 \checkmark$	

Note:

Exercise:

Problem: Solve: $5y - 9 = 16$.

Solution:

$$y = 5$$

Note:

Exercise:

Problem: Solve: $3m - 8 = 19$.

Solution:

$$m = 9$$

Solve an Equation with Variables on Both Sides

What if there are variables on both sides of the equation? We will start like we did above—choosing a variable side and a constant side, and then use the Subtraction and Addition Properties of Equality to collect all variables on one side and all constants on the other side. Remember, what you do to the left side of the equation, you must do to the right side too.

Example:

Exercise:

Problem: Solve: $5x = 4x + 7$.

Solution:

Solution

Here the variable, x , is on both sides, but the constants appear only on the right side, so let's make the right side the "constant" side. Then the left side will be the "variable" side.

		variable constant $5x = 4x + 7$
We don't want any variables on the right, so subtract the $4x$.		$5x - 4x = 4x - 4x + 7$
Simplify.		$x = 7$
We have all the variables on one side and the constants on the other. We have solved the equation.		
Check:	$5x = 4x + 7$	
Substitute 7 for x .	$5(7) \stackrel{?}{=} 4(7) + 7$	
	$35 \stackrel{?}{=} 28 + 7$	
	$35 = 35 \checkmark$	

Note:

Exercise:

Problem: Solve: $6n = 5n + 10$.

Solution:

$$n = 10$$

Note:

Exercise:

Problem: Solve: $-6c = -7c + 1$.

Solution:

$$c = 1$$

Example:

Exercise:

Problem: Solve: $5y - 8 = 7y$.

Solution:

Solution

The only constant, -8 , is on the left side of the equation and variable, y , is on both sides. Let's leave the constant on the left and collect the variables to the right.

constant	variable
$5y - 8 = 7y$	

Subtract $5y$ from both sides.

$$5y - 5y - 8 = 7y - 5y$$

Simplify.

$$-8 = 2y$$

We have the variables on the right and the constants on the left. Divide both sides by 2.

$$\frac{-8}{2} = \frac{2y}{2}$$

Simplify.

$$-4 = y$$

Rewrite with the variable on the left.

$$y = -4$$

Check: Let $y = -4$.

$$5y - 8 = 7y$$

$$5(-4) - 8 \stackrel{?}{=} 7(-4)$$

$$-20 - 8 \stackrel{?}{=} -28$$

$$-28 = -28 \checkmark$$

Note:

Exercise:

Problem: Solve: $3p - 14 = 5p$.

Solution:

$$p = -7$$

Note:

Exercise:

Problem: Solve: $8m + 9 = 5m$.

Solution:

$$m = -3$$

Example:

Exercise:

Problem: Solve: $7x = -x + 24$.

Solution:

Solution

The only constant, 24, is on the right, so let the left side be the variable side.

	variable side constant side $7x = -x + 24$
Remove the $-x$ from the right side by adding x to both sides.	$7x + x = -x + x + 24$
Simplify.	$8x = 24$
All the variables are on the left and the constants are on the right. Divide both sides by 8.	$\frac{8x}{8} = \frac{24}{8}$
Simplify.	$x = 3$
Check: Substitute $x = 3$.	
$7x = -x + 24$ $7(3) \stackrel{?}{=} -(3) + 24$ $21 = 21 \checkmark$	

Note:

Exercise:

Problem: Solve: $12j = -4j + 32$.

Solution:

$$j = 2$$

Note:

Exercise:

Problem: Solve: $8h = -4h + 12$.

Solution:

$$h = 1$$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables *and* constants on both sides of the equation. As we did before, we'll collect the variable terms to one side and the constants to the other side.

Example:

Exercise:

Problem: Solve: $7x + 5 = 6x + 2$.

Solution:

Solution

Start by choosing which side will be the variable side and which side will be the constant side. The variable terms are $7x$ and $6x$. Since 7 is greater than 6, make the left side the variable side and so the right side will be the constant side.

	$7x + 5 = 6x + 2$
Collect the variable terms to the left side by subtracting $6x$ from both sides.	$7x - 6x + 5 = 6x - 6x + 2$
Simplify.	$x + 5 = 2$
Now, collect the constants to the right side by subtracting 5 from both sides.	$x + 5 - 5 = 2 - 5$
Simplify.	$x = -3$
The solution is $x = -3$.	
Check: Let $x = -3$.	
$7x + 5 = 6x + 2$ $7(-3) + 5 \stackrel{?}{=} 6(-3) + 2$ $-21 + 5 \stackrel{?}{=} -18 + 2$ $-16 = -16 \checkmark$	

Note:

Exercise:

Problem: Solve: $12x + 8 = 6x + 2$.

Solution:

$$x = -1$$

Note:**Exercise:**

Problem: Solve: $9y + 4 = 7y + 12$.

Solution:

$$y = 4$$

We'll summarize the steps we took so you can easily refer to them.

Note:

Solve an equation with variables and constants on both sides.

Choose one side to be the variable side and then the other will be the constant side.

Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.

Collect the constants to the other side, using the Addition or Subtraction Property of Equality.

Make the coefficient of the variable 1, using the Multiplication or Division Property of Equality.

Check the solution by substituting it into the original equation.

It is a good idea to make the variable side the one in which the variable has the larger coefficient. This usually makes the arithmetic easier.

Example:

Exercise:

Problem: Solve: $6n - 2 = -3n + 7$.

Solution:
Solution

We have $6n$ on the left and $-3n$ on the right. Since $6 > -3$, make the left side the “variable” side.

	$6n - 2 = -3n + 7$
We don't want variables on the right side—add $3n$ to both sides to leave only constants on the right.	$6n + 3n - 2 = -3n + 3n + 7$
Combine like terms.	$9n - 2 = 7$
We don't want any constants on the left side, so add 2 to both sides.	$9n - 2 + 2 = 7 + 2$
Simplify.	$9n = 9$

The variable term is on the left and the constant term is on the right.

To get the coefficient of n to be one, divide both sides by 9.

$$\frac{9n}{9} = \frac{9}{9}$$

Simplify.

$$n = 1$$

Check: Substitute 1 for n .

$$\begin{aligned} 6n - 2 &= -3n + 7 \\ 6(1) - 2 &\stackrel{?}{=} -3(1) + 7 \\ 4 &= 4 \checkmark \end{aligned}$$

Note:

Exercise:

Problem: Solve: $8q - 5 = -4q + 7$.

Solution:

$$q = 1$$

Note:

Exercise:

Problem: Solve: $7n - 3 = n + 3$.

Solution:

$$n = 1$$

Example:

Exercise:

Problem: Solve: $2a - 7 = 5a + 8$.

Solution:

Solution

This equation has $2a$ on the left and $5a$ on the right. Since $5 > 2$, make the right side the variable side and the left side the constant side.

	$2a - 7 = 5a + 8$
Subtract $2a$ from both sides to remove the variable term from the left.	$2a - 2a - 7 = 5a - 2a + 8$
Combine like terms.	$-7 = 3a + 8$
Subtract 8 from both sides to remove the constant from the right.	$-7 - 8 = 3a + 8 - 8$
Simplify.	$-15 = 3a$

Divide both sides by 3 to make 1 the coefficient of a .

$$\frac{-15}{3} = \frac{3a}{3}$$

Simplify.

$$-5 = a$$

Check: Let $a = -5$.

$$\begin{aligned} 2a - 7 &= 5a + 8 \\ 2(-5) - 7 &\stackrel{?}{=} 5(-5) + 8 \\ -10 - 7 &\stackrel{?}{=} -25 + 8 \\ -17 &= -17 \checkmark \end{aligned}$$

Note that we could have made the left side the variable side instead of the right side, but it would have led to a negative coefficient on the variable term. While we could work with the negative, there is less chance of error when working with positives. The strategy outlined above helps avoid the negatives!

Note:

Exercise:

Problem: Solve: $2a - 2 = 6a + 18$.

Solution:

$$a = -5$$

Note:

Exercise:

Problem: Solve: $4k - 1 = 7k + 17$.

Solution:

$$k = -6$$

To solve an equation with fractions, we still follow the same steps to get the solution.

Example:

Exercise:

Problem: Solve: $\frac{3}{2}x + 5 = \frac{1}{2}x - 3$.

Solution:

Solution

Since $\frac{3}{2} > \frac{1}{2}$, make the left side the variable side and the right side the constant side.

$\frac{3}{2}x + 5 = \frac{1}{2}x - 3$

Subtract $\frac{1}{2}x$ from both sides.	$\frac{3}{2}x - \frac{1}{2}x + 5 = \frac{1}{2}x - \frac{1}{2}x - 3$
Combine like terms.	$x + 5 = -3$
Subtract 5 from both sides.	$x + 5 - 5 = -3 - 5$
Simplify.	$x = -8$
Check: Let $x = -8$.	
$\frac{3}{2}x + 5 = \frac{1}{2}x - 3$ $\frac{3}{2}(-8) + 5 \stackrel{?}{=} \frac{1}{2}(-8) - 3$ $-12 + 5 \stackrel{?}{=} -4 - 3$ $-7 = -7 \checkmark$	

Note:

Exercise:

Problem: Solve: $\frac{7}{8}x - 12 = -\frac{1}{8}x - 2$.

Solution:

$$x = 10$$

Note:

Exercise:

Problem: Solve: $\frac{7}{6}y + 11 = \frac{1}{6}y + 8$.

Solution:

$$y = -3$$

We follow the same steps when the equation has decimals, too.

Example:

Exercise:

Problem: Solve: $3.4x + 4 = 1.6x - 5$.

Solution:

Solution

Since $3.4 > 1.6$, make the left side the variable side and the right side the constant side.

$3.4x + 4 = 1.6x - 5$

Subtract $1.6x$ from both sides.	$3.4x - 1.6x + 4 = 1.6x - 1.6x - 5$
Combine like terms.	$1.8x + 4 = -5$
Subtract 4 from both sides.	$1.8x + 4 - 4 = -5 - 4$
Simplify.	$1.8x = -9$
Use the Division Property of Equality.	$\frac{1.8x}{1.8} = \frac{-9}{1.8}$
Simplify.	$x = -5$
Check: Let $x = -5$.	
$ \begin{aligned} 3.4x + 4 &= 1.6x - 5 \\ 3.4(-5) + 4 &\stackrel{?}{=} 1.6(-5) - 5 \\ -17 + 4 &\stackrel{?}{=} -8 - 5 \\ -13 &= -13 \checkmark \end{aligned} $	

Note:

Exercise:

Problem: Solve: $2.8x + 12 = -1.4x - 9$.

Solution:

$$x = -5$$

Note:

Exercise:

Problem: Solve: $3.6y + 8 = 1.2y - 4$.

Solution:

$$y = -5$$

Solve Equations Using a General Strategy

Each of the first few sections of this chapter has dealt with solving one specific form of a linear equation. It's time now to lay out an overall strategy that can be used to solve *any* linear equation. We call this the *general strategy*. Some equations won't require all the steps to solve, but many will. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

Note:

Use a general strategy for solving linear equations.

Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms. Collect all the variable terms to one side of the equation. Use the Addition

or Subtraction Property of Equality.

Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.

Make the coefficient of the variable term to equal 1. Use the Multiplication or Division Property of Equality. State the solution to the equation.

Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

Example:

Exercise:

Problem: Solve: $3(x + 2) = 18$.

Solution:

Solution

	$3(x + 2) = 18$
Simplify each side of the equation as much as possible. Use the Distributive Property.	$3x + 6 = 18$
Collect all variable terms on one side of the equation—all x s are already on the left side.	
Collect constant terms on the other side of the equation.	$3x + 6 - 6 = 18 - 6$

Subtract 6 from each side	
Simplify.	$3x = 12$
Make the coefficient of the variable term equal to 1. Divide each side by 3.	$\frac{3x}{3} = \frac{12}{3}$
Simplify.	$x = 4$
Check: Let $x = 4$.	
$3(x + 2) = 18$ $3(\textcolor{red}{4} + 2) \stackrel{?}{=} 18$ $3(6) \stackrel{?}{=} 18$ $18 = 18 \checkmark$	

Note:

Exercise:

Problem: Solve: $5(x + 3) = 35$.

Solution:

$$x = 4$$

Note:

Exercise:

Problem: Solve: $6(y - 4) = -18$.

Solution:

$$y = 1$$

Example:

Exercise:

Problem: Solve: $-(x + 5) = 7$.

Solution:

Solution

	$-(x + 5) = 7$
Simplify each side of the equation as much as possible by distributing. The only x term is on the left side, so all variable terms are on the left side of the equation.	$-x - 5 = 7$
Add 5 to both sides to get all constant terms on the right side of the equation.	$-x - 5 + 5 = 7 + 5$
Simplify.	

	$-x = 12$
Make the coefficient of the variable term equal to 1 by multiplying both sides by -1.	$-1(-x) = -1(12)$
Simplify.	$x = -12$
Check: Let $x = -12$.	
$-(x + 5) = 7$ $-(-12 + 5) \stackrel{?}{=} 7$ $-(-7) \stackrel{?}{=} 7$ $7 = 7 \checkmark$	

Note:

Exercise:

Problem: Solve: $-(y + 8) = -2$.

Solution:

$$y = -6$$

Note:

Exercise:

Problem: Solve: $-(z + 4) = -12$.

Solution:

$$z = 8$$

Example:

Exercise:

Problem: Solve: $4(x - 2) + 5 = -3$.

Solution:

Solution

	$4(x - 2) + 5 = -3$
Simplify each side of the equation as much as possible. Distribute.	$4x - 8 + 5 = -3$
Combine like terms	$4x - 3 = -3$

The only x is on the left side, so all variable terms are on one side of the equation.	
Add 3 to both sides to get all constant terms on the other side of the equation.	$4x - 3 + 3 = -3 + 3$
Simplify.	$4x = 0$
Make the coefficient of the variable term equal to 1 by dividing both sides by 4.	$\frac{4x}{4} = \frac{0}{4}$
Simplify.	$x = 0$
Check: Let $x = 0$.	
$ \begin{aligned} 4(x - 2) + 5 &= -3 \\ 4(0 - 2) + 5 &\stackrel{?}{=} -3 \\ 4(-2) + 5 &\stackrel{?}{=} -3 \\ -8 + 5 &\stackrel{?}{=} -3 \\ -3 &= -3 \checkmark \end{aligned} $	

Note:

Exercise:

Problem: Solve: $2(a - 4) + 3 = -1$.

Solution:

$$a = 2$$

Note:

Exercise:

Problem: Solve: $7(n - 3) - 8 = -15$.

Solution:

$$n = 2$$

Example:

Exercise:

Problem: Solve: $8 - 2(3y + 5) = 0$.

Solution:

Solution

Be careful when distributing the negative.

	$8 - 2(3y + 5) = 0$

Simplify—use the Distributive Property.

$$8 - 6y - 10 = 0$$

Combine like terms.

$$-6y - 2 = 0$$

Add 2 to both sides to collect constants on the right.

$$-6y - 2 + 2 = 0 + 2$$

Simplify.

$$-6y = 2$$

Divide both sides by -6 .

$$\frac{-6y}{-6} = \frac{2}{-6}$$

Simplify.

$$y = -\frac{1}{3}$$

Check: Let $y = -\frac{1}{3}$.

$$\begin{aligned} 8 - 2(3y + 5) &= 0 \\ 8 - 2\left[3\left(-\frac{1}{3}\right) + 5\right] &= 0 \\ 8 - 2(-1 + 5) &\stackrel{?}{=} 0 \\ 8 - 2(4) &\stackrel{?}{=} 0 \\ 8 - 8 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

Note:

Exercise:

Problem: Solve: $12 - 3(4j + 3) = -17$.

Solution:

$$j = \frac{5}{3}$$

Note:

Exercise:

Problem: Solve: $-6 - 8(k - 2) = -10$.

Solution:

$$k = \frac{5}{2}$$

Example:

Exercise:

Problem: Solve: $3(x - 2) - 5 = 4(2x + 1) + 5$.

Solution:

Solution

--	--

	$3(x - 2) - 5 = 4(2x + 1) + 5$
Distribute.	$3x - 6 - 5 = 8x + 4 + 5$
Combine like terms.	$3x - 11 = 8x + 9$
Subtract $3x$ to get all the variables on the right since $8 > 3$.	$3x - 3x - 11 = 8x - 3x + 9$
Simplify.	$-11 = 5x + 9$
Subtract 9 to get the constants on the left.	$-11 - 9 = 5x + 9 - 9$
Simplify.	$-20 = 5x$
Divide by 5.	$\frac{-20}{5} = \frac{5x}{5}$
Simplify.	$-4 = x$
Check: Substitute: $-4 = x$.	

$$\begin{aligned}
 3(x - 2) - 5 &= 4(2x + 1) + 5 \\
 3(-4 - 2) - 5 &\stackrel{?}{=} 4[2(-4) + 1] + 5 \\
 3(-6) - 5 &\stackrel{?}{=} 4(-8 + 1) + 5 \\
 -18 - 5 &\stackrel{?}{=} 4(-7) + 5 \\
 -23 &\stackrel{?}{=} -28 + 5 \\
 -23 &= -23 \checkmark
 \end{aligned}$$

Note:

Exercise:

Problem: Solve: $6(p - 3) - 7 = 5(4p + 3) - 12$.

Solution:

$$p = -2$$

Note:

Exercise:

Problem: Solve: $8(q + 1) - 5 = 3(2q - 4) - 1$.

Solution:

$$q = -8$$

Example:

Exercise:

Problem: Solve: $\frac{1}{2}(6x - 2) = 5 - x$.

Solution:
Solution

	$\frac{1}{2}(6x - 2) = 5 - x$
Distribute.	$3x - 1 = 5 - x$
Add x to get all the variables on the left.	$3x - 1 + x = 5 - x + x$
Simplify.	$4x - 1 = 5$
Add 1 to get constants on the right.	$4x - 1 + 1 = 5 + 1$
Simplify.	$4x = 6$
Divide by 4.	$\frac{4x}{4} = \frac{6}{4}$

Simplify.

$$x = \frac{3}{2}$$

Check: Let $x = \frac{3}{2}$.

$$\begin{aligned}\frac{1}{2}(6x - 2) &= 5 - x \\ \frac{1}{2}\left(6 \cdot \frac{3}{2} - 2\right) &\stackrel{?}{=} 5 - \frac{3}{2} \\ \frac{1}{2}(9 - 2) &\stackrel{?}{=} \frac{10}{2} - \frac{3}{2} \\ \frac{1}{2}(7) &\stackrel{?}{=} \frac{7}{2} \\ \frac{7}{2} &= \frac{7}{2} \checkmark\end{aligned}$$

Note:

Exercise:

Problem: Solve: $\frac{1}{3}(6u + 3) = 7 - u$.

Solution:

$$u = 2$$

Note:

Exercise:

Problem: Solve: $\frac{2}{3}(9x - 12) = 8 + 2x$.

Solution:

$$x = 4$$

In many applications, we will have to solve equations with decimals. The same general strategy will work for these equations.

Example:

Exercise:

Problem: Solve: $0.24(100x + 5) = 0.4(30x + 15)$.

Solution:

Solution

	$0.24(100x + 5) = 0.4(30x + 15)$
Distribute.	$24x + 1.2 = 12x + 6$
Subtract $12x$ to get all the x s to the left.	$24x + 1.2 - 12x = 12x + 6 - 12x$

Simplify.	$12x + 1.2 = 6$
Subtract 1.2 to get the constants to the right.	$12x + 1.2 - 1.2 = 6 - 1.2$
Simplify.	$12x = 4.8$
Divide.	$\frac{12x}{12} = \frac{4.8}{12}$
Simplify.	$x = 0.4$
Check: Let $x = 0.4$.	
$0.24(100x + 5) = 0.4(30x + 15)$ $0.24(100(\textcolor{red}{0.4}) + 5) \stackrel{?}{=} 0.4(30(\textcolor{red}{0.4}) + 15)$ $0.24(40 + 5) \stackrel{?}{=} 0.4(12 + 15)$ $0.24(45) \stackrel{?}{=} 0.4(27)$ $10.8 = 10.8 \checkmark$	

Note:

Exercise:

Problem: Solve: $0.55(100n + 8) = 0.6(85n + 14)$.

Solution:

Note:**Exercise:**

Problem: Solve: $0.15(40m - 120) = 0.5(60m + 12)$.

Solution:

–1

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solving Multi-Step Equations](#)
- [Solve an Equation with Variable Terms on Both Sides](#)
- [Solving Multi-Step Equations \(L5.4\)](#)
- [Solve an Equation with Variables and Parentheses on Both Sides](#)

Key Concepts

- **Solve an equation with variables and constants on both sides**

Choose one side to be the variable side and then the other will be the constant side.

Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.

Collect the constants to the other side, using the Addition or Subtraction Property of Equality.

Make the coefficient of the variable 1, using the Multiplication or

Division Property of Equality.

Check the solution by substituting into the original equation.

- **General strategy for solving linear equations**

Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms. Collect all the variable terms to one side of the equation. Use the Addition or Subtraction Property of Equality.

Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.

Make the coefficient of the variable term to equal to 1. Use the Multiplication or Division Property of Equality. State the solution to the equation.

Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

Practice Makes Perfect

Solve an Equation with Constants on Both Sides

In the following exercises, solve the equation for the variable.

Exercise:

Problem: $6x - 2 = 40$

Exercise:

Problem: $7x - 8 = 34$

Solution:

6

Exercise:

Problem: $11w + 6 = 93$

Exercise:

Problem: $14y + 7 = 91$

Solution:

6

Exercise:

Problem: $3a + 8 = -46$

Exercise:

Problem: $4m + 9 = -23$

Solution:

-8

Exercise:

Problem: $-50 = 7n - 1$

Exercise:

Problem: $-47 = 6b + 1$

Solution:

-8

Exercise:

Problem: $25 = -9y + 7$

Exercise:

Problem: $29 = -8x - 3$

Solution:

-4

Exercise:

Problem: $-12p - 3 = 15$

Exercise:

Problem: $-14q - 15 = 13$

Solution:

-2

Solve an Equation with Variables on Both Sides

In the following exercises, solve the equation for the variable.

Exercise:

Problem: $8z = 7z - 7$

Exercise:

Problem: $9k = 8k - 11$

Solution:

-11

Exercise:

Problem: $4x + 36 = 10x$

Exercise:

Problem: $6x + 27 = 9x$

Solution:

9

Exercise:

Problem: $c = -3c - 20$

Exercise:

Problem: $b = -4b - 15$

Solution:

-3

Exercise:

Problem: $5q = 44 - 6q$

Exercise:

Problem: $7z = 39 - 6z$

Solution:

3

Exercise:

Problem: $3y + \frac{1}{2} = 2y$

Exercise:

Problem: $8x + \frac{3}{4} = 7x$

Solution:

$-3/4$

Exercise:

Problem: $-12a - 8 = -16a$

Exercise:

Problem: $-15r - 8 = -11r$

Solution:

-2

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the equations for the variable.

Exercise:

Problem: $6x - 15 = 5x + 3$

Exercise:

Problem: $4x - 17 = 3x + 2$

Solution:

19

Exercise:

Problem: $26 + 8d = 9d + 11$

Exercise:

Problem: $21 + 6f = 7f + 14$

Solution:

$$7$$

Exercise:

Problem: $3p - 1 = 5p - 33$

Exercise:

Problem: $8q - 5 = 5q - 20$

Solution:

$$-5$$

Exercise:

Problem: $4a + 5 = -a - 40$

Exercise:

Problem: $9c + 7 = -2c - 37$

Solution:

$$-4$$

Exercise:

Problem: $8y - 30 = -2y + 30$

Exercise:

Problem: $12x - 17 = -3x + 13$

Solution:

2

Exercise:

Problem: $2z - 4 = 23 - z$

Exercise:

Problem: $3y - 4 = 12 - y$

Solution:

4

Exercise:

Problem: $\frac{5}{4}c - 3 = \frac{1}{4}c - 16$

Exercise:

Problem: $\frac{4}{3}m - 7 = \frac{1}{3}m - 13$

Solution:

-6

Exercise:

Problem: $8 - \frac{2}{5}q = \frac{3}{5}q + 6$

Exercise:

Problem: $11 - \frac{1}{4}a = \frac{3}{4}a + 4$

Solution:

7

Exercise:

Problem: $\frac{4}{3}n + 9 = \frac{1}{3}n - 9$

Exercise:

Problem: $\frac{5}{4}a + 15 = \frac{3}{4}a - 5$

Solution:

-40

Exercise:

Problem: $\frac{1}{4}y + 7 = \frac{3}{4}y - 3$

Exercise:

Problem: $\frac{3}{5}p + 2 = \frac{4}{5}p - 1$

Solution:

15

Exercise:

Problem: $14n + 8.25 = 9n + 19.60$

Exercise:

Problem: $13z + 6.45 = 8z + 23.75$

Solution:

3.46

Exercise:

Problem: $2.4w - 100 = 0.8w + 28$

Exercise:

Problem: $2.7w - 80 = 1.2w + 10$

Solution:

60

Exercise:

Problem: $5.6r + 13.1 = 3.5r + 57.2$

Exercise:

Problem: $6.6x - 18.9 = 3.4x + 54.7$

Solution:

23

Solve an Equation Using the General Strategy

In the following exercises, solve the linear equation using the general strategy.

Exercise:

Problem: $5(x + 3) = 75$

Exercise:

Problem: $4(y + 7) = 64$

Solution:

9

Exercise:

Problem: $8 = 4(x - 3)$

Exercise:

Problem: $9 = 3(x - 3)$

Solution:

6

Exercise:

Problem: $20(y - 8) = -60$

Exercise:

Problem: $14(y - 6) = -42$

Solution:

3

Exercise:

Problem: $-4(2n + 1) = 16$

Exercise:

Problem: $-7(3n + 4) = 14$

Solution:

-2

Exercise:

Problem: $3(10 + 5r) = 0$

Exercise:

Problem: $8(3 + 3p) = 0$

Solution:

-1

Exercise:

Problem: $\frac{2}{3}(9c - 3) = 22$

Exercise:

Problem: $\frac{3}{5}(10x - 5) = 27$

Solution:

5

Exercise:

Problem: $5(1.2u - 4.8) = -12$

Exercise:

Problem: $4(2.5v - 0.6) = 7.6$

Solution:

0.52

Exercise:

Problem: $0.2(30n + 50) = 28$

Exercise:

Problem: $0.5(16m + 34) = -15$

Solution:

$$0.25$$

Exercise:

Problem: $-(w - 6) = 24$

Exercise:

Problem: $-(t - 8) = 17$

Solution:

$$-9$$

Exercise:

Problem: $9(3a + 5) + 9 = 54$

Exercise:

Problem: $8(6b - 7) + 23 = 63$

Solution:

$$2$$

Exercise:

Problem: $10 + 3(z + 4) = 19$

Exercise:

Problem: $13 + 2(m - 4) = 17$

Solution:

$$6$$

Exercise:

Problem: $7 + 5(4 - q) = 12$

Exercise:

Problem: $-9 + 6(5 - k) = 12$

Solution:

$$3/2$$

Exercise:

Problem: $15 - (3r + 8) = 28$

Exercise:

Problem: $18 - (9r + 7) = -16$

Solution:

$$3$$

Exercise:

Problem: $11 - 4(y - 8) = 43$

Exercise:

Problem: $18 - 2(y - 3) = 32$

Solution:

$$-4$$

Exercise:

Problem: $9(p - 1) = 6(2p - 1)$

Exercise:

Problem: $3(4n - 1) - 2 = 8n + 3$

Solution:

$$2$$

Exercise:

Problem: $9(2m - 3) - 8 = 4m + 7$

Exercise:

Problem: $5(x - 4) - 4x = 14$

Solution:

$$34$$

Exercise:

Problem: $8(x - 4) - 7x = 14$

Exercise:

Problem: $5 + 6(3s - 5) = -3 + 2(8s - 1)$

Solution:

$$10$$

Exercise:

Problem: $-12 + 8(x - 5) = -4 + 3(5x - 2)$

Exercise:

Problem: $4(x - 1) - 8 = 6(3x - 2) - 7$

Solution:

2

Exercise:

Problem: $7(2x - 5) = 8(4x - 1) - 9$

Everyday Math**Exercise:****Problem:**

Making a fence Jovani has a fence around the rectangular garden in his backyard. The perimeter of the fence is 150 feet. The length is 15 feet more than the width. Find the width, w , by solving the equation $150 = 2(w + 15) + 2w$.

Solution:

30 feet

Exercise:

Problem:

Concert tickets At a school concert, the total value of tickets sold was \$1,506. Student tickets sold for \$6 and adult tickets sold for \$9. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, s , by solving the equation $6s + 9(3s - 5) = 1506$.

Exercise:**Problem:**

Coins Rhonda has \$1.90 in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n , by solving the equation $0.05n + 0.10(2n - 1) = 1.90$.

Solution:

8 nickels

Exercise:**Problem:**

Fencing Micah has 74 feet of fencing to make a rectangular dog pen in his yard. He wants the length to be 25 feet more than the width. Find the length, L , by solving the equation $2L + 2(L - 25) = 74$.

Writing Exercises**Exercise:****Problem:**

When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient as the variable side?

Solution:

Answers will vary.

Exercise:**Problem:**

Solve the equation $10x + 14 = -2x + 38$, explaining all the steps of your solution.

Exercise:**Problem:**

What is the first step you take when solving the equation $3 - 7(y - 4) = 38$? Explain why this is your first step.

Solution:

Answers will vary.

Exercise:**Problem:**

Solve the equation $\frac{1}{4}(8x + 20) = 3x - 4$ explaining all the steps of your solution as in the examples in this section.

Exercise:**Problem:**

Using your own words, list the steps in the General Strategy for Solving Linear Equations.

Solution:

Answers will vary.

Exercise:

Problem:

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

Ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Solve Equations with Fraction or Decimal Coefficients
By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Note:

Before you get started, take this readiness quiz.

1. Multiply: $8 \cdot \frac{3}{8}$.

If you missed this problem, review [\[link\]](#)

2. Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.

If you missed this problem, review [\[link\]](#)

3. Multiply: 4.78 by 100.

If you missed this problem, review [\[link\]](#)

Solve Equations with Fraction Coefficients

Let's use the General Strategy for Solving Linear Equations introduced earlier to solve the equation $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

	$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x + \frac{1}{2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2}$

Simplify the left side.	$\frac{1}{8}x = \frac{1}{4} - \frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x = \frac{1}{4} - \frac{2}{4}$
Subtract.	$\frac{1}{8}x = -\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1} \cdot \frac{1}{8}x = \frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x = -2$

This method worked fine, but many students don't feel very confident when they see all those fractions. So we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of *all* the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but with no fractions. This process is called *clearing the equation of fractions*. Let's solve the same equation again, but this time use the method that clears the fractions.

Example:

Exercise:

Problem: Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Solution:
Solution

Find the least common denominator of *all* the fractions in the equation.

$$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4} \quad \text{LCD} = 8$$

Multiply both sides of the equation by that LCD, 8. This clears the fractions.

$$8\left(\frac{1}{8}x + \frac{1}{2}\right) = 8\left(\frac{1}{4}\right)$$

Use the Distributive Property.

$$8 \cdot \frac{1}{8}x + 8 \cdot \frac{1}{2} = 8 \cdot \frac{1}{4}$$

Simplify — and notice, no more fractions!

$$x + 4 = 2$$

Solve using the General Strategy for Solving Linear Equations.

$$x + 4 - 4 = 2 - 4$$

Simplify.

$$x = -2$$

Check: Let $x = -2$

$$\begin{aligned}\frac{1}{8}x + \frac{1}{2} &= \frac{1}{4} \\ \frac{1}{8}(-2) + \frac{1}{2} &\stackrel{?}{=} \frac{1}{4} \\ -\frac{2}{8} + \frac{1}{2} &\stackrel{?}{=} \frac{1}{4} \\ -\frac{2}{8} + \frac{4}{8} &\stackrel{?}{=} \frac{1}{4} \\ \frac{2}{8} &\stackrel{?}{=} \frac{1}{4} \\ \frac{1}{4} &= \frac{1}{4} \checkmark\end{aligned}$$

Note:

Exercise:

Problem: Solve: $\frac{1}{4}x + \frac{1}{2} = \frac{5}{8}$.

Solution:

$$x = \frac{1}{2}$$

Note:

Exercise:

Problem: Solve: $\frac{1}{6}y - \frac{1}{3} = \frac{1}{6}$.

Solution:

$$y = 3$$

Notice in [\[link\]](#) that once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

Note:

Solve equations with fraction coefficients by clearing the fractions.

Find the least common denominator of *all* the fractions in the equation. Multiply both sides of the equation by that LCD. This clears the fractions. Solve using the General Strategy for Solving Linear Equations.

Example:**Exercise:**

Problem: Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x$.

Solution:**Solution**

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the least common denominator of *all* the fractions in the equation.

$$7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x \quad \text{LCD} = 12$$

Multiply both sides of the equation by 12.

$$12(7) = 12 \cdot \frac{1}{2}x + 12 \cdot \frac{3}{4}x - 12 \cdot \frac{2}{3}x$$

Distribute.

$$12(7) = 12 \cdot \frac{1}{2}x + 12 \cdot \frac{3}{4}x - 12 \cdot \frac{2}{3}x$$

Simplify — and notice, no more fractions!

$$84 = 6x + 9x - 8x$$

Combine like terms.

$$84 = 7x$$

Divide by 7.

$$\frac{84}{7} = \frac{7x}{7}$$

Simplify.

$$12 = x$$

Check: Let $x = 12$.

$$\begin{aligned} 7 &= \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x \\ 7 &\stackrel{?}{=} \frac{1}{2}(\textcolor{red}{12}) + \frac{3}{4}(\textcolor{red}{12}) - \frac{2}{3}(\textcolor{red}{12}) \\ 7 &\stackrel{?}{=} 6 + 9 - 8 \\ 7 &= 7 \checkmark \end{aligned}$$

Note:

Exercise:

Problem: Solve: $6 = \frac{1}{2}v + \frac{2}{5}v - \frac{3}{4}v$.

Solution:

$$v = 40$$

Note:

Exercise:

Problem: Solve: $-1 = \frac{1}{2}u + \frac{1}{4}u - \frac{2}{3}u$.

Solution:

$$u = -12$$

In the next example, we'll have variables and fractions on both sides of the equation.

Example:

Exercise:

Problem: Solve: $x + \frac{1}{3} = \frac{1}{6}x - \frac{1}{2}$.

Solution:

Solution

Find the LCD of all the fractions in the equation.

$$x + \frac{1}{3} = \frac{1}{6}x - \frac{1}{2}, \text{ LCD} = 6$$

Multiply both sides by the LCD.

$$6\left(x + \frac{1}{3}\right) = 6\left(\frac{1}{6}x - \frac{1}{2}\right)$$

Distribute.

$$6 \cdot x + 6 \cdot \frac{1}{3} = 6 \cdot \frac{1}{6}x - 6 \cdot \frac{1}{2}$$

Simplify — no more fractions!

$$6x + 2 = x - 3$$

Subtract x from both sides.

$$6x - x + 2 = x - x - 3$$

Simplify.

$$5x + 2 = -3$$

Subtract 2 from both sides.

$$5x + 2 - 2 = -3 - 2$$

Simplify.

$$5x = -5$$

Divide by 5.

$$\frac{5x}{5} = \frac{-5}{5}$$

Simplify.

$$x = -1$$

Check: Substitute $x = -1$.

$$\begin{aligned}x + \frac{1}{3} &= \frac{1}{6}x - \frac{1}{2} \\(-1) + \frac{1}{3} &\stackrel{?}{=} \frac{1}{6}(-1) - \frac{1}{2} \\(-1) + \frac{1}{3} &\stackrel{?}{=} -\frac{1}{6} - \frac{1}{2} \\-\frac{3}{3} + \frac{1}{3} &\stackrel{?}{=} -\frac{1}{6} - \frac{3}{6} \\-\frac{2}{3} &\stackrel{?}{=} -\frac{4}{6} \\-\frac{2}{3} &= -\frac{2}{3} \checkmark\end{aligned}$$

Note:

Exercise:

Problem: Solve: $a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$.

Solution:

$$a = -2$$

Note:

Exercise:

Problem: Solve: $c + \frac{3}{4} = \frac{1}{2}c - \frac{1}{4}$.

Solution:

$$c = -2$$

In [\[link\]](#), we'll start by using the Distributive Property. This step will clear the fractions right away!

Example:**Exercise:**

Problem: Solve: $1 = \frac{1}{2}(4x + 2)$.

Solution:**Solution**

	$1 = \frac{1}{2}(4x + 2)$
Distribute.	$1 = \frac{1}{2} \cdot 4x + \frac{1}{2} \cdot 2$
Simplify. Now there are no fractions to clear!	$1 = 2x + 1$
Subtract 1 from both sides.	

	$1 - 1 = 2x + 1 - 1$
Simplify.	$0 = 2x$
Divide by 2.	$\frac{0}{2} = \frac{2x}{2}$
Simplify.	$0 = x$
Check: Let $x = 0$.	
$1 = \frac{1}{2}(4x + 2)$ $1 \stackrel{?}{=} \frac{1}{2}(4(\textcolor{red}{0}) + 2)$ $1 \stackrel{?}{=} \frac{1}{2}(2)$ $1 \stackrel{?}{=} \frac{2}{2}$ $1 = 1 \checkmark$	

Note:

Exercise:

Problem: Solve: $-11 = \frac{1}{2}(6p + 2)$.

Solution:

$$p = -4$$

Note:

Exercise:

Problem: Solve: $8 = \frac{1}{3}(9q + 6)$.

Solution:

$$q = 2$$

Many times, there will still be fractions, even after distributing.

Example:

Exercise:

Problem: Solve: $\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$.

Solution:

Solution

--	--

$$\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$$

Distribute.

$$\frac{1}{2} \cdot y - \frac{1}{2} \cdot 5 = \frac{1}{4} \cdot y - \frac{1}{4} \cdot 1$$

Simplify.

$$\frac{1}{2}y - \frac{5}{2} = \frac{1}{4}y - \frac{1}{4}$$

Multiply by the LCD, 4.

$$4\left(\frac{1}{2}y - \frac{5}{2}\right) = 4\left(\frac{1}{4}y - \frac{1}{4}\right)$$

Distribute.

$$4 \cdot \frac{1}{2}y - 4 \cdot \frac{5}{2} = 4 \cdot \frac{1}{4}y - 4 \cdot \frac{1}{4}$$

Simplify.

$$2y - 10 = y - 1$$

Collect the y terms to the left.

$$2y - 10 - y = y - 1 - y$$

Simplify.

$$y - 10 = -1$$

Collect the constants to the right.

$$y - 10 + 10 = -1 + 10$$

Simplify.

$$y = 9$$

Check: Substitute 9 for y .

$$\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$$

$$\frac{1}{2}(\textcolor{red}{9} - 5) \stackrel{?}{=} \frac{1}{4}(\textcolor{red}{9} - 1)$$

$$\frac{1}{2}(4) \stackrel{?}{=} \frac{1}{4}(8)$$

$$2 = 2 \checkmark$$

Note:

Exercise:

Problem: Solve: $\frac{1}{5}(n + 3) = \frac{1}{4}(n + 2)$.

Solution:

$$n = 2$$

Note:

Exercise:

Problem: Solve: $\frac{1}{2}(m - 3) = \frac{1}{4}(m - 7)$.

Solution:

$$m = -1$$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money and percent. But decimals are really another way to represent fractions. For example, $0.3 = \frac{3}{10}$ and $0.17 = \frac{17}{100}$. So, when we have an equation with decimals, we can use the same process we used to clear fractions—multiply both sides of the equation by the least common denominator.

Example:

Exercise:

Problem: Solve: $0.8x - 5 = 7$.

Solution:

Solution

The only decimal in the equation is 0.8. Since $0.8 = \frac{8}{10}$, the LCD is 10. We can multiply both sides by 10 to clear the decimal.

	$0.8x - 5 = 7$
Multiply both sides by the LCD.	$10(0.8x - 5) = 10(7)$
Distribute.	$10(0.8x) - 10(5) = 10(7)$

Multiply, and notice, no more decimals!	$8x - 50 = 70$
Add 50 to get all constants to the right.	$8x - 50 + 50 = 70 + 50$
Simplify.	$8x = 120$
Divide both sides by 8.	$\frac{8x}{8} = \frac{120}{8}$
Simplify.	$x = 15$
Check: Let $x = 15$.	
$0.8(15) - 5 \stackrel{?}{=} 7$ $12 - 5 \stackrel{?}{=} 7$ $7 = 7 \checkmark$	

Note:

Exercise:

Problem: Solve: $0.6x - 1 = 11$.

Solution:

$$x = 20$$

Note:

Exercise:

Problem: Solve: $1.2x - 3 = 9$.

Solution:

$$x = 10$$

Example:

Exercise:

Problem: Solve: $0.06x + 0.02 = 0.25x - 1.5$.

Solution:

Solution

Look at the decimals and think of the equivalent fractions.

$$0.06 = \frac{6}{100}, \quad 0.02 = \frac{2}{100}, \quad 0.25 = \frac{25}{100}, \quad 1.5 = 1\frac{5}{10}$$

Notice, the LCD is 100.

By multiplying by the LCD we will clear the decimals.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both sides by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the constants to the left.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let $x = 8$.	

$$0.06(8) + 0.02 = 0.25(8) - 1.5$$

$$0.48 + 0.02 = 2.00 - 1.5$$

$$0.50 = 0.50 \checkmark$$

Note:

Exercise:

Problem: Solve: $0.14h + 0.12 = 0.35h - 2.4$.

Solution:

$$h = 12$$

Note:

Exercise:

Problem: Solve: $0.65k - 0.1 = 0.4k - 0.35$.

Solution:

$$k = -1$$

The next example uses an equation that is typical of the ones we will see in the money applications in the next chapter. Notice that we will distribute the decimal first before we clear all decimals in the equation.

Example:

Exercise:**Problem:** Solve: $0.25x + 0.05(x + 3) = 2.85$.**Solution:****Solution**

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$

Divide by 30.

$$\frac{30x}{30} = \frac{270}{30}$$

Simplify.

$$x = 9$$

Check: Let $x = 9$.

$$0.25x + 0.05(x + 3) = 2.85$$

$$0.25(9) + 0.05(9 + 3) \stackrel{?}{=} 2.85$$

$$2.25 + 0.05(12) \stackrel{?}{=} 2.85$$

$$2.25 + 0.60 \stackrel{?}{=} 2.85$$

$$2.85 = 2.85 \checkmark$$

Note:

Exercise:

Problem: Solve: $0.25n + 0.05(n + 5) = 2.95$.

Solution:

$$n = 9$$

Note:

Exercise:

Problem: Solve: $0.10d + 0.05(d - 5) = 2.15$.

Solution:

$$d = 16$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Solve an Equation with Fractions with Variable Terms on Both Sides](#)
- [Ex 1: Solve an Equation with Fractions with Variable Terms on Both Sides](#)
- [Ex 2: Solve an Equation with Fractions with Variable Terms on Both Sides](#)
- [Solving Multiple Step Equations Involving Decimals](#)
- [Ex: Solve a Linear Equation With Decimals and Variables on Both Sides](#)
- [Ex: Solve an Equation with Decimals and Parentheses](#)

Key Concepts

- **Solve equations with fraction coefficients by clearing the fractions.**

Find the least common denominator of *all* the fractions in the equation. Multiply both sides of the equation by that LCD. This clears the fractions.

Solve using the General Strategy for Solving Linear Equations.

Section Exercises

Practice Makes Perfect

Solve equations with fraction coefficients

In the following exercises, solve the equation by clearing the fractions.

Exercise:

Problem: $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

Solution:

$$x = -1$$

Exercise:

Problem: $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

Exercise:

Problem: $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

Solution:

$$y = -1$$

Exercise:

Problem: $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

Exercise:

Problem: $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

Solution:

$$a = \frac{3}{4}$$

Exercise:

Problem: $\frac{5}{8}b + \frac{1}{2} = -\frac{3}{4}$

Exercise:

Problem: $2 = \frac{1}{3}x - \frac{1}{2}x + \frac{2}{3}x$

Solution:

$$x = 4$$

Exercise:

Problem: $2 = \frac{3}{5}x - \frac{1}{3}x + \frac{2}{5}x$

Exercise:

Problem: $\frac{1}{4}m - \frac{4}{5}m + \frac{1}{2}m = -1$

Solution:

$$m = 20$$

Exercise:

Problem: $\frac{5}{6}n - \frac{1}{4}n - \frac{1}{2}n = -2$

Exercise:

Problem: $x + \frac{1}{2} = \frac{2}{3}x - \frac{1}{2}$

Solution:

$$x = -3$$

Exercise:

Problem: $x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$

Exercise:

Problem: $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

Solution:

$$w = \frac{9}{4}$$

Exercise:

Problem: $\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

Exercise:

Problem: $\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$

Solution:

$$x = 1$$

Exercise:

Problem: $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

Exercise:

Problem: $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

Solution:

$$b = 12$$

Exercise:

Problem: $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

Exercise:

Problem: $1 = \frac{1}{6}(12x - 6)$

Solution:

$$x = 1$$

Exercise:

Problem: $1 = \frac{1}{5}(15x - 10)$

Exercise:

Problem: $\frac{1}{4}(p - 7) = \frac{1}{3}(p + 5)$

Solution:

$$p = -41$$

Exercise:

Problem: $\frac{1}{5}(q + 3) = \frac{1}{2}(q - 3)$

Exercise:

Problem: $\frac{1}{2}(x + 4) = \frac{3}{4}$

Solution:

$$x = -\frac{5}{2}$$

Exercise:

Problem: $\frac{1}{3}(x + 5) = \frac{5}{6}$

Solve Equations with Decimal Coefficients

In the following exercises, solve the equation by clearing the decimals.

Exercise:

Problem: $0.6y + 3 = 9$

Solution:

$$y = 10$$

Exercise:

Problem: $0.4y - 4 = 2$

Exercise:

Problem: $3.6j - 2 = 5.2$

Solution:

$$j = 2$$

Exercise:

Problem: $2.1k + 3 = 7.2$

Exercise:

Problem: $0.4x + 0.6 = 0.5x - 1.2$

Solution:

$$x = 18$$

Exercise:

Problem: $0.7x + 0.4 = 0.6x + 2.4$

Exercise:

Problem: $0.23x + 1.47 = 0.37x - 1.05$

Solution:

$$x = 18$$

Exercise:

Problem: $0.48x + 1.56 = 0.58x - 0.64$

Exercise:

Problem: $0.9x - 1.25 = 0.75x + 1.75$

Solution:

$$x = 20$$

Exercise:

Problem: $1.2x - 0.91 = 0.8x + 2.29$

Exercise:

Problem: $0.05n + 0.10(n + 8) = 2.15$

Solution:

$$n = 9$$

Exercise:

Problem: $0.05n + 0.10(n + 7) = 3.55$

Exercise:

Problem: $0.10d + 0.25(d + 5) = 4.05$

Solution:

$$d = 8$$

Exercise:

Problem: $0.10d + 0.25(d + 7) = 5.25$

Exercise:

Problem: $0.05(q - 5) + 0.25q = 3.05$

Solution:

$$q = 11$$

Exercise:

Problem: $0.05(q - 8) + 0.25q = 4.10$

Everyday Math

Exercise:

Problem:

Coins Taylor has \$2.00 in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d + 0.01(d + 2) = 2$ for d , the number of dimes.

Solution:

$$d = 18$$

Exercise:**Problem:**

Stamps Travis bought \$9.45 worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation $0.49s + 0.21(s - 5) = 9.45$ for s , to find the number of 49-cent stamps Travis bought.

Writing Exercises**Exercise:****Problem:**

Explain how to find the least common denominator of $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{3}$.

Solution:

Answers will vary.

Exercise:**Problem:**

If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

Exercise:**Problem:**

If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Solution:

Answers will vary.

Exercise:**Problem:**

In the equation $0.35x + 2.1 = 3.85$, what is the LCD? How do you know?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
solve equations using a general strategy.			
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

Ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises**Solve Equations using the Subtraction and Addition Properties of Equality.**

In the following exercises, determine whether the given number is a solution to the equation.

Exercise:

Problem: $x + 16 = 31$, $x = 15$

Solution:

yes

Exercise:

Problem: $w - 8 = 5$, $w = 3$

Exercise:

Problem: $-9n = 45$, $n = 54$

Solution:

no

Exercise:

Problem: $4a = 72$, $a = 18$

In the following exercises, solve the equation using the Subtraction Property of Equality.

Exercise:

Problem: $x + 7 = 19$

Solution:

12

Exercise:

Problem: $y + 2 = -6$

Exercise:

Problem: $a + \frac{1}{3} = \frac{5}{3}$

Solution:

$$a = \frac{4}{3}$$

Exercise:

Problem: $n + 3.6 = 5.1$

In the following exercises, solve the equation using the Addition Property of Equality.

Exercise:

Problem: $u - 7 = 10$

Solution:

$$u = 17$$

Exercise:

Problem: $x - 9 = -4$

Exercise:

Problem: $c - \frac{3}{11} = \frac{9}{11}$

Solution:

$$c = \frac{12}{11}$$

Exercise:

Problem: $p - 4.8 = 14$

In the following exercises, solve the equation.

Exercise:

Problem: $n - 12 = 32$

Solution:

$$n = 44$$

Exercise:

Problem: $y + 16 = -9$

Exercise:

Problem: $f + \frac{2}{3} = 4$

Solution:

$$f = \frac{10}{3}$$

Exercise:

Problem: $d - 3.9 = 8.2$

Exercise:

Problem: $y + 8 - 15 = -3$

Solution:

$$y = 4$$

Exercise:

Problem: $7x + 10 - 6x + 3 = 5$

Exercise:

Problem: $6(n - 1) - 5n = -14$

Solution:

$$n = -8$$

Exercise:

Problem: $8(3p + 5) - 23(p - 1) = 35$

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise:

Problem: The sum of -6 and m is 25.

Solution:

$$-6 + m = 25; m = 31$$

Exercise:

Problem: Four less than n is 13.

In the following exercises, translate into an algebraic equation and solve.

Exercise:

Problem:

Rochelle's daughter is 11 years old. Her son is 3 years younger. How old is her son?

Solution:

$$s = 11 - 3; 8 \text{ years old}$$

Exercise:

Problem:

Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?

Exercise:**Problem:**

Peter paid \$9.75 to go to the movies, which was \$46.25 less than he paid to go to a concert. How much did he pay for the concert?

Solution:

$$c - 46.25 = 9.75; \$56.00$$

Exercise:**Problem:**

Elissa earned \$152.84 this week, which was \$21.65 more than she earned last week. How much did she earn last week?

Solve Equations using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the Division Property of Equality.

Exercise:

Problem: $8x = 72$

Solution:

$$x = 9$$

Exercise:

Problem: $13a = -65$

Exercise:

Problem: $0.25p = 5.25$

Solution:

$$p = 21$$

Exercise:

Problem: $-y = 4$

In the following exercises, solve each equation using the Multiplication Property of Equality.

Exercise:

Problem: $\frac{n}{6} = 18$

Solution:

$$n = 108$$

Exercise:

Problem: $\frac{y}{-10} = 30$

Exercise:

Problem: $36 = \frac{3}{4}x$

Solution:

$$x = 48$$

Exercise:

Problem: $\frac{5}{8}u = \frac{15}{16}$

In the following exercises, solve each equation.

Exercise:

Problem: $-18m = -72$

Solution:

$$m = 4$$

Exercise:

Problem: $\frac{c}{9} = 36$

Exercise:

Problem: $0.45x = 6.75$

Solution:

$$x = 15$$

Exercise:

Problem: $\frac{11}{12} = \frac{2}{3}y$

Exercise:

Problem: $5r - 3r + 9r = 35 - 2$

Solution:

$$r = 3$$

Exercise:

Problem: $24x + 8x - 11x = -7 - 14$

[Solve Equations with Variables and Constants on Both Sides](#)

In the following exercises, solve the equations with constants on both sides.

Exercise:

Problem: $8p + 7 = 47$

Solution:

$$p = 5$$

Exercise:

Problem: $10w - 5 = 65$

Exercise:

Problem: $3x + 19 = -47$

Solution:

$$x = -22$$

Exercise:

Problem: $32 = -4 - 9n$

In the following exercises, solve the equations with variables on both sides.

Exercise:

Problem: $7y = 6y - 13$

Solution:

$$y = -13$$

Exercise:

Problem: $5a + 21 = 2a$

Exercise:

Problem: $k = -6k - 35$

Solution:

$$k = -5$$

Exercise:

Problem: $4x - \frac{3}{8} = 3x$

In the following exercises, solve the equations with constants and variables on both sides.

Exercise:

Problem: $12x - 9 = 3x + 45$

Solution:

$$x = 6$$

Exercise:

Problem: $5n - 20 = -7n - 80$

Exercise:

Problem: $4u + 16 = -19 - u$

Solution:

$$u = -7$$

Exercise:

Problem: $\frac{5}{8}c - 4 = \frac{3}{8}c + 4$

In the following exercises, solve each linear equation using the general strategy.

Exercise:

Problem: $6(x + 6) = 24$

Solution:

$$x = -2$$

Exercise:

Problem: $9(2p - 5) = 72$

Exercise:

Problem: $-(s + 4) = 18$

Solution:

$$s = -22$$

Exercise:

Problem: $8 + 3(n - 9) = 17$

Exercise:

Problem: $23 - 3(y - 7) = 8$

Solution:

$$y = 12$$

Exercise:

Problem: $\frac{1}{3}(6m + 21) = m - 7$

Exercise:

Problem: $8(r - 2) = 6(r + 10)$

Solution:

$$r = 38$$

Exercise:

Problem: $5 + 7(2 - 5x) = 2(9x + 1) - (13x - 57)$

Exercise:

Problem: $4(3.5y + 0.25) = 365$

Solution:

$$y = 26$$

Exercise:

Problem: $0.25(q - 8) = 0.1(q + 7)$

Solve Equations with Fraction or Decimal Coefficients

In the following exercises, solve each equation by clearing the fractions.

Exercise:

Problem: $\frac{2}{5}n - \frac{1}{10} = \frac{7}{10}$

Solution:

$$n = 2$$

Exercise:

Problem: $\frac{1}{3}x + \frac{1}{5}x = 8$

Exercise:

Problem: $\frac{3}{4}a - \frac{1}{3} = \frac{1}{2}a + \frac{5}{6}$

Solution:

$$a = \frac{14}{3}$$

Exercise:

Problem: $\frac{1}{2}(k + 3) = \frac{1}{3}(k + 16)$

In the following exercises, solve each equation by clearing the decimals.

Exercise:

Problem: $0.8x - 0.3 = 0.7x + 0.2$

Solution:

$$x = 5$$

Exercise:

Problem: $0.36u + 2.55 = 0.41u + 6.8$

Exercise:

Problem: $0.6p - 1.9 = 0.78p + 1.7$

Solution:

$$p = -20$$

Exercise:

Problem: $0.10d + 0.05(d - 4) = 2.05$

Chapter Practice Test

Exercise:

Problem:

Determine whether each number is a solution to the equation.

$$3x + 5 = 23.$$

- Ⓐ 6
- Ⓑ $\frac{23}{5}$

Solution:

- Ⓐ yes
- Ⓑ no

In the following exercises, solve each equation.

Exercise:

Problem: $n - 18 = 31$

Exercise:

Problem: $9c = 144$

Solution:

$$c = 16$$

Exercise:

Problem: $4y - 8 = 16$

Exercise:

Problem: $-8x - 15 + 9x - 1 = -21$

Solution:

$$x = -5$$

Exercise:

Problem: $-15a = 120$

Exercise:

Problem: $\frac{2}{3}x = 6$

Solution:

$$x = 9$$

Exercise:

Problem: $x + 3.8 = 8.2$

Exercise:

Problem: $10y = -5y + 60$

Solution:

$$y = 4$$

Exercise:

Problem: $8n + 2 = 6n + 12$

Exercise:

Problem: $9m - 2 - 4m + m = 42 - 8$

Solution:

$$m = 6$$

Exercise:

Problem: $-5(2x + 1) = 45$

Exercise:

Problem: $-(d + 9) = 23$

Solution:

$$d = -32$$

Exercise:

Problem: $\frac{1}{3}(6m + 21) = m - 7$

Exercise:

Problem: $2(6x + 5) - 8 = -22$

Solution:

$$x = -2$$

Exercise:

Problem: $8(3a + 5) - 7(4a - 3) = 20 - 3a$

Exercise:

Problem: $\frac{1}{4}p + \frac{1}{3} = \frac{1}{2}$

Solution:

$$p = \frac{2}{3}$$

Exercise:

Problem: $0.1d + 0.25(d + 8) = 4.1$

Exercise:

Problem: Translate and solve: The difference of twice x and 4 is 16.

Solution:

$$2x - 4 = 16; x = 10$$

Exercise:

Problem:

Samuel paid \$25.82 for gas this week, which was \$3.47 less than he paid last week. How much did he pay last week?

Graphs

class="introduction"

Cyclists
speed
toward
the
finish
line.
(credit:
ewan
traveler
, Flickr)



Which cyclist will win the race? What will the winning time be? How many seconds will separate the winner from the runner-up? One way to summarize the information from the race is by creating a graph. In this chapter, we will discuss the basic concepts of graphing. The applications of graphing go far beyond races. They are used to present information in almost every field, including healthcare, business, and entertainment.

Use the Rectangular Coordinate System

By the end of this section, you will be able to:

- Plot points on a rectangular coordinate system
- Identify points on a graph
- Verify solutions to an equation in two variables
- Complete a table of solutions to a linear equation
- Find solutions to linear equations in two variables

Note:

Before you get started, take this readiness quiz.

1. Evaluate: $x + 3$ when $x = -1$.
If you missed this problem, review [\[link\]](#).
2. Evaluate: $2x - 5y$ when $x = 3$, $y = -2$.
If you missed this problem, review [\[link\]](#).
3. Solve for y : $40 - 4y = 20$.
If you missed this problem, review [\[link\]](#).

Plot Points on a Rectangular Coordinate System

Many maps, such as the Campus Map shown in [\[link\]](#), use a grid system to identify locations. Do you see the numbers 1, 2, 3, and 4 across the top and bottom of the map and the letters A, B, C, and D along the sides? Every location on the map can be identified by a number and a letter.

For example, the Student Center is in section 2B. It is located in the grid section above the number 2 and next to the letter B. In which grid section is the Stadium? The Stadium is in section 4D.

A	Parking Garage			Residence Halls
B		Student Center	Engineering Building	
C	Taylor Hall	Library		Tiger Field
D		Administration		Stadium
	1	2	3	4

Example:

Exercise:

Problem: Use the map in [\[link\]](#).

- Ⓐ Find the grid section of the Residence Halls.
- Ⓑ What is located in grid section 4C?

Solution:

Solution

- Ⓐ Read the number below the Residence Halls, 4, and the letter to the side, A. So the Residence Halls are in grid section 4A.
- Ⓑ Find 4 across the bottom of the map and C along the side. Look below the 4 and next to the C. Tiger Field is in grid section 4C.

Note:

Exercise:

Problem: Use the map in [\[link\]](#).

- Ⓐ Find the grid section of Taylor Hall.
- Ⓑ What is located in section 3B?

Solution:

- Ⓐ 1C
- Ⓑ Engineering Building

Note:

Exercise:

Problem: Use the map in [\[link\]](#).

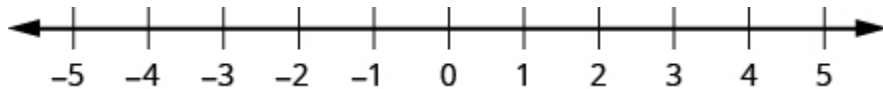
- Ⓐ Find the grid section of the Parking Garage.
- Ⓑ What is located in section 2C?

Solution:

- Ⓐ 1A
- Ⓑ Library

Just as maps use a grid system to identify locations, a grid system is used in algebra to show a relationship between two variables in a rectangular coordinate system. To create a rectangular coordinate system, start with a

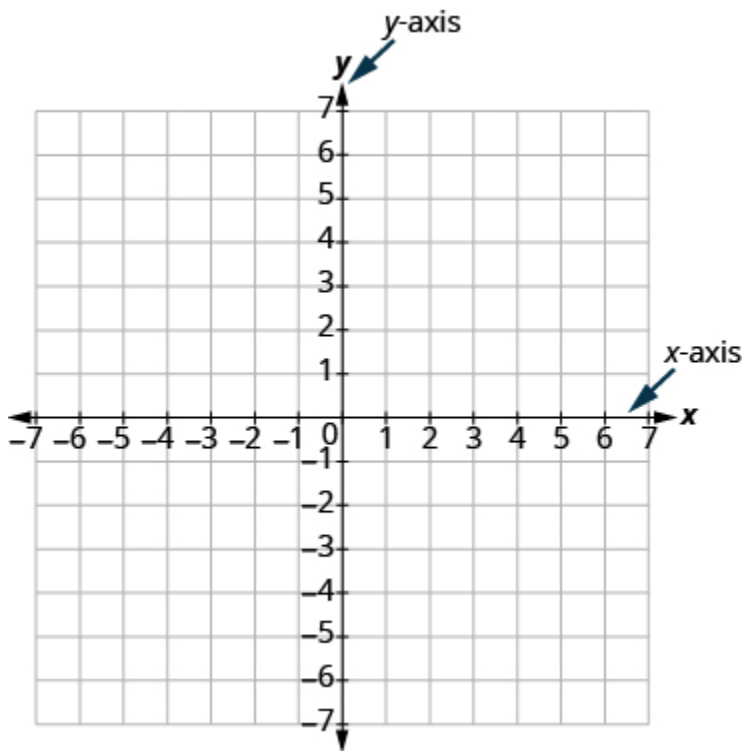
horizontal number line. Show both positive and negative numbers as you did before, using a convenient scale unit. This horizontal number line is called the ***x*-axis**.



Now, make a vertical number line passing through the *x*-axis at 0. Put the positive numbers above 0 and the negative numbers below 0. See [\[link\]](#). This vertical line is called the ***y*-axis**.

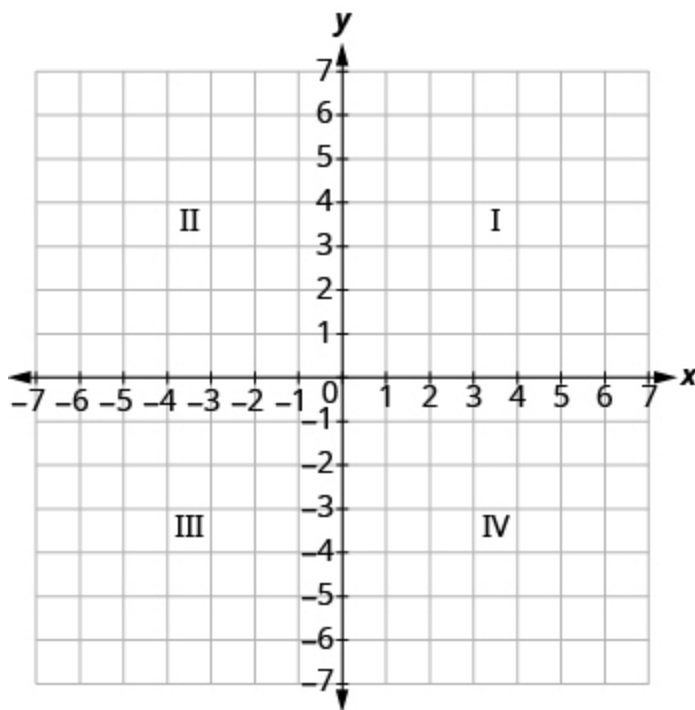
Vertical grid lines pass through the integers marked on the *x*-axis. Horizontal grid lines pass through the integers marked on the *y*-axis. The resulting grid is the rectangular coordinate system.

The rectangular coordinate system is also called the *x-y* plane, the coordinate plane, or the Cartesian coordinate system (since it was developed by a mathematician named René Descartes.)



The rectangular coordinate system.

The x -axis and the y -axis form the rectangular coordinate system. These axes divide a plane into four areas, called **quadrants**. The quadrants are identified by Roman numerals, beginning on the upper right and proceeding counterclockwise. See [\[link\]](#).



The four quadrants of the rectangular coordinate system

In the rectangular coordinate system, every point is represented by an **ordered pair**. The first number in the ordered pair is the x -coordinate of the point, and the second number is the y -coordinate of the point.

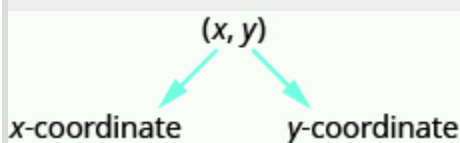
Note:**Ordered Pair**

An ordered pair, (x, y) gives the coordinates of a point in a rectangular coordinate system.

Equation:

The first number is the x -coordinate.

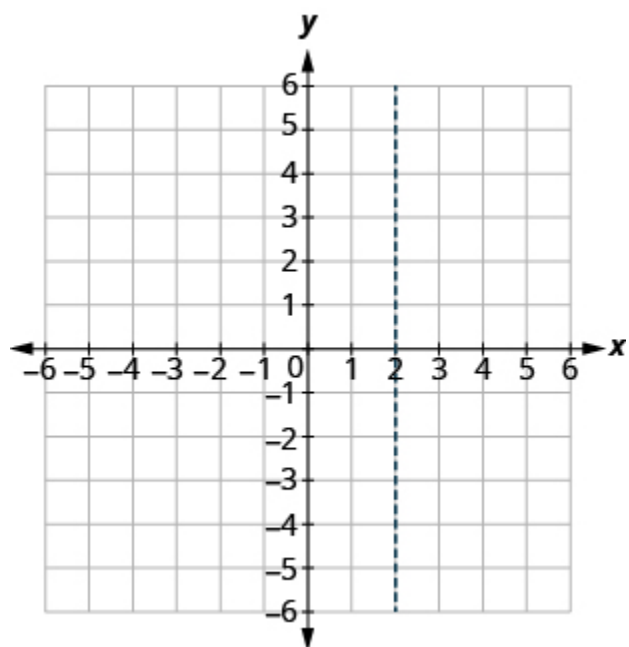
The second number is the y -coordinate.



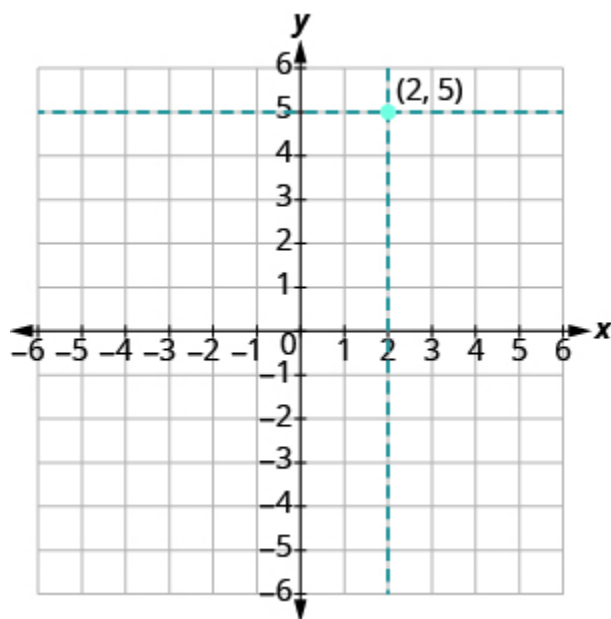
So how do the coordinates of a point help you locate a point on the x - y plane?

Let's try locating the point $(2, 5)$. In this ordered pair, the x -coordinate is 2 and the y -coordinate is 5.

We start by locating the x value, 2, on the x -axis. Then we lightly sketch a vertical line through $x = 2$, as shown in [\[link\]](#).



Now we locate the y value, 5, on the y -axis and sketch a horizontal line through $y = 5$. The point where these two lines meet is the point with coordinates $(2, 5)$. We plot the point there, as shown in [\[link\]](#).

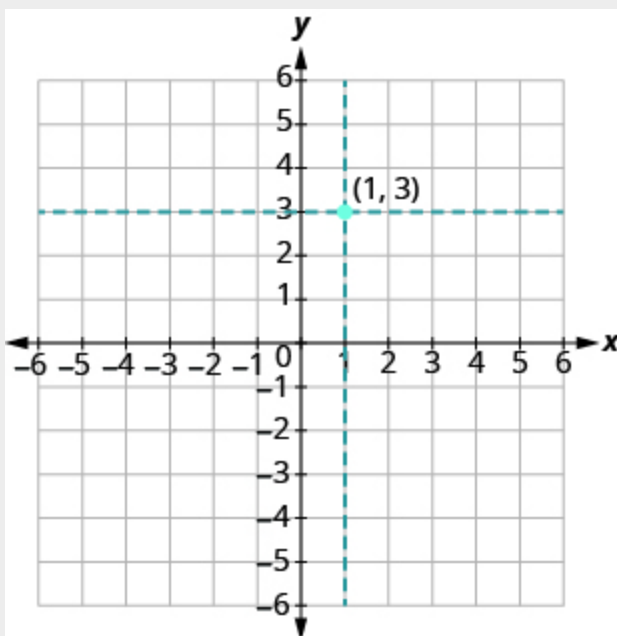


Example:**Exercise:****Problem:**

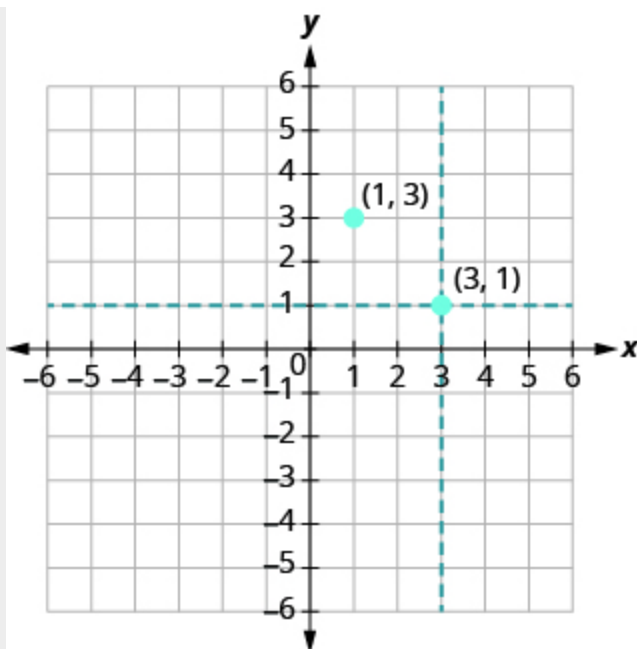
Plot $(1, 3)$ and $(3, 1)$ in the same rectangular coordinate system.

Solution:**Solution**

The coordinate values are the same for both points, but the x and y values are reversed. Let's begin with point $(1, 3)$. The x -coordinate is 1 so find 1 on the x -axis and sketch a vertical line through $x = 1$. The y -coordinate is 3 so we find 3 on the y -axis and sketch a horizontal line through $y = 3$. Where the two lines meet, we plot the point $(1, 3)$.



To plot the point $(3, 1)$, we start by locating 3 on the x -axis and sketch a vertical line through $x = 3$. Then we find 1 on the y -axis and sketch a horizontal line through $y = 1$. Where the two lines meet, we plot the point $(3, 1)$.



Notice that the order of the coordinates does matter, so, $(1, 3)$ is not the same point as $(3, 1)$.

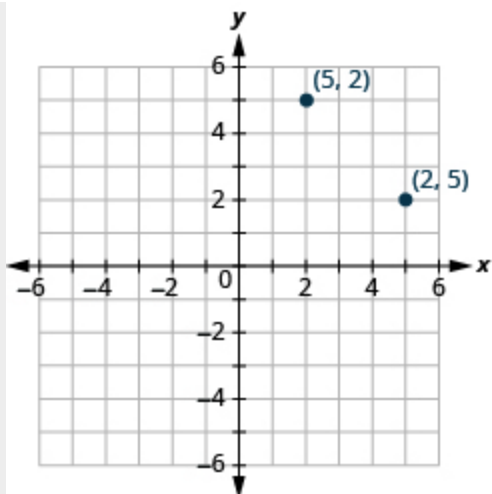
Note:

Exercise:

Problem:

Plot each point on the same rectangular coordinate system:
 $(2, 5)$, $(5, 2)$.

Solution:



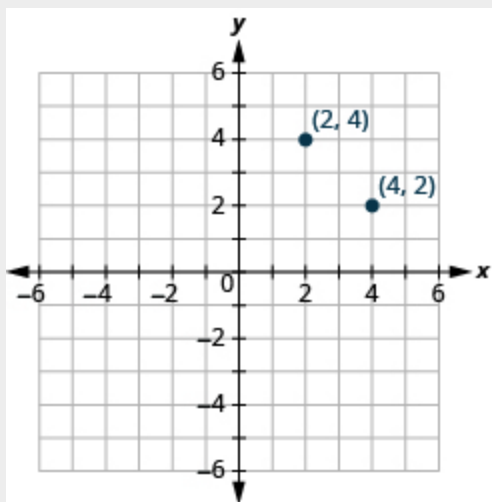
Note:

Exercise:

Problem:

Plot each point on the same rectangular coordinate system:
 $(4, 2)$, $(2, 4)$.

Solution:



Example:**Exercise:****Problem:**

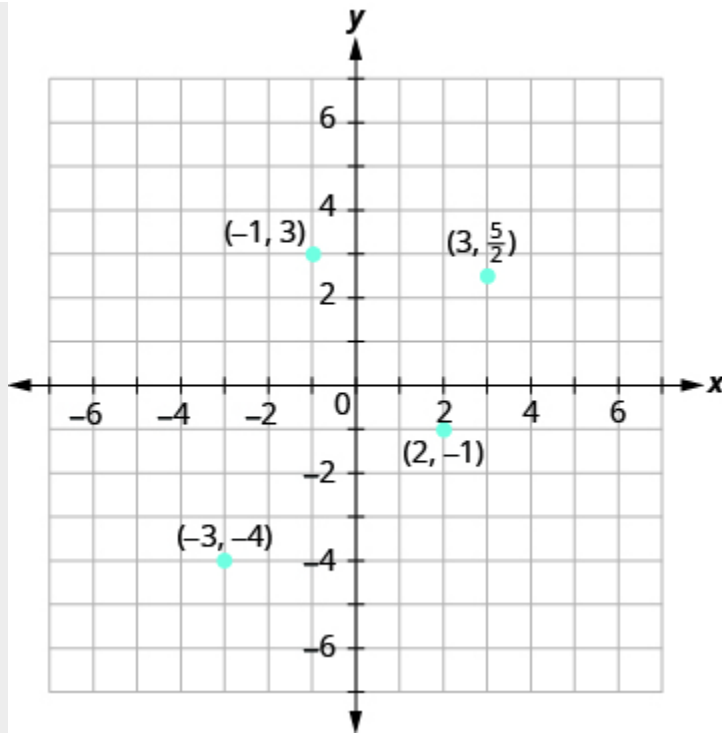
Plot each point in the rectangular coordinate system and identify the quadrant in which the point is located:

- Ⓐ $(-1, 3)$
- Ⓑ $(-3, -4)$
- Ⓒ $(2, -3)$
- Ⓓ $(3, \frac{5}{2})$

Solution:**Solution**

The first number of the coordinate pair is the x -coordinate, and the second number is the y -coordinate.

- Ⓐ Since $x = -1$, $y = 3$, the point $(-1, 3)$ is in Quadrant II.
- Ⓑ Since $x = -3$, $y = -4$, the point $(-3, -4)$ is in Quadrant III.
- Ⓒ Since $x = 2$, $y = -1$, the point $(2, -1)$ is in Quadrant IV.
- Ⓓ Since $x = 3$, $y = \frac{5}{2}$, the point $(3, \frac{5}{2})$ is in Quadrant I. It may be helpful to write $\frac{5}{2}$ as the mixed number, $2\frac{1}{2}$, or decimal, 2.5. Then we know that the point is halfway between 2 and 3 on the y -axis.



Note:

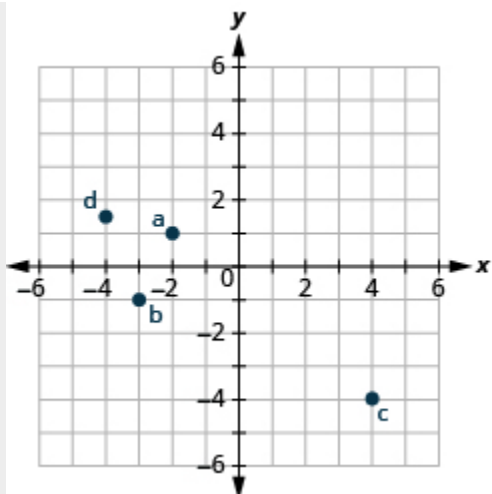
Exercise:

Problem:

Plot each point on a rectangular coordinate system and identify the quadrant in which the point is located:

- Ⓐ $(-2, 1)$
- Ⓑ $(-3, -1)$
- Ⓒ $(4, -4)$
- Ⓓ $(-4, \frac{3}{2})$

Solution:



Note:

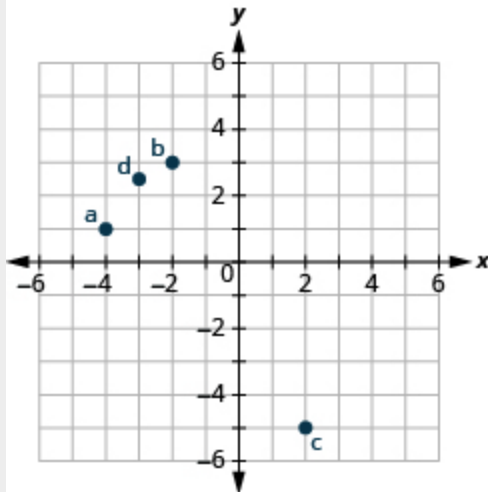
Exercise:

Problem:

Plot each point on a rectangular coordinate system and identify the quadrant in which the point is located

- Ⓐ $(-4, 1)$
- Ⓑ $(-2, 3)$
- Ⓒ $(2, -5)$
- Ⓓ $(-3, \frac{5}{2})$

Solution:



How do the signs affect the location of the points?

Example:

Exercise:

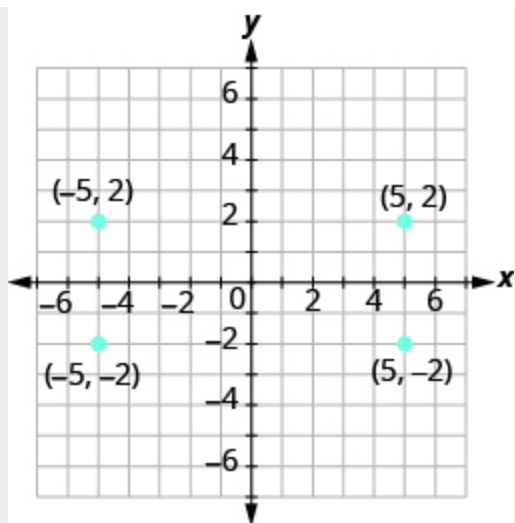
Problem: Plot each point:

- Ⓐ $(-5, 2)$
- Ⓑ $(-5, -2)$
- Ⓒ $(5, 2)$
- Ⓓ $(5, -2)$

Solution:

Solution

As we locate the x -coordinate and the y -coordinate, we must be careful with the signs.



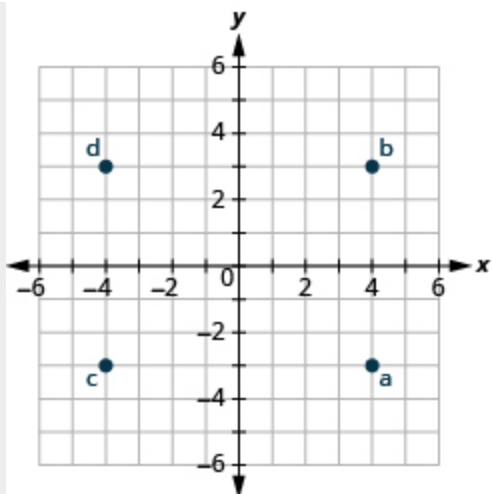
Note:

Exercise:

Problem: Plot each point:

- Ⓐ $(4, -3)$
- Ⓑ $(4, 3)$
- Ⓒ $(-4, -3)$
- Ⓓ $(-4, 3)$

Solution:



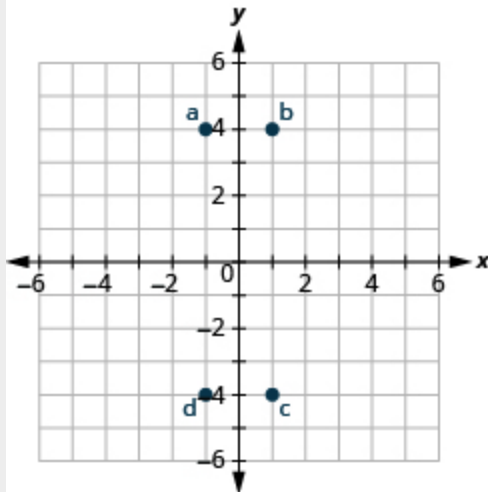
Note:

Exercise:

Problem: Plot each point:

- Ⓐ $(-1, 4)$
- Ⓑ $(1, 4)$
- Ⓒ $(1, -4)$
- Ⓓ $(-1, -4)$

Solution:

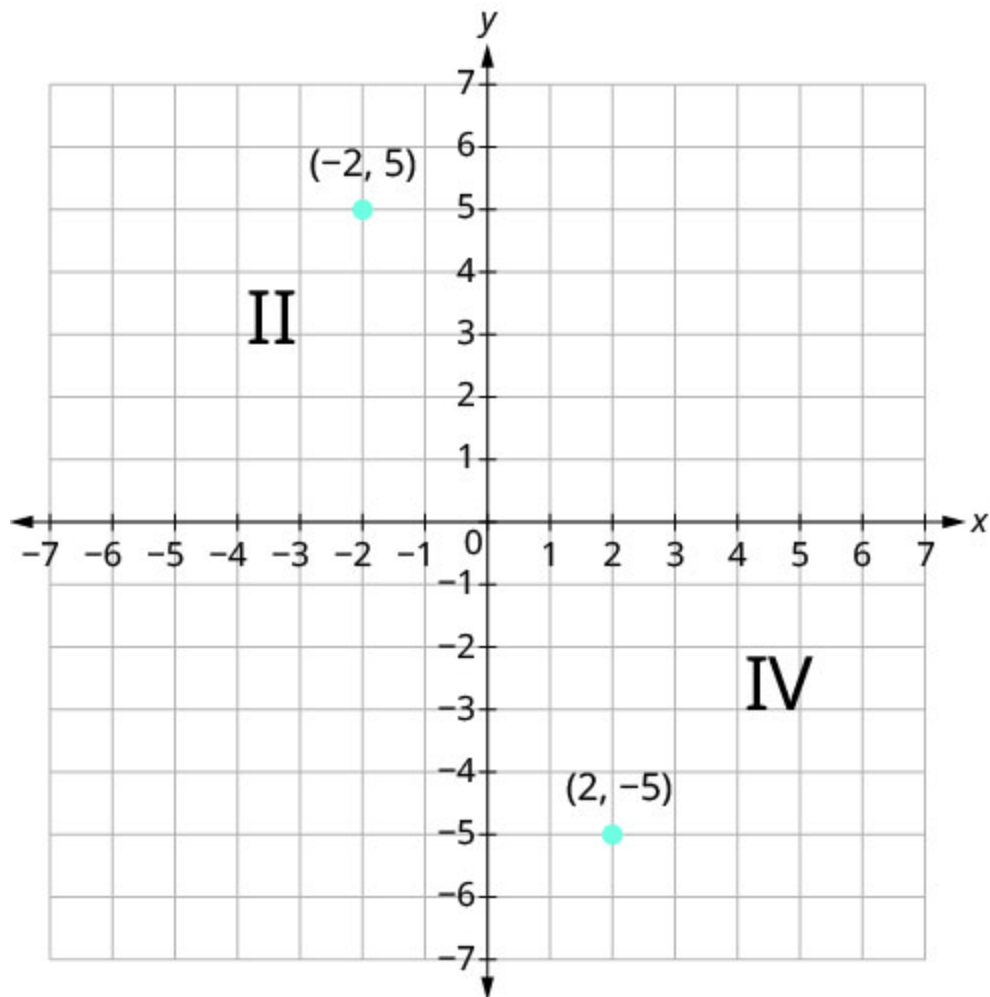


You may have noticed some patterns as you graphed the points in the two previous examples.

For each point in Quadrant IV, what do you notice about the signs of the coordinates?

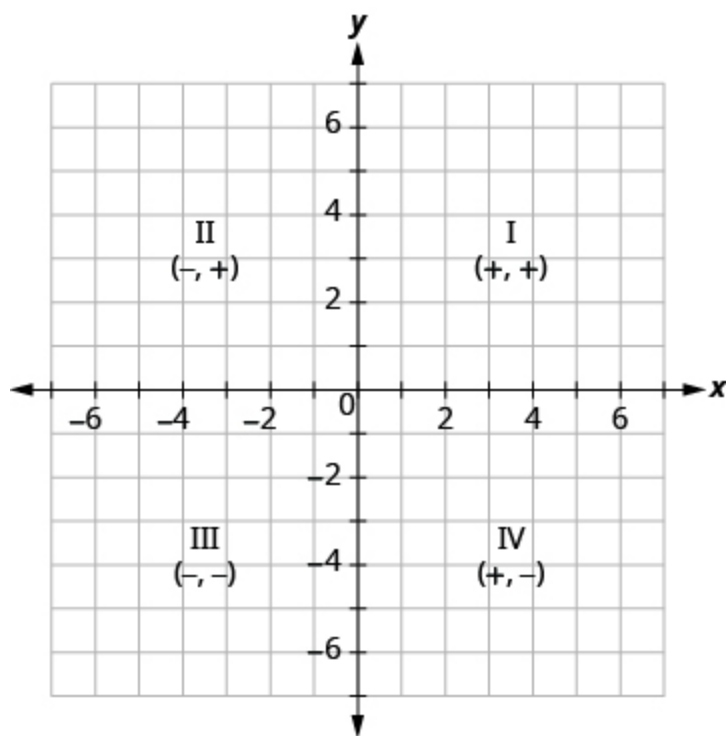
What about the signs of the coordinates of the points in the third quadrant? The second quadrant? The first quadrant?

Can you tell just by looking at the coordinates in which quadrant the point $(-2, 5)$ is located? In which quadrant is $(2, -5)$ located?



We can summarize sign patterns of the quadrants as follows. Also see [\[link\]](#).

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x,y)	(x,y)	(x,y)	(x,y)
$(+,+)$	$(-,+)$	$(-,-)$	$(+,-)$



What if one coordinate is zero? Where is the point $(0, 4)$ located? Where is the point $(-2, 0)$ located? The point $(0, 4)$ is on the y -axis and the point $(-2, 0)$ is on the x -axis.

Note:

Points on the Axes

Points with a y -coordinate equal to 0 are on the x -axis, and have coordinates $(a, 0)$.

Points with an x -coordinate equal to 0 are on the y -axis, and have coordinates $(0, b)$.

What is the ordered pair of the point where the axes cross? At that point both coordinates are zero, so its ordered pair is $(0, 0)$. The point has a special name. It is called the *origin*.

Note:

The Origin

The point $(0, 0)$ is called the **origin**. It is the point where the x -axis and y -axis intersect.

Example:

Exercise:

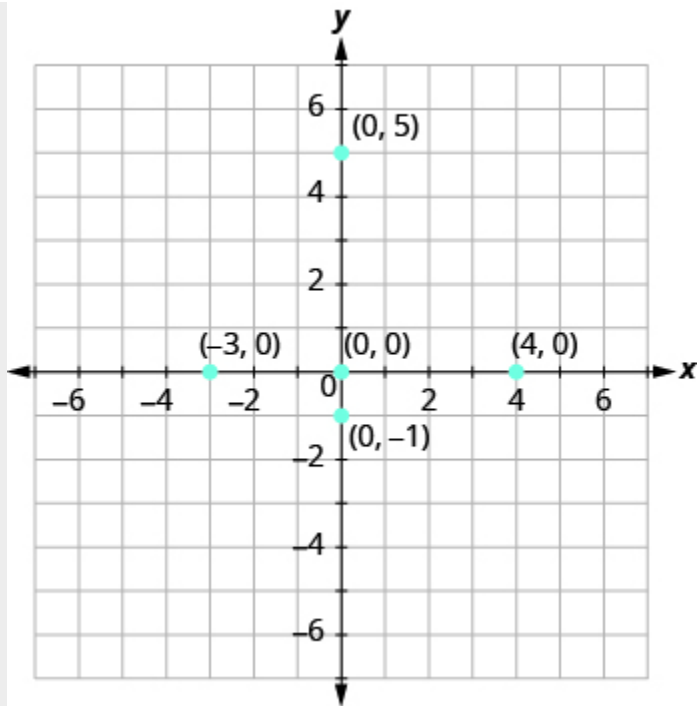
Problem: Plot each point on a coordinate grid:

- Ⓐ $(0, 5)$
- Ⓑ $(4, 0)$
- Ⓒ $(-3, 0)$
- Ⓓ $(0, 0)$
- Ⓔ $(0, -1)$

Solution:

Solution

- Ⓐ Since $x = 0$, the point whose coordinates are $(0, 5)$ is on the y -axis.
- Ⓑ Since $y = 0$, the point whose coordinates are $(4, 0)$ is on the x -axis.
- Ⓒ Since $y = 0$, the point whose coordinates are $(-3, 0)$ is on the x -axis.
- Ⓓ Since $x = 0$ and $y = 0$, the point whose coordinates are $(0, 0)$ is the origin.
- Ⓔ Since $x = 0$, the point whose coordinates are $(0, -1)$ is on the y -axis.



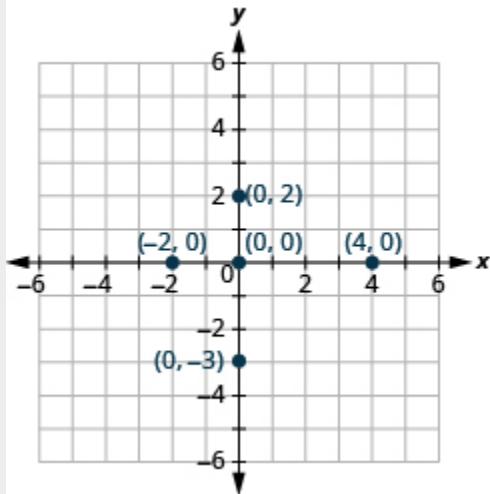
Note:

Exercise:

Problem: Plot each point on a coordinate grid:

- Ⓐ $(4, 0)$
- Ⓑ $(-2, 0)$
- Ⓒ $(0, 0)$
- Ⓓ $(0, 2)$
- Ⓔ $(0, -3)$

Solution:



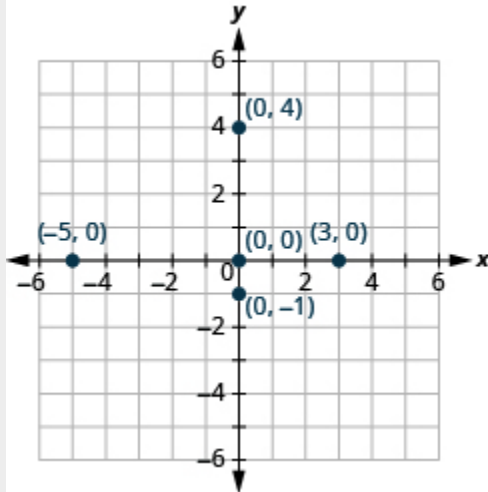
Note:

Exercise:

Problem: Plot each point on a coordinate grid:

- Ⓐ $(-5, 0)$
- Ⓑ $(3, 0)$
- Ⓒ $(0, 0)$
- Ⓓ $(0, -1)$
- Ⓔ $(0, 4)$

Solution:



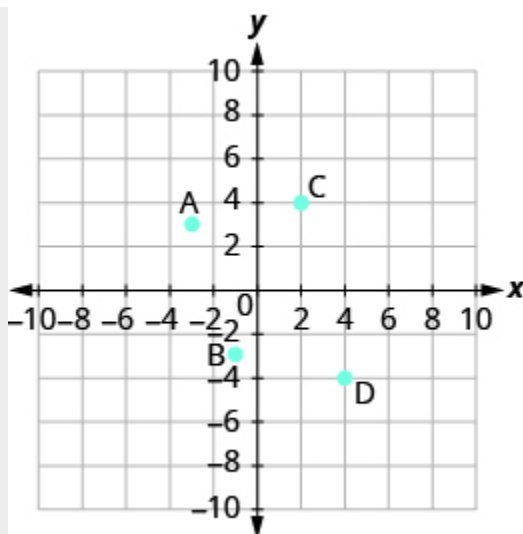
Identify Points on a Graph

In algebra, being able to identify the coordinates of a point shown on a graph is just as important as being able to plot points. To identify the x -coordinate of a point on a graph, read the number on the x -axis directly above or below the point. To identify the y -coordinate of a point, read the number on the y -axis directly to the left or right of the point. Remember, to write the ordered pair using the correct order (x, y) .

Example:

Exercise:

Problem: Name the ordered pair of each point shown:



Solution:
Solution

Point A is above -3 on the x -axis, so the x -coordinate of the point is -3 . The point is to the left of 3 on the y -axis, so the y -coordinate of the point is 3 . The coordinates of the point are $(-3, 3)$.

Point B is below -1 on the x -axis, so the x -coordinate of the point is -1 . The point is to the left of -3 on the y -axis, so the y -coordinate of the point is -3 . The coordinates of the point are $(-1, -3)$.

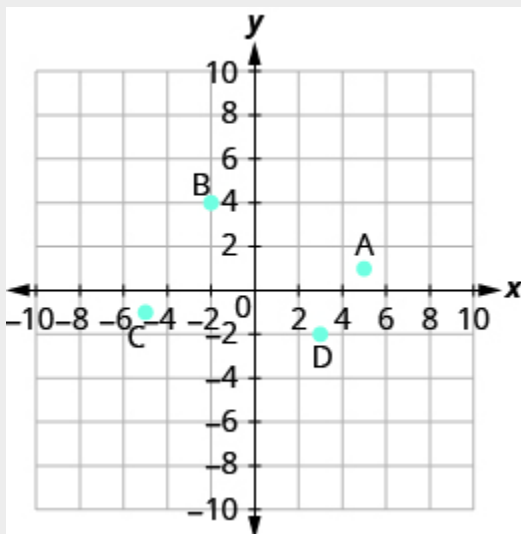
Point C is above 2 on the x -axis, so the x -coordinate of the point is 2 . The point is to the right of 4 on the y -axis, so the y -coordinate of the point is 4 . The coordinates of the point are $(2, 4)$.

Point D is below 4 on the x - axis, so the x -coordinate of the point is 4 . The point is to the right of -4 on the y -axis, so the y -coordinate of the point is -4 . The coordinates of the point are $(4, -4)$.

Note:

Exercise:

Problem: Name the ordered pair of each point shown:



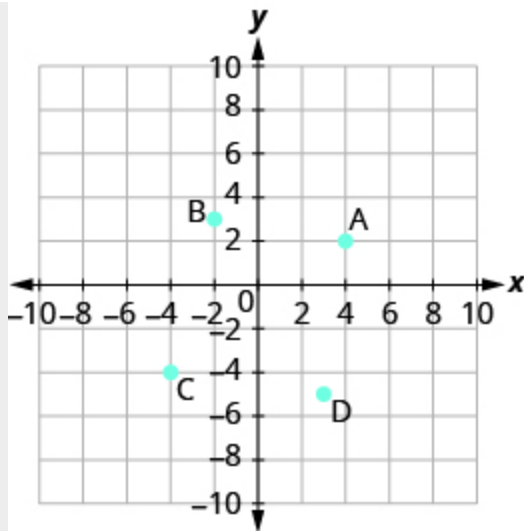
Solution:

1. A: (5,1)
2. B: (-2,4)
3. C: (-5,-1)
4. D: (3,-2)

Note:

Exercise:

Problem: Name the ordered pair of each point shown:



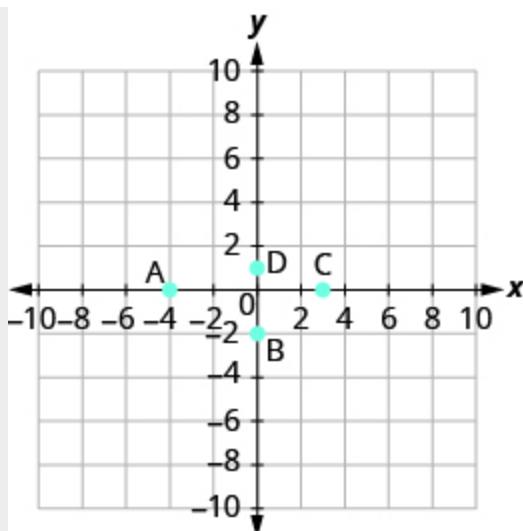
Solution:

1. A: (4,2)
2. B: (-2,3)
3. C: (-4,-4)
4. D: (3,-5)

Example:

Exercise:

Problem: Name the ordered pair of each point shown:



Solution:
Solution

Point A is on the x -axis at
 $x = -4$.

The coordinates of point A are
 $(-4, 0)$.

Point B is on the y -axis at
 $y = -2$

The coordinates of point B are
 $(0, -2)$.

Point C is on the x -axis at
 $x = 3$.

The coordinates of point C are
 $(3, 0)$.

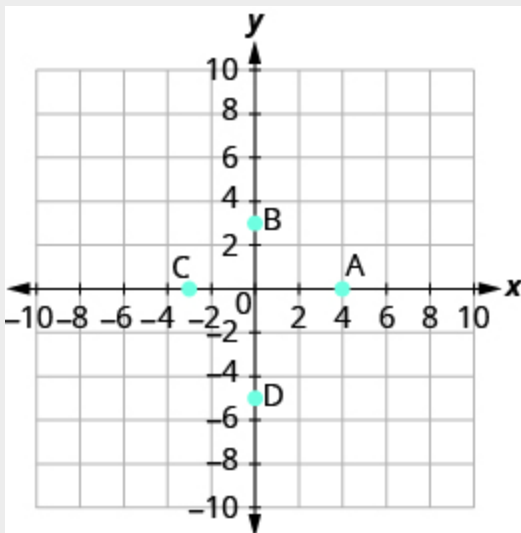
Point D is on the y -axis at
 $y = 1$.

The coordinates of point D are
 $(0, 1)$.

Note:

Exercise:

Problem: Name the ordered pair of each point shown:



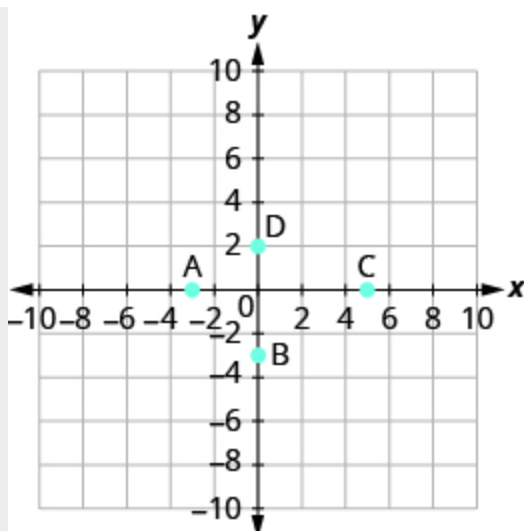
Solution:

1. A: (4,0)
2. B: (0,3)
3. C: (-3,0)
4. D: (0,-5)

Note:

Exercise:

Problem: Name the ordered pair of each point shown:



Solution:

1. A: $(-3, 0)$
2. B: $(0, -3)$
3. C: $(5, 0)$
4. D: $(0, 2)$

Verify Solutions to an Equation in Two Variables

All the equations we solved so far have been equations with one variable. In almost every case, when we solved the equation we got exactly one solution. The process of solving an equation ended with a statement such as $x = 4$. Then we checked the solution by substituting back into the equation.

Here's an example of a linear equation in one variable, and its one solution.

Equation:

$$3x + 5 = 17$$

$$3x = 12$$

$$x = 4$$

But equations can have more than one variable. Equations with two variables can be written in the general form $Ax + By = C$. An equation of this form is called a linear equation in two variables.

Note:

Linear Equation

An equation of the form $Ax + By = C$, where A and B are not both zero, is called a linear equation in two variables.

Notice that the word “line” is in linear.

Here is an example of a linear equation in two variables, x and y :

$$Ax + By = C$$

$$x + 4y = 8$$

$$A = 1, B = 4, C = 8$$

Is $y = -5x + 1$ a linear equation? It does not appear to be in the form $Ax + By = C$. But we could rewrite it in this form.

	$y = -5x + 1$
Add $5x$ to both sides.	$y + 5x = -5x + 1 + 5x$

Simplify.	$y + 5x = 1$
Use the Commutative Property to put it in $Ax + By = C$.	$Ax + By = C$ $5x + y = 1$

By rewriting $y = -5x + 1$ as $5x + y = 1$, we can see that it is a linear equation in two variables because it can be written in the form $Ax + By = C$.

Linear equations in two variables have infinitely many solutions. For every number that is substituted for x , there is a corresponding y value. This pair of values is a **solution to the linear equation** and is represented by the ordered pair (x, y) . When we substitute these values of x and y into the equation, the result is a true statement because the value on the left side is equal to the value on the right side.

Note:

Solution to a Linear Equation in Two Variables

An ordered pair (x, y) is a solution to the linear equation $Ax + By = C$, if the equation is a true statement when the x - and y -values of the ordered pair are substituted into the equation.

Example:

Exercise:

Problem:

Determine which ordered pairs are solutions of the equation $x + 4y = 8$:

- Ⓐ $(0, 2)$
- Ⓑ $(2, -4)$

Ⓒ $(-4, 3)$

Solution:
Solution

Substitute the x - and y -values from each ordered pair into the equation and determine if the result is a true statement.

Ⓐ $(0, 2)$	Ⓑ $(2, -4)$	Ⓒ $(-4, 3)$
$x = 0, y = 2$ $x + 4y = 8$ $0 + 4 \cdot 2 \stackrel{?}{=} 8$ $0 + 8 \stackrel{?}{=} 8$ $8 = 8 \checkmark$	$x = 2, y = -4$ $x + 4y = 8$ $2 + 4(-4) \stackrel{?}{=} 8$ $2 + (-16) \stackrel{?}{=} 8$ $-14 \neq 8$	$x = -4, y = 3$ $x + 4y = 8$ $-4 + 4 \cdot 3 \stackrel{?}{=} 8$ $-4 + 12 \stackrel{?}{=} 8$ $8 = 8 \checkmark$
$(0, 2)$ is a solution.	$(2, -4)$ is not a solution.	$(-4, 3)$ is a solution.

Note:

Exercise:

Problem:

Determine which ordered pairs are solutions to the given equation:

$$2x + 3y = 6$$

- Ⓐ (3, 0)
- Ⓑ (2, 0)
- Ⓒ (6, -2)

Solution:

- Ⓐ, Ⓒ

Note:**Exercise:****Problem:**

Determine which ordered pairs are solutions to the given equation:

$$4x - y = 8$$

- Ⓐ (0, 8)
- Ⓑ (2, 0)
- Ⓒ (1, -4)

Solution:

- Ⓑ, Ⓒ

Example:**Exercise:**

Problem:

Determine which ordered pairs are solutions of the equation.

$$y = 5x - 1$$

- Ⓐ $(0, -1)$
- Ⓑ $(1, 4)$
- Ⓒ $(-2, -7)$

Solution:**Solution**

Substitute the x - and y -values from each ordered pair into the equation and determine if it results in a true statement.

Ⓐ $(0, -1)$	Ⓑ $(1, 4)$	Ⓒ $(-2, -7)$
$x = 0, y = -1$ $y = 5x - 1$ $-1 \stackrel{?}{=} 5(0) - 1$ $-1 \stackrel{?}{=} 0 - 1$ $-1 = -1 \checkmark$	$x = 1, y = 4$ $y = 5x - 1$ $4 \stackrel{?}{=} 5(1) - 1$ $4 \stackrel{?}{=} 5 - 1$ $4 = 4 \checkmark$	$x = -2, y = -7$ $y = 5x - 1$ $-7 \stackrel{?}{=} 5(-2) - 1$ $-7 \stackrel{?}{=} -10 - 1$ $-7 \neq -11$
$(0, -1)$ is a solution.	$(1, 4)$ is a solution.	$(-2, -7)$ is not a solution.

Note:

Exercise:

Problem:

Determine which ordered pairs are solutions of the given equation:

$$y = 4x - 3$$

Ⓐ $(0, 3)$

Ⓑ $(1, 1)$

Ⓒ $(1, 1)$

Solution:

Ⓑ

Note:

Exercise:

Problem:

Determine which ordered pairs are solutions of the given equation:

$$y = -2x + 6$$

Ⓐ $(0, 6)$

Ⓑ $(1, 4)$

Ⓒ $(-2, -2)$

Solution:

Ⓐ, Ⓑ

Complete a Table of Solutions to a Linear Equation

In the previous examples, we substituted the x - and y -values of a given ordered pair to determine whether or not it was a solution to a linear equation. But how do we find the ordered pairs if they are not given? One way is to choose a value for x and then solve the equation for y . Or, choose a value for y and then solve for x .

We'll start by looking at the solutions to the equation $y = 5x - 1$ we found in [\[link\]](#). We can summarize this information in a table of solutions.

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$

To find a third solution, we'll let $x = 2$ and solve for y .

	$y = 5x - 1$
Substitute $x = 2$.	$y = 5(2) - 1$

Multiply.	$y = 10 - 1$
Simplify.	$y = 9$

The ordered pair is a solution to $y = 5x - 1$. We will add it to the table.

$y = 5x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	4	$(1, 4)$
2	9	$(2, 9)$

We can find more solutions to the equation by substituting any value of x or any value of y and solving the resulting equation to get another ordered pair that is a solution. There are an infinite number of solutions for this equation.

Example:

Exercise:

Problem:

Complete the table to find three solutions to the equation $y = 4x - 2$:

$$y = 4x - 2$$

x	y	(x, y)
0		
-1		
2		

Solution:
Solution

Substitute $x = 0$, $x = -1$, and $x = 2$ into $y = 4x - 2$.

$x = 0$	$x = -1$	$x = 2$
$y = 4x - 2$	$y = 4x - 2$	$y = 4x - 2$
$y = 4 \cdot 0 - 2$	$y = 4(-1) - 2$	$y = 4 \cdot 2 - 2$
$y = 0 - 2$	$y = -4 - 2$	$y = 8 - 2$
$y = -2$	$y = -6$	$y = 6$
$(0, -2)$	$(-1, -6)$	$(2, 6)$

The results are summarized in the table.

$y = 4x - 2$		
x	y	(x, y)
0	-2	$(0, -2)$
-1	-6	$(-1, -6)$
2	6	$(2, 6)$

Note:

Exercise:

Problem:

Complete the table to find three solutions to the equation:

$$y = 3x - 1.$$

$y = 3x - 1$		
x	y	(x, y)

$$y = 3x - 1$$

x	y	(x, y)
0		
-1		
2		

Solution:

$$y = 3x - 1$$

x	y	(x, y)
0	-1	$(0, -1)$
-1	-4	$(-1, -4)$
2	5	$(2, 5)$

Note:

Exercise:

Problem:

Complete the table to find three solutions to the equation: $y = 6x + 1$

$y = 6x + 1$		
x	y	(x, y)
0		
1		
-2		

Solution:

$y = 6x + 1$		
x	y	(x, y)
0	1	(0, 1)
1	7	(1, 7)
-2	-11	(-2, -11)

Example:

Exercise:

Problem:

Complete the table to find three solutions to the equation
 $5x - 4y = 20$:

$5x - 4y = 20$		
x	y	(x, y)
0		
	0	
	5	

Solution:

Solution

$x = 0$	$y = 0$	$y = 5$
$5x - 4y = 20$	$5x - 4y = 20$	$5x - 4y = 20$
$5 \cdot 0 - 4y = 20$	$5x - 4 \cdot 0 = 20$	$5x - 4 \cdot 5 = 20$
$0 - 4y = 20$	$5x - 0 = 20$	$5x - 20 = 20$
$-4y = 20$	$5x = 20$	$5x = 40$
$y = -5$	$x = 4$	$x = 8$
$(0, -5)$	$(4, 0)$	$(8, 5)$

The results are summarized in the table.

$5x - 4y = 20$		
x	y	(x, y)
0	-5	$(0, -5)$
4	0	$(4, 0)$
8	5	$(8, 5)$

Note:

Exercise:

Problem:

Complete the table to find three solutions to the equation:
 $2x - 5y = 20$.

$$2x - 5y = 20$$

x	y	(x, y)
0		
	0	
-5		

Solution:

$$2x - 5y = 20$$

x	y	(x, y)
0	-4	$(0, -4)$
10	0	$(10, 0)$
-5	-6	$(-5, -6)$

Note:

Exercise:

Problem:

Complete the table to find three solutions to the equation:

$$3x - 4y = 12.$$

$3x - 4y = 12$		
x	y	(x, y)
0		
	0	
-4		

Solution:

$3x - 4y = 12$		
x	y	(x, y)
0	-3	$(0, -3)$
4	0	$(4, 0)$

$$3x - 4y = 12$$

x	y	(x, y)
-4	-6	$(-4, -6)$

Find Solutions to Linear Equations in Two Variables

To find a solution to a linear equation, we can choose any number we want to substitute into the equation for either x or y . We could choose 1, 100, 1,000, or any other value we want. But it's a good idea to choose a number that's easy to work with. We'll usually choose 0 as one of our values.

Example:

Exercise:

Problem: Find a solution to the equation $3x + 2y = 6$.

Solution:

Solution

Step 1: Choose any value for one of the variables in the equation.

We can substitute any value we want for x or any

		value for y. Let's pick $x = 0$. What is the value of y if $x = 0$?
Step 2: Substitute that value into the equation. Solve for the other variable.	Substitute 0 for x. Simplify. Divide both sides by 2.	$\begin{aligned} 3x + 2y &= 6 \\ 3 \cdot 0 + 2y &= 6 \\ 0 + 2y &= 6 \\ 2y &= 6 \\ y &= 3 \end{aligned}$
Step 3: Write the solution as an ordered pair.	So, when $x = 0, y = 3$.	This solution is represented by the ordered pair $(0, 3)$.
Step 4: Check.	Substitute $x = 0, y = 3$ into the equation $3x + 2y = 6$. Is the result a true equation? Yes!	$\begin{aligned} 3x + 2y &= 6 \\ 3 \cdot 0 + 2 \cdot 3 &\stackrel{?}{=} 6 \\ 0 + 6 &\stackrel{?}{=} 6 \\ 6 &= 6 \checkmark \end{aligned}$

Note:

Exercise:

Problem: Find a solution to the equation: $4x + 3y = 12$.

Solution:

Answers will vary.

Note:**Exercise:**

Problem: Find a solution to the equation: $2x + 4y = 8$.

Solution:

Answers will vary.

We said that linear equations in two variables have infinitely many solutions, and we've just found one of them. Let's find some other solutions to the equation $3x + 2y = 6$.

Example:**Exercise:**

Problem: Find three more solutions to the equation $3x + 2y = 6$.

Solution:**Solution**

To find solutions to $3x + 2y = 6$, choose a value for x or y . Remember, we can choose any value we want for x or y . Here we chose 1 for x , and 0 and -3 for y .

Substitute it into the equation.	$y = 0$ $3x + 2y = 6$ $3x + 2(0) = 6$	$x = 1$ $3x + 2y = 6$ $3(1) + 2y = 6$	$y = -3$ $3x + 2y = 6$ $3x + 2(-3) = 6$
Simplify. Solve.	$3x + 0 = 6$ $3x = 6$	$3 + 2y = 6$ $2y = 3$	$3x - 6 = 6$ $3x = 12$
	$x = 2$	$y = \frac{3}{2}$	$x = 4$
Write the ordered pair.	$(2, 0)$	$(1, \frac{3}{2})$	$(4, -3)$

Check your answers.

$(2, 0)$	$(1, \frac{3}{2})$	$(4, -3)$
$3x + 2y = 6$ $3 \cdot 2 + 2 \cdot 0 \stackrel{?}{=} 6$ $6 + 0 \stackrel{?}{=} 6$ $6 = 6 \checkmark$	$3x + 2y = 6$ $3 \cdot 1 + 2 \cdot \frac{3}{2} \stackrel{?}{=} 6$ $3 + 3 \stackrel{?}{=} 6$ $6 = 6 \checkmark$	$3x + 2y = 6$ $3 \cdot 4 + 2(-3) \stackrel{?}{=} 6$ $12 + (-6) \stackrel{?}{=} 6$ $6 = 6 \checkmark$

So $(2, 0)$, $(1, \frac{3}{2})$ and $(4, -3)$ are all solutions to the equation $3x + 2y = 6$. In the previous example, we found that $(0, 3)$ is a solution, too. We can list these solutions in a table.

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$(1, \frac{3}{2})$
4	-3	$(4, -3)$

Note:

Exercise:

Problem: Find three solutions to the equation: $2x + 3y = 6$.

Solution:

Answers will vary.

Note:
Exercise:

Problem: Find three solutions to the equation: $4x + 2y = 8$.

Solution:
 Answers will vary.

Let’s find some solutions to another equation now.

Example:
Exercise:

Problem: Find three solutions to the equation $x - 4y = 8$.

Solution:
Solution

	$x - 4y = 8$	$x - 4y = 8$	$x - 4y = 8$
Choose a value for x or y .	$x = 0$	$y = 0$	$y = 3$
Substitute it into the equation.	$0 - 4y = 8$	$x - 4 \cdot 0 = 8$	$x - 4 \cdot 3 = 8$

Solve.	$-4y = 8$ $y = -2$	$x - 0 = 8$ $x = 8$	$x - 12 = 8$ $x = 20$
Write the ordered pair.	$(0, -2)$	$(8, 0)$	$(20, 3)$

So $(0, -2)$, $(8, 0)$, and $(20, 3)$ are three solutions to the equation $x - 4y = 8$.

$x - 4y = 8$		
x	y	(x, y)
0	-2	$(0, -2)$
8	0	$(8, 0)$
20	3	$(20, 3)$

Remember, there are an infinite number of solutions to each linear equation. Any point you find is a solution if it makes the equation true.

Note:

Exercise:

Problem: Find three solutions to the equation: $4x + y = 8$.

Solution:

Answers will vary.

Note:

Exercise:

Problem: Find three solutions to the equation: $x + 5y = 10$.

Solution:

Answers will vary.

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Plotting Points](#)
- [Identifying Quadrants](#)
- [Verifying Solution to Linear Equation](#)

Key Concepts

- Sign Patterns of the Quadrants

Quadrant I	Quadrant II	Quadrant III	Quadrant IV
(x,y)	(x,y)	(x,y)	(x,y)
$(+,+)$	$(-,+)$	$(-,-)$	$(+,-)$

- **Coordinates of Zero**

- Points with a y -coordinate equal to 0 are on the x -axis, and have coordinates $(a, 0)$.
- Points with a x -coordinate equal to 0 are on the y -axis, and have coordinates $(0, b)$.
- The point $(0, 0)$ is called the origin. It is the point where the x -axis and y -axis intersect.

Practice Makes Perfect

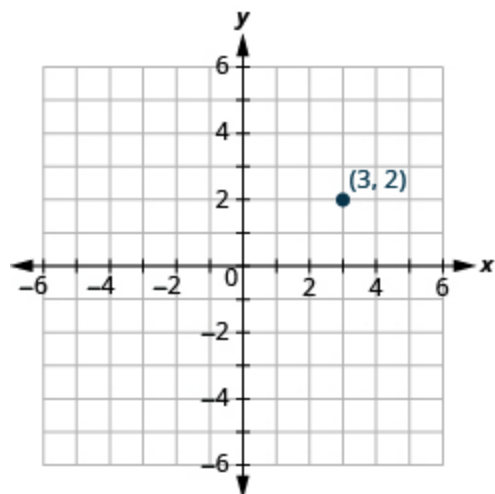
Plot Points on a Rectangular Coordinate System

In the following exercises, plot each point on a coordinate grid.

Exercise:

Problem: $(3, 2)$

Solution:



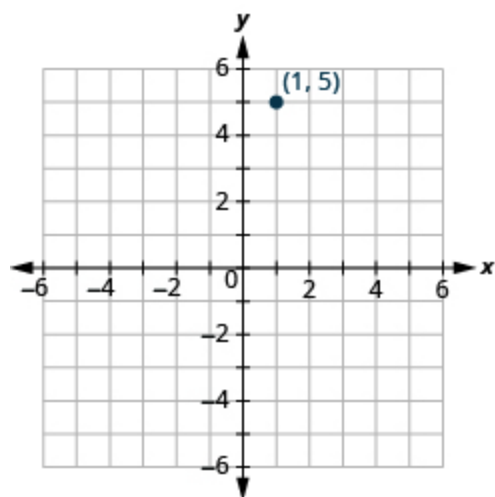
Exercise:

Problem: $(4, 1)$

Exercise:

Problem: $(1, 5)$

Solution:



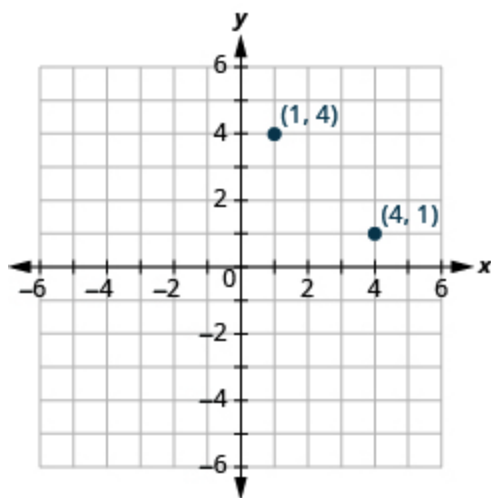
Exercise:

Problem: $(3, 4)$

Exercise:

Problem: $(4, 1), (1, 4)$

Solution:



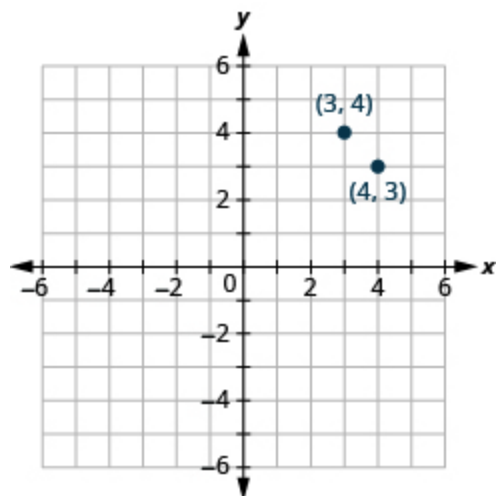
Exercise:

Problem: $(3, 2), (2, 3)$

Exercise:

Problem: $(3, 4), (4, 3)$

Solution:



In the following exercises, plot each point on a coordinate grid and identify the quadrant in which the point is located.

Exercise:

Problem:

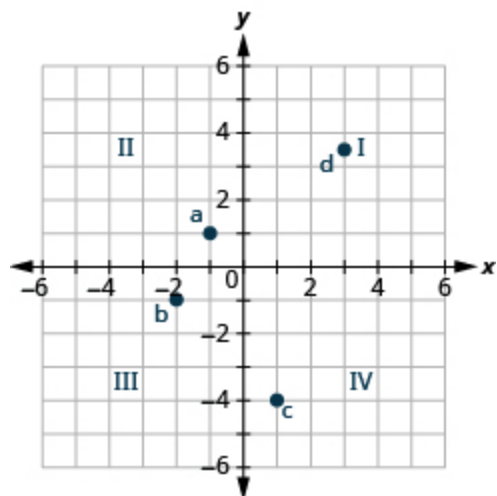
- Ⓐ $(-4, 2)$
- Ⓑ $(-1, -2)$
- Ⓒ $(3, -5)$
- Ⓓ $(2, \frac{5}{2})$

Exercise:

Problem:

- Ⓐ $(-2, -3)$
- Ⓑ $(3, -3)$
- Ⓒ $(-4, 1)$
- Ⓓ $(1, \frac{3}{2})$

Solution:



Exercise:

Problem:

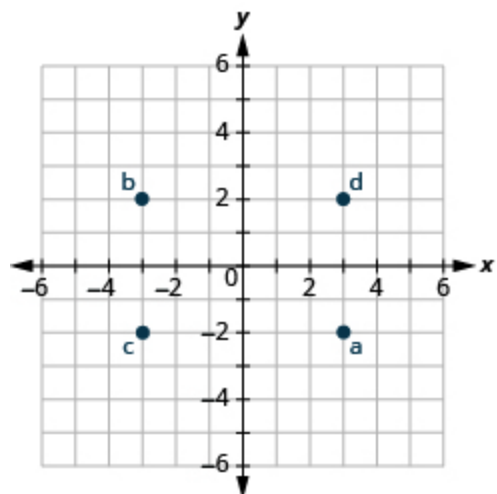
- Ⓐ $(-1, 1)$
- Ⓑ $(-2, -1)$
- Ⓒ $(1, -4)$
- Ⓓ $(3, \frac{7}{2})$

Exercise:

Problem:

- Ⓐ $(3, -2)$
- Ⓑ $(-3, 2)$
- Ⓒ $(-3, -2)$
- Ⓓ $(3, 2)$

Solution:



Exercise:

Problem:

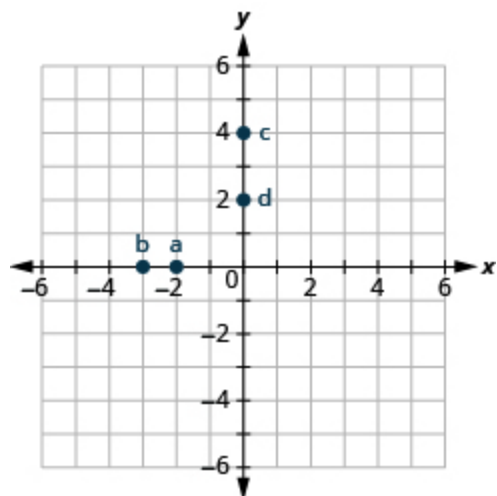
- Ⓐ $(4, -1)$
- Ⓑ $(-4, 1)$
- Ⓒ $(-4, -1)$
- Ⓓ $(4, 1)$

Exercise:

Problem:

- Ⓐ $(-2, 0)$
- Ⓑ $(-3, 0)$
- Ⓒ $(0, 4)$
- Ⓓ $(0, 2)$

Solution:

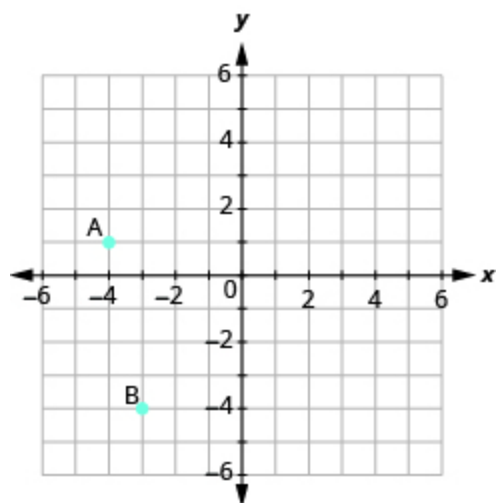


Identify Points on a Graph

In the following exercises, name the ordered pair of each point shown.

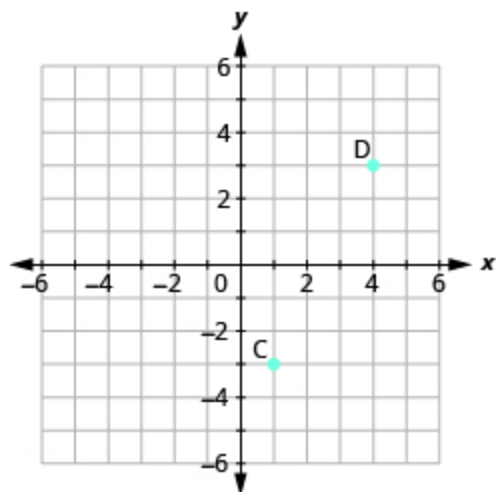
Exercise:

Problem:



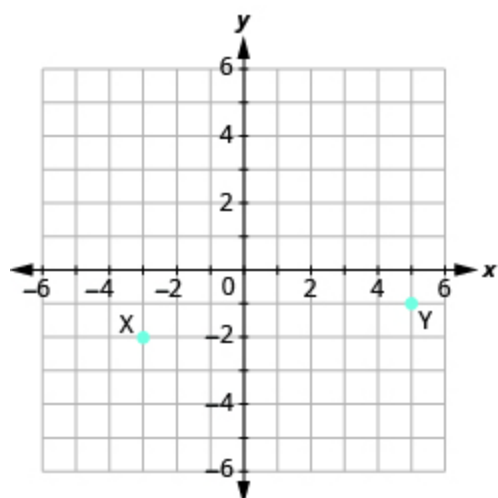
Exercise:

Problem:



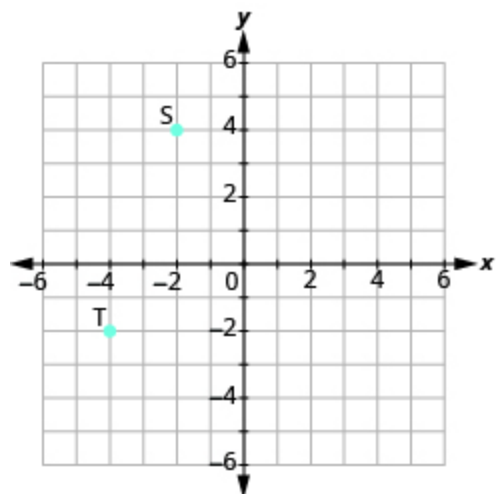
Exercise:

Problem:



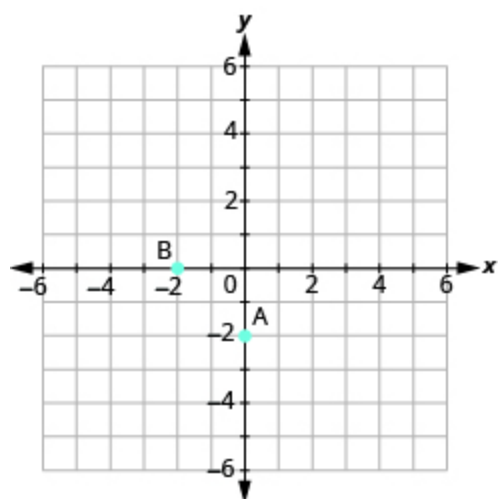
Exercise:

Problem:



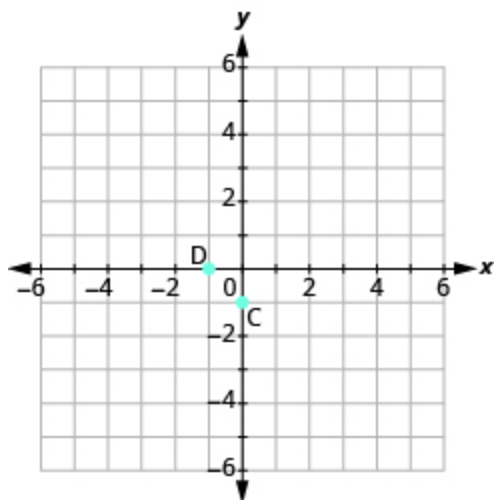
Exercise:

Problem:



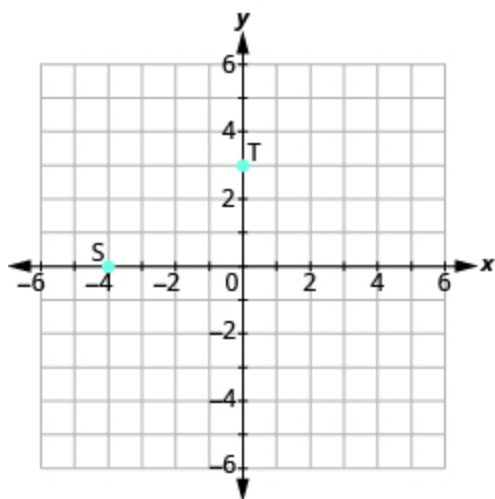
Exercise:

Problem:



Exercise:

Problem:



Verify Solutions to an Equation in Two Variables

In the following exercises, determine which ordered pairs are solutions to the given equation.

Exercise:

Problem: $2x + y = 6$

Ⓐ (1, 4)

Ⓑ (3, 0)

Ⓒ (2, 3)

Solution:

Ⓐ, Ⓑ

Exercise:

Problem: $x + 3y = 9$

Ⓐ (0, 3)

Ⓑ (6, 1)

Ⓒ (-3, -3)

Exercise:

Problem: $4x - 2y = 8$

Ⓐ (3, 2)

Ⓑ (1, 4)

Ⓒ (0, -4)

Solution:

Ⓐ, Ⓒ

Exercise:

Problem: $3x - 2y = 12$

Ⓐ (4, 0)

Ⓑ (2, -3)

Ⓒ (1, 6)

Exercise:

Problem: $y = 4x + 3$

- Ⓐ $(4, 3)$
- Ⓑ $(-1, -1)$
- Ⓒ $(\frac{1}{2}, 5)$

Solution:

- Ⓑ, Ⓒ

Exercise:

Problem: $y = 2x - 5$

- Ⓐ $(0, -5)$
- Ⓑ $(2, 1)$
- Ⓒ $(\frac{1}{2}, -4)$

Exercise:

Problem: $y = \frac{1}{2}x - 1$

- Ⓐ $(2, 0)$
- Ⓑ $(-6, -4)$
- Ⓒ $(-4, -1)$

Solution:

- Ⓐ, Ⓑ

Exercise:

Problem: $y = \frac{1}{3}x + 1$

- Ⓐ $(-3, 0)$
- Ⓑ $(9, 4)$
- Ⓒ $(-6, -1)$

Find Solutions to Linear Equations in Two Variables

In the following exercises, complete the table to find solutions to each linear equation.

Exercise:

Problem: $y = 2x - 4$

x	y	(x, y)
-1		
0		
2		

Solution:

x	y	(x, y)
-1	-6	$(-1, -6)$
0	-4	$(0, -4)$
2	0	$(2, 0)$

Exercise:

Problem: $y = 3x - 1$

x	y	(x, y)
-1		
0		
2		

Exercise:

Problem: $y = -x + 5$

x	y	(x, y)
-2		
0		
3		

Solution:

x	y	(x, y)
-2	7	$(-2, 7)$
0	5	$(0, 5)$
3	2	$(3, 2)$

Exercise:

Problem: $y = \frac{1}{3}x + 1$

x	y	(x, y)
-----	-----	----------

x	y	(x, y)
0		
3		
6		

Exercise:

Problem: $y = -\frac{3}{2}x - 2$

x	y	(x, y)
-2		
0		
2		

Solution:

x	y	(x, y)
-----	-----	----------

x	y	(x, y)
-2	1	$(-2, 1)$
0	-2	$(0, -2)$
2	-5	$(2, -5)$

Exercise:

Problem: $x + 2y = 8$

x	y	(x, y)
0		
4		
	0	

Everyday Math

Exercise:

Problem:

Weight of a baby Mackenzie recorded her baby's weight every two months. The baby's age, in months, and weight, in pounds, are listed in the table, and shown as an ordered pair in the third column.

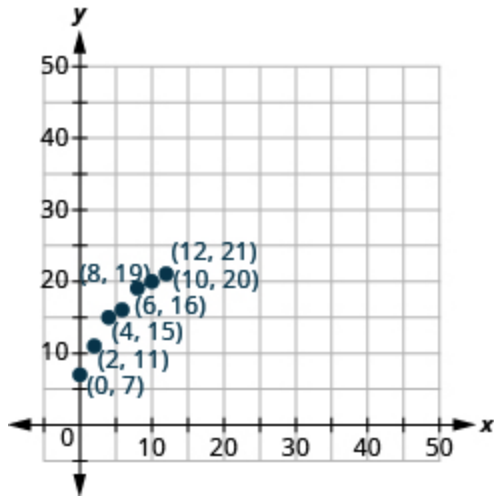
- Ⓐ Plot the points on a coordinate grid.

Age	Weight	(x, y)
0	7	$(0, 7)$
2	11	$(2, 11)$
4	15	$(4, 15)$
6	16	$(6, 16)$
8	19	$(8, 19)$
10	20	$(10, 20)$
12	21	$(12, 21)$

- Ⓑ Why is only Quadrant I needed?
-

Solution:

- Ⓐ



ⓑ Age and weight are only positive.

Exercise:

Problem:

Weight of a child Latresha recorded her son's height and weight every year. His height, in inches, and weight, in pounds, are listed in the table, and shown as an ordered pair in the third column.

ⓐ Plot the points on a coordinate grid.

Height x	Weight y	(x, y)
28	22	(28, 22)
31	27	(31, 27)
33	33	(33, 33)

37	35	(37, 35)
40	41	(40, 41)
42	45	(42, 45)

ⓑ Why is only Quadrant I needed?

Writing Exercises

Exercise:

Problem:

Have you ever used a map with a rectangular coordinate system?
Describe the map and how you used it.

Solution:

Answers may vary.

Exercise:

Problem:

How do you determine if an ordered pair is a solution to a given equation?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
plot points on a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			
find solutions to linear equations in two variables.			

⑥ If most of your checks were:

...confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

...with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

...no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

linear equation

An equation of the form $Ax + By = C$, where A and B are not both zero, is called a linear equation in two variables.

ordered pair

An ordered pair (x, y) gives the coordinates of a point in a rectangular coordinate system. The first number is the x -coordinate. The second number is the y -coordinate.

Equation:

$$\begin{array}{c} (x, y) \\ x\text{-coordinate}, y\text{-coordinate} \end{array}$$

origin

The point $(0, 0)$ is called the origin. It is the point where the x -axis and y -axis intersect.

quadrants

The x -axis and y -axis divide a rectangular coordinate system into four areas, called quadrants.

solution to a linear equation in two variables

An ordered pair (x, y) is a solution to the linear equation $Ax + By = C$, if the equation is a true statement when the x - and y -values of the ordered pair are substituted into the equation.

x -axis

The x -axis is the horizontal axis in a rectangular coordinate system.

y -axis

The y -axis is the vertical axis on a rectangular coordinate system.

Graphing Linear Equations

By the end of this section, you will be able to:

- Recognize the relation between the solutions of an equation and its graph
- Graph a linear equation by plotting points
- Graph vertical and horizontal lines

Note:

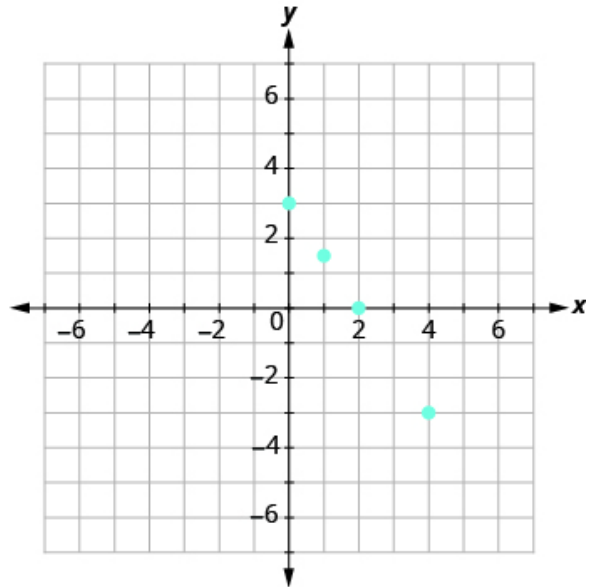
Before you get started, take this readiness quiz.

1. Evaluate: $3x + 2$ when $x = -1$.
If you missed this problem, review [\[link\]](#).
2. Solve the formula: $5x + 2y = 20$ for y .
If you missed this problem, review [\[link\]](#).
3. Simplify: $\frac{3}{8}(-24)$.
If you missed this problem, review [\[link\]](#).

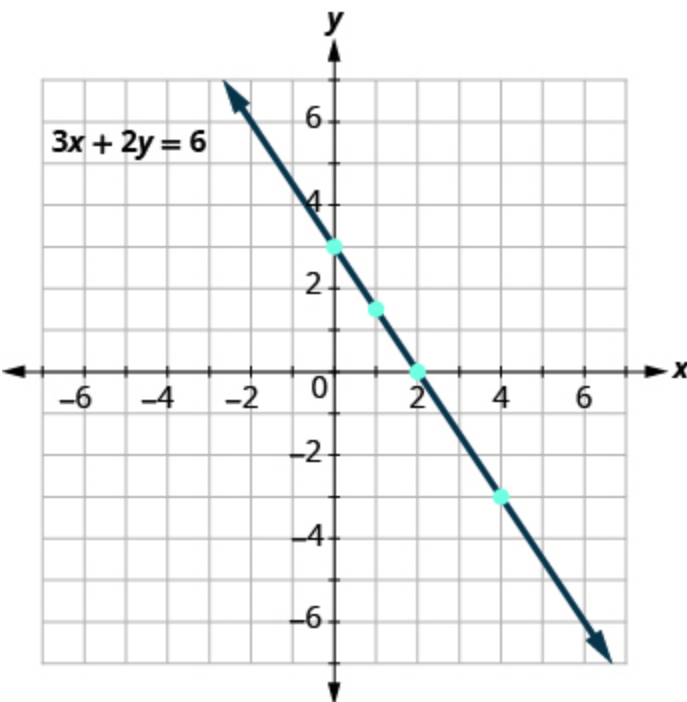
Recognize the Relation Between the Solutions of an Equation and its Graph

In [Use the Rectangular Coordinate System](#), we found a few solutions to the equation $3x + 2y = 6$. They are listed in the table below. So, the ordered pairs $(0, 3)$, $(2, 0)$, $(1, \frac{3}{2})$, $(4, -3)$, are some solutions to the equation $3x + 2y = 6$. We can plot these solutions in the rectangular coordinate system as shown on the graph at right.

$3x + 2y = 6$		
x	y	(x, y)
0	3	$(0, 3)$
2	0	$(2, 0)$
1	$\frac{3}{2}$	$(1, \frac{3}{2})$
4	-3	$(4, -3)$



Notice how the points line up perfectly? We connect the points with a straight line to get the graph of the equation $3x + 2y = 6$. Notice the arrows on the ends of each side of the line. These arrows indicate the line continues.



Every point on the line is a solution of the equation. Also, every solution of this equation is a point on this line. Points not on the line are *not* solutions!

Notice that the point whose coordinates are $(-2, 6)$ is on the line shown in [\[link\]](#). If you substitute $x = -2$ and $y = 6$ into the equation, you find that it is a solution to the equation.

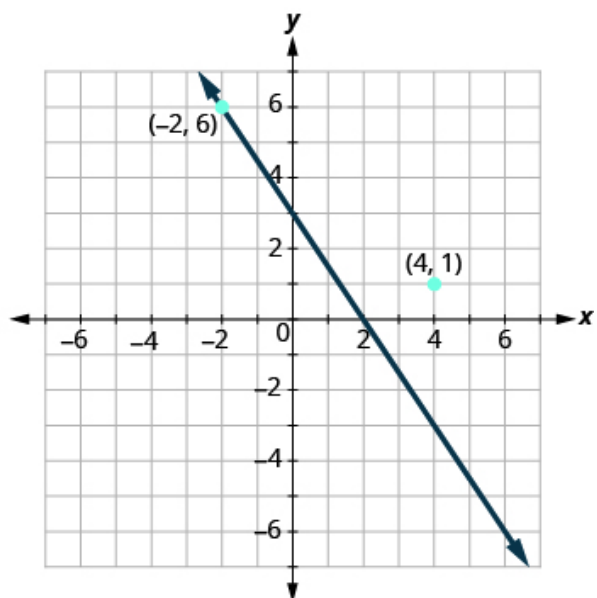
Test $(-2, 6)$:

$$\begin{aligned} 3x + 2y &= 6 \\ 3(-2) + 2(6) &\stackrel{?}{=} 6 \\ -6 + 12 &\stackrel{?}{=} 6 \\ 6 &= 6 \checkmark \end{aligned}$$

So $(-2, 6)$ is a solution to the equation.

What about $(4, 1)$?

$$\begin{aligned} 3x + 2y &= 6 \\ 3 \cdot 4 + 2 \cdot 1 &\stackrel{?}{=} 6 \\ 12 + 2 &\stackrel{?}{=} 6 \\ 14 &\neq 6 \end{aligned}$$



So $(4, 1)$ is not a solution to the equation $3x + 2y = 6$. Therefore the point $(4, 1)$ is not on the line.

This is an example of the saying, "A picture is worth a thousand words." The line shows you all the solutions to the equation. Every point on the line is a solution of the equation. And, every solution of this equation is on this line. This line is called the *graph* of the equation $3x + 2y = 6$.

Note:**Graph of a Linear Equation**

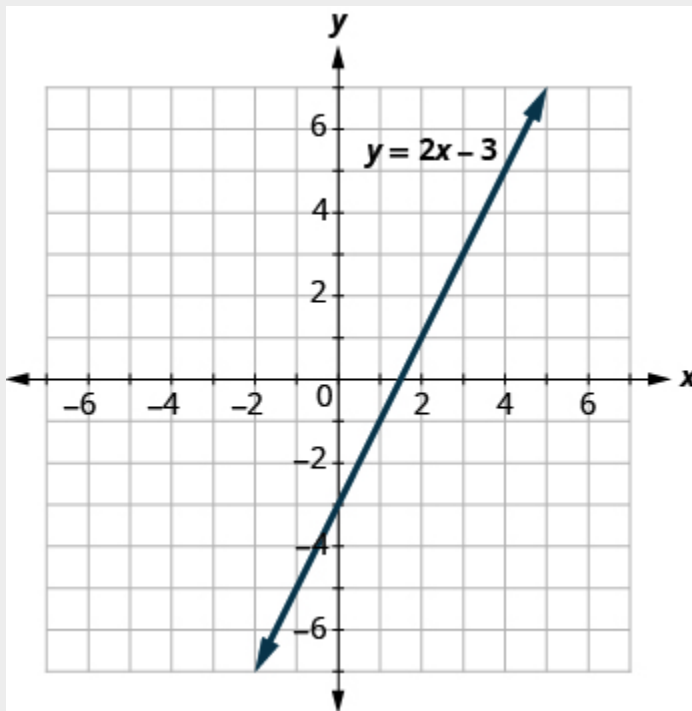
The graph of a linear equation $Ax + By = C$ is a straight line.

- Every point on the line is a solution of the equation.
- Every solution of this equation is a point on this line.

Example:

Exercise:

Problem: The graph of $y = 2x - 3$ is shown below.



For each ordered pair decide

- Ⓐ Is the ordered pair a solution to the equation?
 - Ⓑ Is the point on the line?
-
- Ⓐ $(0, 3)$
 - Ⓑ $(3, -3)$
 - Ⓒ $(2, -3)$
 - Ⓓ $(-1, -5)$

Solution:

Substitute the x - and y -values into the equation to check if the ordered pair is a solution to the equation.

(a)

(a) $(0, -3)$

$$y = 2x - 3$$

$$-3 \stackrel{?}{=} 2(0) - 3$$

$$-3 = -3 \checkmark$$

$(0, -3)$ is a solution.

(b) $(3, 3)$

$$y = 2x - 3$$

$$3 \stackrel{?}{=} 2(3) - 3$$

$$3 = 3 \checkmark$$

$(3, 3)$ is a solution.

(c) $(2, -3)$

$$y = 2x - 3$$

$$-3 \stackrel{?}{=} 2(2) - 3$$

$$-3 \neq 1$$

$(2, -3)$ is not a solution.

(d) $(-1, -5)$

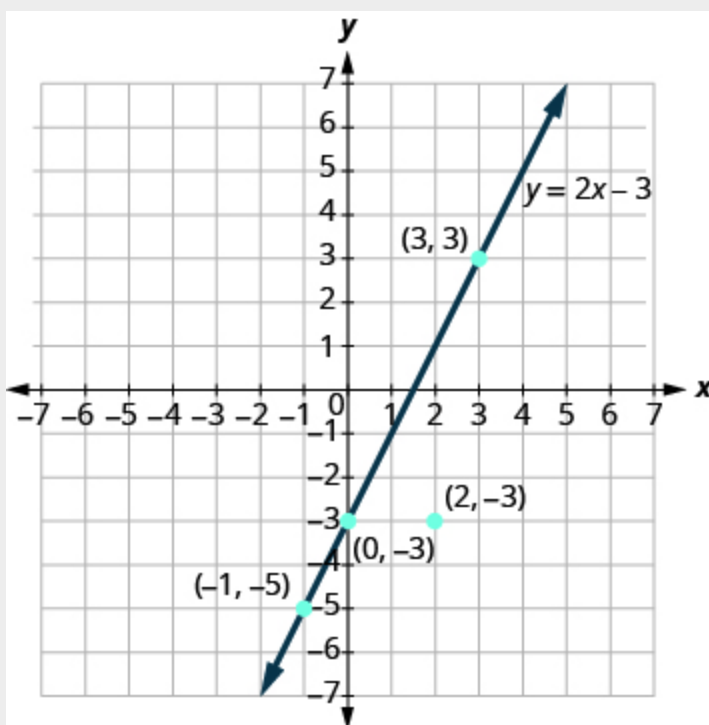
$$y = 2x - 3$$

$$-5 \stackrel{?}{=} 2(-1) - 3$$

$$-5 = -5 \checkmark$$

$(-1, -5)$ is a solution.

(b) Plot the points A: $(0, -3)$ B: $(3, 3)$ C: $(2, -3)$ and D: $(-1, -5)$.
The points $(0, -3)$, $(3, 3)$, and $(-1, -5)$ are on the line $y = 2x - 3$,
and the point $(2, -3)$ is not on the line.



The points which are solutions to $y = 2x - 3$ are on the line, but the point which is not a solution is not on the line.

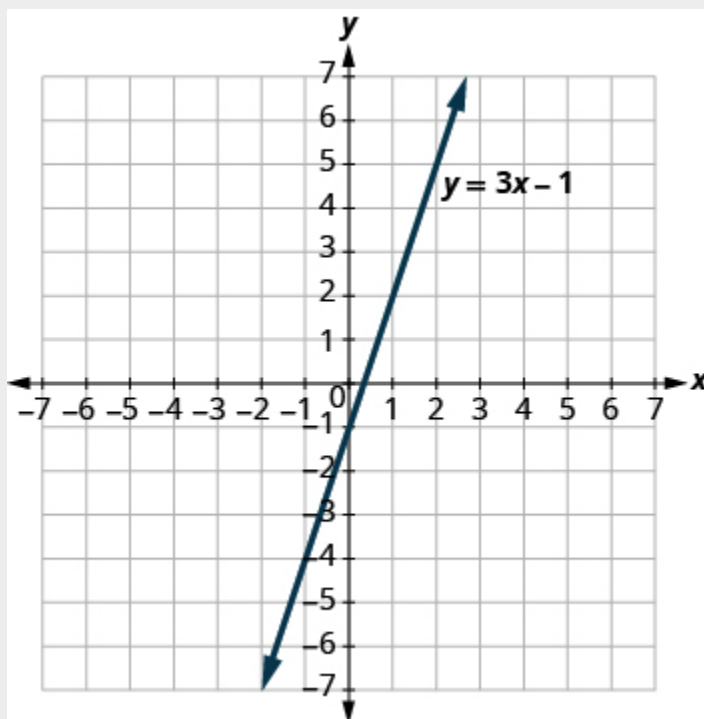
Note:

Exercise:

Problem: The graph of $y = 3x - 1$ is shown.

For each ordered pair, decide

- Ⓐ is the ordered pair a solution to the equation?
- Ⓑ is the point on the line?



1. $(0, -1)$
2. $(2, 2)$
3. $(3, -1)$
4. $(-1, -4)$

Solution:

1. Ⓐ yes Ⓑ yes
2. Ⓐ no Ⓑ no
3. Ⓐ no Ⓑ no
4. Ⓐ yes Ⓑ yes

Graph a Linear Equation by Plotting Points

There are several methods that can be used to graph a linear equation. The method we used at the start of this section to graph is called plotting points, or the Point-Plotting Method.

Let’s graph the equation $y = 2x + 1$ by plotting points.

We start by finding three points that are solutions to the equation. We can choose any value for x or y , and then solve for the other variable.

Since y is isolated on the left side of the equation, it is easier to choose values for x . We will use 0, 1, and -2 for x for this example. We substitute each value of x into the equation and solve for y .

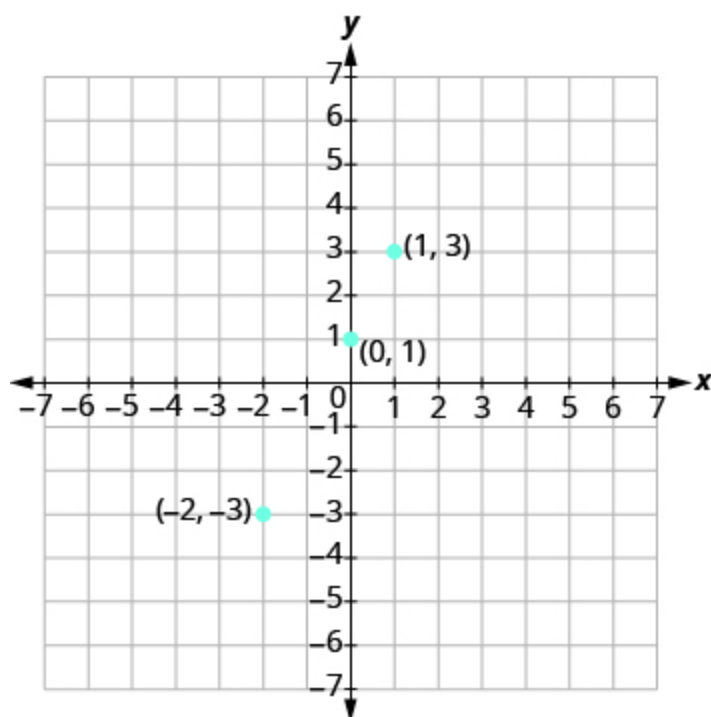
$x = -2$	$x = 0$	$x = 1$
$y = 2x + 1$	$y = 2x + 1$	$y = 2x + 1$
$y = 2(-2) + 1$	$y = 2(0) + 1$	$y = 2(1) + 1$
$y = -4 + 1$	$y = 0 + 1$	$y = 2 + 1$
$y = -3$	$y = 1$	$y = 3$
$(-2, -3)$	$(0, 1)$	$(1, 3)$

We can organize the solutions in a table. See [\[link\]](#).

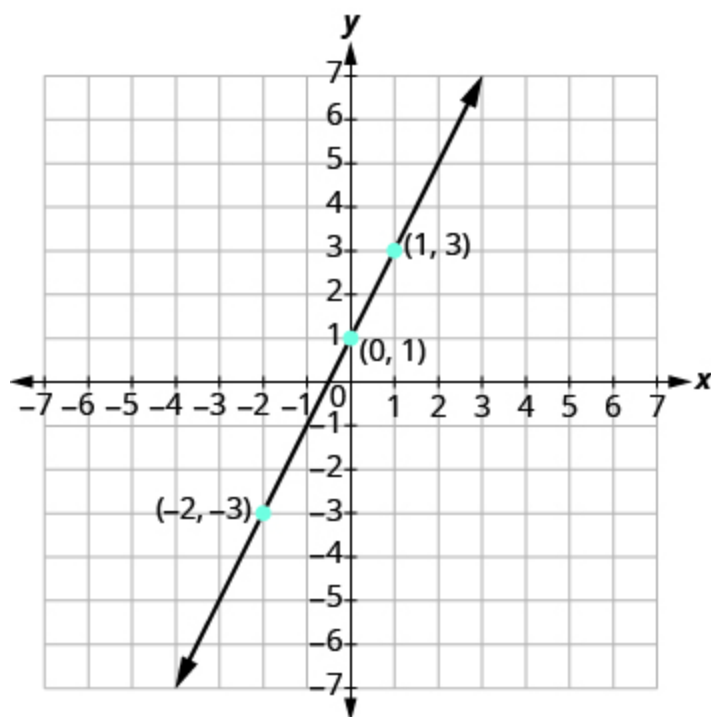
$y = 2x + 1$		
x	y	(x, y)
0	1	$(0, 1)$

$y = 2x + 1$		
x	y	(x, y)
1	3	$(1, 3)$
-2	-3	$(-2, -3)$

Now we plot the points on a rectangular coordinate system. Check that the points line up. If they did not line up, it would mean we made a mistake and should double-check all our work. See [\[link\]](#).



Draw the line through the three points. Extend the line to fill the grid and put arrows on both ends of the line. The line is the graph of $y = 2x + 1$.

**Note:**

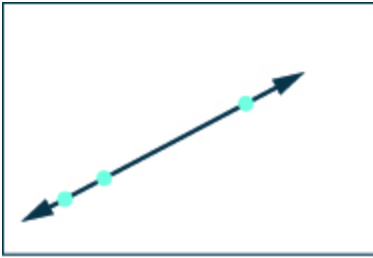
Graph a linear equation by plotting points.

Find three points whose coordinates are solutions to the equation. Organize them in a table.

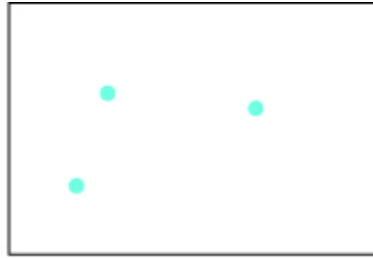
Plot the points on a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.

Draw the line through the points. Extend the line to fill the grid and put arrows on both ends of the line.

It is true that it only takes two points to determine a line, but it is a good habit to use three points. If you plot only two points and one of them is incorrect, you can still draw a line but it will not represent the solutions to the equation. It will be the wrong line. If you use three points, and one is incorrect, the points will not line up. This tells you something is wrong and you need to check your work. See [\[link\]](#).



(a)



(b)

Look at the difference between (a) and (b). All three points in (a) line up so we can draw one line through them. The three points in (b) do not line up. We cannot draw a single straight line through all three points.

Example:

Exercise:

Problem: Graph the equation $y = -3x$.

Solution:

Solution

Find three points that are solutions to the equation. It's easier to choose values for x , and solve for y . Do you see why?

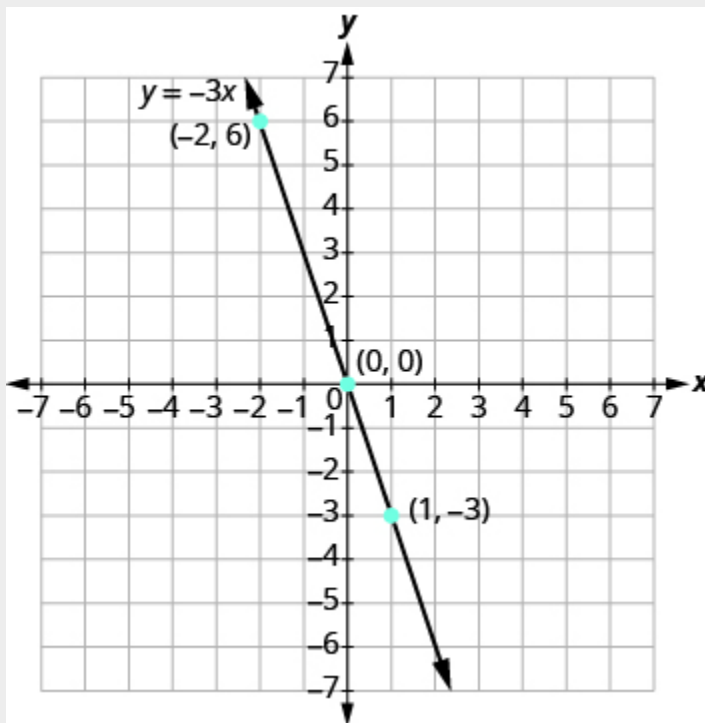
$x = 0$	$x = 1$	$x = -2$
$y = -3x$	$y = -3x$	$y = -3x$
$y = -3(0)$	$y = -3(1)$	$y = -3(-2)$
$y = 0$	$y = -3$	$y = 6$
$(0, 0)$	$(1, 3)$	$(-2, 6)$

List the points in a table.

$$y = -3x$$

x	y	(x, y)
0	0	$(0, 0)$
1	3	$(1, -3)$
-2	6	$(-2, 6)$

Plot the points, check that they line up, and draw the line as shown.

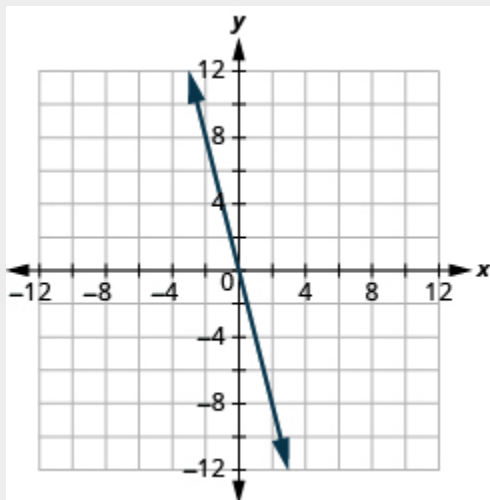


Note:

Exercise:

Problem: Graph the equation by plotting points: $y = -4x$.

Solution:

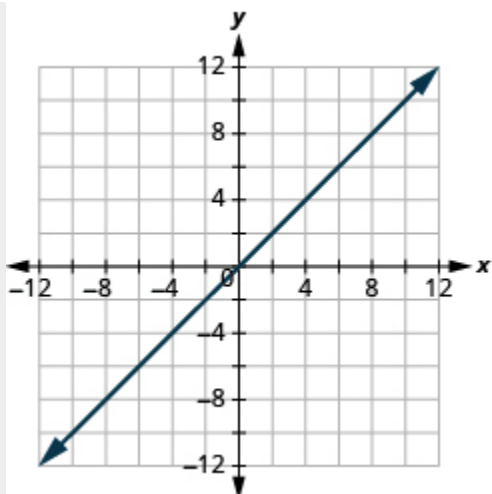


Note:

Exercise:

Problem: Graph the equation by plotting points: $y = x$.

Solution:



When an equation includes a fraction as the coefficient of x , we can substitute any numbers for x . But the math is easier if we make ‘good’ choices for the values of x . This way we will avoid fraction answers, which are hard to graph precisely.

Example:

Exercise:

Problem: Graph the equation $y = \frac{1}{2}x + 3$.

Solution:
Solution

Find three points that are solutions to the equation. Since this equation has the fraction $\frac{1}{2}$ as a coefficient of x , we will choose values of x carefully. We will use zero as one choice and multiples of 2 for the other choices.

$x = 0$

$y = \frac{1}{2}x + 3$

$y = \frac{1}{2}(0) + 3$

$y = 3$

$(0, 3)$

$x = 2$

$y = \frac{1}{2}x + 3$

$y = \frac{1}{2}(2) + 3$

$y = 4$

$(2, 4)$

$x = 4$

$y = \frac{1}{2}x + 3$

$y = \frac{1}{2}(4) + 3$

$y = 5$

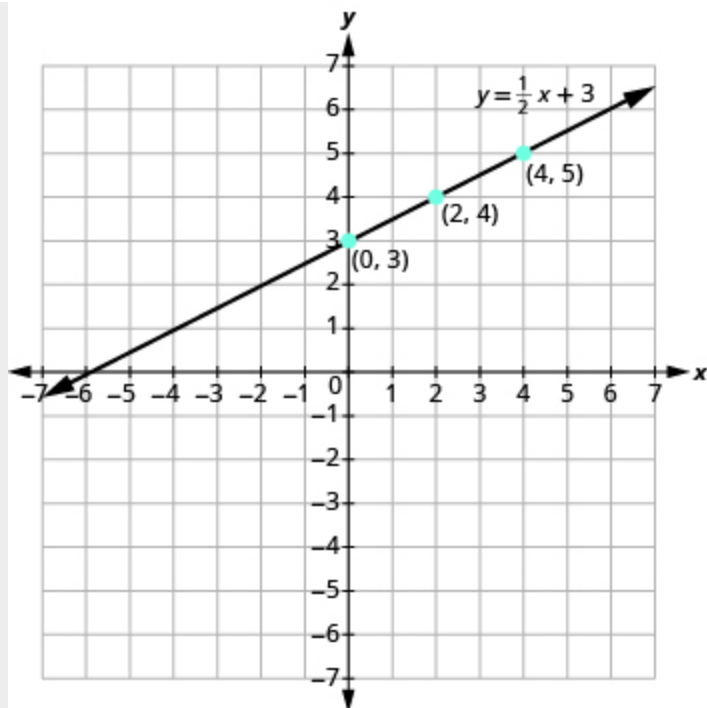
$(4, 5)$

The points are shown in the table.

$$y = \frac{1}{2}x + 3$$

x	y	(x, y)
0	3	$(0, 3)$
2	4	$(2, 4)$
4	5	$(4, 5)$

Plot the points, check that they line up, and draw the line as shown.

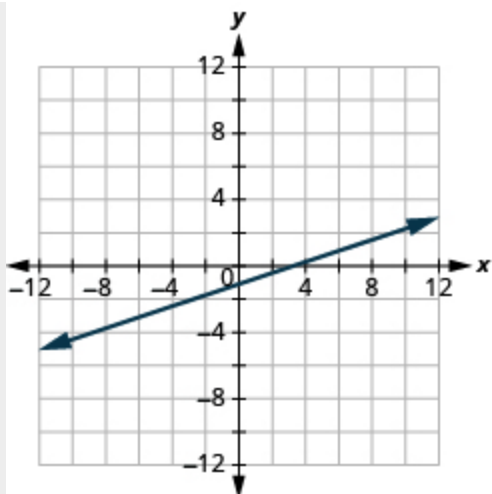


Note:

Exercise:

Problem: Graph the equation: $y = \frac{1}{3}x - 1$.

Solution:

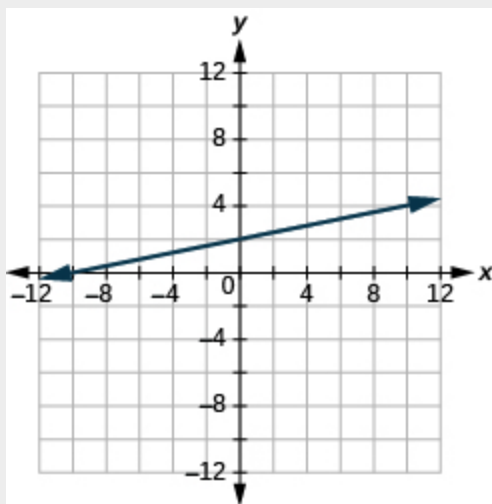


Note:

Exercise:

Problem: Graph the equation: $y = \frac{1}{4}x + 2$.

Solution:



So far, all the equations we graphed had y given in terms of x . Now we'll graph an equation with x and y on the same side.

Example:

Exercise:

Problem: Graph the equation $x + y = 5$.

Solution:

Solution

Find three points that are solutions to the equation. Remember, you can start with *any* value of x or y .

$x = 0$	$x = 1$	$x = 4$
$x + y = 5$	$x + y = 5$	$x + y = 5$
$0 + y = 5$	$1 + y = 5$	$4 + y = 5$
$y = 5$	$y = 4$	$y = 1$
$(0, 5)$	$(1, 4)$	$(4, 1)$

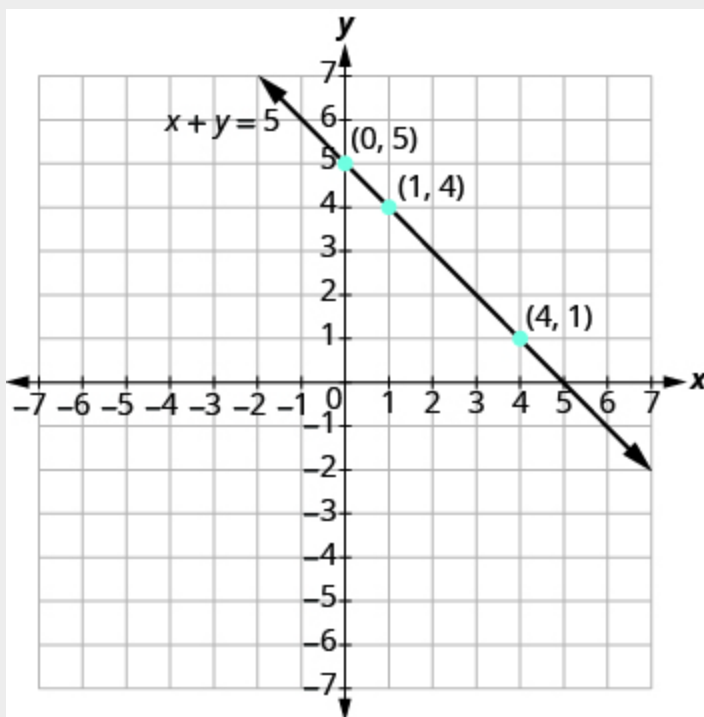
We list the points in a table.

$x + y = 5$		
x	y	(x, y)
0	5	$(0, 5)$
1	4	$(1, 4)$

$$x + y = 5$$

x	y	(x, y)
4	1	$(4, 1)$

Then plot the points, check that they line up, and draw the line.

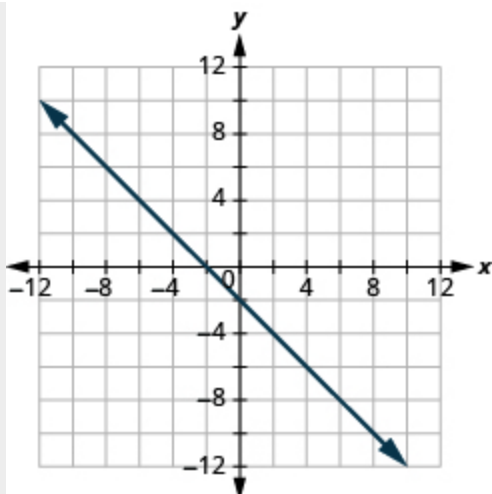


Note:

Exercise:

Problem: Graph the equation: $x + y = -2$.

Solution:

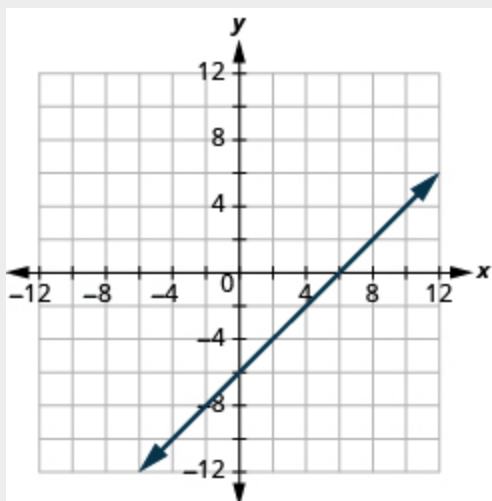


Note:

Exercise:

Problem: Graph the equation: $x - y = 6$.

Solution:



In the previous example, the three points we found were easy to graph. But this is not always the case. Let's see what happens in the equation $2x + y = 3$. If y is 0, what is the value of x ?

$$2x + y = 3$$

$$2x + 0 = 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

The solution is the point $(\frac{3}{2}, 0)$. This point has a fraction for the x -coordinate. While we could graph this point, it is hard to be precise graphing fractions. Remember in the example $y = \frac{1}{2}x + 3$, we carefully chose values for x so as not to graph fractions at all. If we solve the equation $2x + y = 3$ for y , it will be easier to find three solutions to the equation.

Equation:

$$2x + y = 3$$

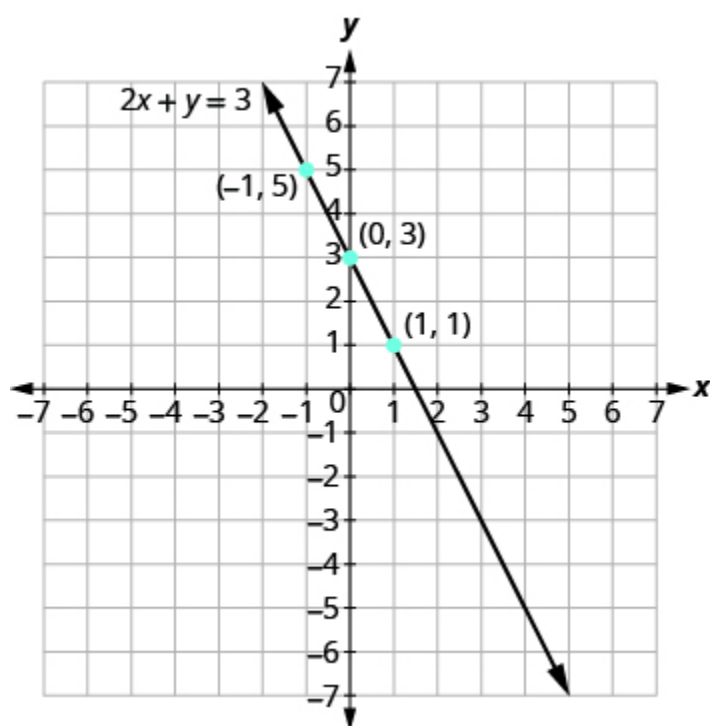
Equation:

$$y = -2x + 3$$

Now we can choose values for x that will give coordinates that are integers. The solutions for $x = 0$, $x = 1$, and $x = -1$ are shown.

$y = -2x + 3$		
x	y	(x, y)

$y = -2x + 3$		
x	y	(x, y)
0	5	$(-1, 5)$
1	3	$(0, 3)$
-1	1	$(1, 1)$



Example:

Exercise:

Problem: Graph the equation $3x + y = -1$.

Solution:
Solution

Find three points that are solutions to the equation.

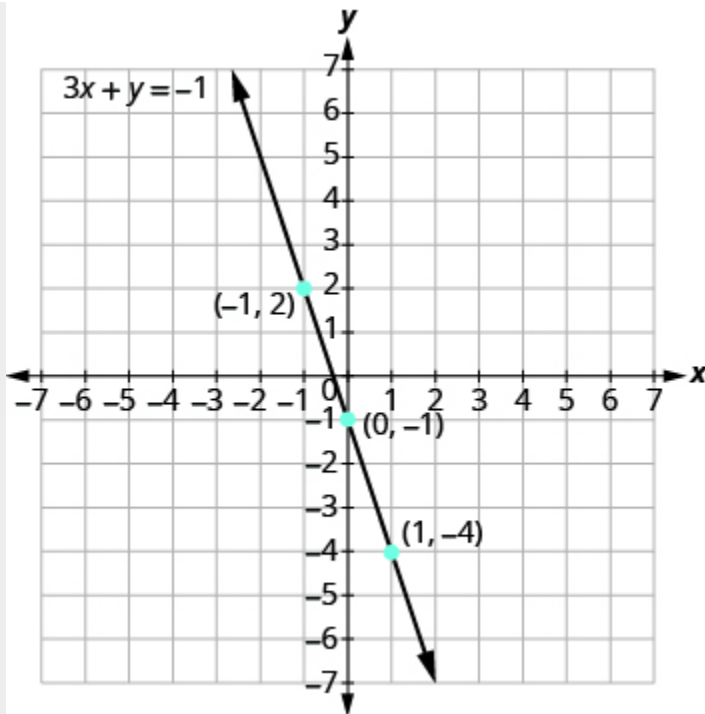
First, solve the equation for y .

$$3x + y = -1$$

$$y = -3x - 1$$

We'll let x be 0, 1, and -1 to find three points. The ordered pairs are shown in the table. Plot the points, check that they line up, and draw the line.

$y = -3x - 1$		
x	y	(x, y)
0	-1	$(0, -1)$
1	-4	$(1, -4)$
-1	2	$(-1, 2)$



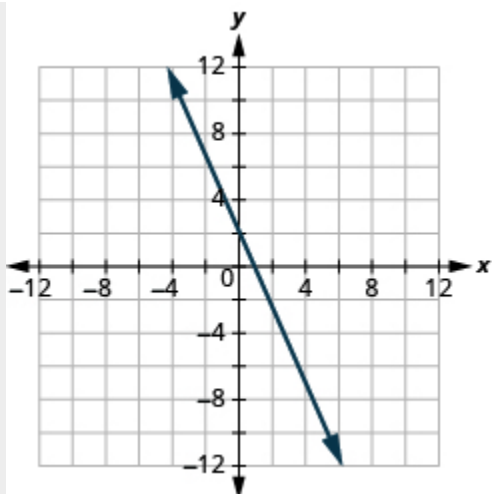
If you can choose any three points to graph a line, how will you know if your graph matches the one shown in the answers in the book? If the points where the graphs cross the x - and y -axes are the same, the graphs match.

Note:

Exercise:

Problem: Graph each equation: $2x + y = 2$.

Solution:

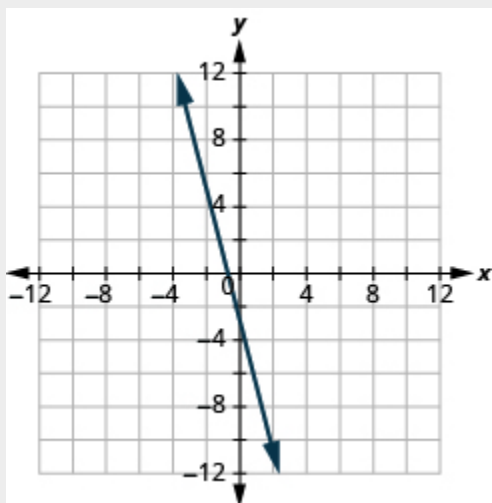


Note:

Exercise:

Problem: Graph each equation: $4x + y = -3$.

Solution:



Graph Vertical and Horizontal Lines

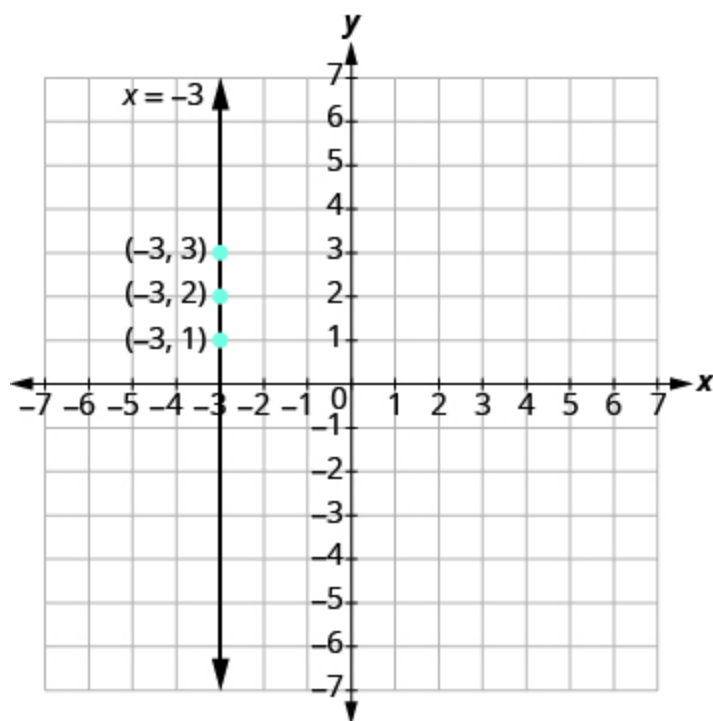
Can we graph an equation with only one variable? Just x and no y , or just y without an x ? How will we make a table of values to get the points to plot?

Let's consider the equation $x = -3$. The equation says that x is always equal to -3 , so its value does not depend on y . No matter what y is, the value of x is always -3 .

To make a table of solutions, we write -3 for all the x values. Then choose any values for y . Since x does not depend on y , you can choose any numbers you like. But to fit the size of our coordinate graph, we'll use 1, 2, and 3 for the y -coordinates as shown in the table.

$x = -3$		
x	y	(x, y)
-3	1	$(-3, 1)$
-3	2	$(-3, 2)$
-3	3	$(-3, 3)$

Then plot the points and connect them with a straight line. Notice in [\[link\]](#) that the graph is a **vertical line**.

**Note:****Vertical Line**

A vertical line is the graph of an equation that can be written in the form $x = a$.

The line passes through the x -axis at $(a, 0)$.

Example:**Exercise:**

Problem: Graph the equation $x = 2$. What type of line does it form?

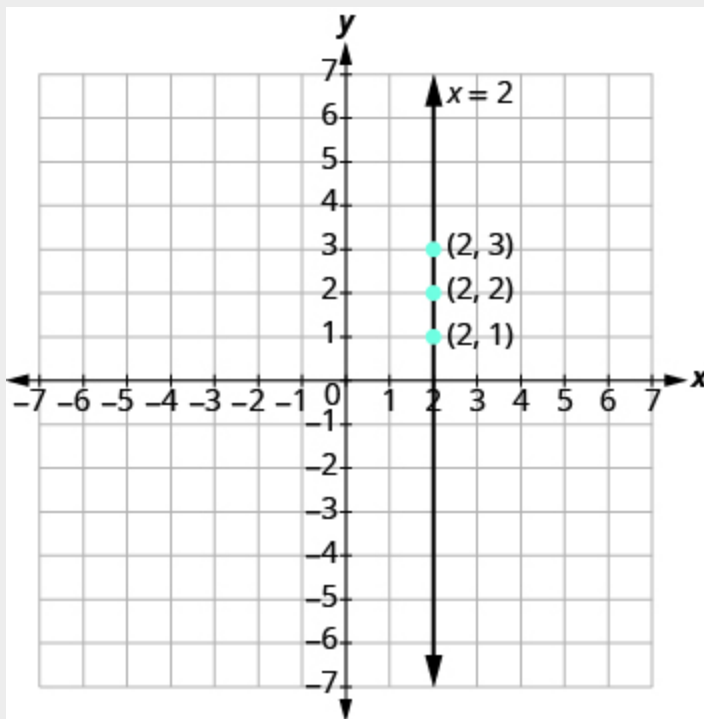
Solution:**Solution**

The equation has only variable, x , and x is always equal to 2. We make a table where x is always 2 and we put in any values for y .

$$x = 2$$

x	y	(x, y)
2	1	$(2, 1)$
2	2	$(2, 2)$
2	3	$(2, 3)$

Plot the points and connect them as shown.



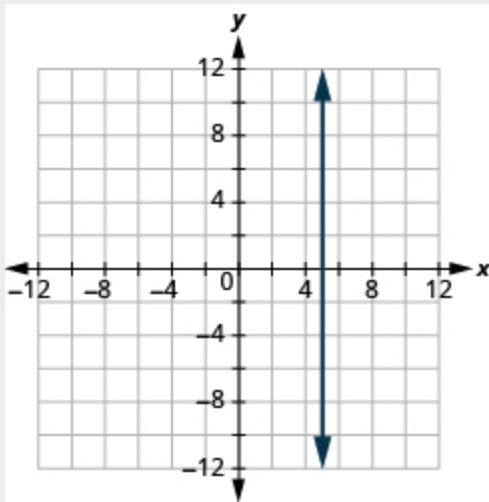
The graph is a vertical line passing through the x -axis at 2.

Note:

Exercise:

Problem: Graph the equation: $x = 5$.

Solution:

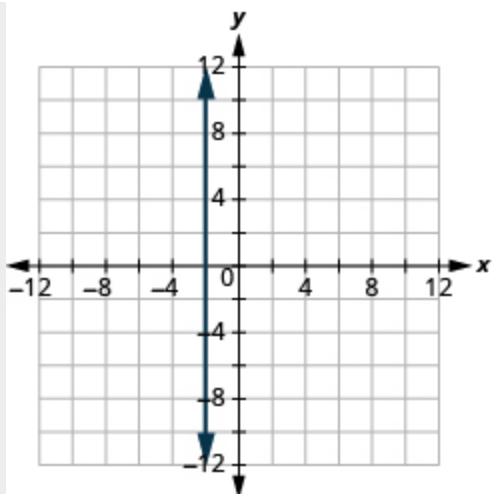


Note:

Exercise:

Problem: Graph the equation: $x = -2$.

Solution:



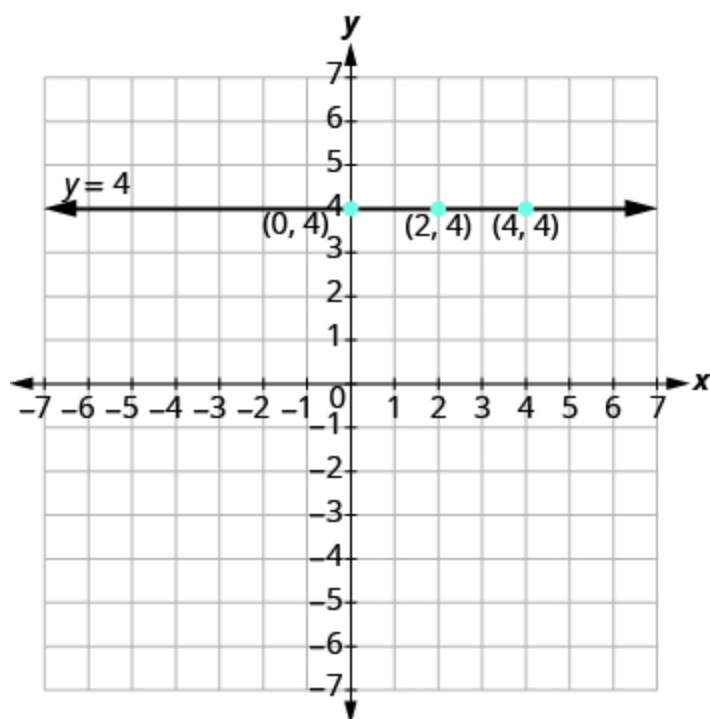
What if the equation has y but no x ? Let's graph the equation $y = 4$. This time the y -value is a constant, so in this equation y does not depend on x .

To make a table of solutions, write 4 for all the y values and then choose any values for x .

We'll use 0, 2, and 4 for the x -values.

$y = 4$		
x	y	(x, y)
0	4	$(0, 4)$
2	4	$(2, 4)$
4	4	$(4, 4)$

Plot the points and connect them, as shown in [\[link\]](#). This graph is a **horizontal line** passing through the y -axis at 4.



Note:

Horizontal Line

A horizontal line is the graph of an equation that can be written in the form $y = b$.

The line passes through the y -axis at $(0, b)$.

Example:

Exercise:

Problem: Graph the equation $y = -1$.

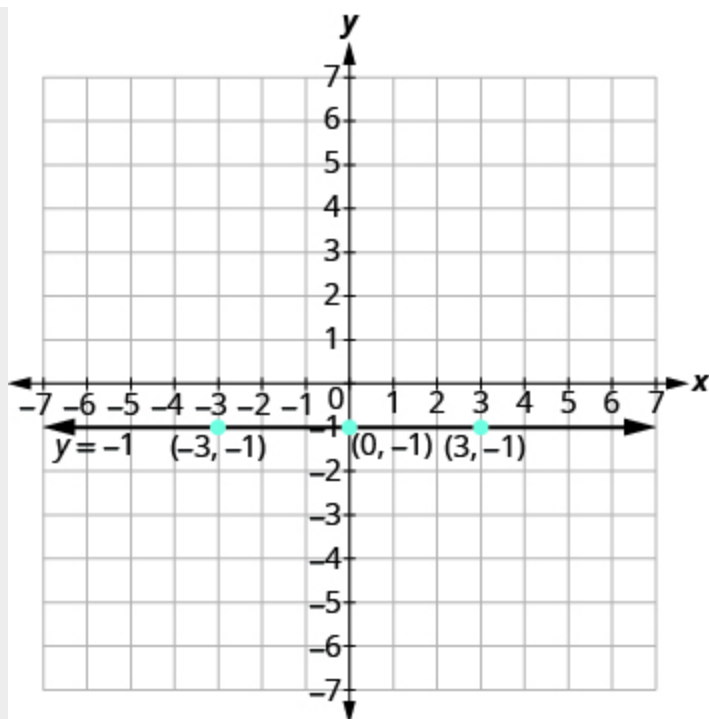
Solution:

Solution

The equation $y = -1$ has only variable, y . The value of y is constant. All the ordered pairs in the table have the same y -coordinate, -1 . We choose $0, 3$, and -3 as values for x .

$y = -1$		
x	y	(x, y)
-3	-1	$(-3, -1)$
0	-1	$(0, -1)$
3	-1	$(3, -1)$

The graph is a horizontal line passing through the y -axis at -1 as shown.

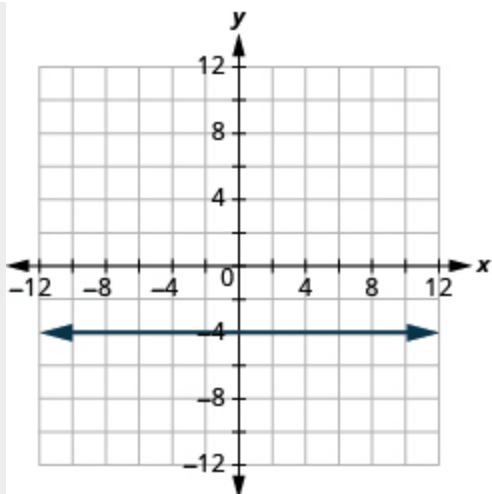


Note:

Exercise:

Problem: Graph the equation: $y = -4$.

Solution:

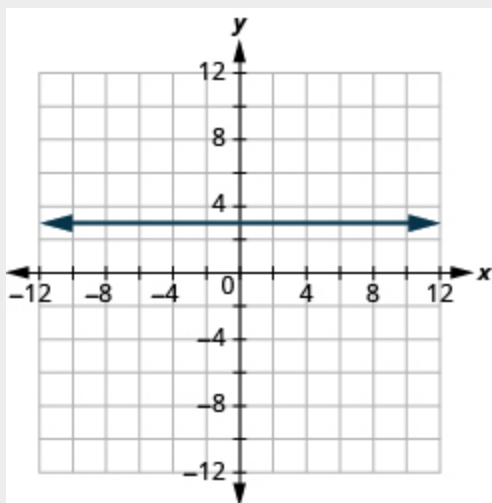


Note:

Exercise:

Problem: Graph the equation: $y = 3$.

Solution:



The equations for vertical and horizontal lines look very similar to equations like $y = 4x$. What is the difference between the equations $y = 4x$ and $y = 4$?

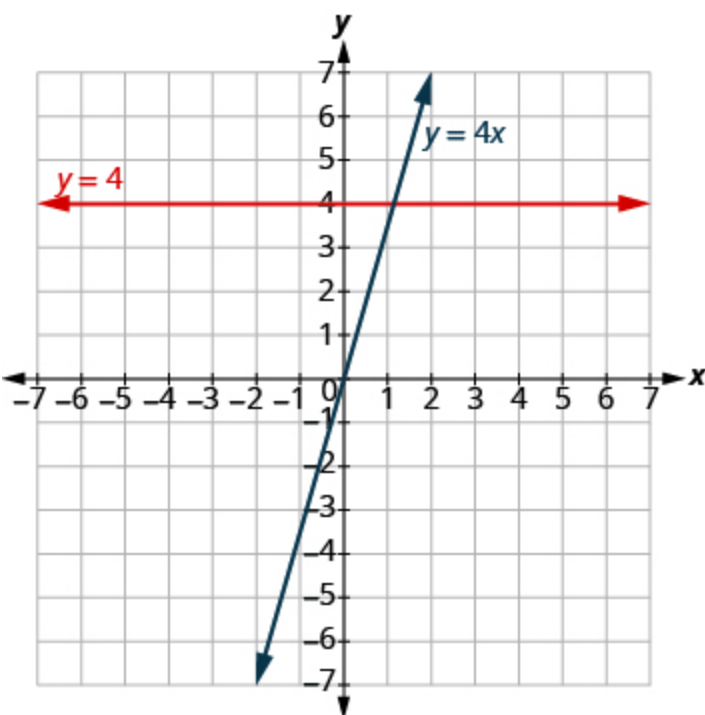
The equation $y = 4x$ has both x and y . The value of y depends on the value of x . The y -coordinate changes according to the value of x .

The equation $y = 4$ has only one variable. The value of y is constant. The y -coordinate is always 4. It does not depend on the value of x .

$y = 4x$		
x	y	(x, y)
0	0	(0, 0)
1	4	(1, 4)
2	8	(2, 8)

$y = 4$		
x	y	(x, y)
0	4	(0, 4)
1	4	(1, 4)
2	4	(2, 4)

The graph shows both equations.



Notice that the equation $y = 4x$ gives a slanted line whereas $y = 4$ gives a horizontal line.

Example:**Exercise:****Problem:**

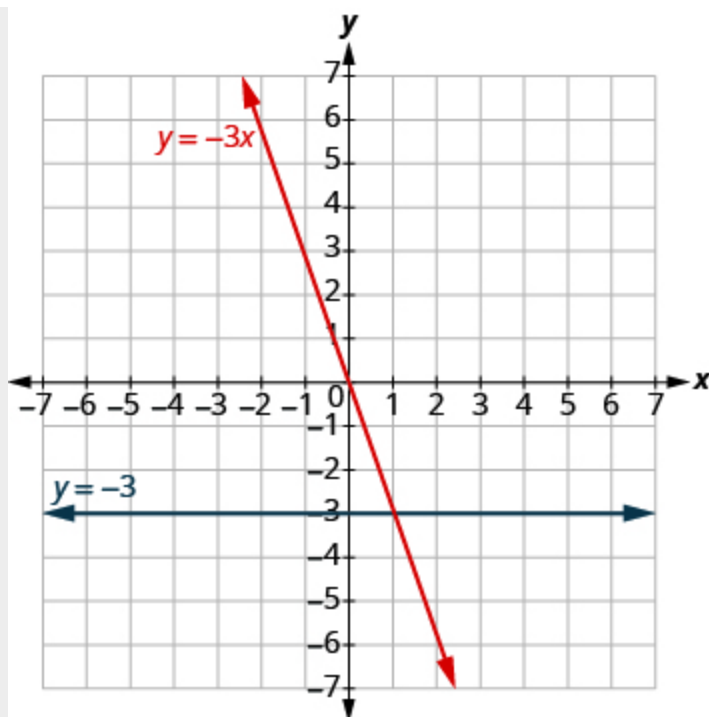
Graph $y = -3x$ and $y = -3$ in the same rectangular coordinate system.

Solution:**Solution**

Find three solutions for each equation. Notice that the first equation has the variable x , while the second does not. Solutions for both equations are listed.

$y = -3x$			$y = -3$		
x	y	(x, y)	x	y	(x, y)
0	0	$(0, 0)$	0	-3	$(0, -3)$
1	-3	$(1, -3)$	1	-3	$(1, -3)$
2	-6	$(2, -6)$	2	-3	$(2, -3)$

The graph shows both equations.



Note:

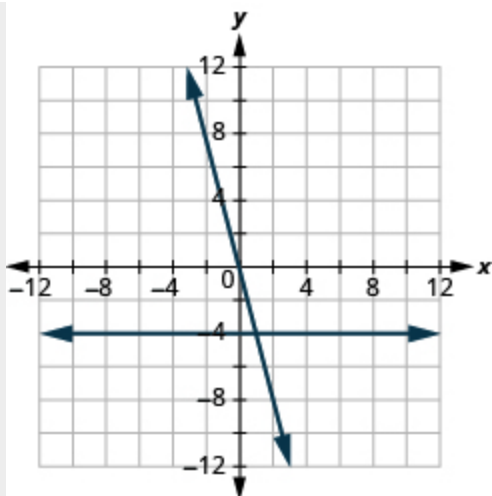
Exercise:

Problem:

Graph the equations in the same rectangular coordinate system:

$$y = -4x \text{ and } y = -4.$$

Solution:



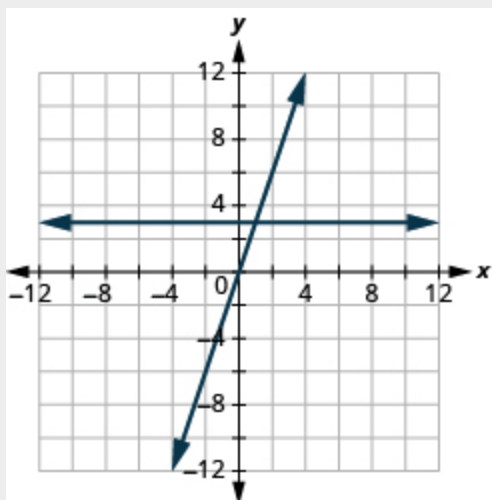
Note:

Exercise:

Problem:

Graph the equations in the same rectangular coordinate system: $y = 3$ and $y = 3x$.

Solution:



Note:**ACCESS ADDITIONAL ONLINE RESOURCES**

- [Use a Table of Values](#)
- [Graph a Linear Equation Involving Fractions](#)
- [Graph Horizontal and Vertical Lines](#)

Key Concepts

- **Graph a linear equation by plotting points.**

Find three points whose coordinates are solutions to the equation.
Organize them in a table.

Plot the points on a rectangular coordinate system. Check that the points line up. If they do not, carefully check your work.

Draw the line through the points. Extend the line to fill the grid and put arrows on both ends of the line.

- **Graph of a Linear Equation:** The graph of a linear equation $ax + by = c$ is a straight line.
 - Every point on the line is a solution of the equation.
 - Every solution of this equation is a point on this line.

Practice Makes Perfect**Recognize the Relation Between the Solutions of an Equation and its Graph**

For each ordered pair, decide

- Ⓐ is the ordered pair a solution to the equation?
Ⓑ is the point on the line?

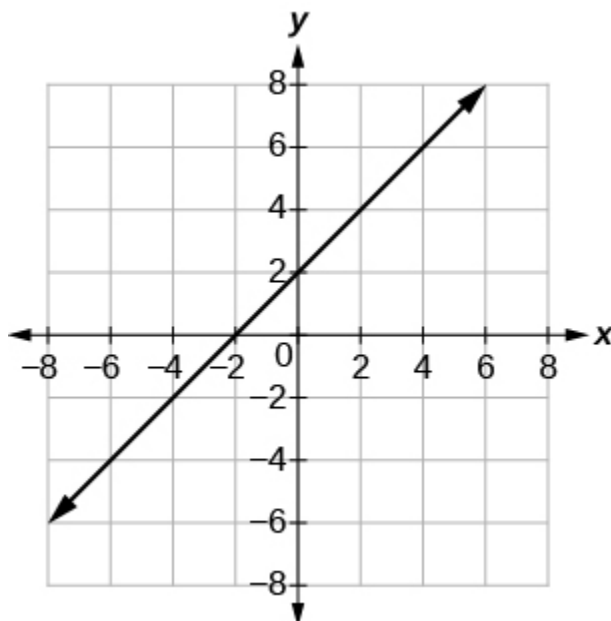
Exercise:

Problem: $y = x + 2$

1. $(0, 2)$
2. $(1, 2)$
3. $(-1, 1)$
4. $(-3, 1)$

Solution:

- Ⓐ yes Ⓑ yes
Ⓐ no Ⓑ no
Ⓐ yes Ⓑ yes
Ⓐ yes Ⓑ yes



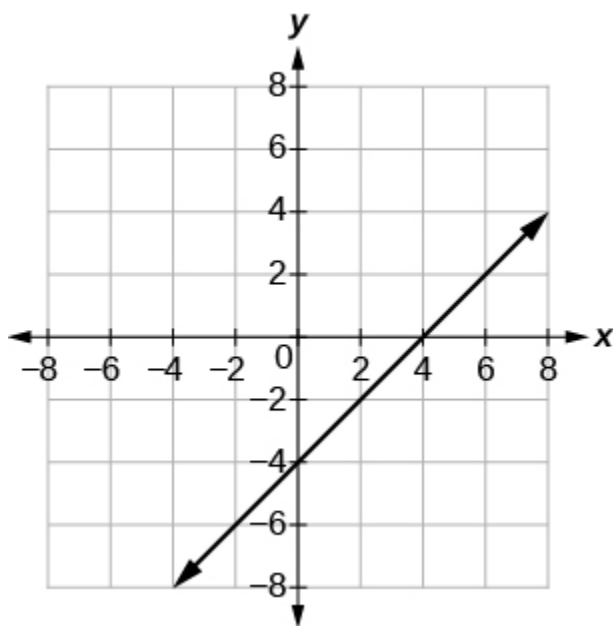
Exercise:

Problem: $y = x - 4$

1. $(0, -4)$
2. $(3, -1)$
3. $(2, 2)$
4. $(1, -5)$

Solution:

1. Ⓐ yes Ⓑ yes
2. Ⓐ yes Ⓑ yes
3. Ⓐ no Ⓑ no
4. Ⓐ no Ⓑ no



Exercise:

Problem: $y = \frac{1}{2}x - 3$

1. $(0, -3)$
2. $(2, -2)$

3. $(-2, -4)$

4. $(4, 1)$

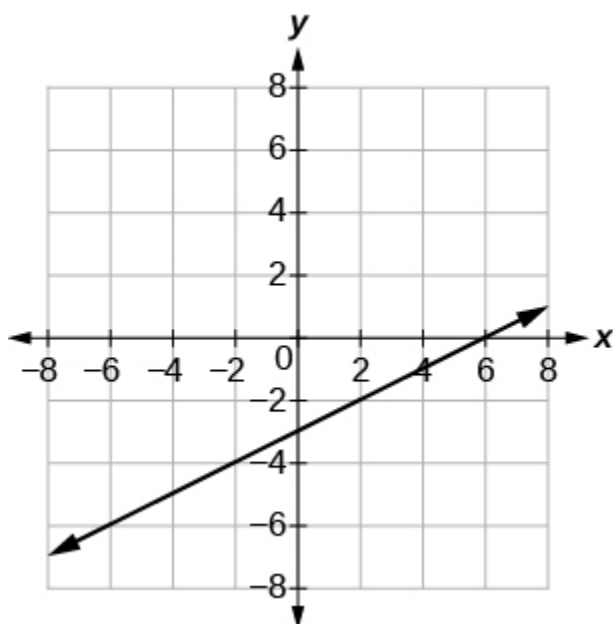
Solution:

☐ a yes ☐ b yes

☐ a yes ☐ b yes

☐ a yes ☐ b yes

☐ a no ☐ b no



Exercise:

Problem: $y = \frac{1}{3}x + 2$

1. $(0, 2)$

2. $(3, 3)$

3. $(-3, 2)$

4. $(-6, 0)$

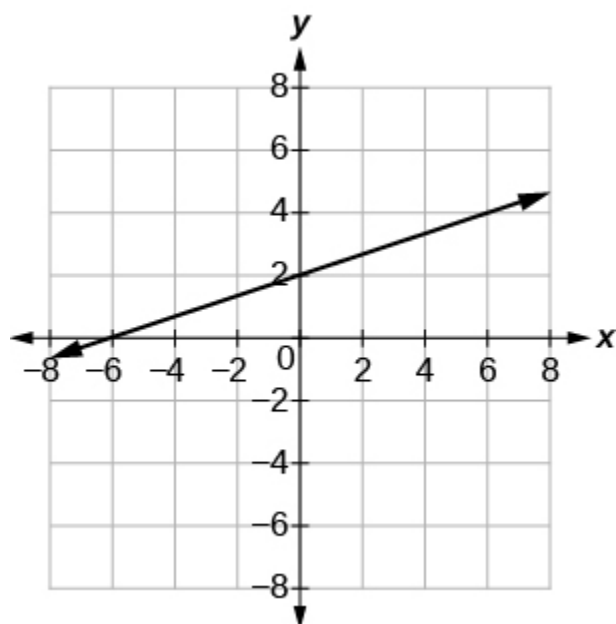
Solution:

Ⓐ yes Ⓑ yes

Ⓐ yes Ⓑ yes

Ⓐ no Ⓑ no

Ⓐ yes Ⓑ yes



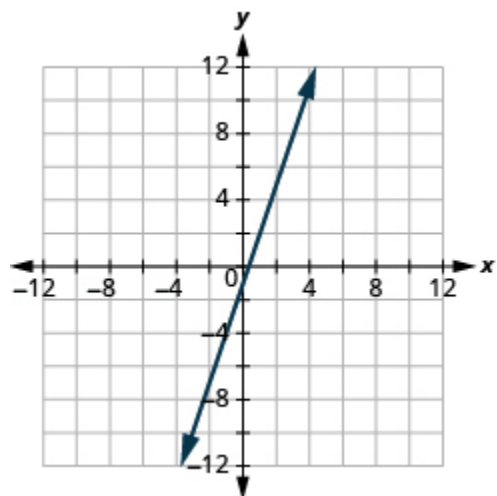
Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

Exercise:

Problem: $y = 3x - 1$

Solution:



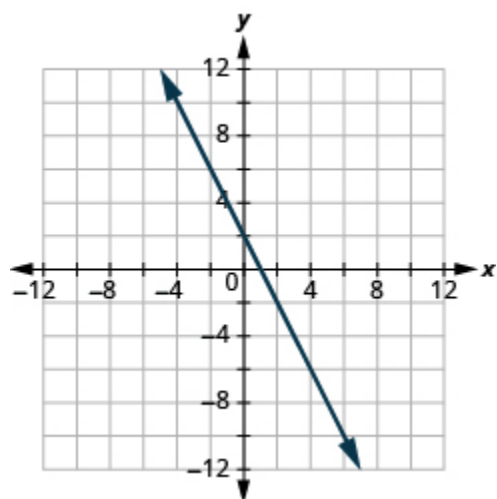
Exercise:

Problem: $y = 2x + 3$

Exercise:

Problem: $y = -2x + 2$

Solution:



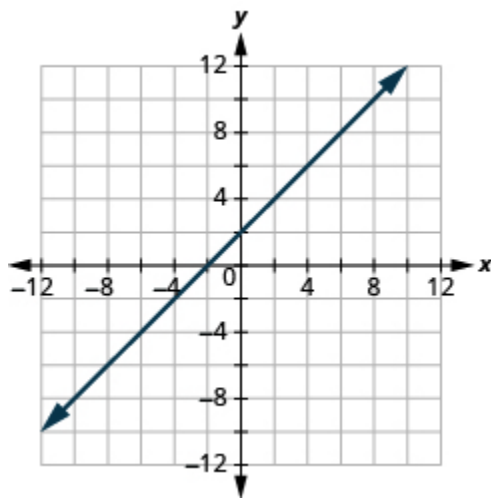
Exercise:

Problem: $y = -3x + 1$

Exercise:

Problem: $y = x + 2$

Solution:



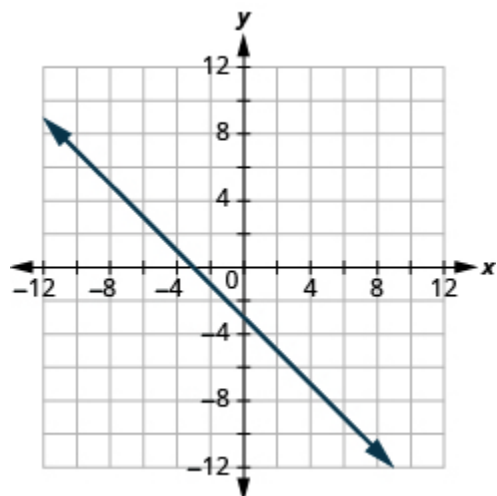
Exercise:

Problem: $y = x - 3$

Exercise:

Problem: $y = -x - 3$

Solution:



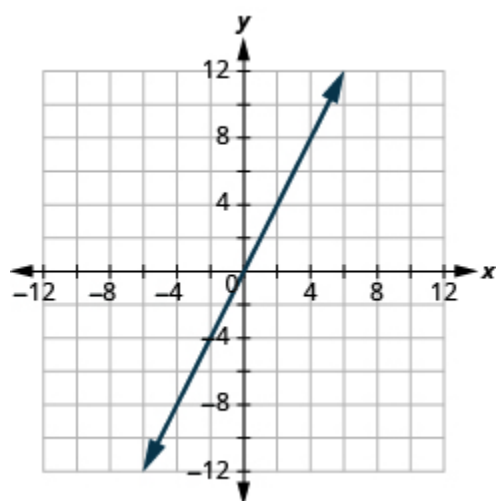
Exercise:

Problem: $y = -x - 2$

Exercise:

Problem: $y = 2x$

Solution:



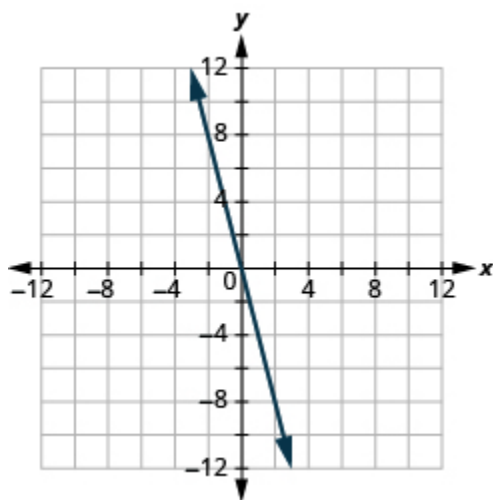
Exercise:

Problem: $y = 3x$

Exercise:

Problem: $y = -4x$

Solution:



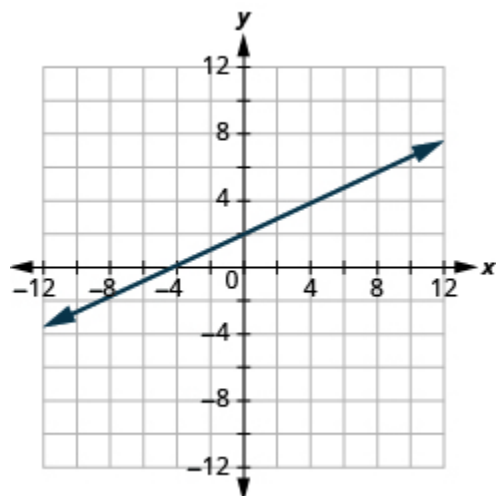
Exercise:

Problem: $y = -2x$

Exercise:

Problem: $y = \frac{1}{2}x + 2$

Solution:



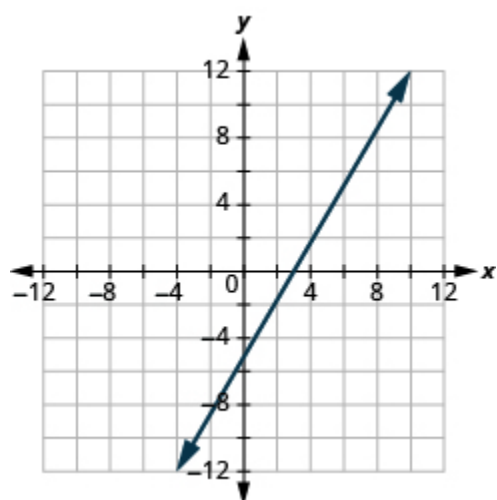
Exercise:

Problem: $y = \frac{1}{3}x - 1$

Exercise:

Problem: $y = \frac{4}{3}x - 5$

Solution:



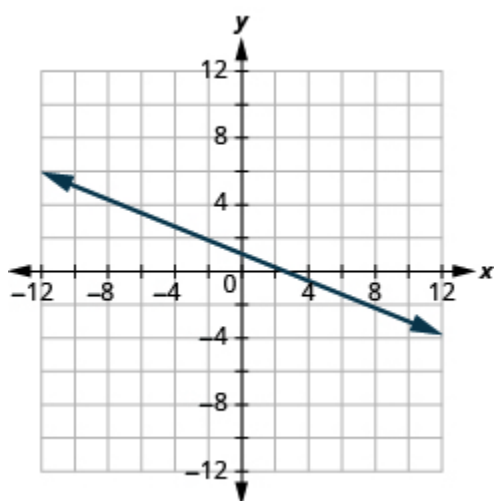
Exercise:

Problem: $y = \frac{3}{2}x - 3$

Exercise:

Problem: $y = -\frac{2}{5}x + 1$

Solution:



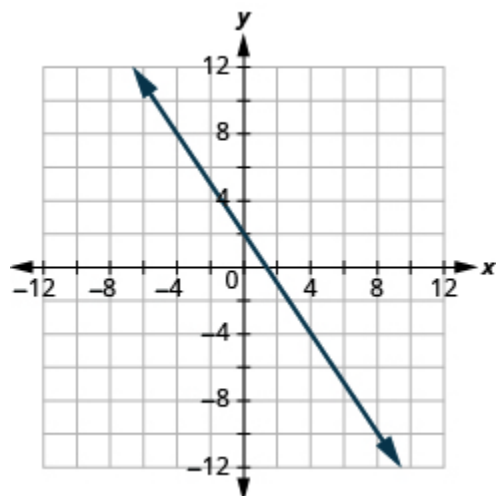
Exercise:

Problem: $y = -\frac{4}{5}x - 1$

Exercise:

Problem: $y = -\frac{3}{2}x + 2$

Solution:



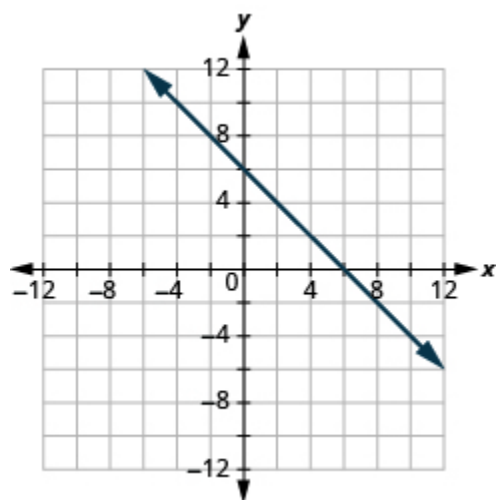
Exercise:

Problem: $y = -\frac{5}{3}x + 4$

Exercise:

Problem: $x + y = 6$

Solution:



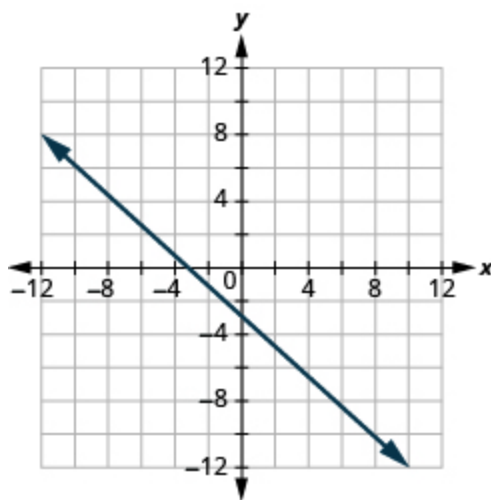
Exercise:

Problem: $x + y = 4$

Exercise:

Problem: $x + y = -3$

Solution:



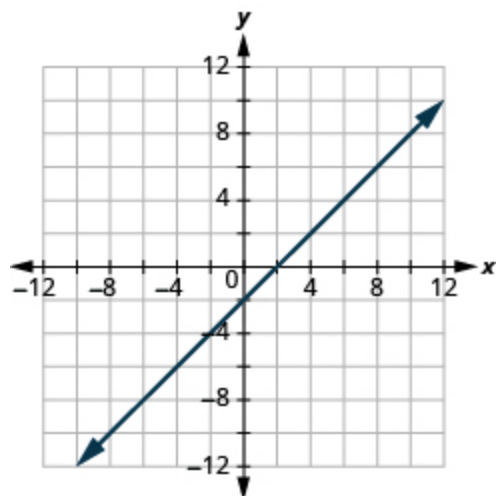
Exercise:

Problem: $x + y = -2$

Exercise:

Problem: $x - y = 2$

Solution:



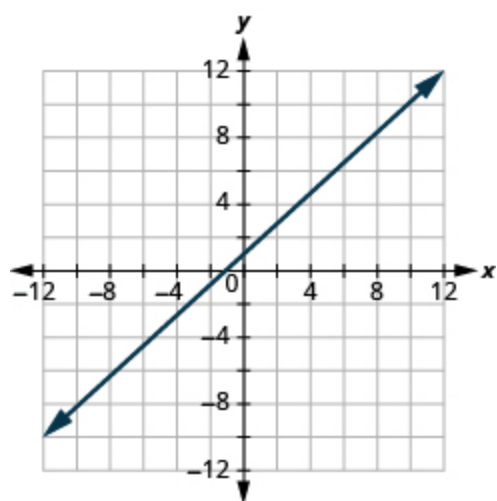
Exercise:

Problem: $x - y = 1$

Exercise:

Problem: $x - y = -1$

Solution:



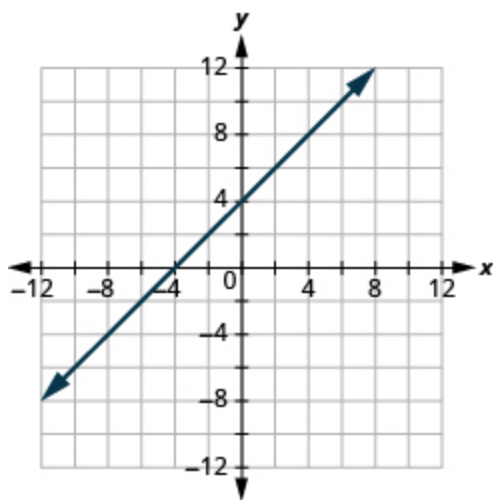
Exercise:

Problem: $x - y = -3$

Exercise:

Problem: $-x + y = 4$

Solution:



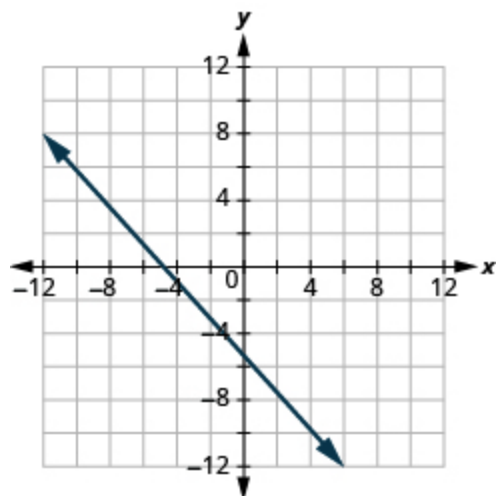
Exercise:

Problem: $-x + y = 3$

Exercise:

Problem: $-x - y = 5$

Solution:



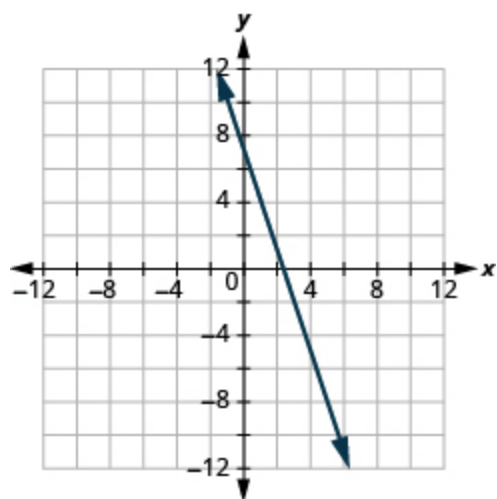
Exercise:

Problem: $-x - y = 1$

Exercise:

Problem: $3x + y = 7$

Solution:



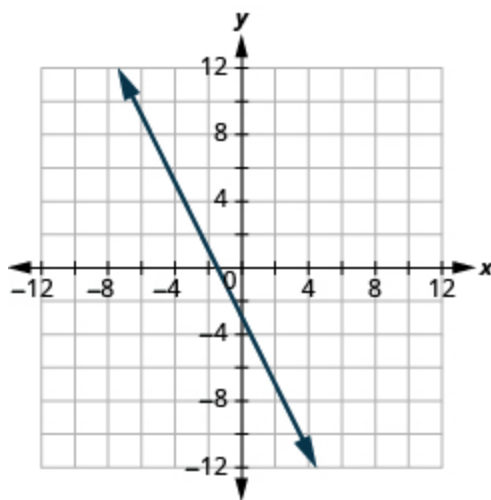
Exercise:

Problem: $5x + y = 6$

Exercise:

Problem: $2x + y = -3$

Solution:



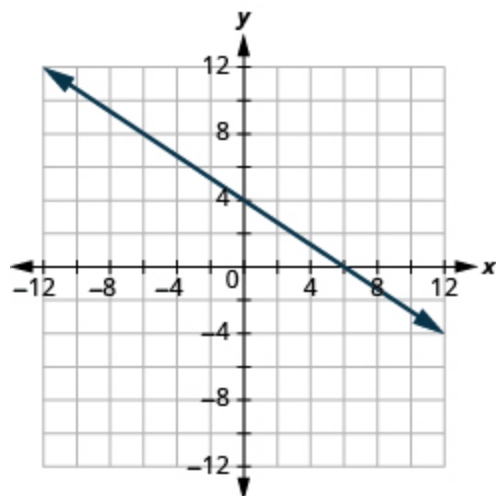
Exercise:

Problem: $4x + y = -5$

Exercise:

Problem: $2x + 3y = 12$

Solution:



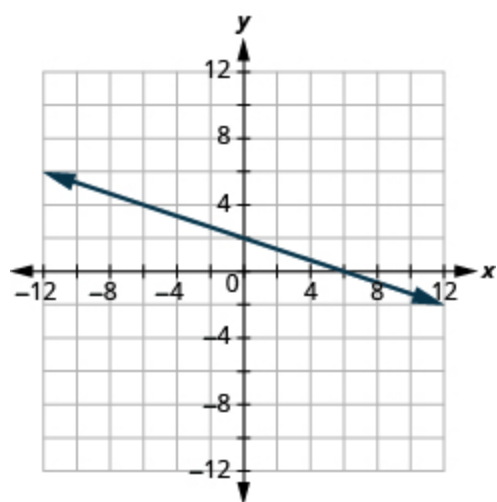
Exercise:

Problem: $3x - 4y = 12$

Exercise:

Problem: $\frac{1}{3}x + y = 2$

Solution:



Exercise:

Problem: $\frac{1}{2}x + y = 3$

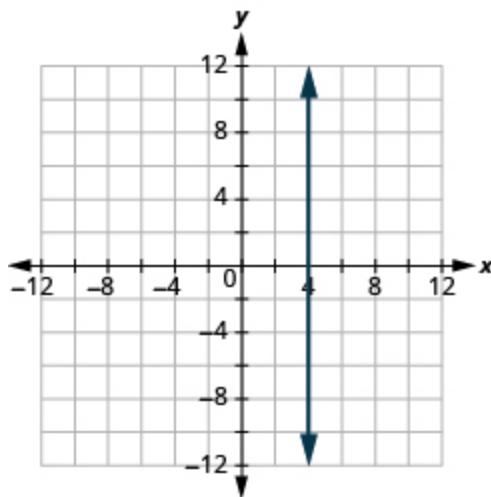
Graph Vertical and Horizontal lines

In the following exercises, graph the vertical and horizontal lines.

Exercise:

Problem: $x = 4$

Solution:



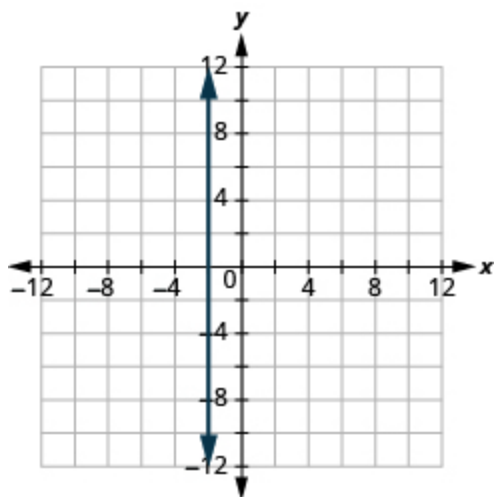
Exercise:

Problem: $x = 3$

Exercise:

Problem: $x = -2$

Solution:



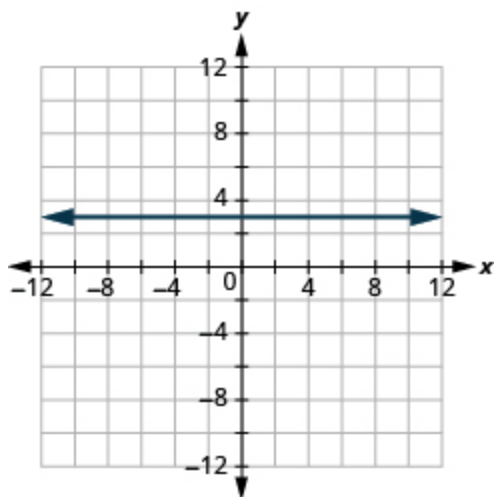
Exercise:

Problem: $x = -5$

Exercise:

Problem: $y = 3$

Solution:



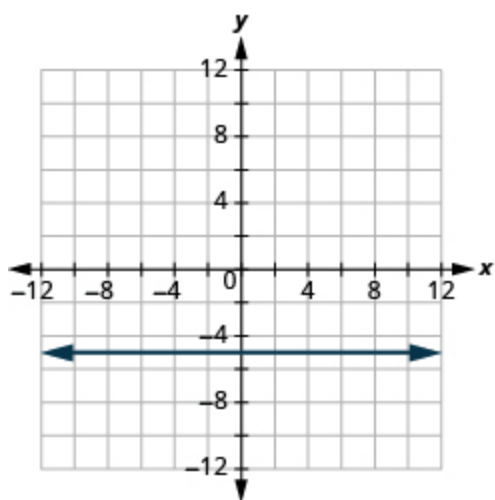
Exercise:

Problem: $y = 1$

Exercise:

Problem: $y = -5$

Solution:



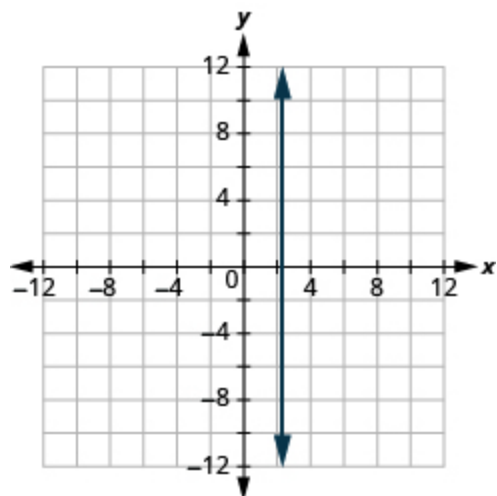
Exercise:

Problem: $y = -2$

Exercise:

Problem: $x = \frac{7}{3}$

Solution:



Exercise:

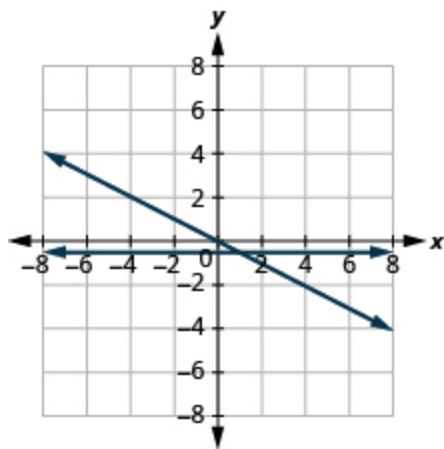
Problem: $x = \frac{5}{4}$

In the following exercises, graph each pair of equations in the same rectangular coordinate system.

Exercise:

Problem: $y = -\frac{1}{2}x$ and $y = -\frac{1}{2}$

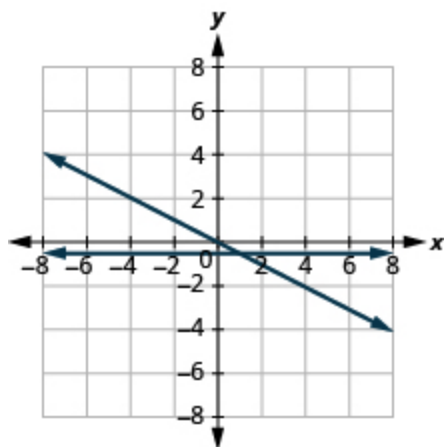
Solution:



Exercise:

Problem: $y = -\frac{1}{3}x$ and $y = -\frac{1}{3}$

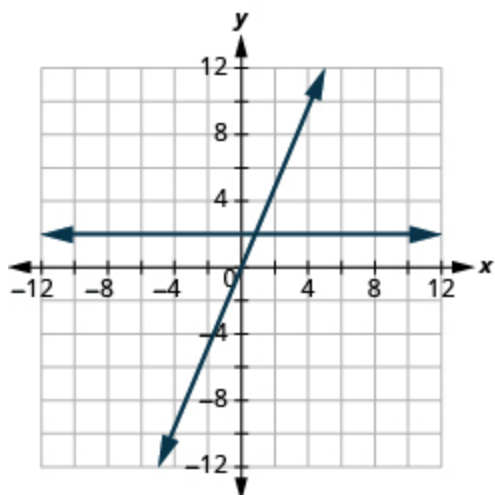
Solution:



Exercise:

Problem: $y = 2x$ and $y = 2$

Solution:



Exercise:

Problem: $y = 5x$ and $y = 5$

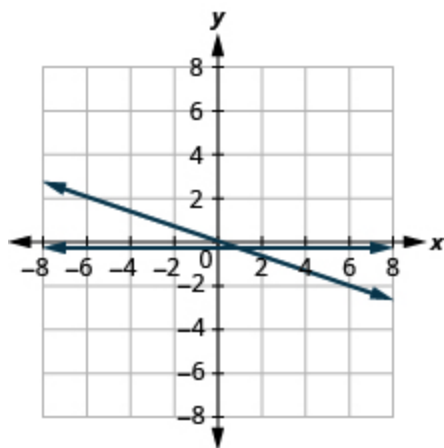
Mixed Practice

In the following exercises, graph each equation.

Exercise:

Problem: $y = 4x$

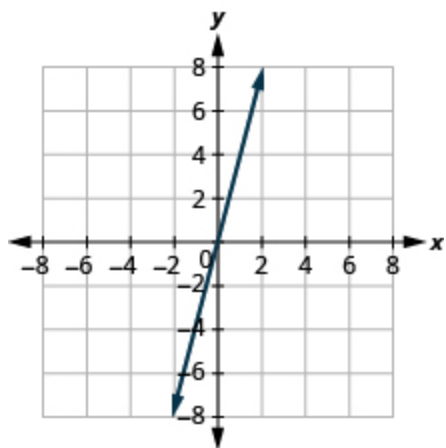
Solution:



Exercise:

Problem: $y = 2x$

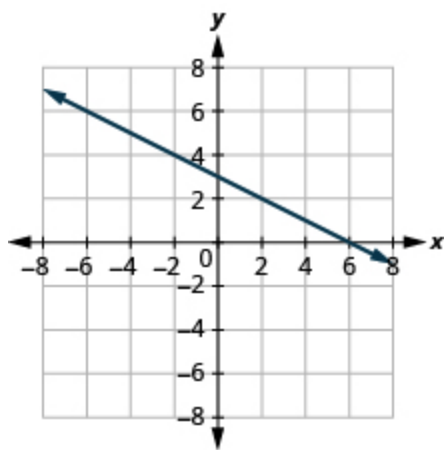
Solution:



Exercise:

Problem: $y = -\frac{1}{2}x + 3$

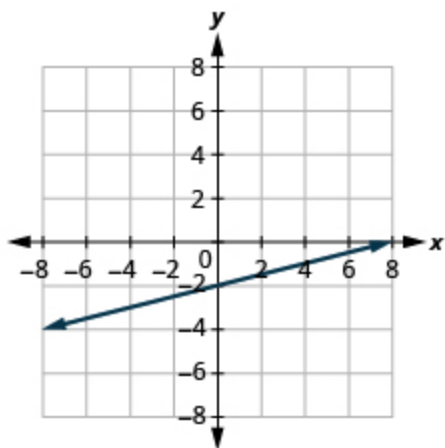
Solution:



Exercise:

Problem: $y = \frac{1}{4}x - 2$

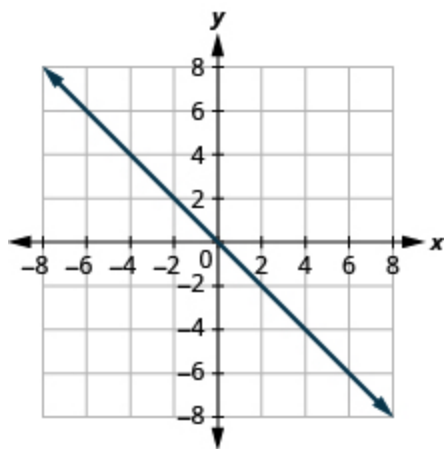
Solution:



Exercise:

Problem: $y = -x$

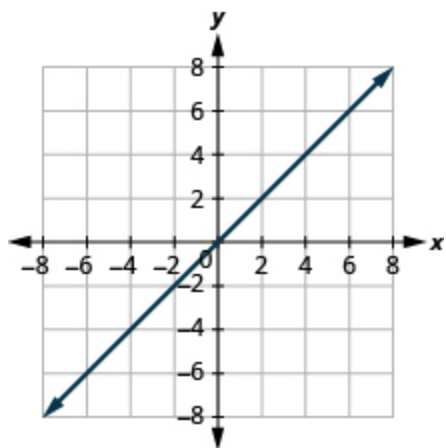
Solution:



Exercise:

Problem: $y = x$

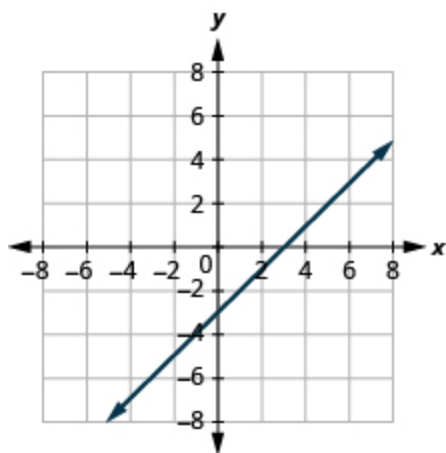
Solution:



Exercise:

Problem: $x - y = 3$

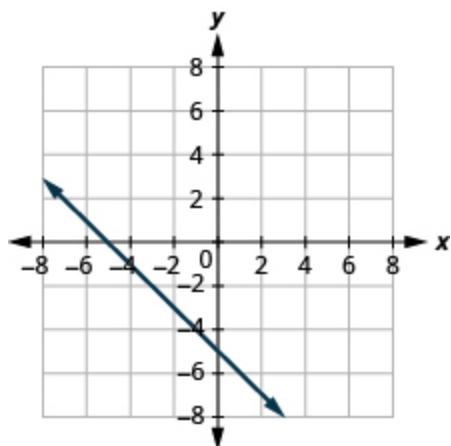
Solution:



Exercise:

Problem: $x + y = -5$

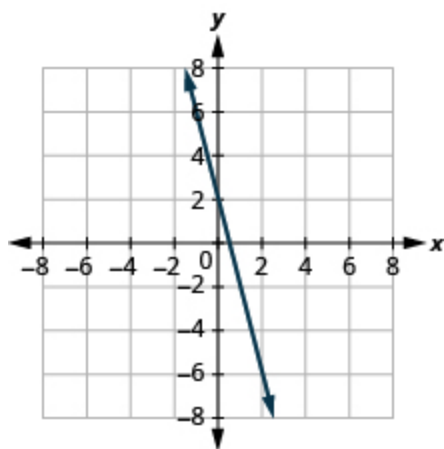
Solution:



Exercise:

Problem: $4x + y = 2$

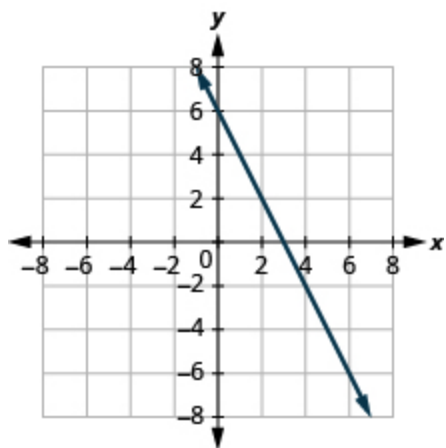
Solution:



Exercise:

Problem: $2x + y = 6$

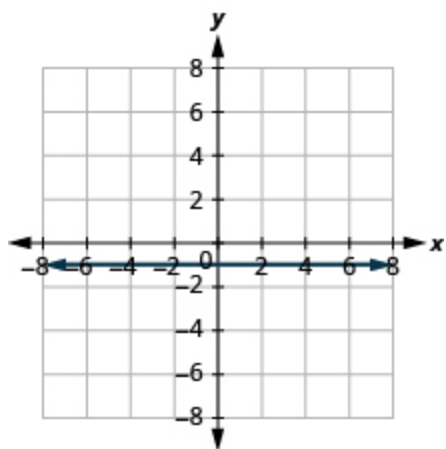
Solution:



Exercise:

Problem: $y = -1$

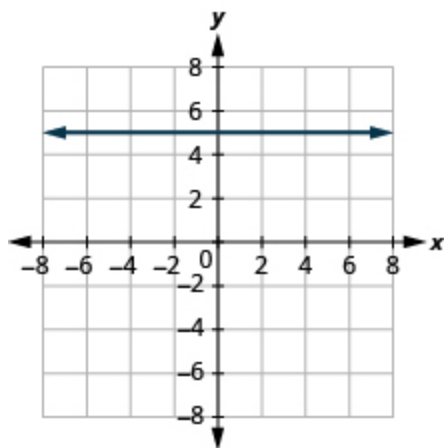
Solution:



Exercise:

Problem: $y = 5$

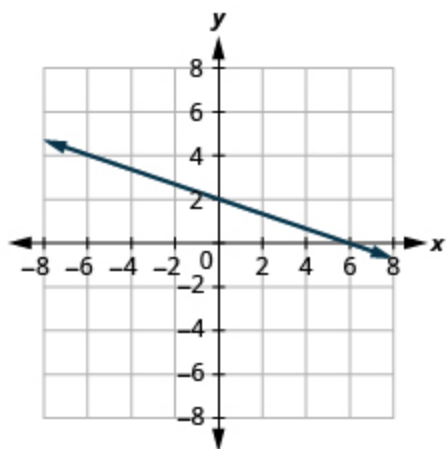
Solution:



Exercise:

Problem: $2x + 6y = 12$

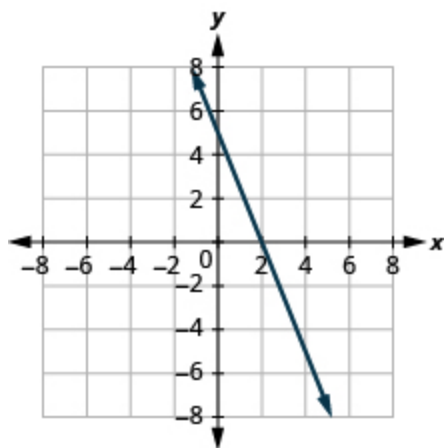
Solution:



Exercise:

Problem: $5x + 2y = 10$

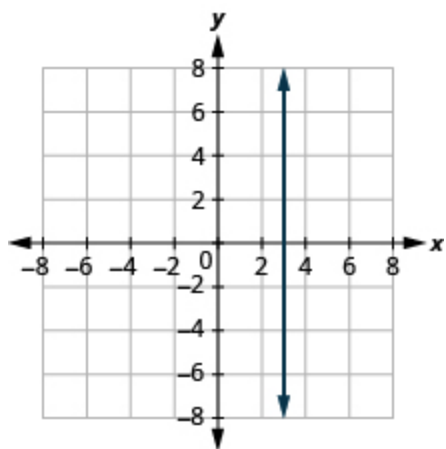
Solution:



Exercise:

Problem: $x = 3$

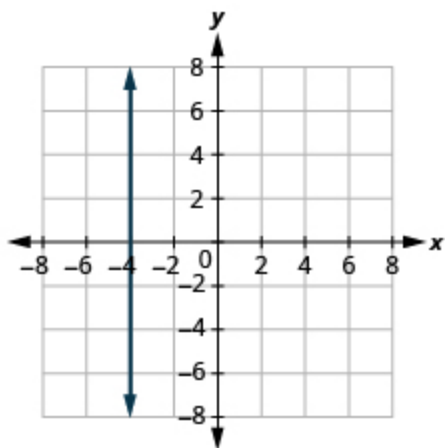
Solution:



Exercise:

Problem: $x = -4$

Solution:



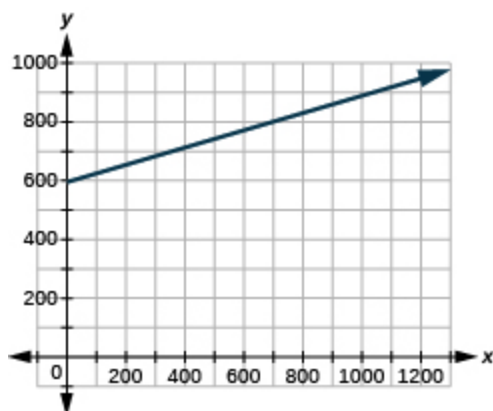
Everyday Math

Exercise:

Problem:

Motor home cost The Robinsons rented a motor home for one week to go on vacation. It cost them \$594 plus \$0.32 per mile to rent the motor home, so the linear equation $y = 594 + 0.32x$ gives the cost, y , for driving x miles. Calculate the rental cost for driving 400, 800, and 1,200 miles, and then graph the line.

Solution:



\$722, \$850, \$978

Exercise:

Problem:

Weekly earning At the art gallery where he works, Salvador gets paid \$200 per week plus 15% of the sales he makes, so the equation $y = 200 + 0.15x$ gives the amount y he earns for selling x dollars of artwork. Calculate the amount Salvador earns for selling \$900, \$1,600, and \$2,000, and then graph the line.

Writing Exercises

Exercise:

Problem:

Explain how you would choose three x -values to make a table to graph the line $y = \frac{1}{5}x - 2$.

Solution:

Answers will vary.

Exercise:

Problem:

What is the difference between the equations of a vertical and a horizontal line?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
graph a linear equation by plotting points.			
graph vertical and horizontal lines.			

Ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

horizontal line

A horizontal line is the graph of an equation that can be written in the form $y = b$. The line passes through the y -axis at $(0, b)$.

vertical line

A vertical line is the graph of an equation that can be written in the form $x = a$. The line passes through the x -axis at $(a, 0)$.

Graphing with Intercepts

By the end of this section, you will be able to:

- Identify the intercepts on a graph
- Find the intercepts from an equation of a line
- Graph a line using the intercepts
- Choose the most convenient method to graph a line

Note:

Before you get started, take this readiness quiz.

1. Solve: $3x + 4y = -12$ for x when $y = 0$.
If you missed this problem, review [\[link\]](#).
2. Is the point $(0, -5)$ on the x -axis or y -axis?
If you missed this problem, review [\[link\]](#).
3. Which ordered pairs are solutions to the equation $2x - y = 6$?
Ⓐ $(6, 0)$ Ⓑ $(0, -6)$ Ⓒ $(4, -2)$.
If you missed this problem, review [\[link\]](#).

Identify the Intercepts on a Graph

Every linear equation has a unique line that represents all the solutions of the equation. When graphing a line by plotting points, each person who graphs the line can choose any three points, so two people graphing the line might use different sets of points.

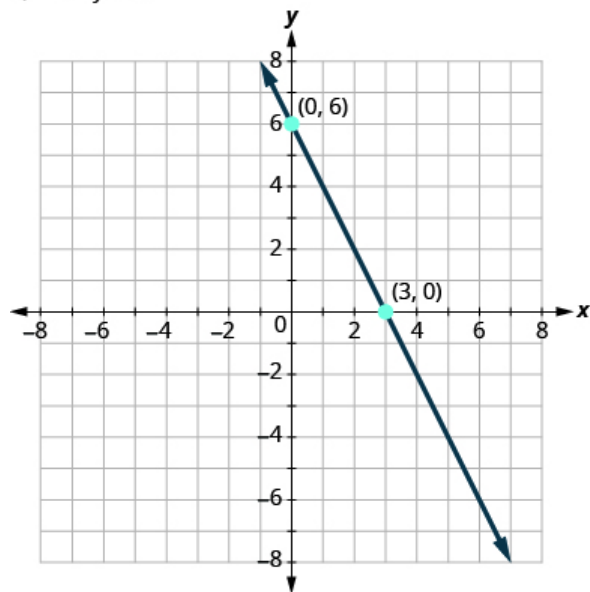
At first glance, their two lines might appear different since they would have different points labeled. But if all the work was done correctly, the lines will be exactly the same line. One way to recognize that they are indeed the same line is to focus on where the line crosses the axes. Each of these points is called an **intercept of the line**.

Note:**Intercepts of a Line**

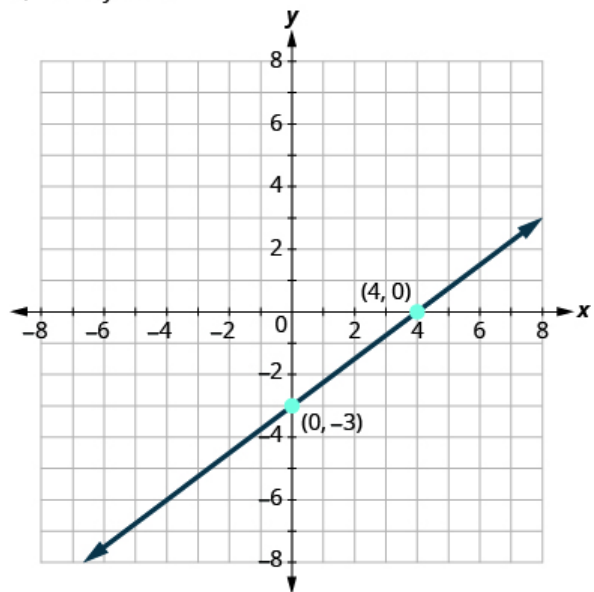
Each of the points at which a line crosses the x -axis and the y -axis is called an intercept of the line.

Let's look at the graph of the lines shown in [\[link\]](#).

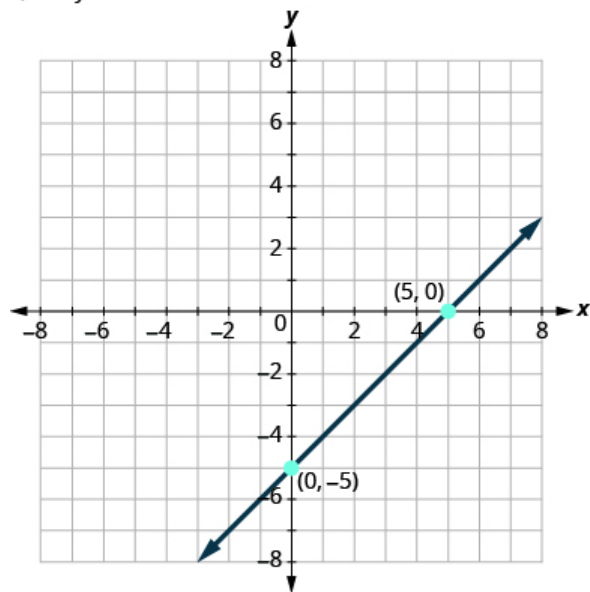
a) $2x + y = 6$



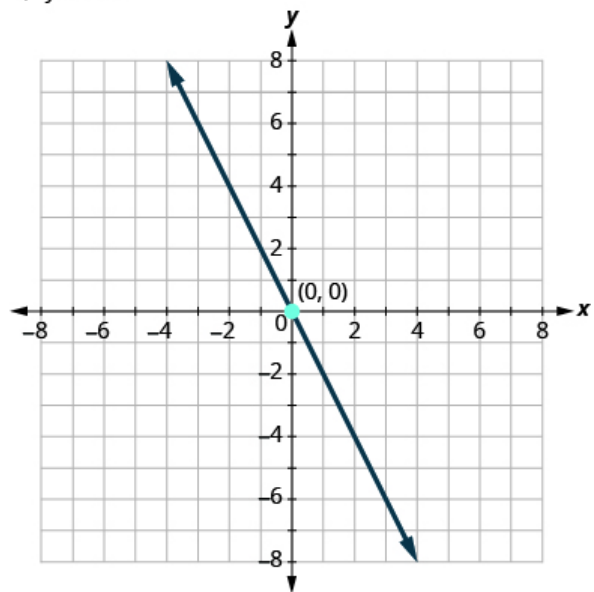
b) $3x - 4y = 12$



c) $x - y = 5$



d) $y = -2x$



First, notice where each of these lines crosses the x - axis:

Figure:	The line crosses the x-axis at:	Ordered pair of this point
42	3	(3,0)
43	4	(4,0)
44	5	(5,0)
45	0	(0,0)

Do you see a pattern?

For each row, the y -coordinate of the point where the line crosses the x -axis is zero. The point where the line crosses the x -axis has the form $(a, 0)$; and is called the *x-intercept* of the line. The x -intercept occurs when y is zero.

Now, let's look at the points where these lines cross the y -axis.

Figure:	The line crosses the y-axis at:	Ordered pair for this point
42	6	(0,6)
43	-3	(0,-3)
44	-5	(0,-5)
45	0	(0,0)

Note:

x - intercept and y - intercept of a line

The x -intercept is the point, $(a, 0)$, where the graph crosses the x -axis.

The x -intercept occurs when y is zero.

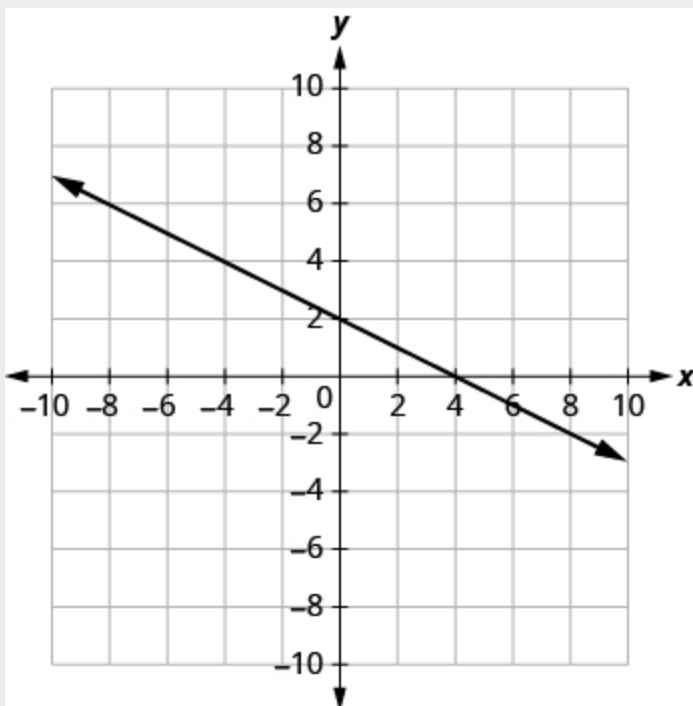
The y -intercept is the point, $(0, b)$, where the graph crosses the y -axis.

The y -intercept occurs when x is zero.

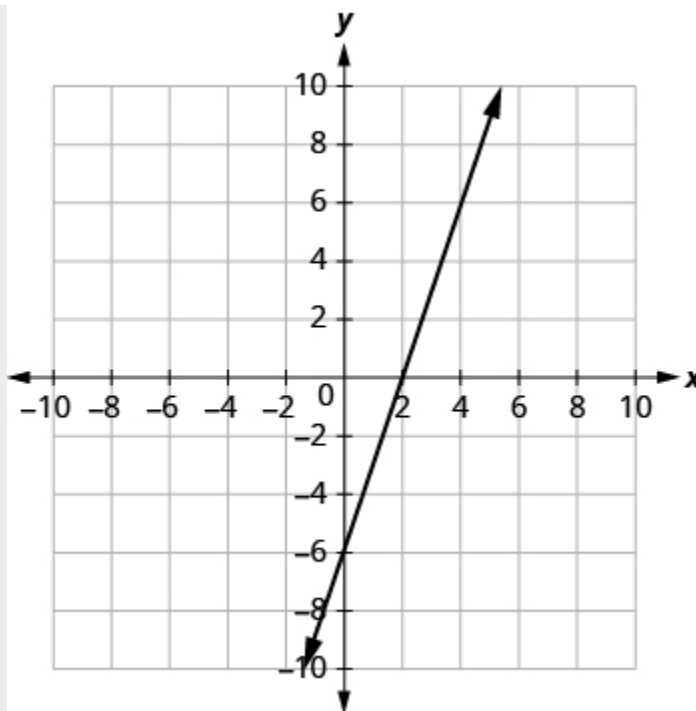
Example:**Exercise:**

Problem: Find the x - and y -intercepts of each line:

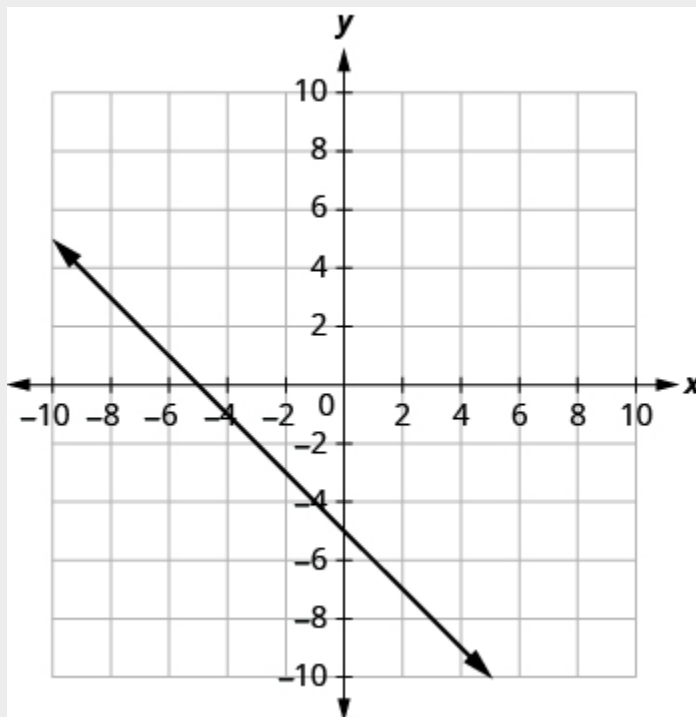
Ⓐ $x + 2y = 4$



ⓑ $3x - y = 6$



ⓒ $x + y = -5$



Solution:
Solution

Ⓐ	
The graph crosses the x -axis at the point $(4, 0)$.	The x -intercept is $(4, 0)$.
The graph crosses the y -axis at the point $(0, 2)$.	The x -intercept is $(0, 2)$.

Ⓑ	
The graph crosses the x -axis at the point $(2, 0)$.	The x -intercept is $(2, 0)$
The graph crosses the y -axis at the point $(0, -6)$.	The y -intercept is $(0, -6)$.

Ⓒ	

The graph crosses the x -axis at the point $(-5, 0)$.

The x -intercept is $(-5, 0)$.

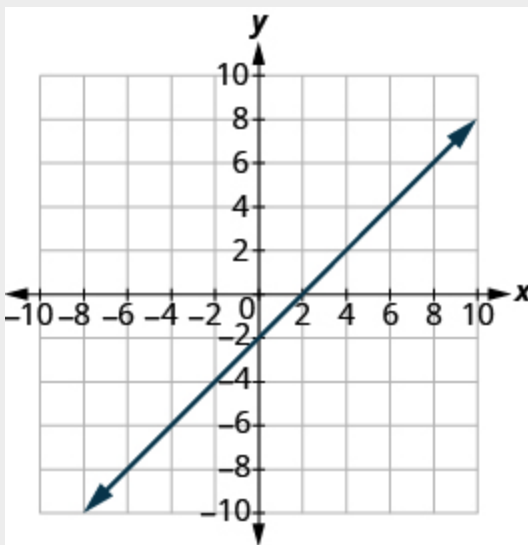
The graph crosses the y -axis at the point $(0, -5)$.

The y -intercept is $(0, -5)$.

Note:

Exercise:

Problem: Find the x - and y -intercepts of the graph: $x - y = 2$.



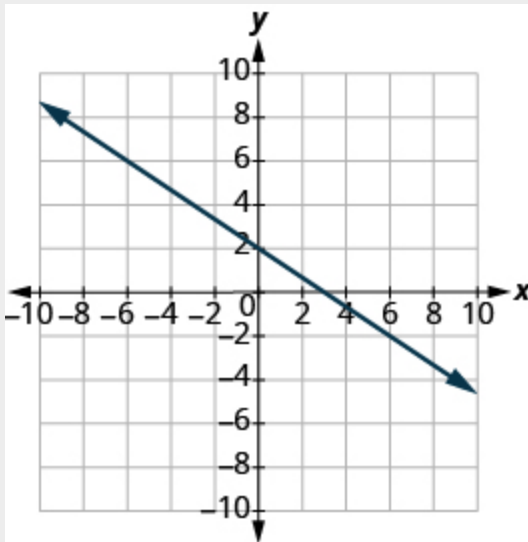
Solution:

x -intercept $(2, 0)$; y -intercept $(0, -2)$

Note:

Exercise:

Problem: Find the x - and y -intercepts of the graph: $2x + 3y = 6$.



Solution:

x -intercept $(3,0)$; y -intercept $(0,2)$

Find the Intercepts from an Equation of a Line

Recognizing that the x -intercept occurs when y is zero and that the y -intercept occurs when x is zero gives us a method to find the intercepts of a line from its equation. To find the x -intercept, let $y = 0$ and solve for x . To find the y -intercept, let $x = 0$ and solve for y .

Note:

Find the x and y from the Equation of a Line

Use the equation to find:

- the x -intercept of the line, let $y = 0$ and solve for x .
- the y -intercept of the line, let $x = 0$ and solve for y .

x	y
	0
0	

Example:

Exercise:

Problem: Find the intercepts of $2x + y = 6$

Solution:

We'll fill in [\[link\]](#).

$2x + y = 6$	
x	y
	0
0	

x-intercept

y-intercept

To find the x- intercept, let $y = 0$:

	$2x + y = 6$
Substitute 0 for y.	$2x + 0 = 6$

Add.	$2x = 6$
Divide by 2.	$x = 3$
The x -intercept is (3, 0).	

To find the y - intercept, let $x = 0$:

	$2x + y = 6$
Substitute 0 for x .	$2 \cdot 0 + y = 6$
Multiply.	$0 + y = 6$
Add.	$y = 6$
The y -intercept is (0, 6).	

$2x + y = 6$	
x	y
3	0
0	6

The intercepts are the points $(3, 0)$ and $(0, 6)$.

Note:

Exercise:

Problem: Find the intercepts: $3x + y = 12$

Solution:

$(4, 0)$ and $(0, 12)$

Note:

Exercise:

Problem: Find the intercepts: $x + 4y = 8$

Solution:

$(8, 0)$ and $(0, 2)$

Example:

Exercise:

Problem: Find the intercepts of $4x - 3y = 12$.

Solution:
Solution

To find the x -intercept, let $y = 0$.

	$4x - 3y = 12$
Substitute 0 for y .	$4x - 3 \cdot 0 = 12$
Multiply.	$4x - 0 = 12$
Subtract.	$4x = 12$
Divide by 4.	$x = 3$

The x -intercept is $(3, 0)$.

To find the y -intercept, let $x = 0$.

	$4x - 3y = 12$
Substitute 0 for x .	$4 \cdot 0 - 3y = 12$
Multiply.	$0 - 3y = 12$

Simplify.

$$-3y = 12$$

Divide by -3 .

$$y = -4$$

The y -intercept is $(0, -4)$.

The intercepts are the points $(-3, 0)$ and $(0, -4)$.

$$4x - 3y = 12$$

x

y

3

0

0

-4

Note:

Exercise:

Problem: Find the intercepts of the line: $3x - 4y = 12$.

Solution:

x-intercept $(4, 0)$; y-intercept: $(0, -3)$

Note:

Exercise:

Problem: Find the intercepts of the line: $2x - 4y = 8$.

Solution:

x -intercept $(4,0)$; y -intercept: $(0,-2)$

Graph a Line Using the Intercepts

To graph a linear equation by plotting points, you can use the intercepts as two of your three points. Find the two intercepts, and then a third point to ensure accuracy, and draw the line. This method is often the quickest way to graph a line.

Example:

Exercise:

Problem: Graph $-x + 2y = 6$ using intercepts.

Solution:

Solution

First, find the x -intercept. Let $y = 0$,

$$-x + 2y = 6$$

$$-x + 2(0) = 6$$

$$-x = 6$$

$$x = -6$$

The x -intercept is $(-6, 0)$.

Now find the y -intercept. Let $x = 0$.

$$-x + 2y = 6$$

$$-0 + 2y = 6$$

$$2y = 6$$

$$y = 3$$

The y -intercept is $(0, 3)$.

Find a third point. We'll use $x = 2$,

$$-x + 2y = 6$$

$$-2 + 2y = 6$$

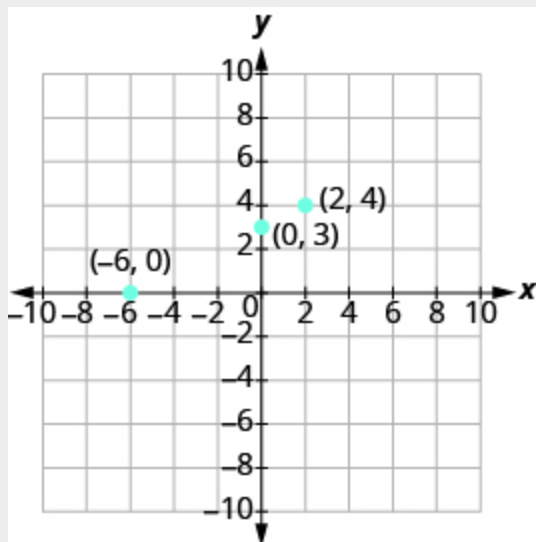
$$2y = 8$$

$$y = 4$$

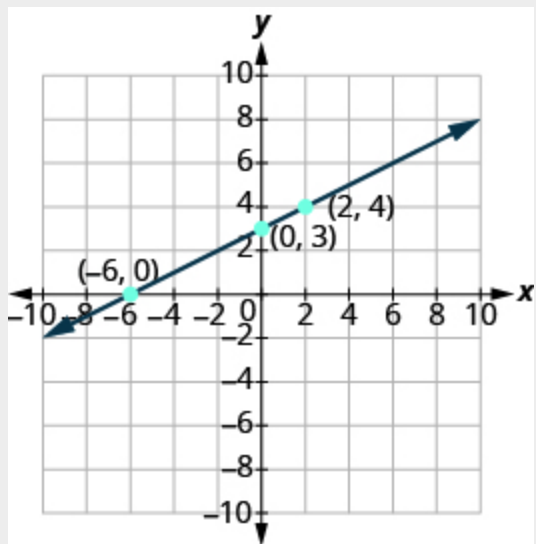
A third solution to the equation is $(2, 4)$.

Summarize the three points in a table and then plot them on a graph.

$-x + 2y = 6$		
x	y	(x,y)
-6	0	$(-6, 0)$
0	3	$(0, 3)$
2	4	$(2, 4)$



Do the points line up? Yes, so draw line through the points.

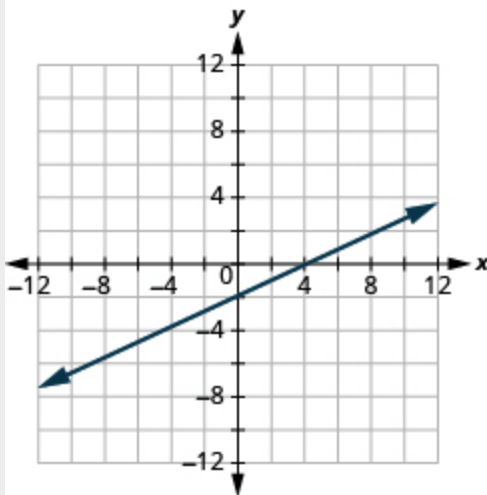


Note:

Exercise:

Problem: Graph the line using the intercepts: $x - 2y = 4$.

Solution:

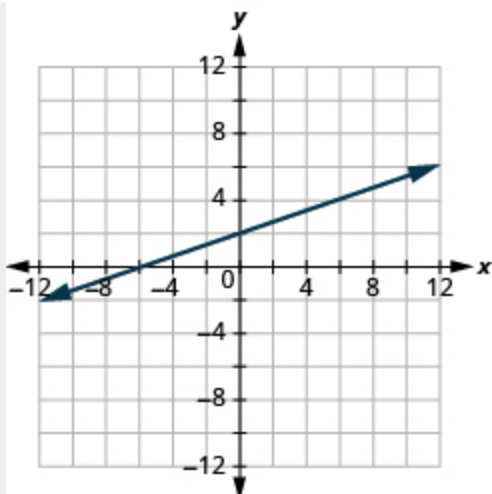


Note:

Exercise:

Problem: Graph the line using the intercepts: $-x + 3y = 6$.

Solution:

**Note:**

Graph a line using the intercepts.

Find the x - and y -intercepts of the line.

- Let $y = 0$ and solve for x
- Let $x = 0$ and solve for y .

Find a third solution to the equation.

Plot the three points and then check that they line up.

Draw the line.

Example:**Exercise:**

Problem: Graph $4x - 3y = 12$ using intercepts.

Solution:**Solution**

Find the intercepts and a third point.

x-intercept, let $y = 0$

$$\begin{aligned}4x - 3y &= 12 \\4x - 3(0) &= 12 \\4x &= 12 \\x &= 3\end{aligned}$$

y-intercept, let $x = 0$

$$\begin{aligned}4x - 3y &= 12 \\4(0) - 3y &= 12 \\-3y &= 12 \\y &= -4\end{aligned}$$

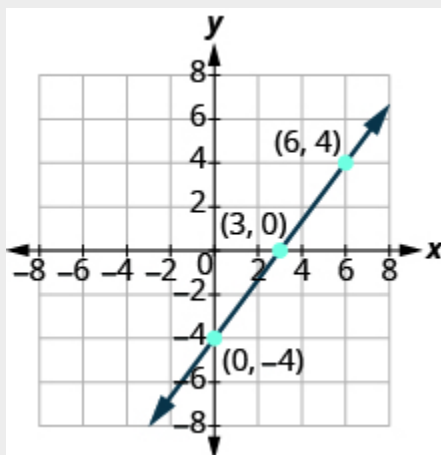
third point, let $y = 4$

$$\begin{aligned}4x - 3y &= 12 \\4x - 3(4) &= 12 \\4x - 12 &= 12 \\4x &= 24 \\x &= 6\end{aligned}$$

We list the points and show the graph.

$$4x - 3y = 12$$

x	y	(x, y)
3	0	$(3, 0)$
0	-4	$(0, -4)$
6	4	$(6, 4)$

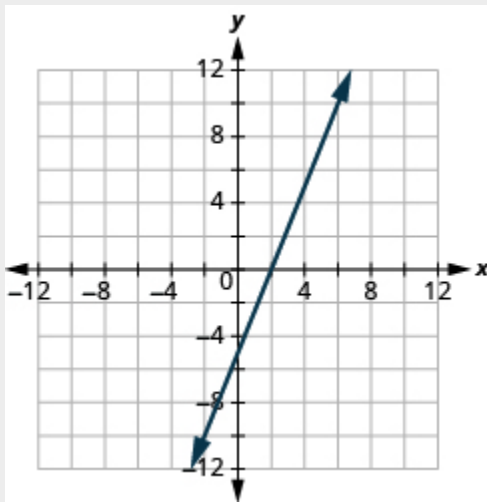


Note:

Exercise:

Problem: Graph the line using the intercepts: $5x - 2y = 10$.

Solution:

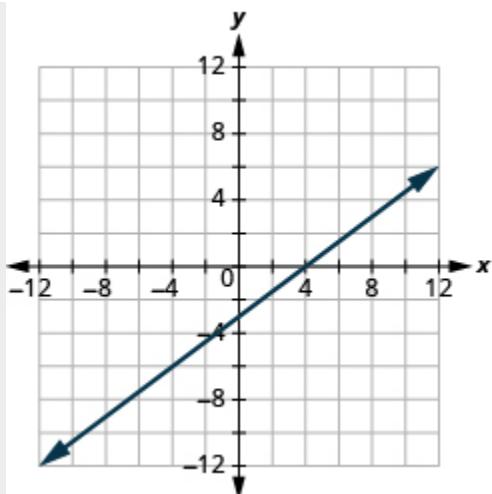


Note:

Exercise:

Problem: Graph the line using the intercepts: $3x - 4y = 12$.

Solution:



Example:

Exercise:

Problem: Graph $y = 5x$ using the intercepts.

Solution:

Solution

x-intercept; Let $y = 0$.

$$y = 5x$$

$$0 = 5x$$

$$0 = x$$

$$x = 0$$

The x-intercept is $(0, 0)$.

y-intercept; Let $x = 0$.

$$y = 5x$$

$$y = 5(0)$$

$$y = 0$$

The y-intercept is $(0, 0)$.

This line has only one intercept! It is the point $(0, 0)$.

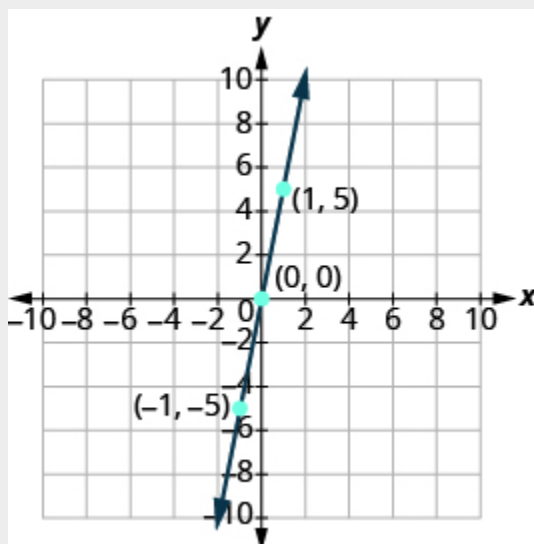
To ensure accuracy, we need to plot three points. Since the intercepts are the same point, we need two more points to graph the line. As always, we can choose any values for x , so we'll let x be 1 and -1 .

$x = 1$	$x = -1$
$y = 5x$	$y = 5x$
$y = 5(1)$	$y = 5(-1)$
$y = 5$	$y = -5$
$(1, 5)$	$(-1, -5)$

Organize the points in a table.

$y = 5x$		
x	y	(x, y)
0	0	$(0, 0)$
1	5	$(1, 5)$
-1	-5	$(-1, -5)$

Plot the three points, check that they line up, and draw the line.

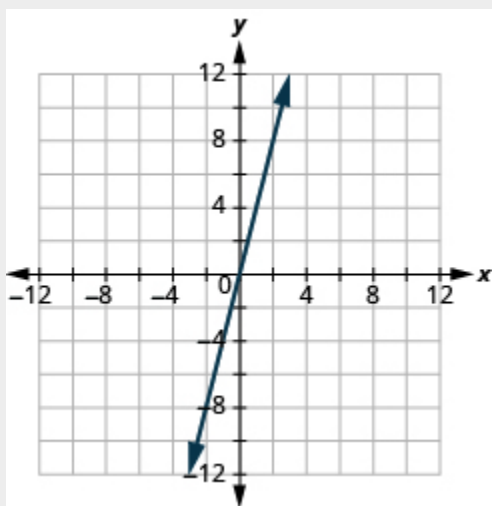


Note:

Exercise:

Problem: Graph using the intercepts: $y = 3x$.

Solution:

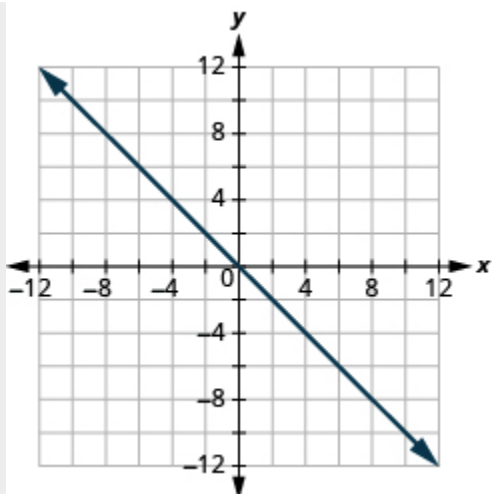


Note:

Exercise:

Problem: Graph using the intercepts: $y = -x$.

Solution:



Choose the Most Convenient Method to Graph a Line

While we could graph any linear equation by plotting points, it may not always be the most convenient method. This table shows six of equations we've graphed in this chapter, and the methods we used to graph them.

	Equation	Method
#1	$y = 2x + 1$	Plotting points
#2	$y = \frac{1}{2}x + 3$	Plotting points
#3	$x = -7$	Vertical line
#4	$y = 4$	Horizontal line
#5	$2x + y = 6$	Intercepts

	Equation	Method
#6	$4x - 3y = 12$	Intercepts

What is it about the form of equation that can help us choose the most convenient method to graph its line?

Notice that in equations #1 and #2, y is isolated on one side of the equation, and its coefficient is 1. We found points by substituting values for x on the right side of the equation and then simplifying to get the corresponding y -values.

Equations #3 and #4 each have just one variable. Remember, in this kind of equation the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #5 and #6, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ and $x = 0$ to find the x - and y - intercepts, and then found a third point by choosing a value for x or y .

This leads to the following strategy for choosing the most convenient method to graph a line.

Note:

Choose the most convenient method to graph a line.

If the equation has only one variable.

It is a vertical or horizontal line.

- $x = a$ is a vertical line passing through the x -axis at a
- $y = b$ is a horizontal line passing through the y -axis at b .

If y is isolated on one side of the

- Choose any three values for x and

equation. Graph by plotting points.

then solve for the corresponding y -values.

If the equation $Ax + By = C$, find the intercepts.

- Find the x - and y -intercepts and then a third point.

Example:

Exercise:

Problem: Identify the most convenient method to graph each line:

- Ⓐ $y = -3$
- Ⓑ $4x - 6y = 12$
- Ⓒ $x = 2$
- Ⓓ $y = \frac{2}{5}x - 1$

Solution:

Solution

Ⓐ $y = -3$

This equation has only one variable, y . Its graph is a horizontal line crossing the y -axis at -3 .

Ⓑ $4x - 6y = 12$

This equation is of the form $Ax + By = C$. Find the intercepts and one more point.

Ⓒ $x = 2$

There is only one variable, x . The graph is a vertical line crossing the x -axis at 2.

④ $y = \frac{2}{5}x - 1$

Since y is isolated on the left side of the equation, it will be easiest to graph this line by plotting three points.

Note:

Exercise:

Problem: Identify the most convenient method to graph each line:

① $3x + 2y = 12$

② $y = 4$

③ $y = \frac{1}{5}x - 4$

④ $x = -7$

Solution:

① intercepts

② horizontal line

③ plotting points

④ vertical line

Note:

Exercise:

Problem: Identify the most convenient method to graph each line:

- Ⓐ $x = 6$
- Ⓑ $y = -\frac{3}{4}x + 1$
- Ⓒ $y = -8$
- Ⓓ $4x - 3y = -1$

Solution:

- Ⓐ vertical line
- Ⓑ plotting points
- Ⓒ horizontal line
- Ⓓ intercepts

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Graph by Finding Intercepts](#)
- [Use Intercepts to Graph](#)
- [State the Intercepts from a Graph](#)

Key Concepts

- **Intercepts**
 - The x -intercept is the point, $(a, 0)$, where the graph crosses the x -axis. The x -intercept occurs when y is zero.
 - The y -intercept is the point, $(0, b)$, where the graph crosses the y -axis. The y -intercept occurs when x is zero.
 - The x -intercept occurs when y is zero.
 - The y -intercept occurs when x is zero.
- **Find the x and y intercepts from the equation of a line**

- To find the x -intercept of the line, let $y = 0$ and solve for x .
- To find the y -intercept of the line, let $x = 0$ and solve for y .

x	y
	0
0	

- **Graph a line using the intercepts**

Find the x - and y -intercepts of the line.

- Let $y = 0$ and solve for x .
- Let $x = 0$ and solve for y .

Find a third solution to the equation.

Plot the three points and then check that they line up.

Draw the line.

- **Choose the most convenient method to graph a line**

Determine if the equation has only one variable. Then it is a vertical or horizontal line.

$x = a$ is a vertical line passing through the x -axis at a .

$y = b$ is a vertical line passing through the y -axis at b .

Determine if x or y is isolated on one side of the equation. The graph is then a line with a positive or negative slope. Choose any three values for x and solve for the corresponding y -values. Plot the points and draw the line.

Determine if the equation is of the form $Ax + By = C$. Find the x - and y -intercepts and then a third point to graph the line.

form

point.

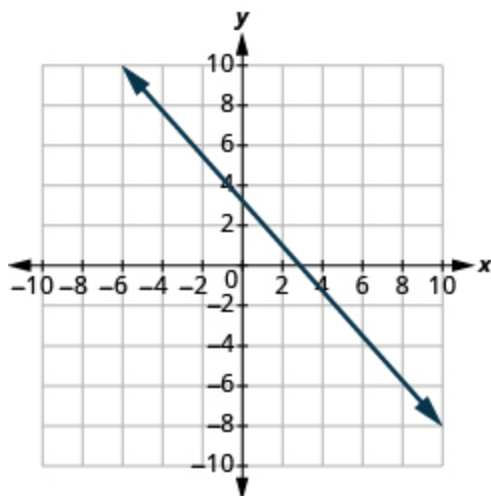
Practice Makes Perfect

Identify the Intercepts on a Graph

In the following exercises, find the x - and y - intercepts.

Exercise:

Problem:

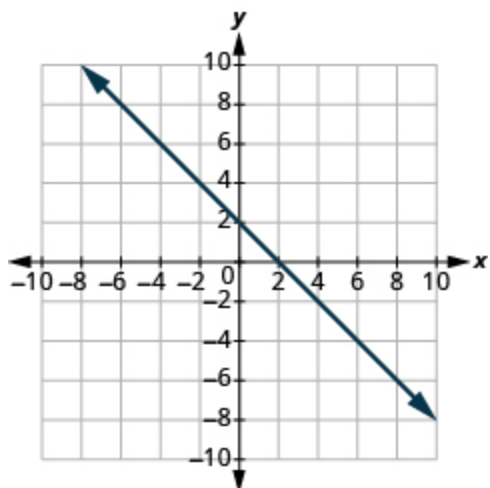


Solution:

$(3,0), (0,3)$

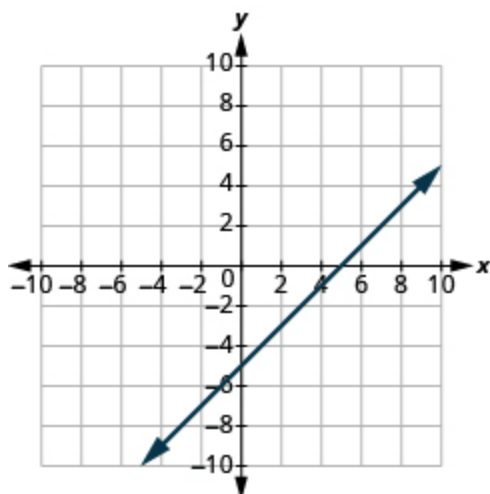
Exercise:

Problem:



Exercise:

Problem:

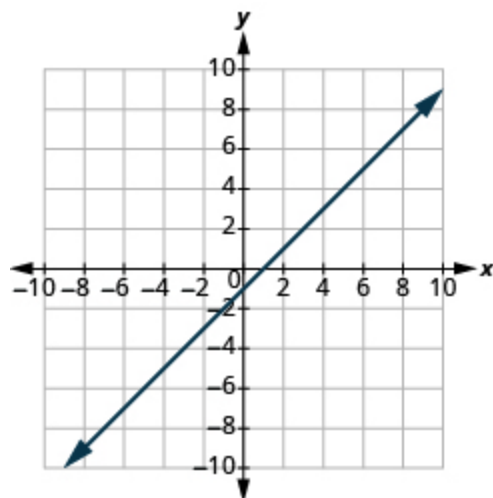


Solution:

$(5, 0), (0, -5)$

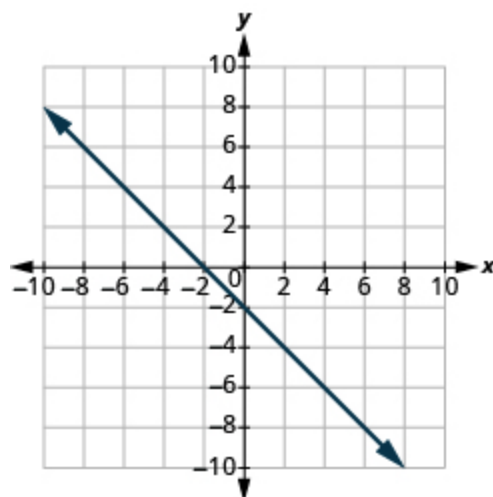
Exercise:

Problem:



Exercise:

Problem:

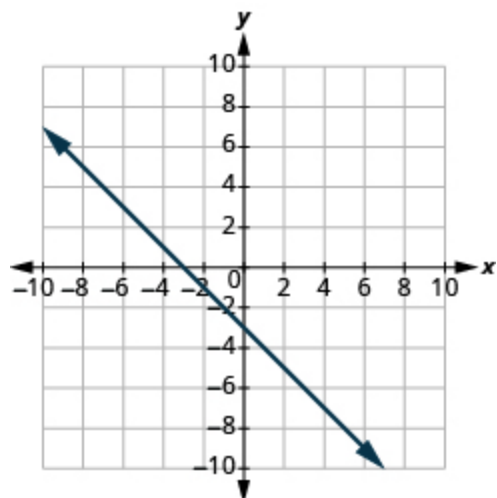


Solution:

$(-2, 0), (0, -2)$

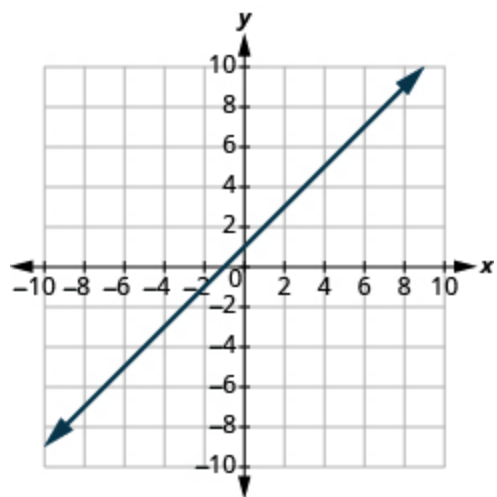
Exercise:

Problem:



Exercise:

Problem:

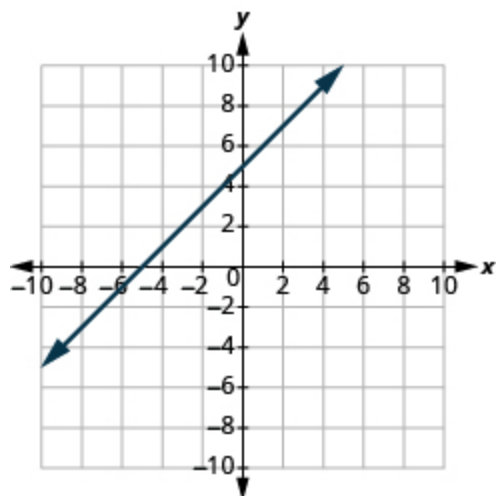


Solution:

$(-1,0),(0,1)$

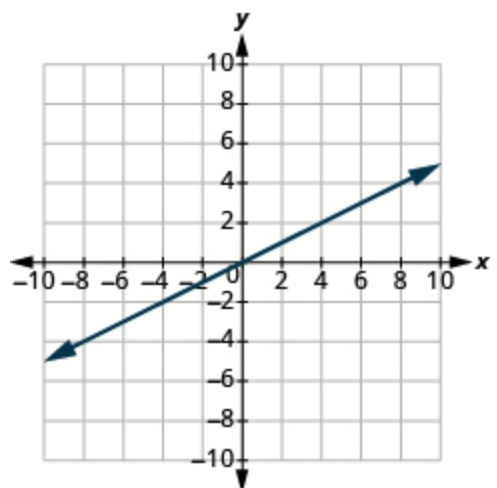
Exercise:

Problem:



Exercise:

Problem:

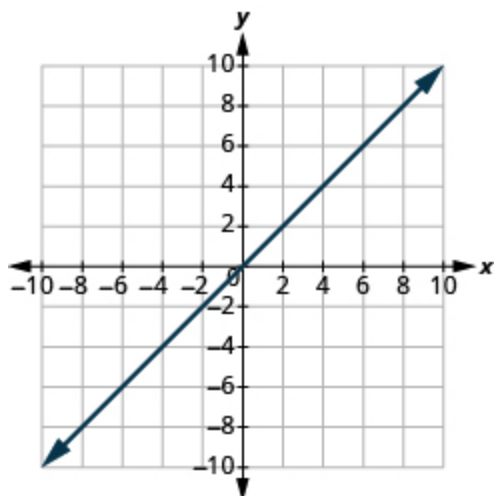


Solution:

$(0,0)$

Exercise:

Problem:



Find the x and y Intercepts from an Equation of a Line

In the following exercises, find the intercepts.

Exercise:

Problem: $x + y = 4$

Solution:

$(4,0), (0,4)$

Exercise:

Problem: $x + y = 3$

Exercise:

Problem: $x + y = -2$

Solution:

$(-2,0), (0,-2)$

Exercise:

Problem: $x + y = -5$

Exercise:

Problem: $x - y = 5$

Solution:

$(5,0), (0,-5)$

Exercise:

Problem: $x - y = 1$

Exercise:

Problem: $x - y = -3$

Solution:

$(-3,0), (0,3)$

Exercise:

Problem: $x - y = -4$

Exercise:

Problem: $x + 2y = 8$

Solution:

$(8,0), (0,4)$

Exercise:

Problem: $x + 2y = 10$

Exercise:

Problem: $3x + y = 6$

Solution:

$(2,0),(0,6)$

Exercise:

Problem: $3x + y = 9$

Exercise:

Problem: $x - 3y = 12$

Solution:

$(12,0),(0,-4)$

Exercise:

Problem: $x - 2y = 8$

Exercise:

Problem: $4x - y = 8$

Solution:

$(2,0),(0,-8)$

Exercise:

Problem: $5x - y = 5$

Exercise:

Problem: $2x + 5y = 10$

Solution:

$(5,0), (0,2)$

Exercise:

Problem: $2x + 3y = 6$

Exercise:

Problem: $3x - 2y = 12$

Solution:

$(4,0), (0,-6)$

Exercise:

Problem: $3x - 5y = 30$

Exercise:

Problem: $y = \frac{1}{3}x - 1$

Solution:

$(3,0), (0,-1)$

Exercise:

Problem: $y = \frac{1}{4}x - 1$

Exercise:

Problem: $y = \frac{1}{5}x + 2$

Solution:

$(-10,0),(0,2)$

Exercise:

Problem: $y = \frac{1}{3}x + 4$

Exercise:

Problem: $y = 3x$

Solution:

$(0,0)$

Exercise:

Problem: $y = -2x$

Exercise:

Problem: $y = -4x$

Solution:

$(0,0)$

Exercise:

Problem: $y = 5x$

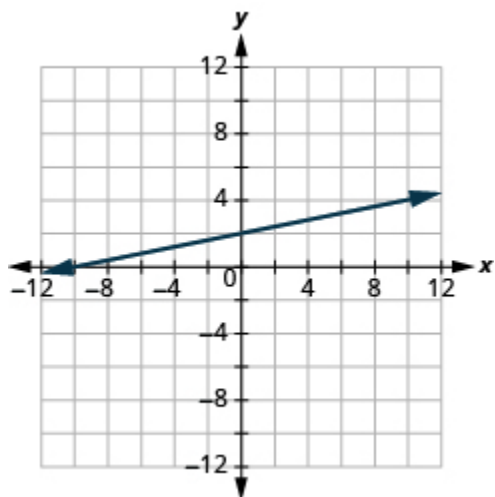
Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

Exercise:

Problem: $-x + 5y = 10$

Solution:



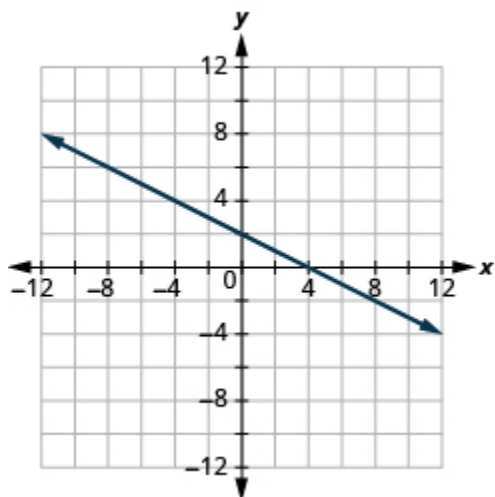
Exercise:

Problem: $-x + 4y = 8$

Exercise:

Problem: $x + 2y = 4$

Solution:



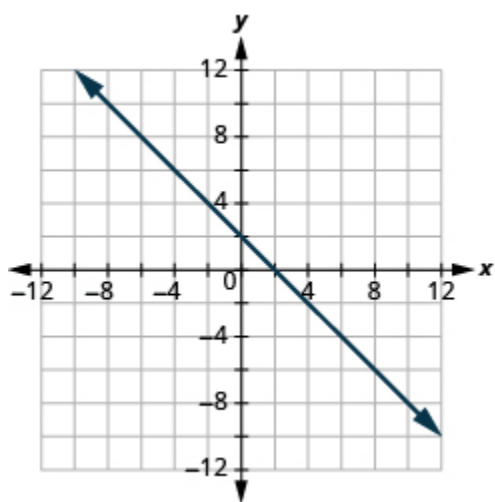
Exercise:

Problem: $x + 2y = 6$

Exercise:

Problem: $x + y = 2$

Solution:



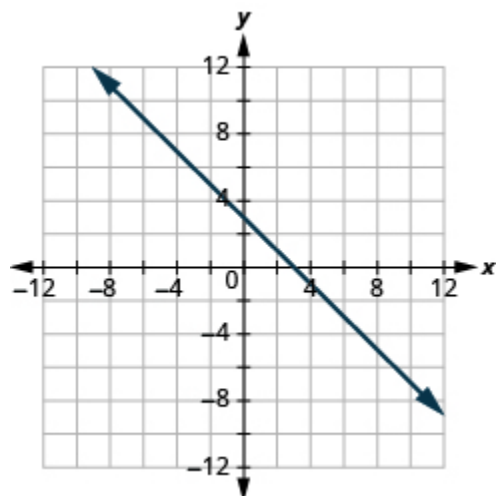
Exercise:

Problem: $x + y = 5$

Exercise:

Problem: $x + y = -3$

Solution:



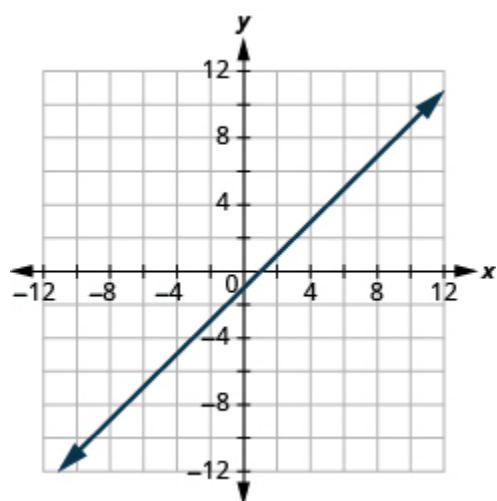
Exercise:

Problem: $x + y = -1$

Exercise:

Problem: $x - y = 1$

Solution:



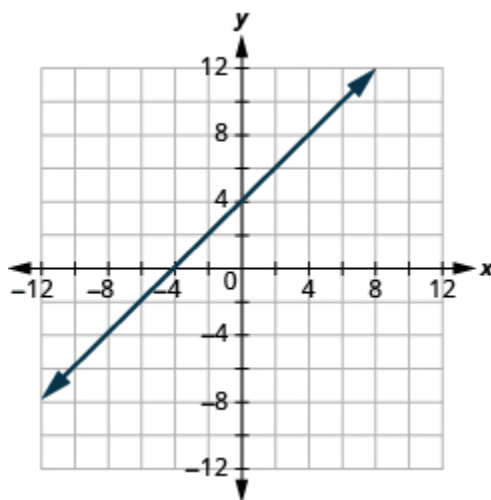
Exercise:

Problem: $x - y = 2$

Exercise:

Problem: $x - y = -4$

Solution:



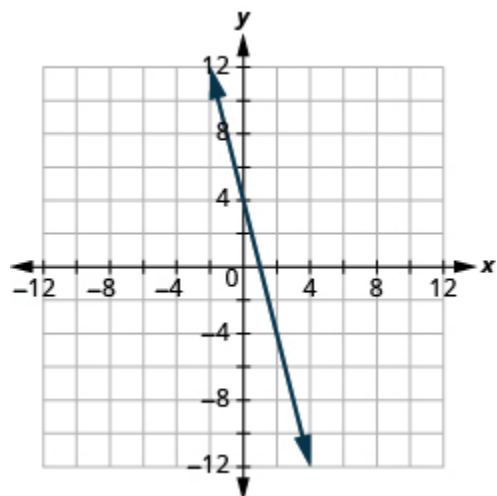
Exercise:

Problem: $x - y = -3$

Exercise:

Problem: $4x + y = 4$

Solution:



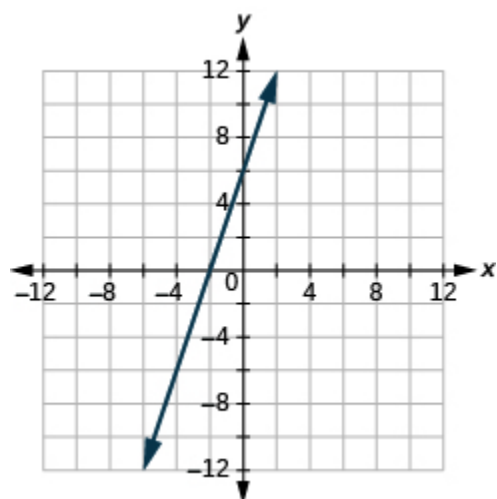
Exercise:

Problem: $3x + y = 3$

Exercise:

Problem: $3x - y = -6$

Solution:



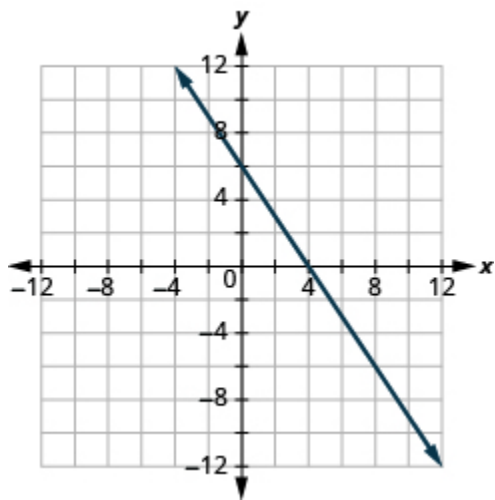
Exercise:

Problem: $2x - y = -8$

Exercise:

Problem: $2x + 4y = 12$

Solution:



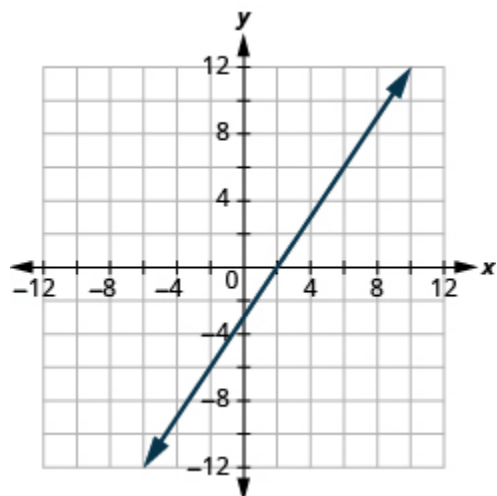
Exercise:

Problem: $3x + 2y = 12$

Exercise:

Problem: $3x - 2y = 6$

Solution:



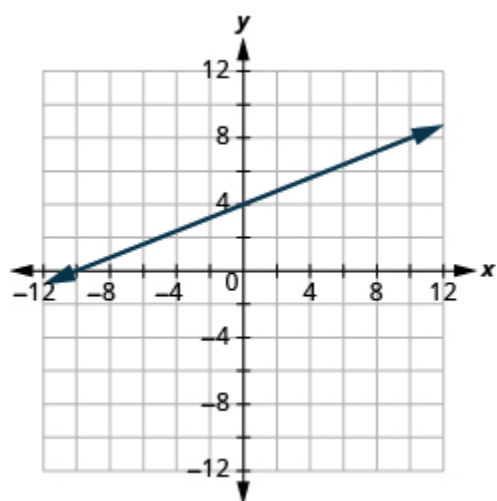
Exercise:

Problem: $5x - 2y = 10$

Exercise:

Problem: $2x - 5y = -20$

Solution:



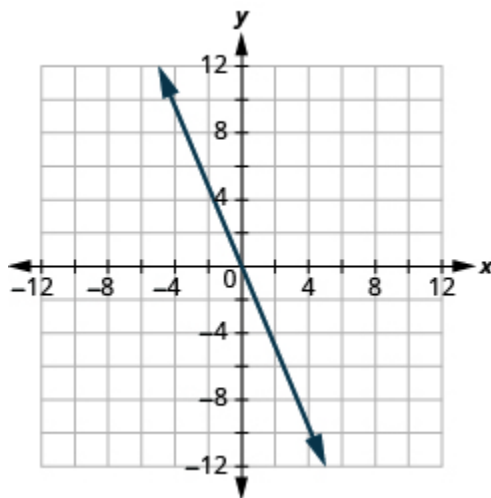
Exercise:

Problem: $3x - 4y = -12$

Exercise:

Problem: $y = -2x$

Solution:



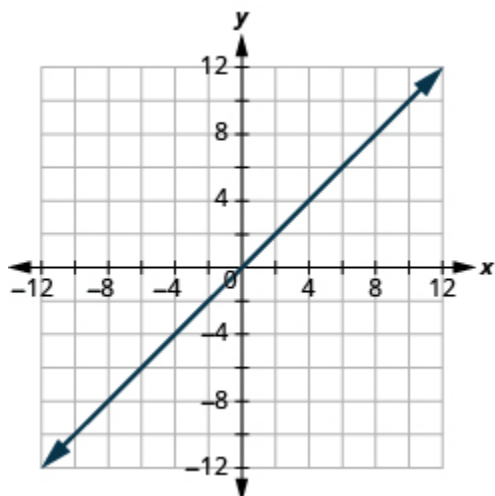
Exercise:

Problem: $y = -4x$

Exercise:

Problem: $y = x$

Solution:



Exercise:

Problem: $y = 3x$

Choose the Most Convenient Method to Graph a Line

In the following exercises, identify the most convenient method to graph each line.

Exercise:

Problem: $x = 2$

Solution:

vertical line

Exercise:

Problem: $y = 4$

Exercise:

Problem: $y = 5$

Solution:

horizontal line

Exercise:

Problem: $x = -3$

Exercise:

Problem: $y = -3x + 4$

Solution:

plotting points

Exercise:

Problem: $y = -5x + 2$

Exercise:

Problem: $x - y = 5$

Solution:

intercepts

Exercise:

Problem: $x - y = 1$

Exercise:

Problem: $y = \frac{2}{3}x - 1$

Solution:

plotting points

Exercise:

Problem: $y = \frac{4}{5}x - 3$

Exercise:

Problem: $y = -3$

Solution:

horizontal line

Exercise:

Problem: $y = -1$

Exercise:

Problem: $3x - 2y = -12$

Solution:

intercepts

Exercise:

Problem: $2x - 5y = -10$

Exercise:

Problem: $y = -\frac{1}{4}x + 3$

Solution:

plotting points

Exercise:

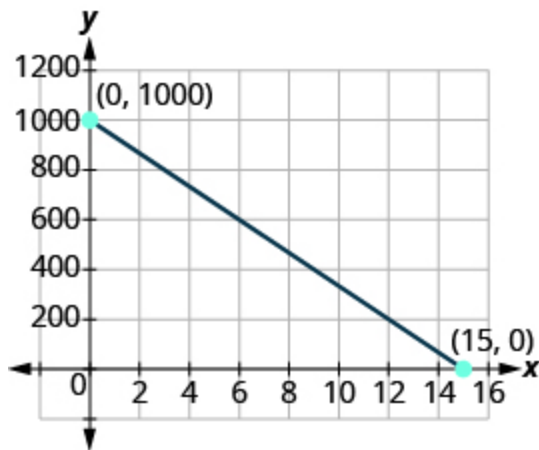
Problem: $y = -\frac{1}{3}x + 5$

Everyday Math

Exercise:

Problem:

Road trip Damien is driving from Chicago to Denver, a distance of 1,000 miles. The x -axis on the graph below shows the time in hours since Damien left Chicago. The y -axis represents the distance he has left to drive.



- Ⓐ Find the x - and y - intercepts
 - Ⓑ Explain what the x - and y - intercepts mean for Damien.
-

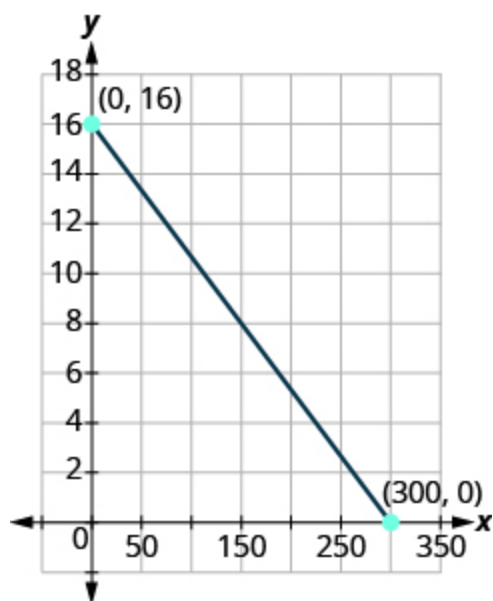
Solution:

- Ⓐ (0,1,000),(15,0). Ⓑ At (0,1,000) he left Chicago 0 hours ago and has 1,000 miles left to drive. At (15,0) he left Chicago 15 hours ago and has 0 miles left to drive.

Exercise:

Problem:

Road trip Ozzie filled up the gas tank of his truck and went on a road trip. The x -axis on the graph shows the number of miles Ozzie drove since filling up. The y -axis represents the number of gallons of gas in the truck's gas tank.



- Ⓐ Find the x - and y - intercepts.
- Ⓑ Explain what the x - and y - intercepts mean for Ozzie.

Writing Exercises

Exercise:

Problem:

How do you find the x -intercept of the graph of $3x - 2y = 6$?

Solution:

Answers will vary.

Exercise:

Problem:

How do you find the y -intercept of the graph of $5x - y = 10$?

Exercise:

Problem:

Do you prefer to graph the equation $4x + y = -4$ by plotting points or intercepts? Why?

Solution:

Answers will vary.

Exercise:**Problem:**

Do you prefer to graph the equation $y = \frac{2}{3}x - 2$ by plotting points or intercepts? Why?

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
identify the intercepts on a graph.			
find the intercepts from an equation of a line.			
graph a line using the intercepts.			
choose the most convenient method to graph a line.			

Ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

intercepts of a line

Each of the points at which a line crosses the x -axis and the y -axis is called an intercept of the line.

Understand Slope of a Line

By the end of this section, you will be able to:

- Use geoboards to model slope
- Find the slope of a line from its graph
- Find the slope of horizontal and vertical lines
- Use the slope formula to find the slope of a line between two points
- Graph a line given a point and the slope
- Solve slope applications

Note:

Before you get started, take this readiness quiz.

1. Simplify: $\frac{1-4}{8-2}$.

If you missed this problem, review [\[link\]](#).

2. Divide: $\frac{0}{4}$, $\frac{4}{0}$.

If you missed this problem, review [\[link\]](#).

3. Simplify: $\frac{15}{-3}$, $\frac{-15}{3}$, $\frac{-15}{-3}$.

If you missed this problem, review [\[link\]](#).

As we've been graphing linear equations, we've seen that some lines slant up as they go from left to right and some lines slant down. Some lines are very steep and some lines are flatter. What determines whether a line slants up or down, and if its slant is steep or flat?

The steepness of the slant of a line is called the **slope of the line**. The concept of slope has many applications in the real world. The pitch of a roof and the grade of a highway or wheelchair ramp are just some examples in which you literally see slopes. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

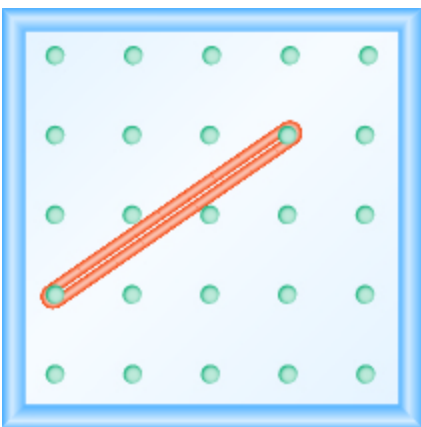
Use Geoboards to Model Slope

In this section, we will explore the concepts of slope.

Using rubber bands on a geoboard gives a concrete way to model lines on a coordinate grid. By stretching a rubber band between two pegs on a geoboard, we can discover how to find the slope of a line. And when you ride a bicycle, you feel the slope as you pump uphill or coast downhill.

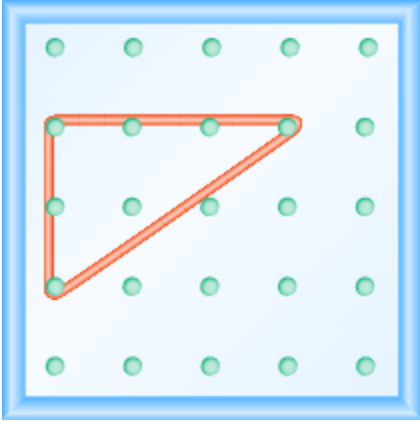
Note: Doing the Manipulative Mathematics activity "Exploring Slope" will help you develop a better understanding of the slope of a line.

We'll start by stretching a rubber band between two pegs to make a line as shown in [\[link\]](#).

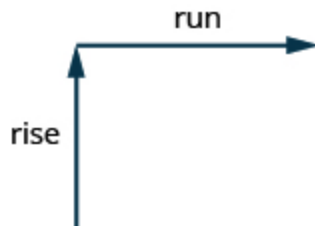


Does it look like a line?

Now we stretch one part of the rubber band straight up from the left peg and around a third peg to make the sides of a right triangle as shown in [\[link\]](#). We carefully make a 90° angle around the third peg, so that one side is vertical and the other is horizontal.



To find the slope of the line, we measure the distance along the vertical and horizontal legs of the triangle. The vertical distance is called the ***rise*** and the horizontal distance is called the ***run***, as shown in [\[link\]](#).



To help remember the terms, it may help to think of the images shown in [\[link\]](#).



It goes straight up,
as if along the y -axis.

RISE ↑

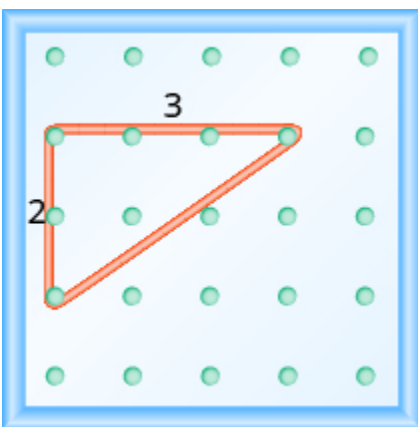


A jogger runs straight across,
as if along the x -axis.

RUN ↔

On our geoboard, the rise is 2 units because the rubber band goes up 2 spaces on the vertical leg. See [\[link\]](#).

What is the run? Be sure to count the spaces between the pegs rather than the pegs themselves! The rubber band goes across 3 spaces on the horizontal leg, so the run is 3 units.



The slope of a line is the ratio of the rise to the run. So the slope of our line is $\frac{2}{3}$. In mathematics, the slope is always represented by the letter m .

Note:

Slope of a line

The slope of a line is $m = \frac{\text{rise}}{\text{run}}$.

The **rise** measures the vertical change and the **run** measures the horizontal change.

What is the slope of the line on the geoboard in [\[link\]](#)?

Equation:

$$m = \frac{\text{rise}}{\text{run}}$$

Equation:

$$m = \frac{2}{3}$$

Equation:

$$\text{The line has slope } \frac{2}{3}.$$

When we work with geoboards, it is a good idea to get in the habit of starting at a peg on the left and connecting to a peg to the right. Then we stretch the rubber band to form a right triangle.

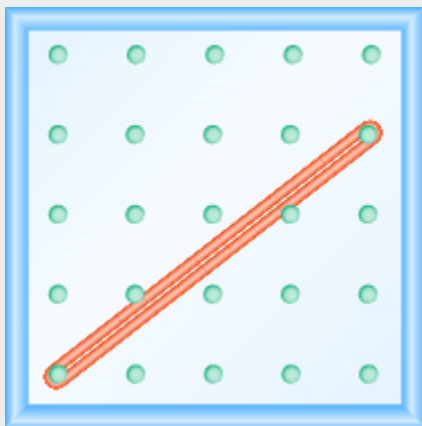
If we start by going up the rise is positive, and if we stretch it down the rise is negative. We will count the run from left to right, just like you read this paragraph, so the run will be positive.

Since the slope formula has rise over run, it may be easier to always count out the rise first and then the run.

Example:

Exercise:

Problem: What is the slope of the line on the geoboard shown?



Solution:

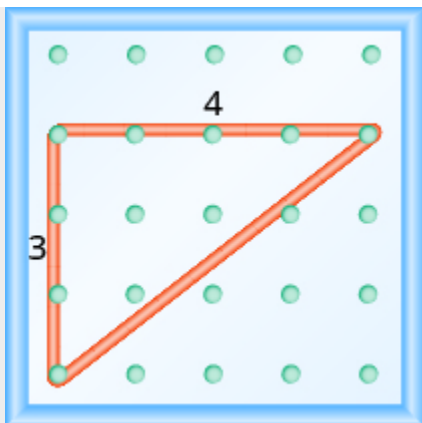
Solution

Use the definition of slope.

$$m = \frac{\text{rise}}{\text{run}}$$

Start at the left peg and make a right triangle by stretching the rubber band up and to the right to reach the second peg.

Count the rise and the run as shown.



The rise is 3 units.

The run is 4 units.

$$m = \frac{3}{\text{run}}$$

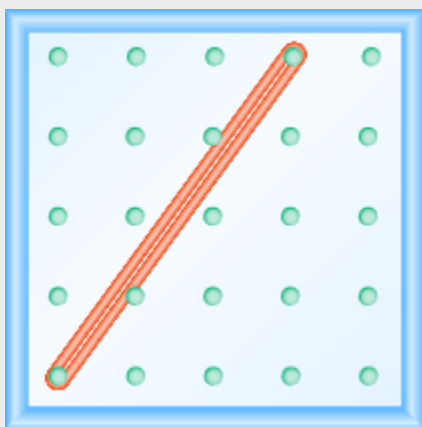
$$m = \frac{3}{4}$$

The slope is $\frac{3}{4}$.

Note:

Exercise:

Problem: What is the slope of the line on the geoboard shown?



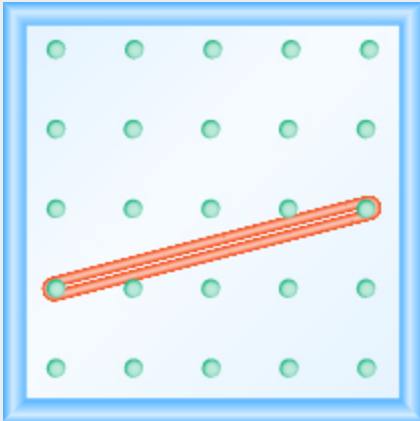
Solution:

$$\frac{4}{3}$$

Note:

Exercise:

Problem: What is the slope of the line on the geoboard shown?



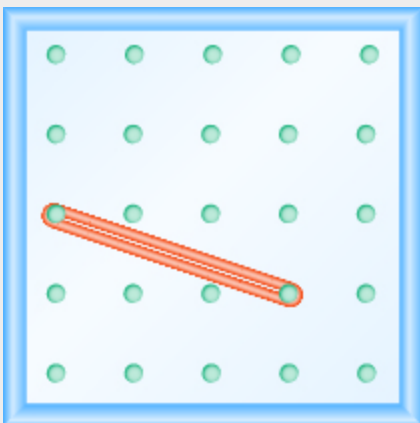
Solution:

$$\frac{1}{3}$$

Example:

Exercise:

Problem: What is the slope of the line on the geoboard shown?

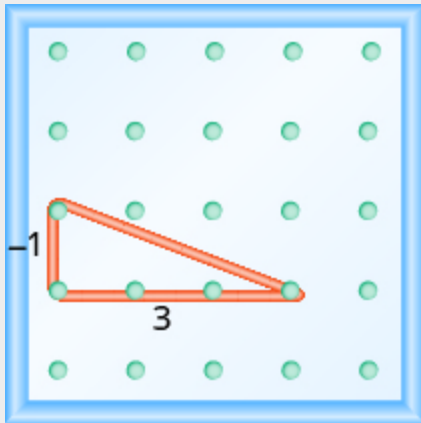


Solution:
Solution

Use the definition of slope.

$$m = \frac{\text{rise}}{\text{run}}$$

Start at the left peg and make a right triangle by stretching the rubber band to the peg on the right. This time we need to stretch the rubber band down to make the vertical leg, so the rise is negative.



The rise is -1 .

$$m = \frac{-1}{\text{run}}$$

The run is 3 .

$$m = \frac{-1}{3}$$

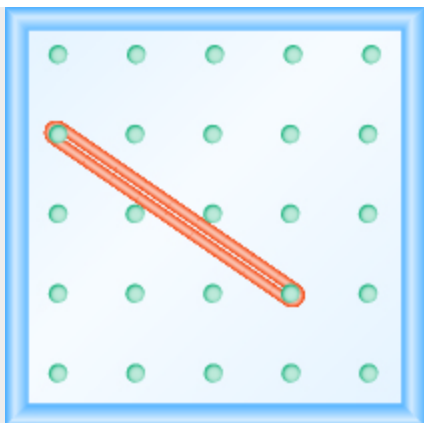
$$m = -\frac{1}{3}$$

The slope is $-\frac{1}{3}$.

Note:

Exercise:

Problem: What is the slope of the line on the geoboard?



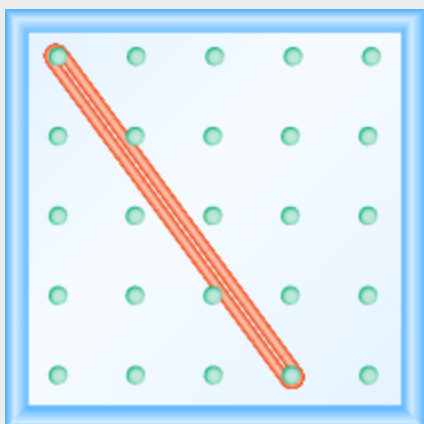
Solution:

$$-\frac{2}{3}$$

Note:

Exercise:

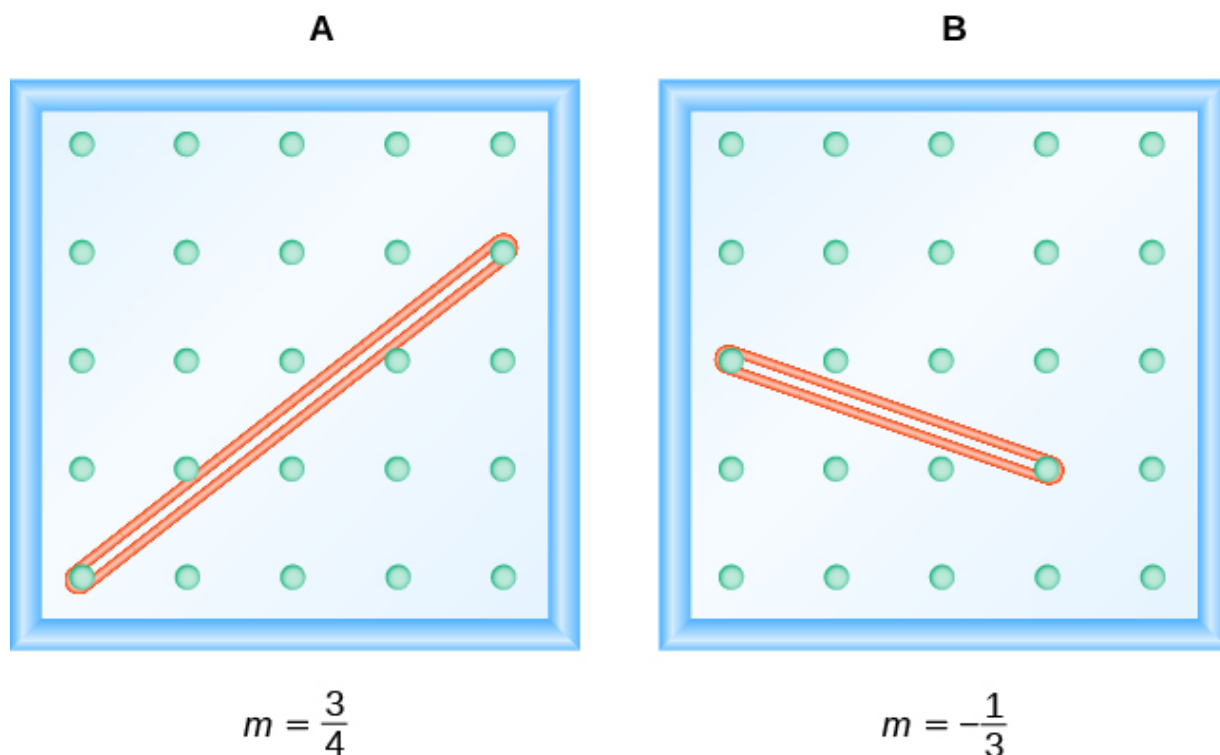
Problem: What is the slope of the line on the geoboard?



Solution:

$$-\frac{4}{3}$$

Notice that in the first example, the slope is positive and in the second example the slope is negative. Do you notice any difference in the two lines shown in [\[link\]](#).



As you read from left to right, the line in Figure A, is going up; it has positive slope. The line Figure B is going down; it has negative slope.



Example:

Exercise:

Problem: Use a geoboard to model a line with slope $\frac{1}{2}$.

Solution: Solution

To model a line with a specific slope on a geoboard, we need to know the rise and the run.

Use the slope formula.

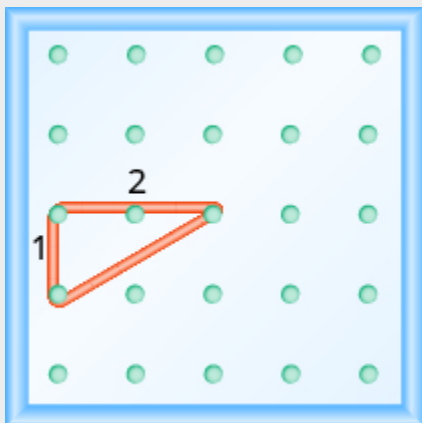
$$m = \frac{\text{rise}}{\text{run}}$$

Replace m with $\frac{1}{2}$.

$$\frac{1}{2} = \frac{\text{rise}}{\text{run}}$$

So, the rise is 1 unit and the run is 2 units.

Start at a peg in the lower left of the geoboard. Stretch the rubber band up 1 unit, and then right 2 units.



The hypotenuse of the right triangle formed by the rubber band represents a line with a slope of $\frac{1}{2}$.

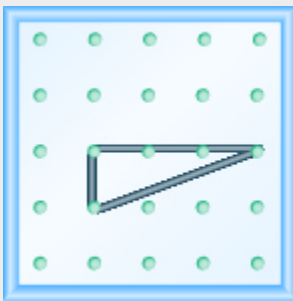
Note:

Exercise:

Problem:

Use a geoboard to model a line with the given slope: $m = \frac{1}{3}$.

Solution:



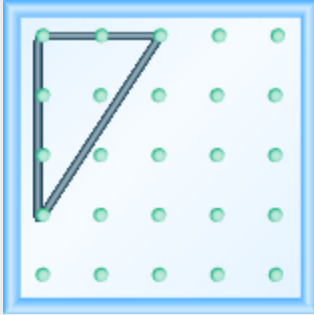
Note:

Exercise:

Problem:

Use a geoboard to model a line with the given slope: $m = \frac{3}{2}$.

Solution:



Example:

Exercise:

Problem: Use a geoboard to model a line with slope $-\frac{1}{4}$,

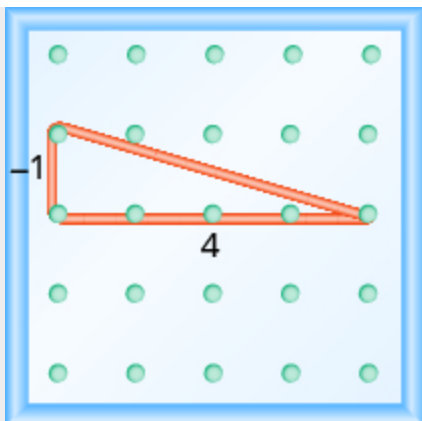
Solution:

Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Replace m with $-\frac{1}{4}$.	$-\frac{1}{4} = \frac{\text{rise}}{\text{run}}$

So, the rise is -1 and the run is 4 .

Since the rise is negative, we choose a starting peg on the upper left that will give us room to count down. We stretch the rubber band down 1 unit, then to the right 4 units.



The hypotenuse of the right triangle formed by the rubber band represents a line whose slope is $-\frac{1}{4}$.

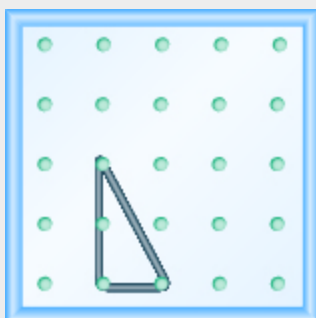
Note:

Exercise:

Problem:

Use a geoboard to model a line with the given slope: $m = \frac{-3}{2}$.

Solution:



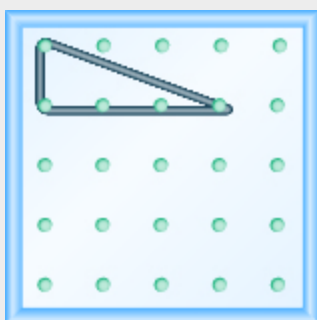
Note:

Exercise:

Problem:

Use a geoboard to model a line with the given slope: $m = \frac{-1}{3}$.

Solution:



Find the Slope of a Line from its Graph

Now we'll look at some graphs on a coordinate grid to find their slopes. The method will be very similar to what we just modeled on our geoboards.

Note: Doing the Manipulative Mathematics activity "Slope of Lines Between Two Points" will help you develop a better understanding of how to find the slope of a line from its graph.

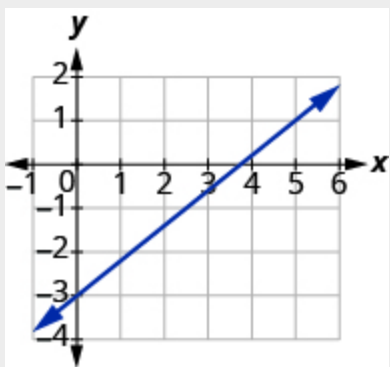
To find the slope, we must count out the rise and the run. But where do we start?

We locate any two points on the line. We try to choose points with coordinates that are integers to make our calculations easier. We then start with the point on the left and sketch a right triangle, so we can count the rise and run.

Example:

Exercise:

Problem: Find the slope of the line shown:

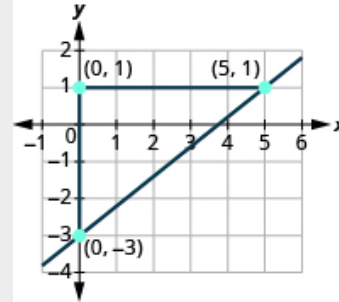


Solution:

Solution

Locate two points on the graph, choosing points whose coordinates are integers. We will use $(0, -3)$ and $(5, 1)$.

Starting with the point on the left, $(0, -3)$, sketch a right triangle, going from the first point to the second point, $(5, 1)$.



Count the rise on the vertical leg of the triangle.

The rise is 4 units.

Count the run on the horizontal leg.

The run is 5 units.

Use the slope formula.

$$m = \frac{\text{rise}}{\text{run}}$$

Substitute the values of the rise and run.

$$m = \frac{4}{5}$$

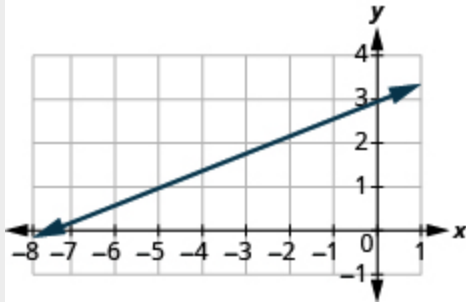
The slope of the line is $\frac{4}{5}$.

Notice that the slope is positive since the line slants upward from left to right.

Note:

Exercise:

Problem: Find the slope of the line:



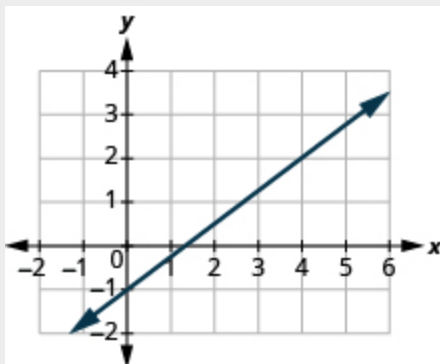
Solution:

$$\frac{2}{5}$$

Note:

Exercise:

Problem: Find the slope of the line:



Solution:

$$\frac{3}{4}$$

Note:

Find the slope from a graph.

Locate two points on the line whose coordinates are integers.

Starting with the point on the left, sketch a right triangle, going from the first point to the second point.

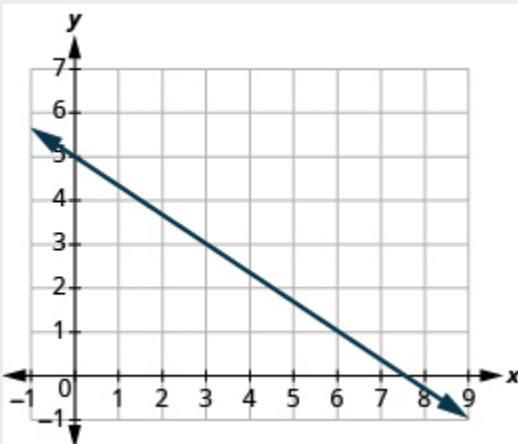
Count the rise and the run on the legs of the triangle.

Take the ratio of rise to run to find the slope. $m = \frac{\text{rise}}{\text{run}}$

Example:

Exercise:

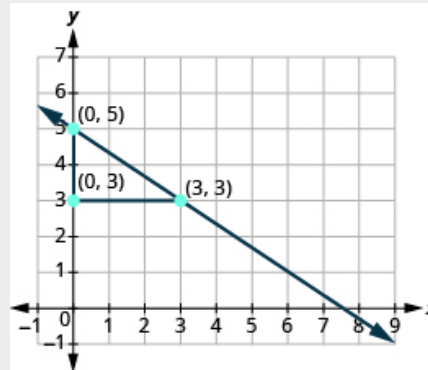
Problem: Find the slope of the line shown:



Solution:

Solution

Locate two points on the graph. Look for points with coordinates that are integers. We can choose any points, but we will use (0, 5) and (3, 3). Starting with the point on the left, sketch a right triangle, going from the first point to the second point.



Count the rise – it is negative.

The rise is -2 .

Count the run.

The run is 3 .

Use the slope formula.

$$m = \frac{\text{rise}}{\text{run}}$$

Substitute the values of the rise and run.

$$m = \frac{-2}{3}$$

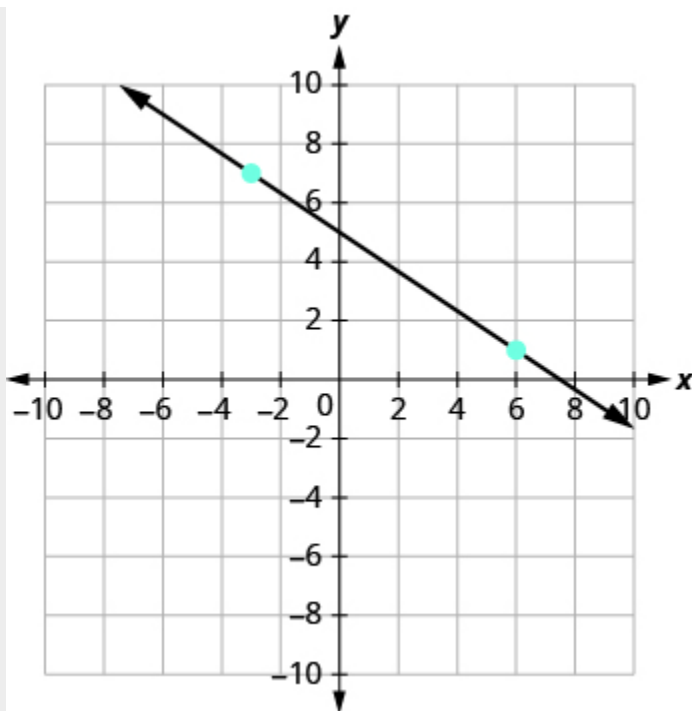
Simplify.

$$m = -\frac{2}{3}$$

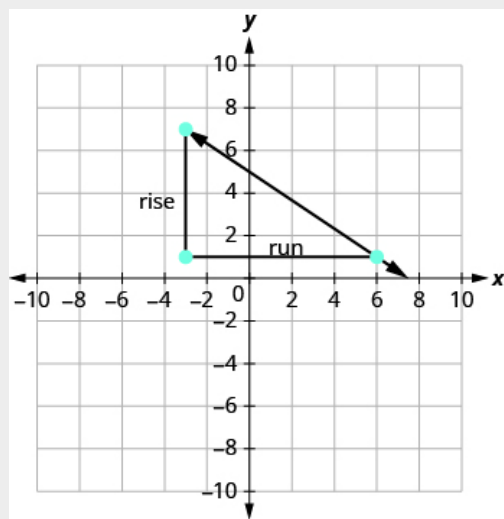
The slope of the line is $-\frac{2}{3}$.

Notice that the slope is negative since the line slants downward from left to right.

What if we had chosen different points? Let's find the slope of the line again, this time using different points. We will use the points $(-3, 7)$ and $(6, 1)$.



Starting at $(-3, 7)$, sketch a right triangle to $(6, 1)$.



Count the rise.

The rise is -6 .

Count the run.

The run is 9 .

Use the slope formula.

$$m = \frac{\text{rise}}{\text{run}}$$

Substitute the values of the rise and run.

$$m = \frac{-6}{9}$$

Simplify the fraction.

$$m = -\frac{2}{3}$$

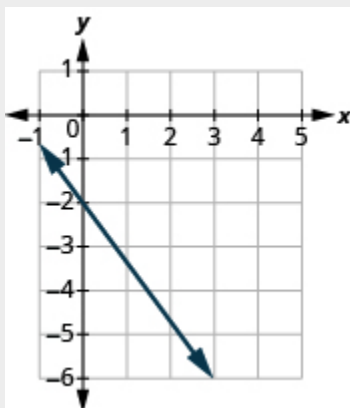
The slope of the line is $-\frac{2}{3}$.

It does not matter which points you use—the slope of the line is always the same. The slope of a line is constant!

Note:

Exercise:

Problem: Find the slope of the line:



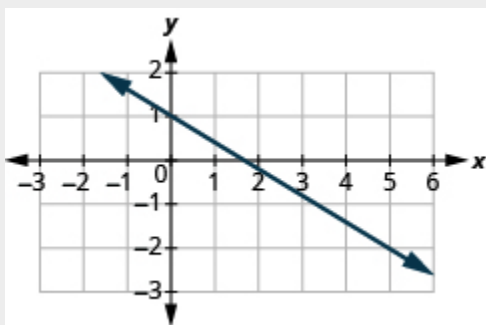
Solution:

$$-\frac{4}{3}$$

Note:

Exercise:

Problem: Find the slope of the line:



Solution:

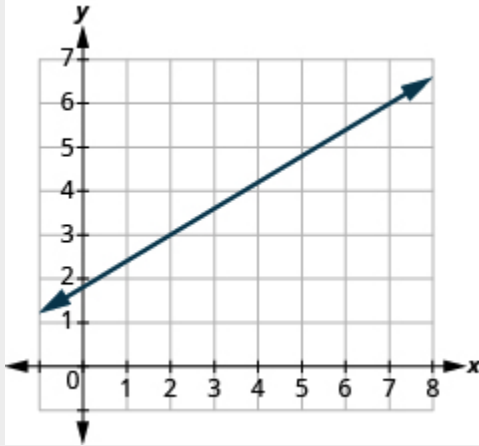
$$-\frac{3}{5}$$

The lines in the previous examples had y -intercepts with integer values, so it was convenient to use the y -intercept as one of the points we used to find the slope. In the next example, the y -intercept is a fraction. The calculations are easier if we use two points with integer coordinates.

Example:

Exercise:

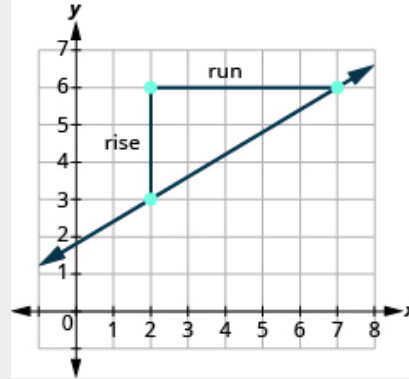
Problem: Find the slope of the line shown:



Solution:
Solution

Locate two points on the graph whose coordinates are integers.	(2, 3) and (7, 6)
Which point is on the left?	(2, 3)
Starting at (2, 3), sketch a right angle to (7, 6) as shown below.	

--	--



Count the rise.

The rise is 3.

Count the run.

The run is 5.

Use the slope formula.

$$m = \frac{\text{rise}}{\text{run}}$$

Substitute the values of the rise and run.

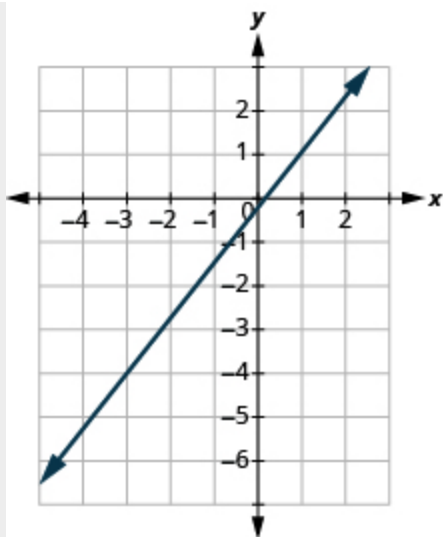
$$m = \frac{3}{5}$$

The slope of the line is $\frac{3}{5}$.

Note:

Exercise:

Problem: Find the slope of the line:



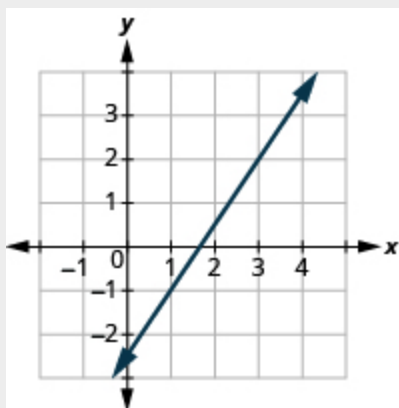
Solution:

$$\frac{5}{2}$$

Note:

Exercise:

Problem: Find the slope of the line:



Solution:

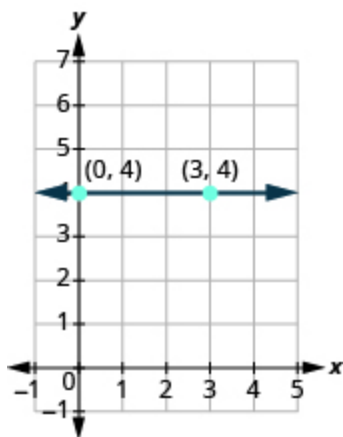
Find the Slope of Horizontal and Vertical Lines

Do you remember what was special about horizontal and vertical lines? Their equations had just one variable.

horizontal line $y = b$; all the y -coordinates are the same.

vertical line $x = a$; all the x -coordinates are the same.

So how do we find the slope of the horizontal line $y = 4$? One approach would be to graph the horizontal line, find two points on it, and count the rise and the run. Let's see what happens in [\[link\]](#). We'll use the two points $(0, 4)$ and $(3, 4)$ to count the rise and run.



What is the rise?	The rise is 0.
What is the run?	The run is 3.

What is the slope?	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{0}{3}$
	$m = 0$

The slope of the horizontal line $y = 4$ is 0.

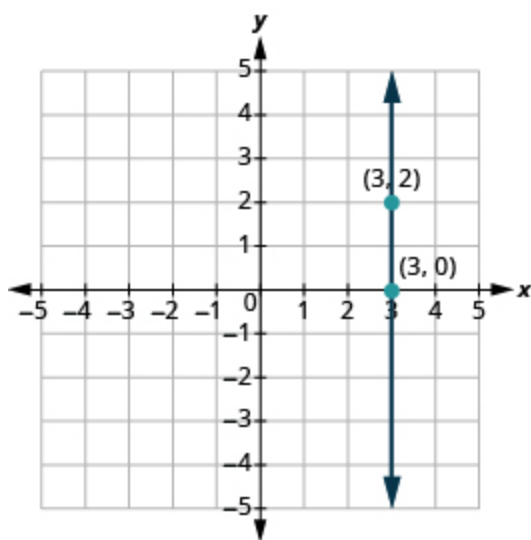
All horizontal lines have slope 0. When the y -coordinates are the same, the rise is 0.

Note:

Slope of a Horizontal Line

The slope of a horizontal line, $y = b$, is 0.

Now we'll consider a vertical line, such as the line $x = 3$, shown in [\[link\]](#). We'll use the two points $(3, 0)$ and $(3, 2)$ to count the rise and run.



What is the rise?	The rise is 2.
What is the run?	The run is 0.
What is the slope?	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{2}{0}$

But we can't divide by 0. Division by 0 is undefined. So we say that the slope of the vertical line $x = 3$ is undefined. The slope of all vertical lines is undefined, because the run is 0.

Note:

Slope of a Vertical Line

The slope of a vertical line, $x = a$, is undefined.

Example:

Exercise:

Problem: Find the slope of each line:

Ⓐ $x = 8$

Ⓑ $y = -5$

Solution:

Solution

Ⓐ $x = 8$

This is a vertical line, so its slope is undefined.

ⓑ $y = -5$

This is a horizontal line, so its slope is 0.

Note:

Exercise:

Problem: Find the slope of the line: $x = -4$.

Solution:

undefined

Note:

Exercise:

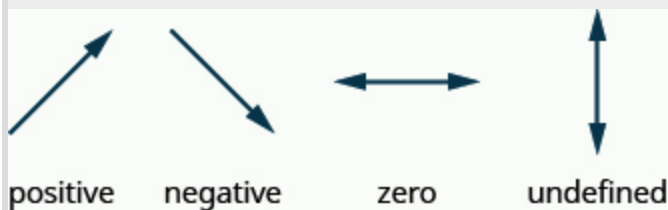
Problem: Find the slope of the line: $y = 7$.

Solution:

0

Note:

Quick Guide to the Slopes of Lines



Use the Slope Formula to find the Slope of a Line between Two Points

Sometimes we need to find the slope of a line between two points and we might not have a graph to count out the rise and the run. We could plot the points on grid paper, then count out the rise and the run, but there is a way to find the slope without graphing.

Before we get to it, we need to introduce some new algebraic notation. We have seen that an ordered pair (x, y) gives the coordinates of a point. But when we work with slopes, we use two points. How can the same symbol (x, y) be used to represent two different points?

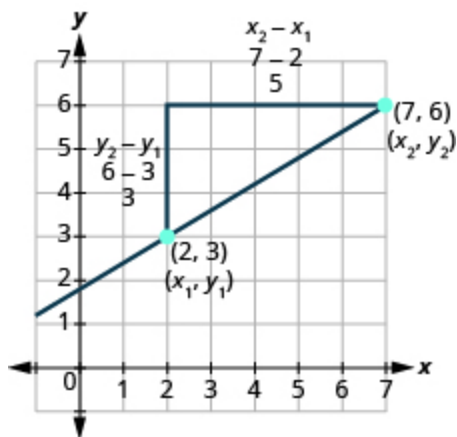
Mathematicians use subscripts to distinguish between the points. A subscript is a small number written to the right of, and a little lower than, a variable.

(x_1, y_1) read x sub 1, y sub 1

(x_2, y_2) read x sub 2, y sub 2

We will use (x_1, y_1) to identify the first point and (x_2, y_2) to identify the second point. If we had more than two points, we could use (x_3, y_3) , (x_4, y_4) , and so on.

To see how the rise and run relate to the coordinates of the two points, let's take another look at the slope of the line between the points $(2, 3)$ and $(7, 6)$ in [\[link\]](#).



Since we have two points, we will use subscript notation.

Equation:

$$\begin{matrix} x_1, y_1 & x_2, y_2 \\ (2, 3) & (7, 6) \end{matrix}$$

On the graph, we counted the rise of 3. The rise can also be found by subtracting the y -coordinates of the points.

Equation:

$$\begin{aligned} y_2 - y_1 \\ 6 - 3 \\ 3 \end{aligned}$$

We counted a run of 5. The run can also be found by subtracting the x -coordinates.

Equation:

$$\begin{aligned} x_2 - x_1 \\ 7 - 2 \\ 5 \end{aligned}$$

We know	$m = \frac{\text{rise}}{\text{run}}$
So	$m = \frac{3}{5}$
We rewrite the rise and run by putting in the coordinates.	$m = \frac{6-3}{7-2}$
But 6 is the y -coordinate of the second point, y_2 and 3 is the y -coordinate of the first point y_1 . So we can rewrite the rise using subscript notation.	$m = \frac{y_2 - y_1}{7 - 2}$
Also 7 is the x -coordinate of the second point, x_2 and 2 is the x -coordinate of the first point x_1 . So we rewrite the run using subscript notation.	$m = \frac{y_2 - y_1}{x_2 - x_1}$

We've shown that $m = \frac{y_2 - y_1}{x_2 - x_1}$ is really another version of $m = \frac{\text{rise}}{\text{run}}$. We can use this formula to find the slope of a line when we have two points on the line.

Note:

Slope Formula

The slope of the line between two points (x_1, y_1) and (x_2, y_2) is

Equation:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Say the formula to yourself to help you remember it:

Equation:

Slope is y of the second point minus y of the first point

Equation:

over

Equation:

x of the second point minus x of the first point.

Note:Doing the Manipulative Mathematics activity “Slope of Lines Between Two Points” will help you develop a better understanding of how to find the slope of a line between two points.

Example:

Exercise:

Problem:

Find the slope of the line between the points $(1, 2)$ and $(4, 5)$.

Solution:

Solution

We'll call $(1, 2)$ point #1 and $(4, 5)$ point #2.	$\overset{x_1,y_1}{(1, 2)}$ and $\overset{x_2,y_2}{(4, 5)}$
Use the slope formula.	$m = \frac{y_2-y_1}{x_2-x_1}$
Substitute the values in the slope formula:	

y of the second point minus y of the first point

$$m = \frac{5-2}{x_2-x_1}$$

x of the second point minus x of the first point

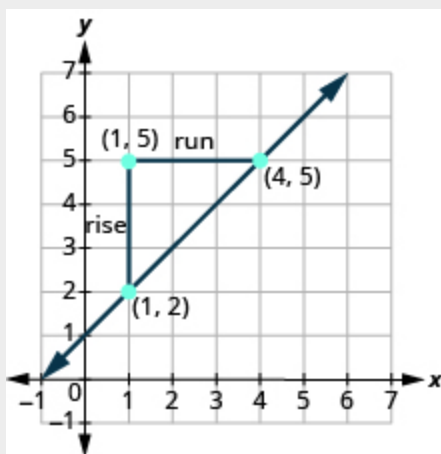
$$m = \frac{5-2}{4-1}$$

Simplify the numerator and the denominator.

$$m = \frac{3}{3}$$

$$m = 1$$

Let's confirm this by counting out the slope on the graph.



The rise is 3 and the run is 3, so

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{3}$$

$$m = 1$$

Note:

Exercise:

Problem:

Find the slope of the line through the given points: (8, 5) and (6, 3).

Solution:

1

Note:**Exercise:****Problem:**

Find the slope of the line through the given points: (1, 5) and (5, 9).

Solution:

1

How do we know which point to call #1 and which to call #2? Let's find the slope again, this time switching the names of the points to see what happens. Since we will now be counting the run from right to left, it will be negative.

We'll call (4, 5) point #1 and (1, 2) point #2.	$\overset{x_1, y_1}{(4, 5)} \text{ and } \overset{x_2, y_2}{(1, 2)}$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute the values in the slope formula:	
y of the second point minus y of the first point	$m = \frac{2-5}{x_2-x_1}$
x of the second point minus x of the first point	$m = \frac{2-5}{1-4}$
Simplify the numerator and the denominator.	$m = \frac{-3}{-3}$
	$m = 1$

The slope is the same no matter which order we use the points.

Example:

Exercise:

Problem:

Find the slope of the line through the points $(-2, -3)$ and $(-7, 4)$.

Solution:

Solution

We'll call $(-2, -3)$ point #1 and $(-7, 4)$ point #2.	$\overset{x_1, y_1}{(-2, -3)}$ and $\overset{x_2, y_2}{(-7, 4)}$
Use the slope formula.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Substitute the values	

y of the second point minus y of the first point

$$m = \frac{4 - (-3)}{x_2 - x_1}$$

x of the second point minus x of the first point

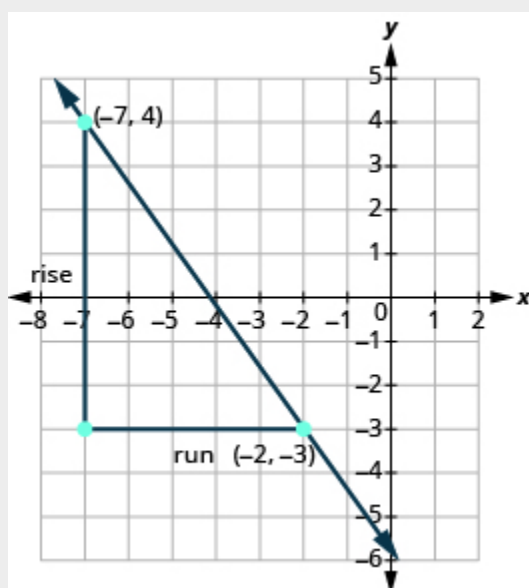
$$m = \frac{4 - (-3)}{-7 - (-2)}$$

Simplify.

$$m = \frac{7}{-5}$$

$$m = -\frac{7}{5}$$

Let's confirm this on the graph shown.



$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{-7}{5}$$

$$m = -\frac{7}{5}$$

Note:

Exercise:

Problem:

Find the slope of the line through the pair of points: $(-3, 4)$ and $(2, -1)$.

Solution:

-1

Note:**Exercise:****Problem:**

Find the slope of the line through the pair of points: $(-2, 6)$ and $(-3, -4)$.

Solution:

10

Graph a Line Given a Point and the Slope

In this chapter, we graphed lines by plotting points, by using intercepts, and by recognizing horizontal and vertical lines.

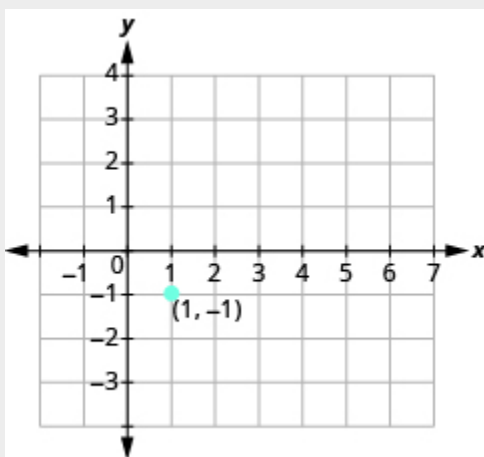
Another method we can use to graph lines is the point-slope method. Sometimes, we will be given one point and the slope of the line, instead of its equation. When this happens, we use the definition of slope to draw the graph of the line.

Example:**Exercise:****Problem:**

Graph the line passing through the point $(1, -1)$ whose slope is $m = \frac{3}{4}$.

Solution:**Solution**

Plot the given point, $(1, -1)$.



Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.

Equation:

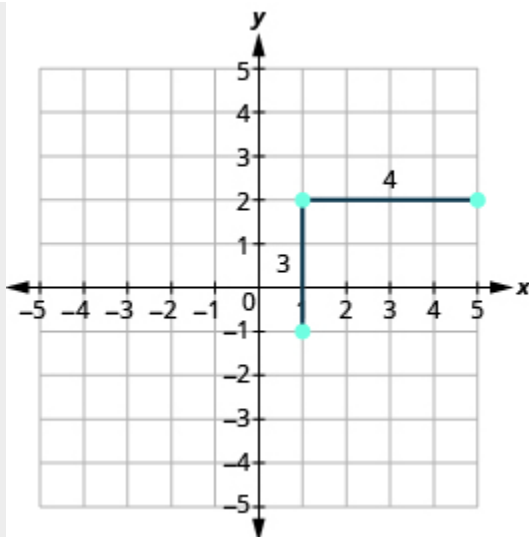
$$m = \frac{3}{4}$$

$$\frac{\text{rise}}{\text{run}} = \frac{3}{4}$$

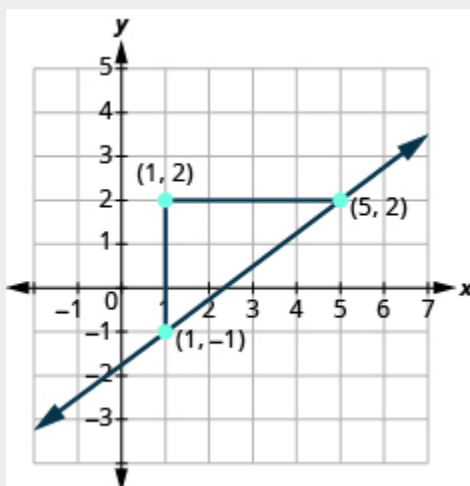
$$\text{rise} = 3$$

$$\text{run} = 4$$

Starting at the point we plotted, count out the rise and run to mark the second point. We count 3 units up and 4 units right.



Then we connect the points with a line and draw arrows at the ends to show it continues.



We can check our line by starting at any point and counting up 3 and to the right 4. We should get to another point on the line.

Note:

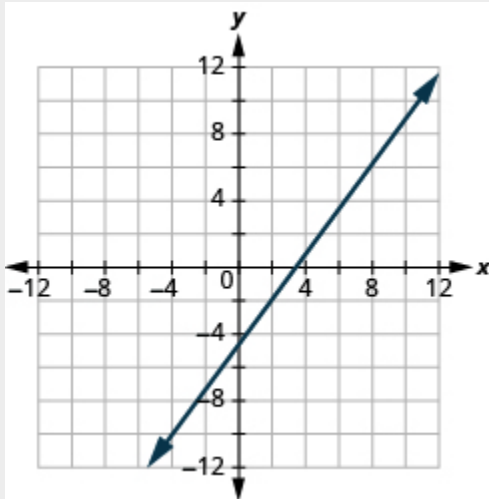
Exercise:

Problem:

Graph the line passing through the point with the given slope:

$$(2, -2), m = \frac{4}{3}$$

Solution:



Note:

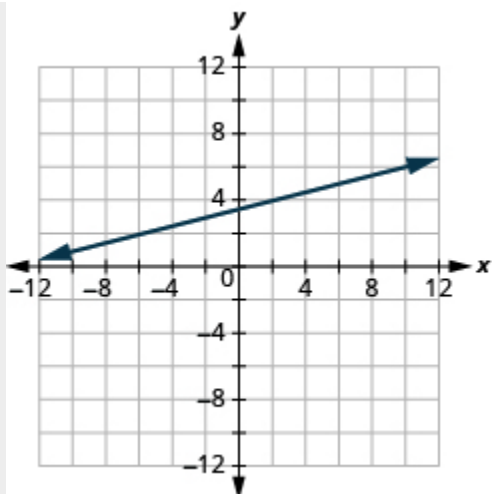
Exercise:

Problem:

Graph the line passing through the point with the given slope:

$$(-2, 3), m = \frac{1}{4}$$

Solution:

**Note:**

Graph a line given a point and a slope.

Plot the given point.

Use the slope formula to identify the rise and the run.

Starting at the given point, count out the rise and run to mark the second point.

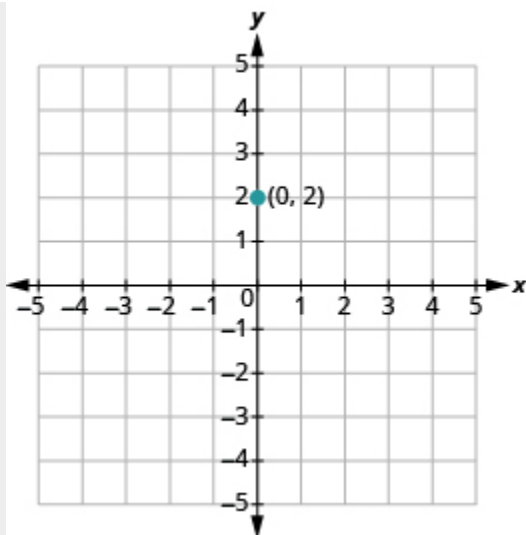
Connect the points with a line.

Example:**Exercise:**

Problem: Graph the line with y -intercept $(0, 2)$ and slope $m = -\frac{2}{3}$.

Solution:**Solution**

Plot the given point, the y -intercept $(0, 2)$.



Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.

Equation:

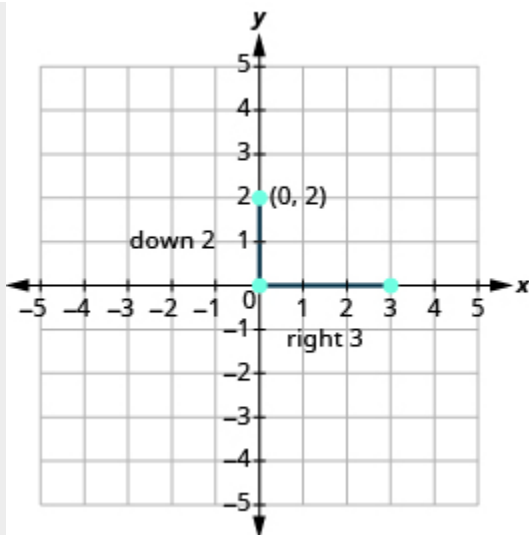
$$m = -\frac{2}{3}$$

$$\frac{\text{rise}}{\text{run}} = \frac{-2}{3}$$

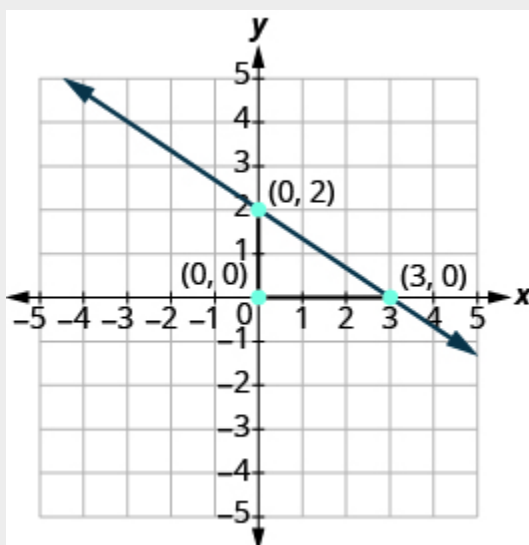
$$\text{rise} = -2$$

$$\text{run} = 3$$

Starting at $(0, 2)$, count the rise and the run and mark the second point.



Connect the points with a line.



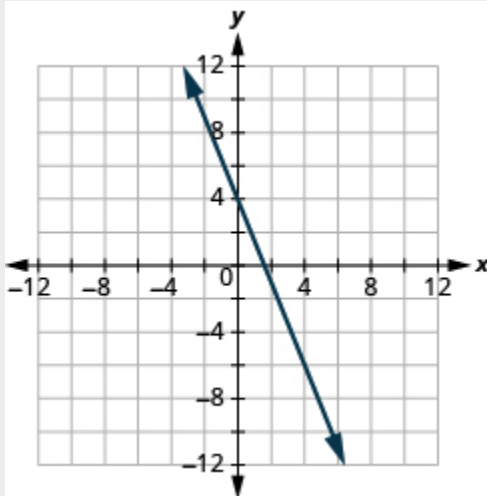
Note:

Exercise:

Problem: Graph the line with the given intercept and slope:

y -intercept 4, $m = -\frac{5}{2}$

Solution:



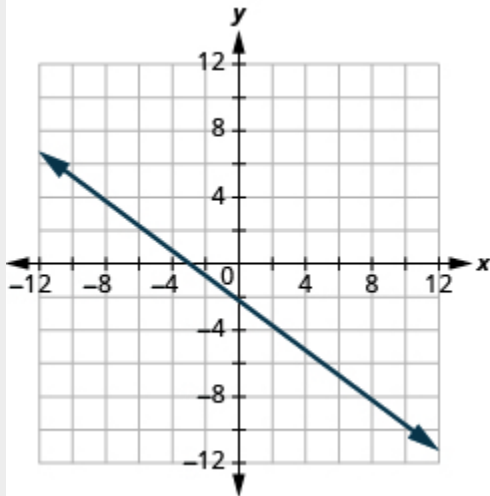
Note:

Exercise:

Problem: Graph the line with the given intercept and slope:

$$x\text{-intercept } -3, m = -\frac{3}{4}$$

Solution:



Example:

Exercise:

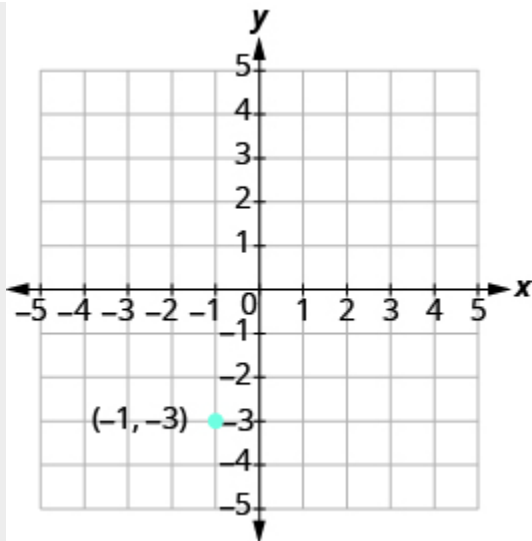
Problem:

Graph the line passing through the point $(-1, -3)$ whose slope is $m = 4$.

Solution:

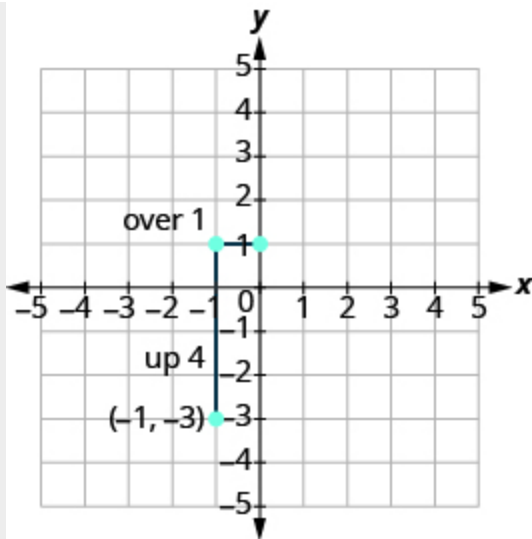
Solution

Plot the given point.

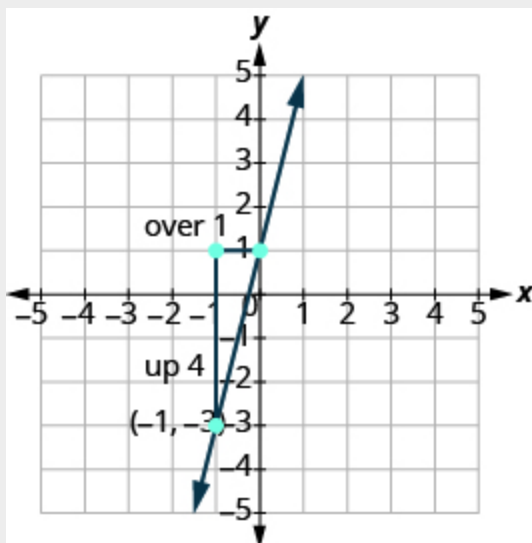


Identify the rise and the run.	$m = 4$
Write 4 as a fraction.	$\frac{\text{rise}}{\text{run}} = \frac{4}{1}$
	rise = 4 run = 1

Count the rise and run.



Mark the second point. Connect the two points with a line.



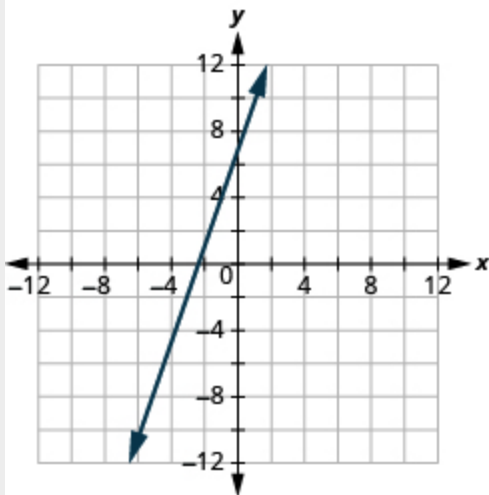
Note:

Exercise:

Problem:

Graph the line with the given intercept and slope: $(-2, 1)$, $m = 3$.

Solution:



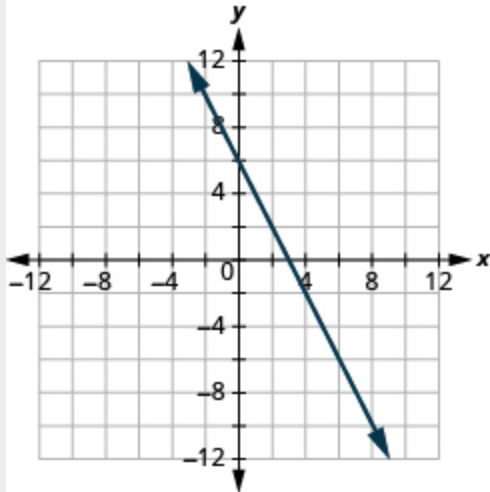
Note:

Exercise:

Problem:

Graph the line with the given intercept and slope: $(4, -2)$, $m = -2$.

Solution:



Solve Slope Applications

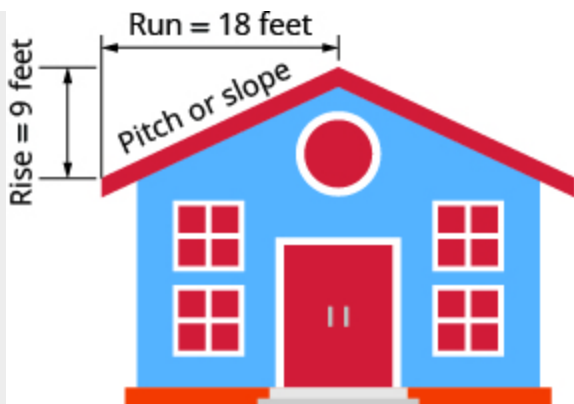
At the beginning of this section, we said there are many applications of slope in the real world. Let's look at a few now.

Example:

Exercise:

Problem:

The pitch of a building's roof is the slope of the roof. Knowing the pitch is important in climates where there is heavy snowfall. If the roof is too flat, the weight of the snow may cause it to collapse. What is the slope of the roof shown?



Solution:
Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
Substitute the values for rise and run.	$m = \frac{9 \text{ ft}}{18 \text{ ft}}$
Simplify.	$m = \frac{1}{2}$
	The slope of the roof is $\frac{1}{2}$.

Note:
Exercise:

Problem:

Find the slope given rise and run: A roof with a rise = 14 and run = 24.

Solution:

$$\frac{7}{12}$$

Note:**Exercise:****Problem:**

Find the slope given rise and run: A roof with a rise = 15 and run = 36.

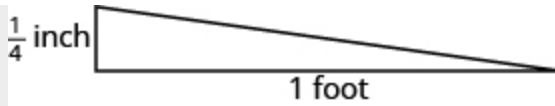
Solution:

$$\frac{5}{12}$$

Have you ever thought about the sewage pipes going from your house to the street? Their slope is an important factor in how they take waste away from your house.

Example:**Exercise:****Problem:**

Sewage pipes must slope down $\frac{1}{4}$ inch per foot in order to drain properly. What is the required slope?



Solution:
Solution

Use the slope formula.	$m = \frac{\text{rise}}{\text{run}}$
	$m = \frac{-\frac{1}{4} \text{ in.}}{1 \text{ ft}}$
	$m = \frac{-\frac{1}{4} \text{ in.}}{1 \text{ ft}}$
Convert 1 foot to 12 inches.	$m = \frac{-\frac{1}{4} \text{ in.}}{12 \text{ in.}}$
Simplify.	$m = -\frac{1}{48}$
	The slope of the pipe is $-\frac{1}{48}$.

Note:

Exercise:

Problem:

Find the slope of the pipe: The pipe slopes down $\frac{1}{3}$ inch per foot.

Solution:

$$-\frac{1}{36}$$

Note:

Exercise:

Problem:

Find the slope of the pipe: The pipe slopes down $\frac{3}{4}$ inch per yard.

Solution:

$$-\frac{1}{48}$$

Note:

ACCESS ADDITIONAL ONLINE RESOURCES

- [Determine Positive slope from a Graph](#)
- [Determine Negative slope from a Graph](#)
- [Determine Slope from Two Points](#)

Key Concepts

- **Find the slope from a graph**

Locate two points on the line whose coordinates are integers.

Starting with the point on the left, sketch a right triangle, going from the first point to the second point.

Count the rise and the run on the legs of the triangle.

Take the ratio of rise to run to find the slope, $m = \frac{\text{rise}}{\text{run}}$

- **Slope of a Horizontal Line**

- The slope of a horizontal line, $y = b$, is 0.

- **Slope of a Vertical Line**

- The slope of a vertical line, $x = a$, is undefined.

- **Slope Formula**

- The slope of the line between two points (x_1, y_1) and (x_2, y_2) is
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- **Graph a line given a point and a slope.**

Plot the given point.

Use the slope formula to identify the rise and the run.

Starting at the given point, count out the rise and run to mark the second point.

Connect the points with a line.

Section Exercises

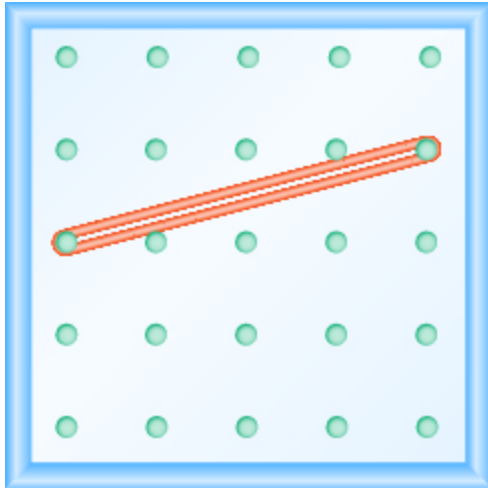
Practice Makes Perfect

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

Exercise:

Problem:

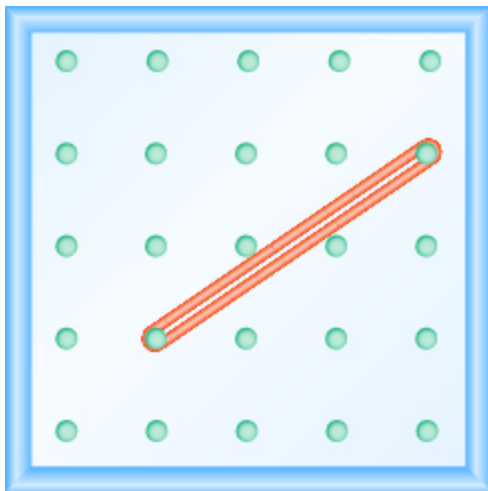


Solution:

$$\frac{1}{4}$$

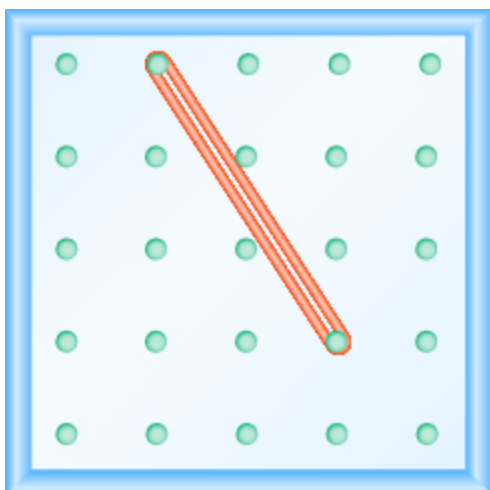
Exercise:

Problem:



Exercise:

Problem:

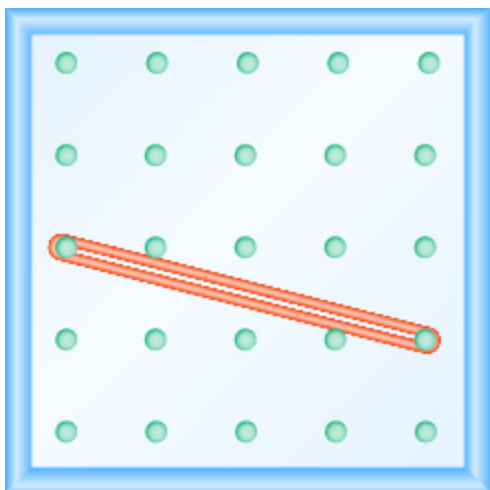


Solution:

$$-\frac{3}{2}$$

Exercise:

Problem:

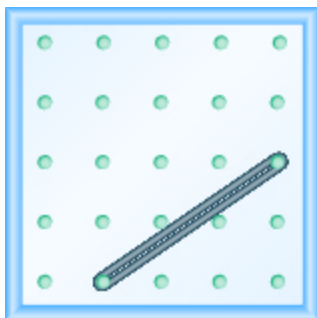


In the following exercises, model each slope. Draw a picture to show your results.

Exercise:

Problem: $\frac{2}{3}$

Solution:



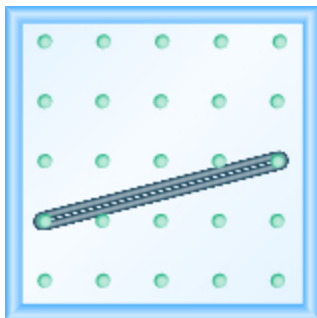
Exercise:

Problem: $\frac{3}{4}$

Exercise:

Problem: $\frac{1}{4}$

Solution:



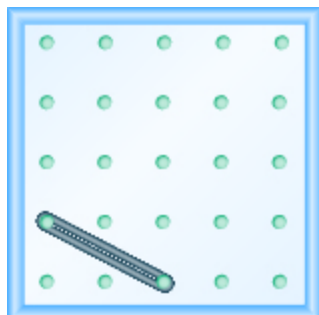
Exercise:

Problem: $\frac{4}{3}$

Exercise:

Problem: $-\frac{1}{2}$

Solution:



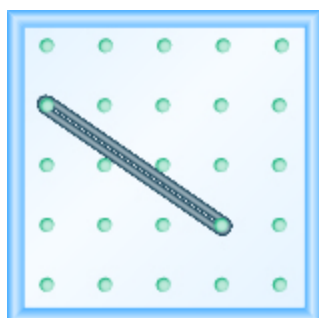
Exercise:

Problem: $-\frac{3}{4}$

Exercise:

Problem: $-\frac{2}{3}$

Solution:



Exercise:

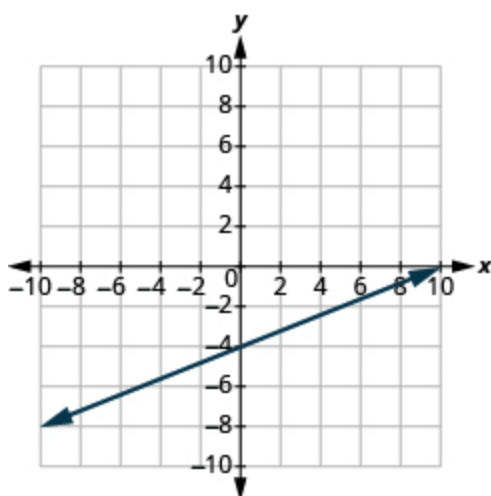
Problem: $-\frac{3}{2}$

Find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

Exercise:

Problem:

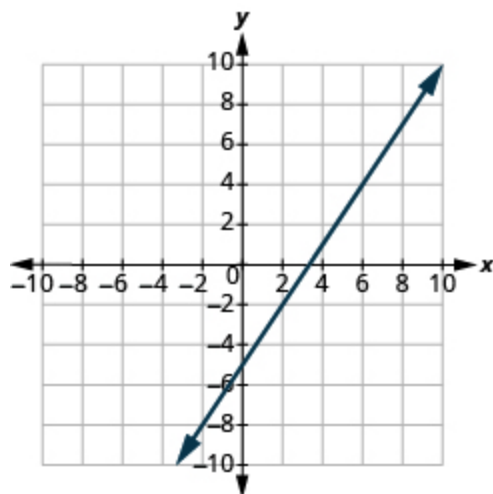


Solution:

$$\frac{2}{5}$$

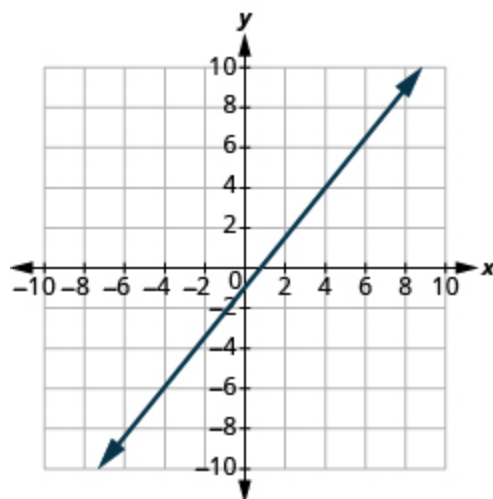
Exercise:

Problem:



Exercise:

Problem:

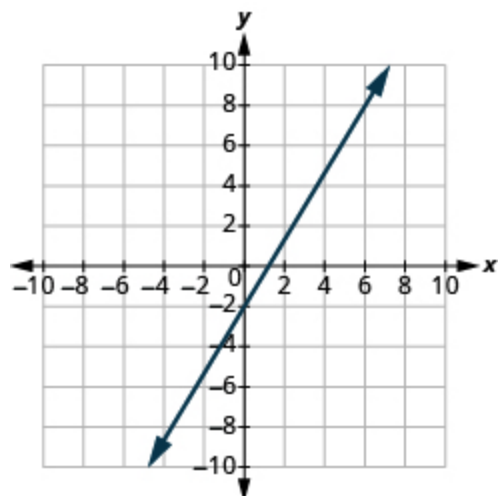


Solution:

$$\frac{5}{4}$$

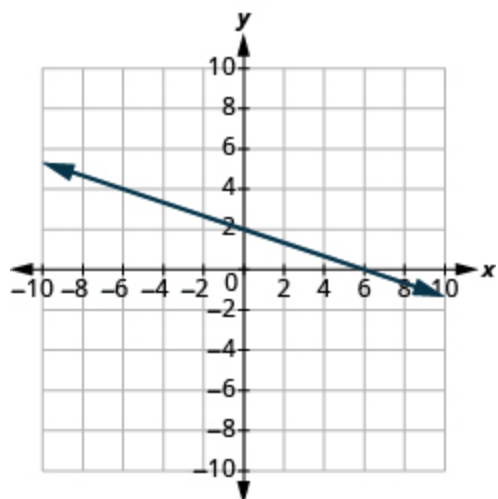
Exercise:

Problem:



Exercise:

Problem:

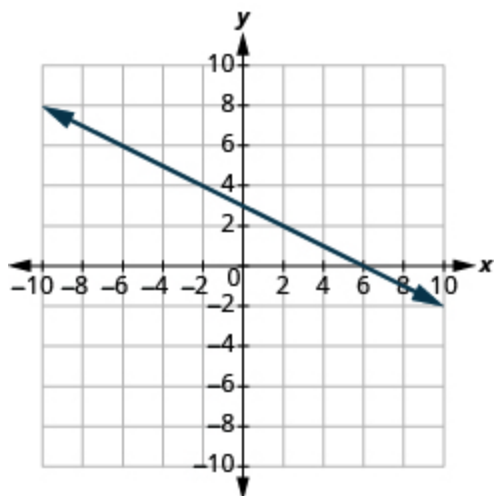


Solution:

$$-\frac{1}{3}$$

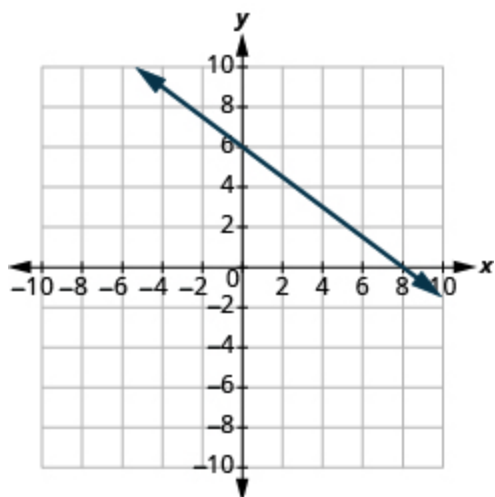
Exercise:

Problem:



Exercise:

Problem:

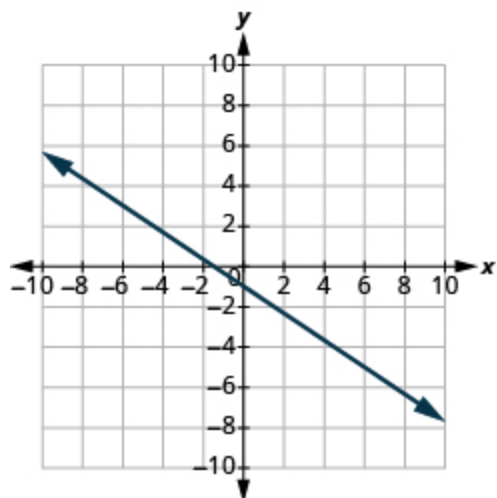


Solution:

$$-\frac{3}{4}$$

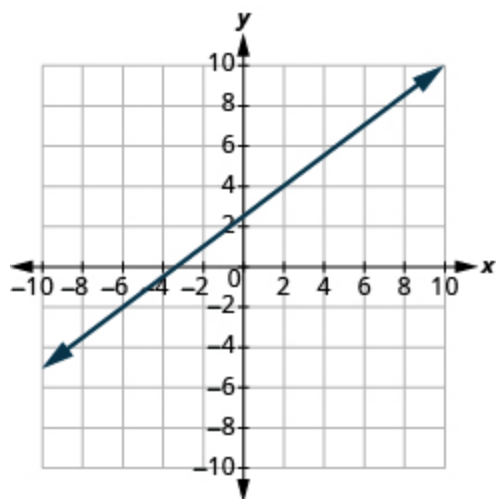
Exercise:

Problem:



Exercise:

Problem:

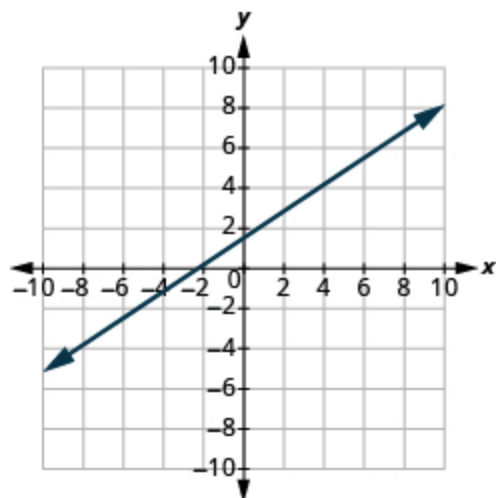


Solution:

$$\frac{3}{4}$$

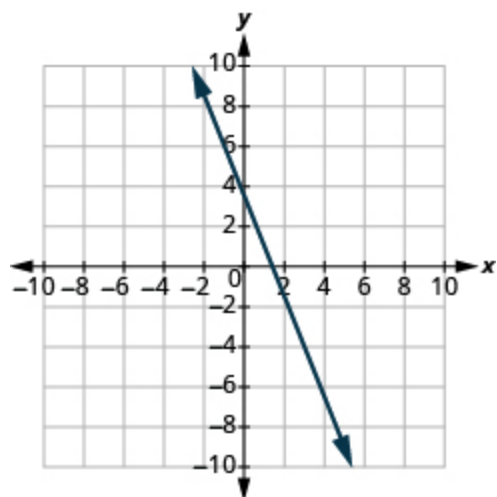
Exercise:

Problem:



Exercise:

Problem:

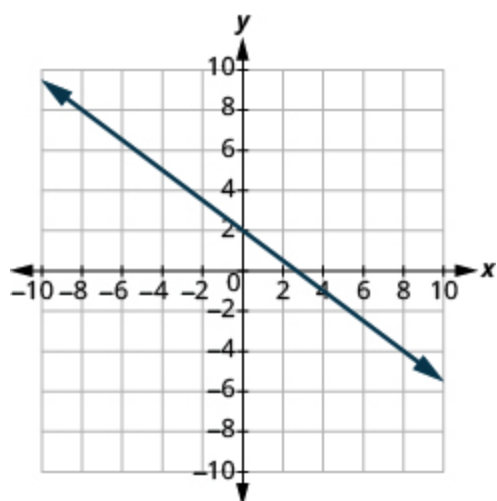


Solution:

$$-\frac{5}{2}$$

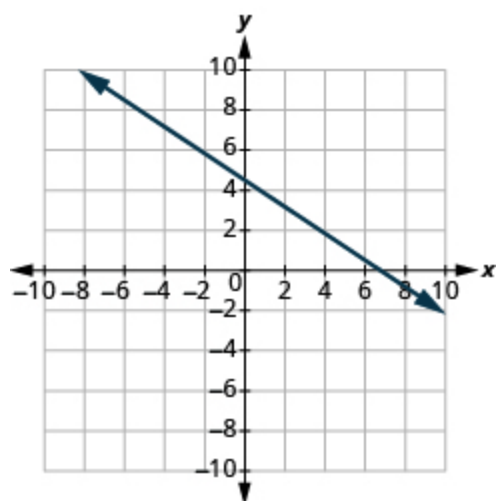
Exercise:

Problem:



Exercise:

Problem:

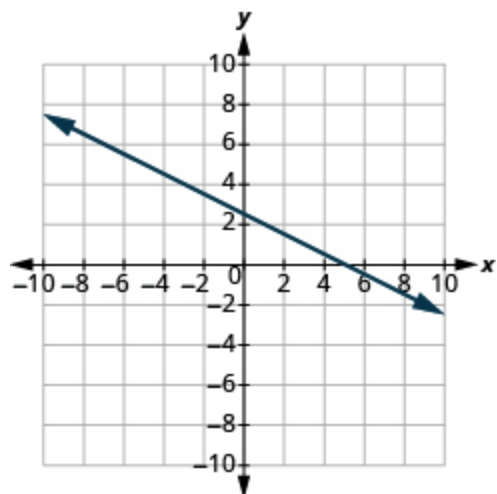


Solution:

$$-\frac{2}{3}$$

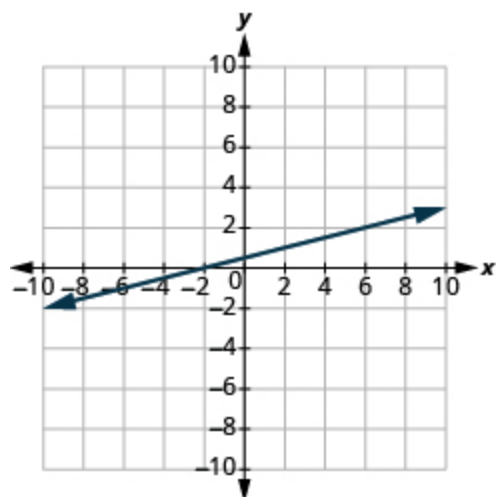
Exercise:

Problem:



Exercise:

Problem:

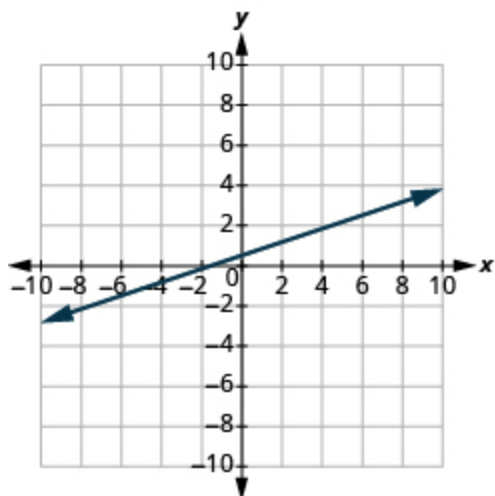


Solution:

$$\frac{1}{4}$$

Exercise:

Problem:



Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise:

Problem: $y = 3$

Solution:

0

Exercise:

Problem: $y = 1$

Exercise:

Problem: $x = 4$

Solution:

undefined

Exercise:

Problem: $x = 2$

Exercise:

Problem: $y = -2$

Solution:

0

Exercise:

Problem: $y = -3$

Exercise:

Problem: $x = -5$

Solution:

undefined

Exercise:

Problem: $x = -4$

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise:

Problem: $(1, 4), (3, 9)$

Solution:

$$\frac{5}{2}$$

Exercise:

Problem: $(2, 3), (5, 7)$

Exercise:

Problem: $(0, 3), (4, 6)$

Solution:

$$\frac{3}{4}$$

Exercise:

Problem: $(0, 1), (5, 4)$

Exercise:

Problem: $(2, 5), (4, 0)$

Solution:

$$-\frac{5}{2}$$

Exercise:

Problem: $(3, 6), (8, 0)$

Exercise:

Problem: $(-3, 3), (2, -5)$

Solution:

$$-\frac{8}{5}$$

Exercise:

Problem: $(-2, 4), (3, -1)$

Exercise:

Problem: $(-1, -2), (2, 5)$

Solution:

$$\frac{7}{3}$$

Exercise:

Problem: $(-2, -1), (6, 5)$

Exercise:

Problem: $(4, -5), (1, -2)$

Solution:

$$-1$$

Exercise:

Problem: $(3, -6), (2, -2)$

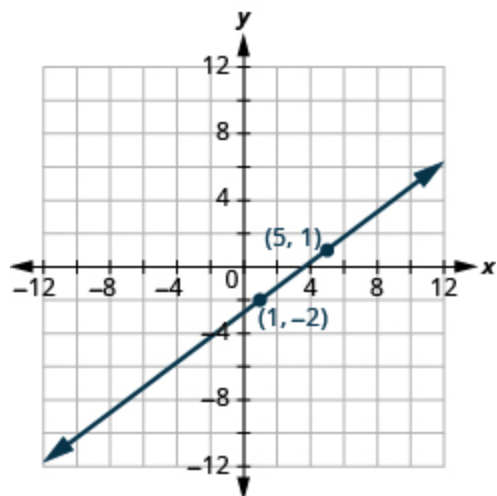
Graph a Line Given a Point and the Slope

In the following exercises, graph the line given a point and the slope.

Exercise:

Problem: $(1, -2); m = \frac{3}{4}$

Solution:



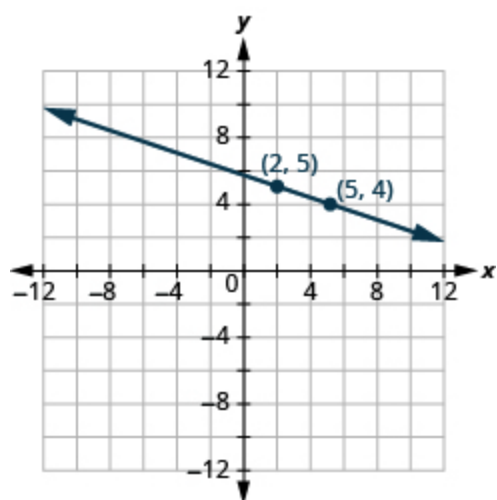
Exercise:

Problem: $(1, -1); m = \frac{1}{2}$

Exercise:

Problem: $(2, 5); m = -\frac{1}{3}$

Solution:



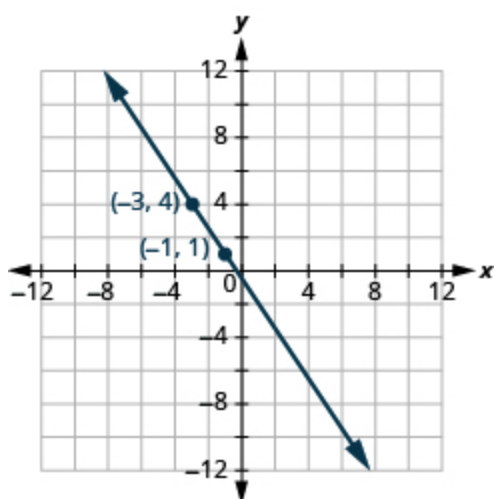
Exercise:

Problem: $(1, 4); m = -\frac{1}{2}$

Exercise:

Problem: $(-3, 4); m = -\frac{3}{2}$

Solution:



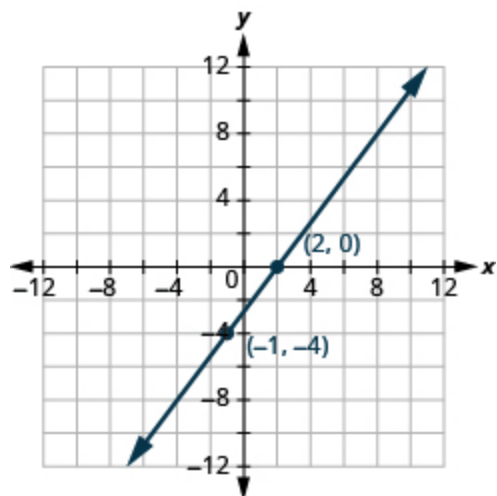
Exercise:

Problem: $(-2, 5); m = -\frac{5}{4}$

Exercise:

Problem: $(-1, -4); m = \frac{4}{3}$

Solution:



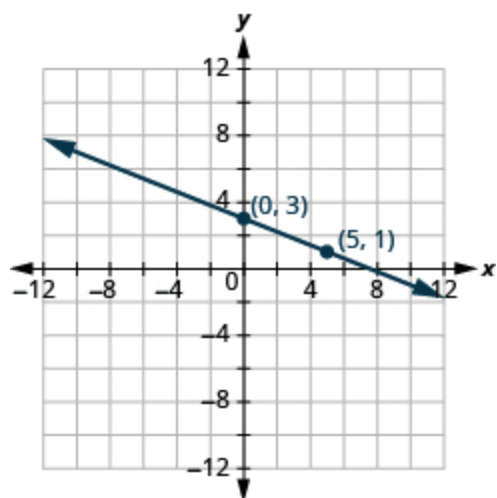
Exercise:

Problem: $(-3, -5); m = \frac{3}{2}$

Exercise:

Problem: $(0, 3); m = -\frac{2}{5}$

Solution:



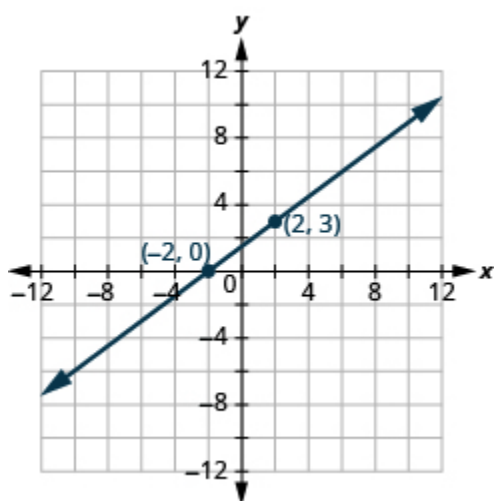
Exercise:

Problem: $(0, 5); m = -\frac{4}{3}$

Exercise:

Problem: $(-2, 0); m = \frac{3}{4}$

Solution:



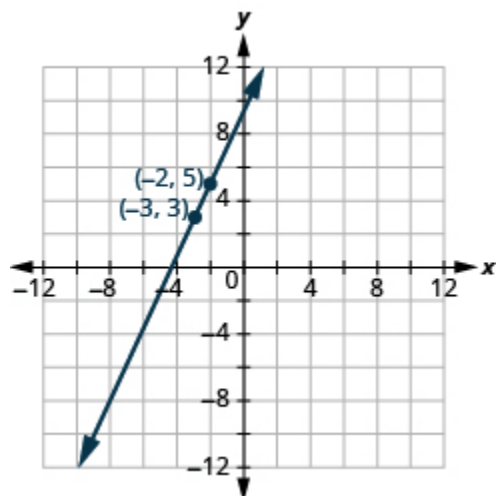
Exercise:

Problem: $(-1, 0); m = \frac{1}{5}$

Exercise:

Problem: $(-3, 3); m = 2$

Solution:



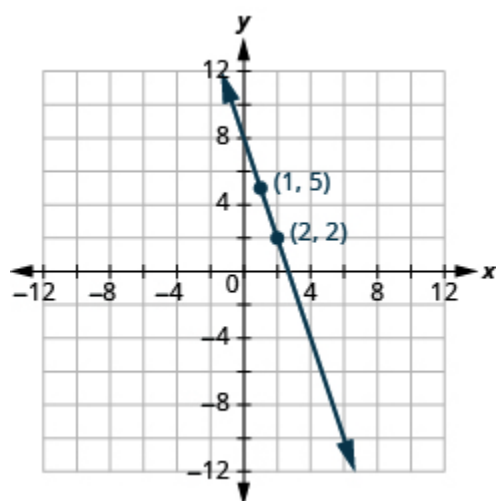
Exercise:

Problem: $(-4, 2); m = 4$

Exercise:

Problem: $(1, 5); m = -3$

Solution:



Exercise:

Problem: $(2, 3); m = -1$

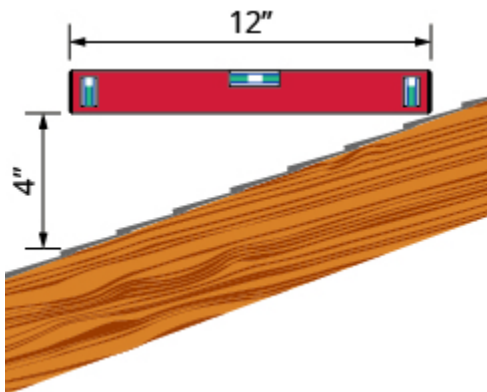
Solve Slope Applications

In the following exercises, solve these slope applications.

Exercise:

Problem:

Slope of a roof A fairly easy way to determine the slope is to take a 12-inch level and set it on one end on the roof surface. Then take a tape measure or ruler, and measure from the other end of the level down to the roof surface. You can use these measurements to calculate the slope of the roof. What is the slope of the roof in this picture?

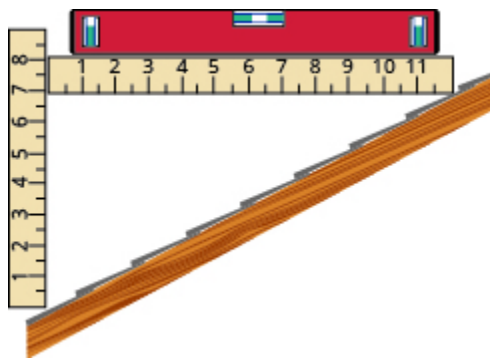


Solution:

$$\frac{1}{3}$$

Exercise:

Problem: What is the slope of the roof shown?



Exercise:

Problem:

Road grade A local road has a grade of 6%. The grade of a road is its slope expressed as a percent.

- Ⓐ Find the slope of the road as a fraction and then simplify the fraction.
- Ⓑ What rise and run would reflect this slope or grade?

Solution:

- Ⓐ $\frac{3}{50}$ Ⓑ rise = 3; run = 50

Exercise:

Problem:

Highway grade A local road rises 2 feet for every 50 feet of highway.

- Ⓐ What is the slope of the highway?
- Ⓑ The grade of a highway is its slope expressed as a percent. What is the grade of this highway?

Exercise:

Problem:

Wheelchair ramp The rules for wheelchair ramps require a maximum 1 inch rise for a 12 inch run.

- Ⓐ How long must the ramp be to accommodate a 24-inch rise to the door?
- Ⓑ Draw a model of this ramp.

Solution:

- Ⓐ 288 inches (24 feet)
- Ⓑ Models will vary.

Exercise:

Problem:

Wheelchair ramp A 1-inch rise for a 16-inch run makes it easier for the wheelchair rider to ascend the ramp.

- Ⓐ How long must the ramp be to easily accommodate a 24-inch rise to the door?
- Ⓑ Draw a model of this ramp.

Writing Exercises

Exercise:

Problem: What does the sign of the slope tell you about a line?

Solution:

Answers will vary.

Exercise:

Problem:

How does the graph of a line with slope $m = \frac{1}{2}$ differ from the graph of a line with slope $m = 2$?

Exercise:

Problem: Why is the slope of a vertical line undefined?

Solution:

Answers will vary.

Exercise:

Problem:

Explain how you can graph a line given a point and its slope.

Self Check

Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
use geoboards to model slope.			
find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Chapter Review Exercises

Use the Rectangular Coordinate System

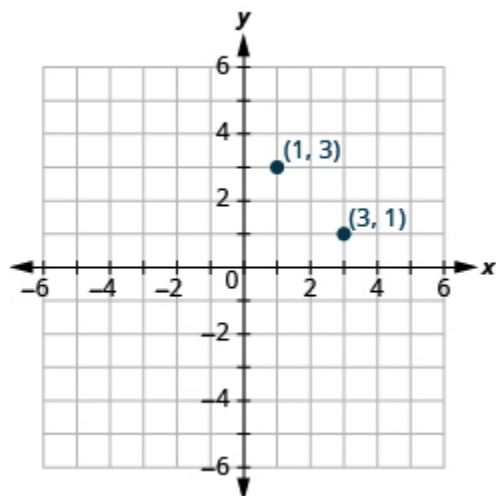
Plot Points in a Rectangular Coordinate System

In the following exercises, plot each point in a rectangular coordinate system.

Exercise:

Problem: $(1, 3)$, $(3, 1)$

Solution:



Exercise:

Problem: $(2, 5)$, $(5, 2)$

In the following exercises, plot each point in a rectangular coordinate system and identify the quadrant in which the point is located.

Exercise:

Problem:

- Ⓐ $(-1, -5)$
- Ⓑ $(-3, 4)$
- Ⓒ $(2, -3)$
- Ⓓ $(1, \frac{5}{2})$

Solution:

- Ⓐ III
- Ⓑ II
- Ⓒ IV
- Ⓓ I

Exercise:

Problem:

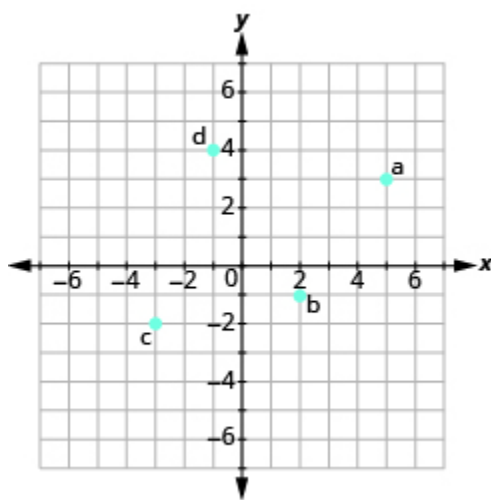
- Ⓐ $(3, -2)$
- Ⓑ $(-4, -1)$
- Ⓒ $(-5, 4)$
- Ⓓ $(2, \frac{10}{3})$

Identify Points on a Graph

In the following exercises, name the ordered pair of each point shown in the rectangular coordinate system.

Exercise:

Problem:

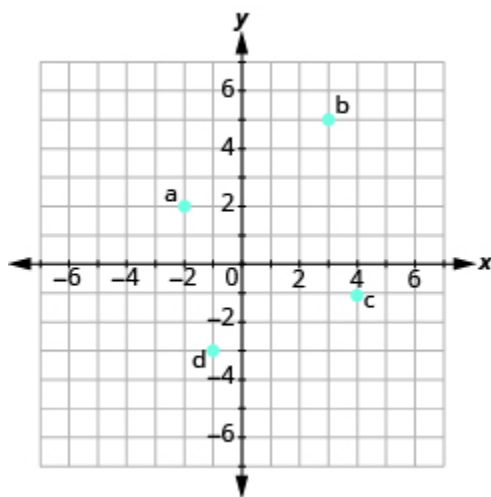


Solution:

- Ⓐ $(5, 3)$
- Ⓑ $(2, -1)$
- Ⓒ $(-3, -2)$
- Ⓓ $(-1, 4)$

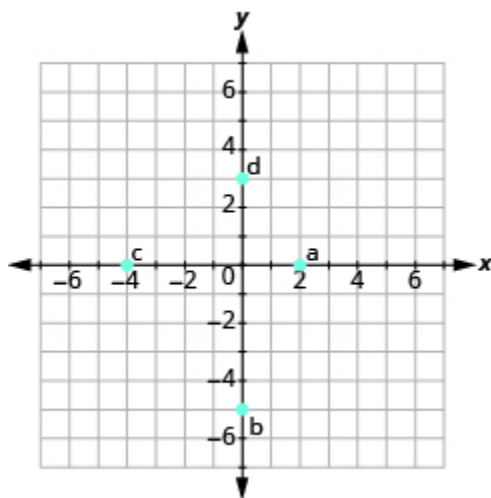
Exercise:

Problem:



Exercise:

Problem:

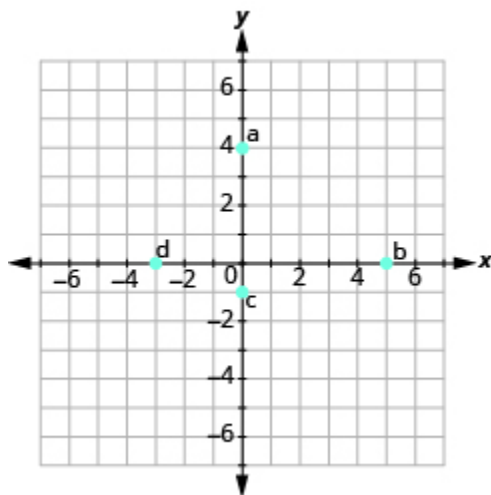


Solution:

- Ⓐ (2,0)
- Ⓑ (0,-5)
- Ⓒ (-4,0)
- Ⓓ (0,3)

Exercise:

Problem:



Verify Solutions to an Equation in Two Variables

In the following exercises, find the ordered pairs that are solutions to the given equation.

Exercise:

Problem: $5x + y = 10$

- Ⓐ (5, 1)
- Ⓑ (2, 0)
- Ⓒ (4, -10)

Solution:

Ⓑ, Ⓒ

Exercise:

Problem: $y = 6x - 2$

- Ⓐ (1, 4)
- Ⓑ (1/3, 0)
- Ⓒ (6, -2)

Complete a Table of Solutions to a Linear Equation in Two Variables

In the following exercises, complete the table to find solutions to each linear equation.

Exercise:

Problem: $y = 4x - 1$

x	y	(x, y)
0		
1		
-2		

Solution:

x	y	(x, y)
-----	-----	----------

x	y	(x, y)
0	-1	$(0, -1)$
1	3	$(1, 3)$
-2	-9	$(-2, -9)$

Exercise:

Problem: $y = -\frac{1}{2}x + 3$

x	y	(x, y)
0		
1		
-2		

Exercise:

Problem: $x + 2y = 5$

x	y	(x, y)
	0	
1		
-1		

Solution:

x	y	(x, y)
5	0	$(5, 0)$
1	2	$(1, 2)$
-1	3	$(-1, 3)$

Exercise:

Problem: $3x - 2y = 6$

x	y	(x, y)
-----	-----	----------

x	y	(x, y)
0		
	0	
-2		

Find Solutions to a Linear Equation in Two Variables

In the following exercises, find three solutions to each linear equation.

Exercise:

Problem: $x + y = 3$

Solution:

Answers will vary.

Exercise:

Problem: $x + y = -4$

Exercise:

Problem: $y = 3x + 1$

Solution:

Answers will vary.

Exercise:

Problem: $y = -x - 1$

Graphing Linear Equations

Recognize the Relation Between the Solutions of an Equation and its Graph

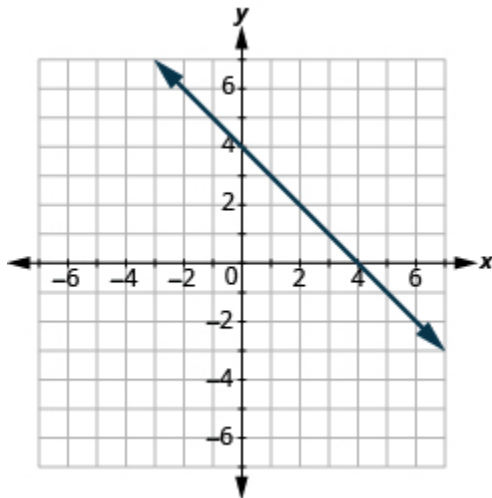
In the following exercises, for each ordered pair, decide

- Ⓐ if the ordered pair is a solution to the equation.
- Ⓑ if the point is on the line.

Exercise:

Problem: $y = -x + 4$

- Ⓐ $(0, 4)$
- Ⓑ $(-1, 3)$
- Ⓒ $(2, 2)$
- Ⓓ $(-2, 6)$



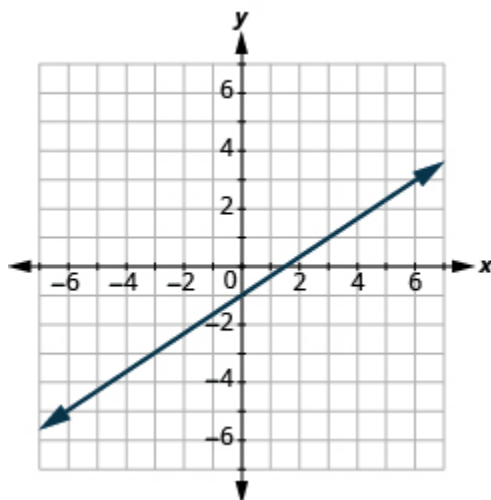
Solution:

- Ⓐ yes Ⓑ no Ⓒ yes Ⓓ yes
- Ⓐ yes Ⓑ no Ⓒ yes Ⓓ yes

Exercise:

Problem: $y = \frac{2}{3}x - 1$

- Ⓐ $(0, -1)$
- Ⓑ $(3, 1)$
- Ⓒ $(-3, -3)$
- Ⓓ $(6, 4)$



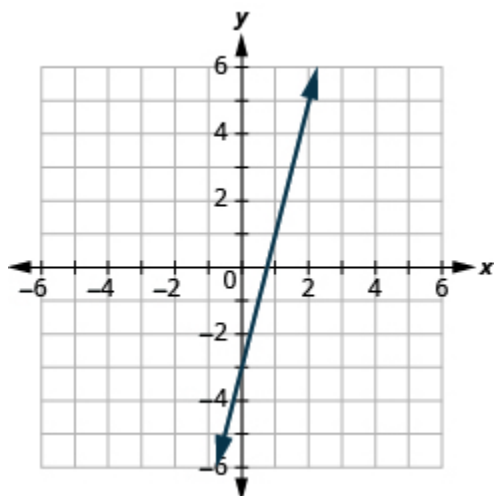
Graph a Linear Equation by Plotting Points

In the following exercises, graph by plotting points.

Exercise:

Problem: $y = 4x - 3$

Solution:



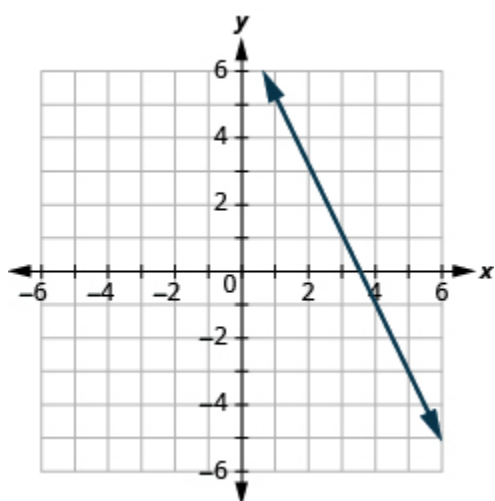
Exercise:

Problem: $y = -3x$

Exercise:

Problem: $2x + y = 7$

Solution:



Graph Vertical and Horizontal lines

In the following exercises, graph the vertical or horizontal lines.

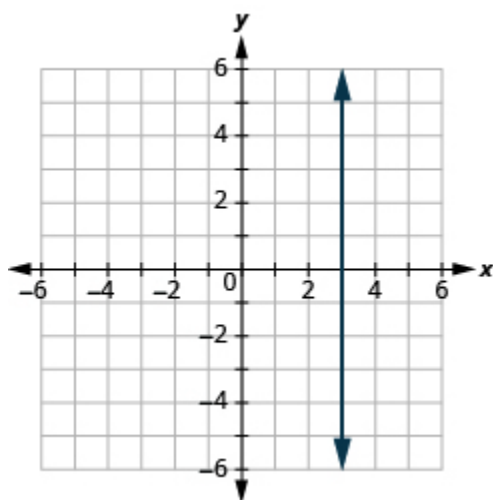
Exercise:

Problem: $y = -2$

Exercise:

Problem: $x = 3$

Solution:



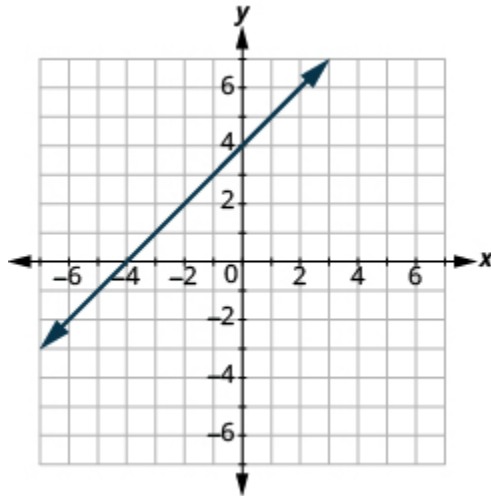
Graphing with Intercepts

Identify the Intercepts on a Graph

In the following exercises, find the x - and y -intercepts.

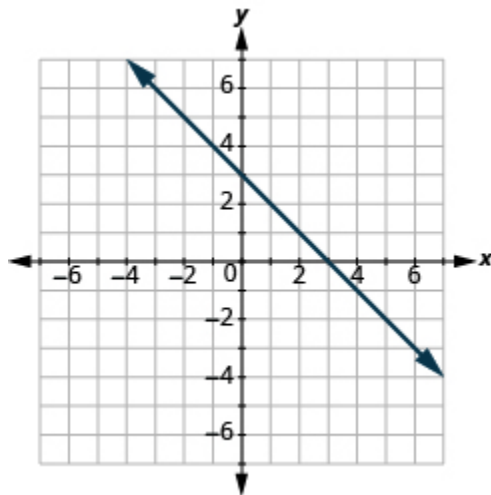
Exercise:

Problem:



Exercise:

Problem:



Solution:

$(0, 3)$ $(3, 0)$

Find the Intercepts from an Equation of a Line

In the following exercises, find the intercepts.

Exercise:

Problem: $x + y = 5$

Exercise:

Problem: $x - y = -1$

Solution:

$(-1,0) (0,1)$

Exercise:

Problem: $y = \frac{3}{4}x - 12$

Exercise:

Problem: $y = 3x$

Solution:

$(0,0)$

Graph a Line Using the Intercepts

In the following exercises, graph using the intercepts.

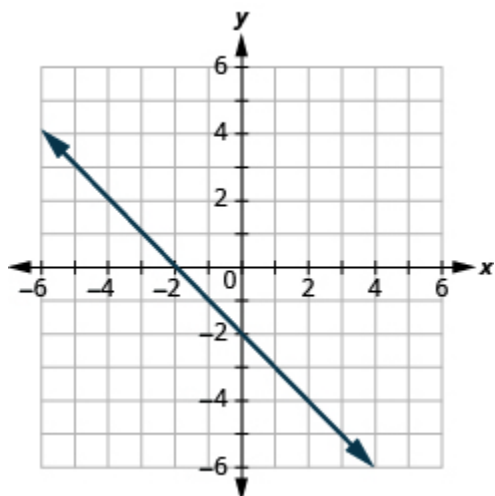
Exercise:

Problem: $-x + 3y = 3$

Exercise:

Problem: $x + y = -2$

Solution:



Choose the Most Convenient Method to Graph a Line

In the following exercises, identify the most convenient method to graph each line.

Exercise:

Problem: $x = 5$

Exercise:

Problem: $y = -3$

Solution:

horizontal line

Exercise:

Problem: $2x + y = 5$

Exercise:

Problem: $x - y = 2$

Solution:

intercepts

Exercise:

Problem: $y = \frac{1}{2}x + 2$

Exercise:

Problem: $y = \frac{3}{4}x - 1$

Solution:

plotting points

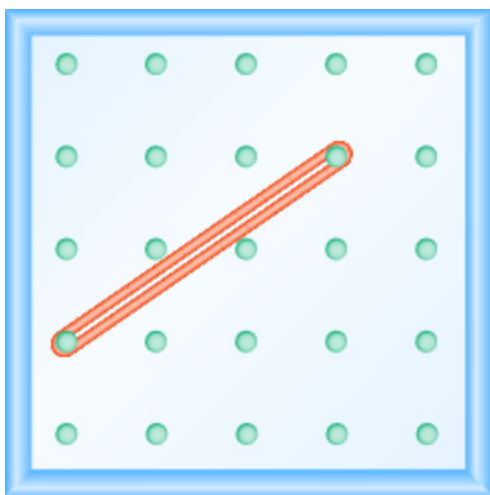
Understand Slope of a Line

Use Geoboards to Model Slope

In the following exercises, find the slope modeled on each geoboard.

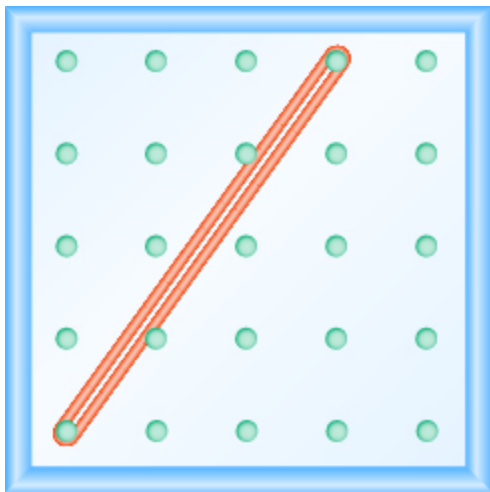
Exercise:

Problem:



Exercise:

Problem:

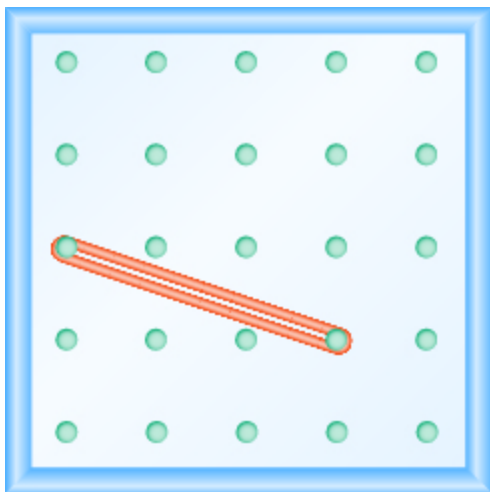


Solution:

$$\frac{4}{3}$$

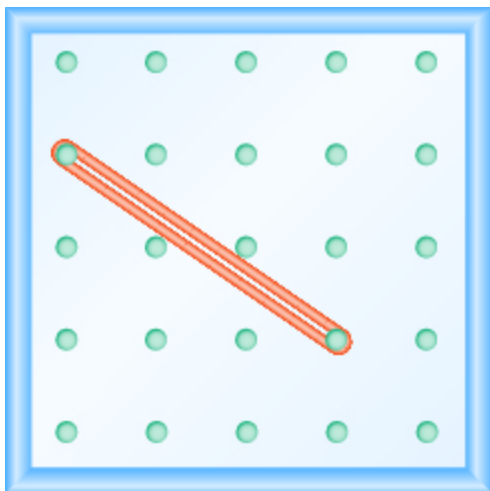
Exercise:

Problem:



Exercise:

Problem:



Solution:

$$-\frac{2}{3}$$

In the following exercises, model each slope. Draw a picture to show your results.

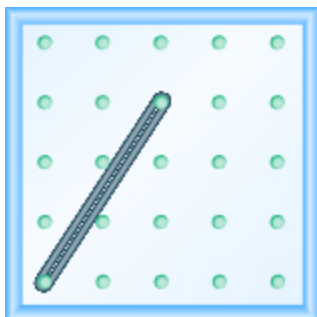
Exercise:

Problem: $\frac{1}{3}$

Exercise:

Problem: $\frac{3}{2}$

Solution:



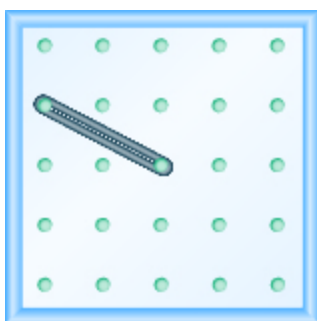
Exercise:

Problem: $-\frac{2}{3}$

Exercise:

Problem: $-\frac{1}{2}$

Solution:

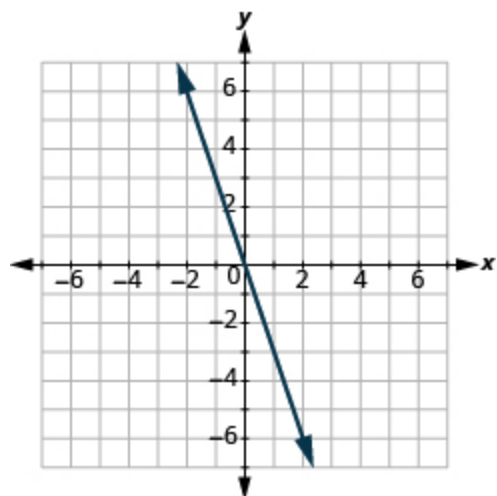


Find the Slope of a Line from its Graph

In the following exercises, find the slope of each line shown.

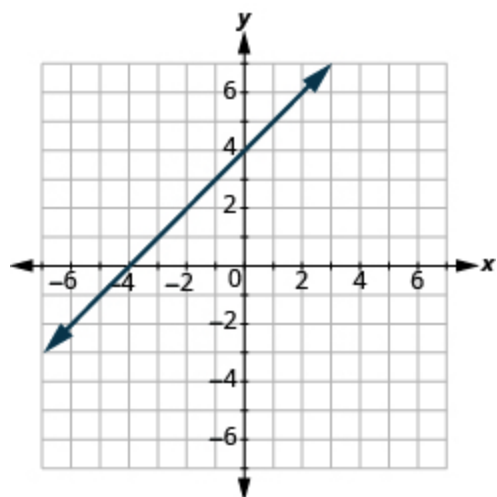
Exercise:

Problem:



Exercise:

Problem:

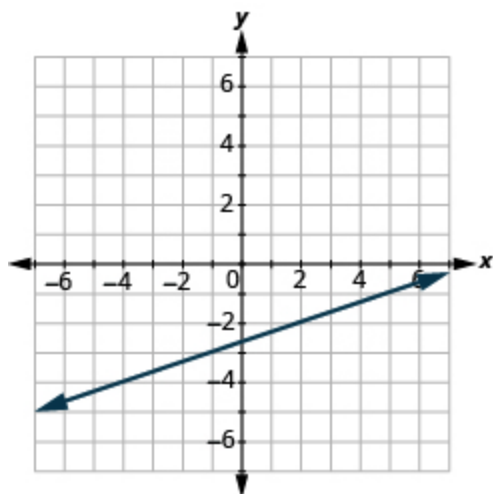


Solution:

1

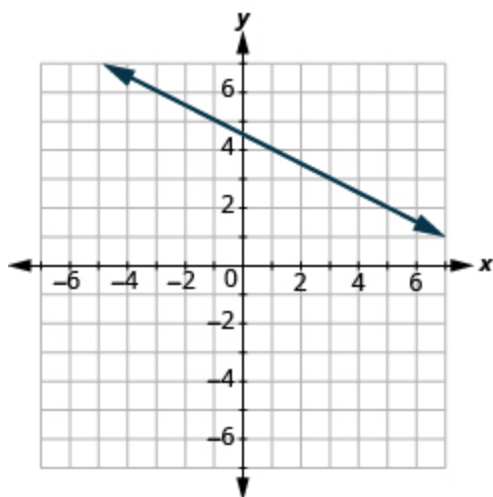
Exercise:

Problem:



Exercise:

Problem:



Solution:

$$-\frac{1}{2}$$

Find the Slope of Horizontal and Vertical Lines

In the following exercises, find the slope of each line.

Exercise:

Problem: $y = 2$

Exercise:

Problem: $x = 5$

Solution:

undefined

Exercise:

Problem: $x = -3$

Exercise:

Problem: $y = -1$

Solution:

0

Use the Slope Formula to find the Slope of a Line between Two Points

In the following exercises, use the slope formula to find the slope of the line between each pair of points.

Exercise:

Problem: $(2, 1), (4, 5)$

Exercise:

Problem: $(-1, -1), (0, -5)$

Solution:

-4

Exercise:

Problem: $(3, 5), (4, -1)$

Exercise:

Problem: $(-5, -2), (3, 2)$

Solution:

$$\frac{1}{2}$$

Graph a Line Given a Point and the Slope

In the following exercises, graph the line given a point and the slope.

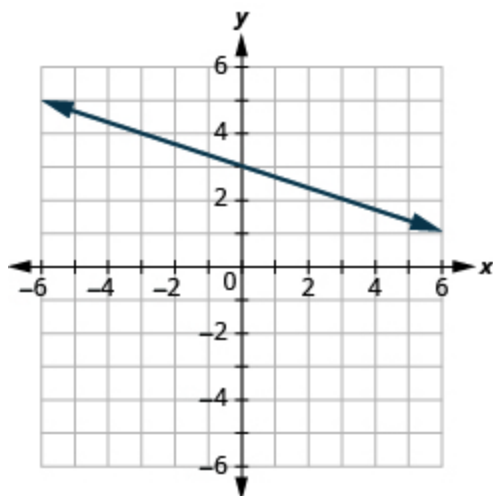
Exercise:

Problem: $(2, -2); m = \frac{5}{2}$

Exercise:

Problem: $(-3, 4); m = -\frac{1}{3}$

Solution:



Solve Slope Applications

In the following exercise, solve the slope application.

Exercise:

Problem: A roof has rise 10 feet and run 15 feet. What is its slope?

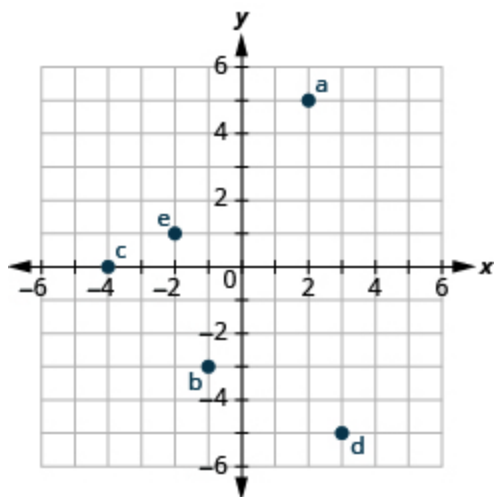
Chapter Practice Test

Exercise:

Problem: Plot and label these points:

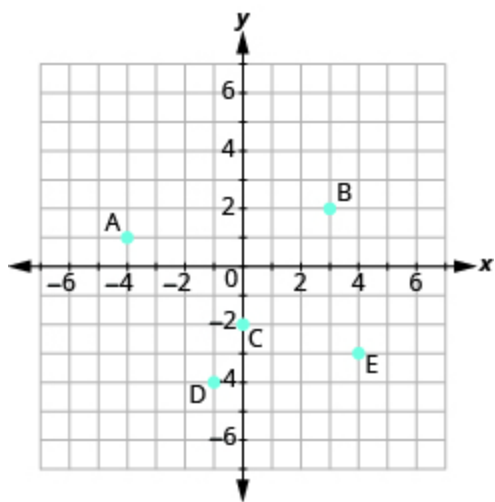
- Ⓐ $(2, 5)$
- Ⓑ $(-1, -3)$
- Ⓒ $(-4, 0)$
- Ⓓ $(3, -5)$
- Ⓔ $(-2, 1)$

Solution:



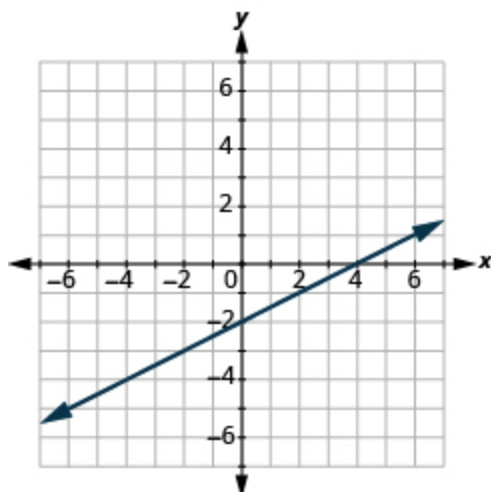
Exercise:

Problem: Name the ordered pair for each point shown.



Exercise:

Problem: Find the x -intercept and y -intercept on the line shown.



Solution:

$(4, 0), (0, -2)$

Exercise:

Problem:

Find the x -intercept and y -intercept of the equation $3x - y = 6$.

Exercise:

Problem:

Is $(1, 3)$ a solution to the equation $x + 4y = 12$? How do you know?

Solution:

no; $1 + 4 \cdot 3 \neq 12$

Exercise:

Problem:

Complete the table to find four solutions to the equation $y = -x + 1$.

x	y	(x, y)
0		
1		
3		
-2		

Exercise:

Problem:

Complete the table to find three solutions to the equation $4x + y = 8$

x	y	(x, y)
0		
	0	
3		

Solution:

--	--	--

x	y	(x, y)
0	8	$(0, 8)$
2	0	$(2, 0)$
3	-4	$(3, -4)$

In the following exercises, find three solutions to each equation and then graph each line.

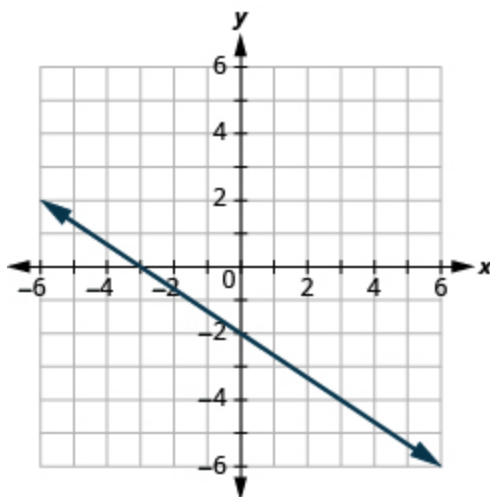
Exercise:

Problem: $y = -3x$

Exercise:

Problem: $2x + 3y = -6$

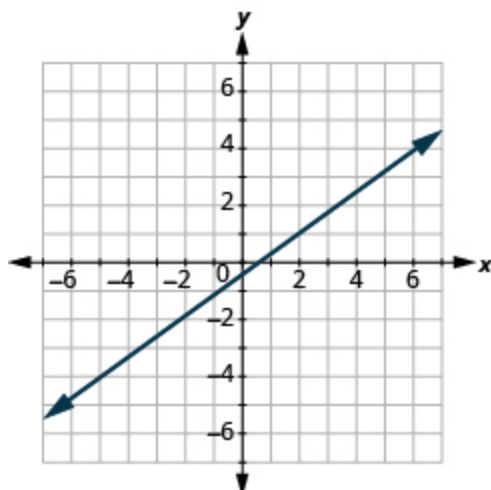
Solution:



In the following exercises, find the slope of each line.

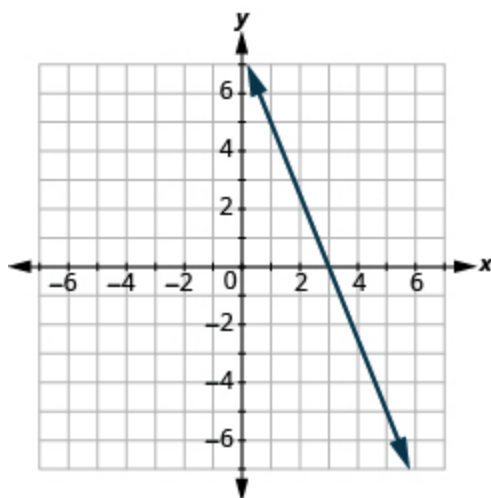
Exercise:

Problem:



Exercise:

Problem:



Solution:

$$-\frac{5}{2}$$

Exercise:

Problem:

Use the slope formula to find the slope of the line between $(0, -4)$ and $(5, 2)$.

Exercise:

Problem: Find the slope of the line $y = 2$.

Solution:

0

Exercise:

Problem: Graph the line passing through $(1, 1)$ with slope $m = \frac{3}{2}$.

Exercise:**Problem:**

A bicycle route climbs 20 feet for 1,000 feet of horizontal distance.
What is the slope of the route?

Solution:

$\frac{1}{50}$

Glossary

slope of a line

The slope of a line is $m = \frac{\text{rise}}{\text{run}}$. The rise measures the vertical change and the run measures the horizontal change.

Use the Slope–Intercept Form of an Equation of a Line

By the end of this section, you will be able to:

- Recognize the relation between the graph and the slope–intercept form of an equation of a line
- Identify the slope and y-intercept form of an equation of a line
- Graph a line using its slope and intercept
- Choose the most convenient method to graph a line
- Graph and interpret applications of slope–intercept
- Use slopes to identify parallel lines
- Use slopes to identify perpendicular lines

Note:

Before you get started, take this readiness quiz.

1. Add: $\frac{x}{4} + \frac{1}{4}$.

If you missed this problem, review [\[link\]](#).

2. Find the reciprocal of $\frac{3}{7}$.

If you missed this problem, review [\[link\]](#).

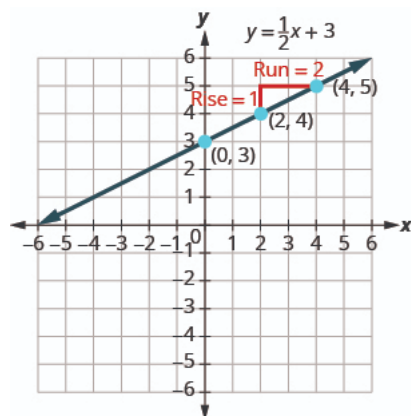
3. Solve $2x - 3y = 12$ for y .

If you missed this problem, review [\[link\]](#).

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

We have graphed linear equations by plotting points, using intercepts, recognizing horizontal and vertical lines, and using the point–slope method. Once we see how an equation in slope–intercept form and its graph are related, we'll have one more method we can use to graph lines.

In [Graph Linear Equations in Two Variables](#), we graphed the line of the equation $y = \frac{1}{2}x + 3$ by plotting points. See [\[link\]](#). Let's find the slope of this line.



The red lines show us the rise is 1 and the run is 2. Substituting into the slope formula:

Equation:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{1}{2}$$

What is the y -intercept of the line? The y -intercept is where the line crosses the y -axis, so y -intercept is $(0, 3)$. The equation of this line is:

$$y = \frac{1}{2}x + 3$$

Notice, the line has:

$$\text{slope } m = \frac{1}{2} \text{ and } y\text{-intercept } (0, 3)$$

When a linear equation is solved for y , the coefficient of the x term is the slope and the constant term is the y -coordinate of the y -intercept. We say that the equation $y = \frac{1}{2}x + 3$ is in slope–intercept form.

$$m = \frac{1}{2}; y\text{-intercept is } (0, 3)$$

$$y = \frac{1}{2}x + 3$$

$$y = mx + b$$

Note:

Slope-Intercept Form of an Equation of a Line

The **slope–intercept form** of an equation of a line with slope m and y -intercept, $(0, b)$ is,

Equation:

$$y = mx + b$$

Sometimes the slope–intercept form is called the “ y -form.”

Example:

Exercise:

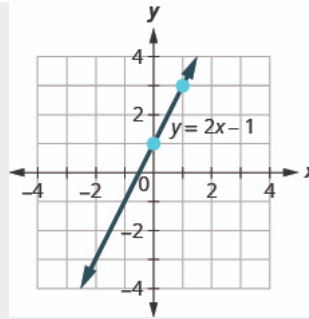
Problem: Use the graph to find the slope and y -intercept of the line, $y = 2x + 1$.

Compare these values to the equation $y = mx + b$.

Solution:

Solution

To find the slope of the line, we need to choose two points on the line. We’ll use the points $(0, 1)$ and $(1, 3)$.



Find the rise and run.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{2}{1}$$

$$m = 2$$

Find the y-intercept of the line.

The y-intercept is the point (0, 1).

We found slope $m = 2$ and y-intercept (0, 1).

$$y = 2x + 1$$

$$y = mx + b$$

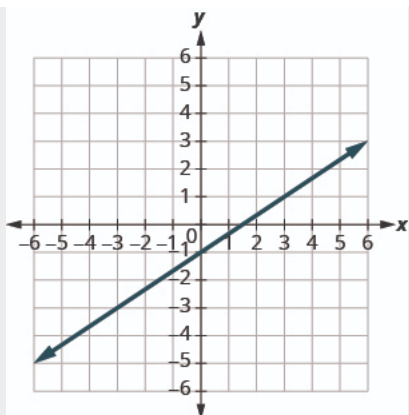
The slope is the same as the coefficient of x and the y -coordinate of the y -intercept is the same as the constant term.

Note:

Exercise:

Problem:

Use the graph to find the slope and y -intercept of the line $y = \frac{2}{3}x - 1$. Compare these values to the equation $y = mx + b$.



Solution:

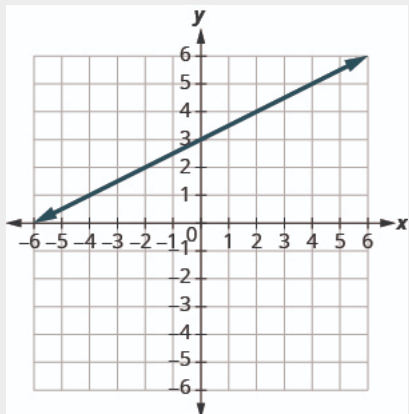
slope $m = \frac{2}{3}$ and y-intercept $(0, -1)$

Note:

Exercise:

Problem:

Use the graph to find the slope and y-intercept of the line $y = \frac{1}{2}x + 3$. Compare these values to the equation $y = mx + b$.



Solution:

slope $m = \frac{1}{2}$ and y-intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In [Understand Slope of a Line](#), we graphed a line using the slope and a point. When we are given an equation in slope-intercept form, we can use the y-intercept as the point, and then count out the slope from there. Let's practice finding the values of the slope and y-intercept from the equation of a line.

Example:

Exercise:

Problem: Identify the slope and y-intercept of the line with equation $y = -3x + 5$.

Solution:

Solution

We compare our equation to the slope–intercept form of the equation.

	$y = mx + b$
Write the equation of the line.	$y = -3x + 5$
Identify the slope.	$m = -3$
Identify the y-intercept.	y-intercept is (0, 5)

Note:

Exercise:

Problem: Identify the slope and y-intercept of the line $y = \frac{2}{5}x - 1$.

Solution:

$\frac{2}{5}; (0, -1)$

Note:

Exercise:

Problem: Identify the slope and y-intercept of the line $y = -\frac{4}{3}x + 1$.

Solution:

$-\frac{4}{3}; (0, 1)$

When an equation of a line is not given in slope–intercept form, our first step will be to solve the equation for y .

Example:

Exercise:

Problem: Identify the slope and y-intercept of the line with equation $x + 2y = 6$.

Solution:

Solution

This equation is not in slope–intercept form. In order to compare it to the slope–intercept form we must first solve the equation for y .

Solve for y .	$x + 2y = 6$
Subtract x from each side.	$2y = -x + 6$
Divide both sides by 2.	$\frac{2y}{2} = \frac{-x + 6}{2}$
Simplify.	$\frac{2y}{2} = \frac{-x}{2} + \frac{6}{2}$
(Remember: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$)	
Simplify.	$y = -\frac{1}{2}x + 3$
Write the slope–intercept form of the equation of the line.	$y = mx + b$
Write the equation of the line.	$y = -\frac{1}{2}x + 3$
Identify the slope.	$m = -\frac{1}{2}$
Identify the y-intercept.	y-intercept is $(0, 3)$

Note:

Exercise:

Problem: Identify the slope and y-intercept of the line $x + 4y = 8$.

Solution:

$$-\frac{1}{4}; (0, 2)$$

Note:

Exercise:

Problem: Identify the slope and y-intercept of the line $3x + 2y = 12$.

Solution:

$$-\frac{3}{2}; (0, 6)$$

Graph a Line Using its Slope and Intercept

Now that we know how to find the slope and y-intercept of a line from its equation, we can graph the line by plotting the y-intercept and then using the slope to find another point.

Example:

How to Graph a Line Using its Slope and Intercept

Exercise:

Problem: Graph the line of the equation $y = 4x - 2$ using its slope and y-intercept.

Solution:

Solution

Step 1. Find the slope-intercept form of the equation.

This equation is in slope-intercept form.

$$y = 4x - 2$$

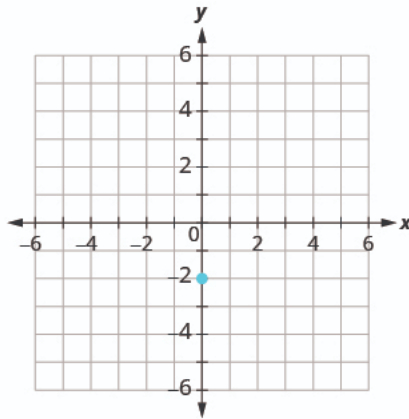
Step 2. Identify the slope and y-intercept.

Use $y = mx + b$
Find the slope.
Find the y-intercept.

$$\begin{aligned}y &= mx + b \\y &= 4x + (-2) \\m &= 4 \\b &= -2, (0, -2)\end{aligned}$$

Step 3. Plot the y-intercept.

Plot $(0, -2)$.



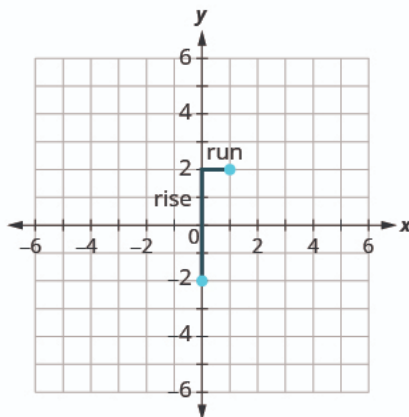
Step 4. Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.

Identify the rise and the run.

$$m = 4$$
$$\frac{\text{rise}}{\text{run}} = \frac{4}{1}$$
$$\text{rise} = 4$$
$$\text{run} = 1$$

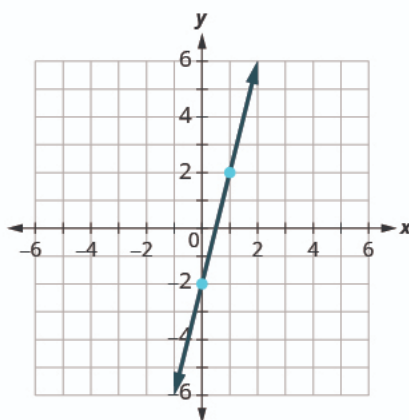
Step 5. Starting at the y-intercept, count out the rise and run to mark the second point.

Start at $(0, -2)$ and count the rise and the run.
Up 4, right 1.



Step 6. Connect the points with a line.

Connect the two points with a line.

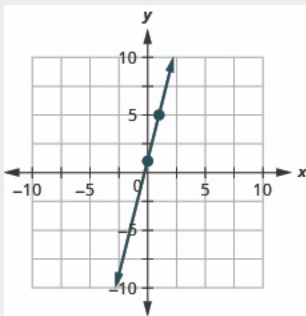


Note:

Exercise:

Problem: Graph the line of the equation $y = 4x + 1$ using its slope and y-intercept.

Solution:

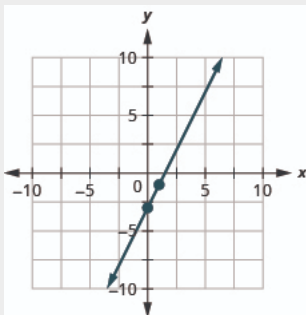


Note:

Exercise:

Problem: Graph the line of the equation $y = 2x - 3$ using its slope and y-intercept.

Solution:



Note:

Graph a line using its slope and y-intercept.

Find the slope-intercept form of the equation of the line.

Identify the slope and y-intercept.

Plot the y-intercept.

Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.

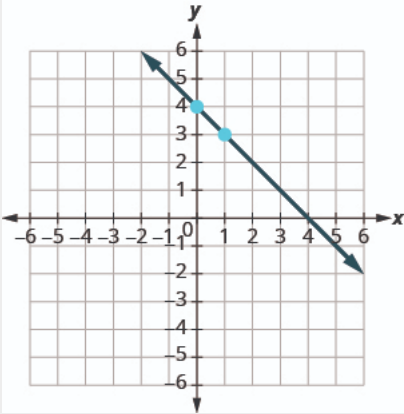
Starting at the y-intercept, count out the rise and run to mark the second point.

Connect the points with a line.

Example:
Exercise:

Problem: Graph the line of the equation $y = -x + 4$ using its slope and y-intercept.

Solution:
Solution

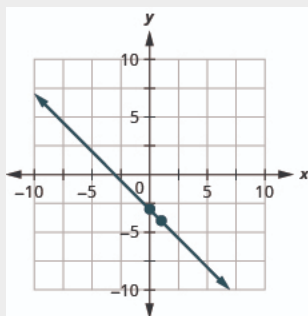
	$y = mx + b$
The equation is in slope–intercept form.	$y = -x + 4$
Identify the slope and y-intercept.	$m = -1$
	y-intercept is (0, 4)
Plot the y-intercept.	See graph below.
Identify the rise and the run.	$m = \frac{-1}{1}$
Count out the rise and run to mark the second point.	rise -1, run 1
Draw the line.	
To check your work, you can find another point on the line and make sure it is a solution of the equation. In the graph we see the line goes through (4, 0).	
Check. $y = -x + 4$ $\quad ?$ $0 = -4 + 4$ $0 = 0 \checkmark$	

Note:

Exercise:

Problem: Graph the line of the equation $y = -x - 3$ using its slope and y-intercept.

Solution:

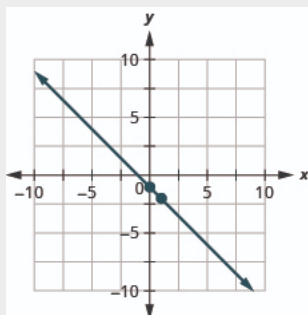


Note:

Exercise:

Problem: Graph the line of the equation $y = -x - 1$ using its slope and y-intercept.

Solution:



Example:

Exercise:

Problem: Graph the line of the equation $y = -\frac{2}{3}x - 3$ using its slope and y-intercept.

Solution:

Solution

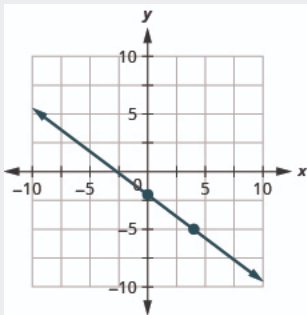
	$y = mx + b$
The equation is in slope–intercept form.	$y = -\frac{2}{3}x - 3$
Identify the slope and y-intercept.	$m = -\frac{2}{3}$; y-intercept is $(0, -3)$
Plot the y-intercept.	See graph below.
Identify the rise and the run.	
Count out the rise and run to mark the second point.	
Draw the line.	

Note:	
Exercise:	
Problem: Graph the line of the equation $y = -\frac{5}{2}x + 1$ using its slope and y-intercept.	
Solution:	

Note:	
Exercise:	

Problem: Graph the line of the equation $y = -\frac{3}{4}x - 2$ using its slope and y-intercept.

Solution:



Example:

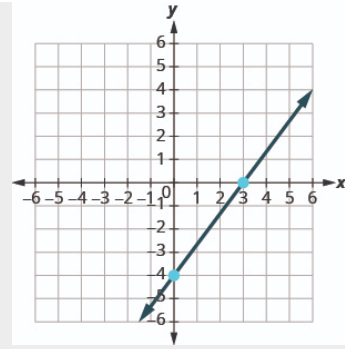
Exercise:

Problem: Graph the line of the equation $4x - 3y = 12$ using its slope and y-intercept.

Solution:

Solution

	$4x - 3y = 12$
Find the slope–intercept form of the equation.	$-3y = -4x + 12$
	$-\frac{3y}{3} = \frac{-4x+12}{-3}$
The equation is now in slope–intercept form.	$y = \frac{4}{3}x - 4$
Identify the slope and y-intercept.	$m = \frac{4}{3}$
	y-intercept is $(0, -4)$
Plot the y-intercept.	See graph below.
Identify the rise and the run; count out the rise and run to mark the second point.	
Draw the line.	

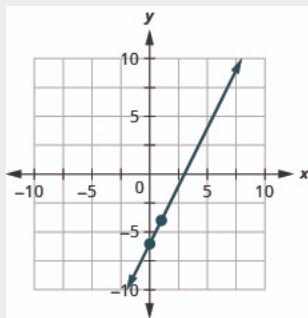


Note:

Exercise:

Problem: Graph the line of the equation $2x - y = 6$ using its slope and y-intercept.

Solution:

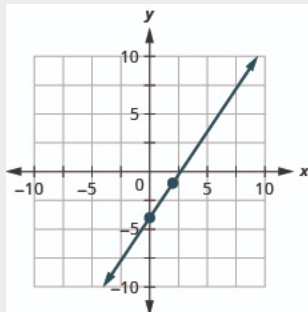


Note:

Exercise:

Problem: Graph the line of the equation $3x - 2y = 8$ using its slope and y-intercept.

Solution:



We have used a grid with x and y both going from about -10 to 10 for all the equations we've graphed so far. Not all linear equations can be graphed on this small grid. Often, especially in applications with real-world data, we'll need to extend the axes to bigger positive or smaller negative numbers.

Example:

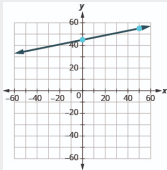
Exercise:

Problem: Graph the line of the equation $y = 0.2x + 45$ using its slope and y-intercept.

Solution:

Solution

We'll use a grid with the axes going from about -80 to 80 .

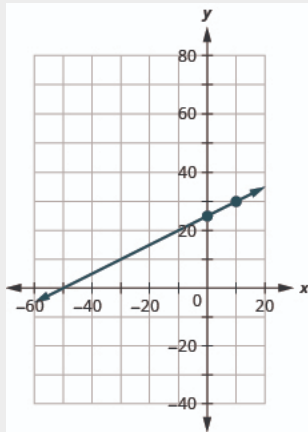
	$y = mx + b$
The equation is in slope–intercept form.	$y = 0.2x + 45$
Identify the slope and y-intercept.	$m = 0.2$
	The y-intercept is $(0, 45)$
Plot the y-intercept.	See graph below.
Count out the rise and run to mark the second point. The slope is $m = 0.2$; in fraction form this means $m = \frac{2}{10}$. Given the scale of our graph, it would be easier to use the equivalent fraction $m = \frac{10}{50}$.	
Draw the line.	

Note:

Exercise:

Problem: Graph the line of the equation $y = 0.5x + 25$ using its slope and y-intercept.

Solution:

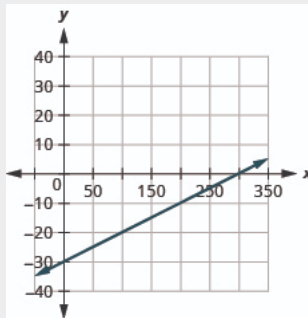


Note:

Exercise:

Problem: Graph the line of the equation $y = 0.1x - 30$ using its slope and y-intercept.

Solution:



Now that we have graphed lines by using the slope and y-intercept, let's summarize all the methods we have used to graph lines. See [\[link\]](#).

Methods to Graph Lines			
Point Plotting  Find three points. Plot the points, make sure they line up, then draw the line.	Slope-Intercept $y = mx + b$ Find the slope and y-intercept. Start at the y-intercept, then count the slope to get a second point.	Intercepts  Find the intercepts and a third point. Plot the points, make sure they line up, then draw the line.	Recognize Vertical and Horizontal Lines The equation has only one variable. $x = a$ vertical $y = b$ horizontal

Choose the Most Convenient Method to Graph a Line

Now that we have seen several methods we can use to graph lines, how do we know which method to use for a given equation?

While we could plot points, use the slope-intercept form, or find the intercepts for *any* equation, if we recognize the most convenient way to graph a certain type of equation, our work will be easier. Generally, plotting points is not the most efficient way to graph a line. We saw better methods in sections 4.3, 4.4, and earlier in this section. Let's look for some patterns to help determine the most convenient method to graph a line.

Here are six equations we graphed in this chapter, and the method we used to graph each of them.

Equation:

	Equation	Method
#1	$x = 2$	Vertical line
#2	$y = 4$	Horizontal line
#3	$-x + 2y = 6$	Intercepts
#4	$4x - 3y = 12$	Intercepts
#5	$y = 4x - 2$	Slope-intercept
#6	$y = -x + 4$	Slope-intercept

Equations #1 and #2 each have just one variable. Remember, in equations of this form the value of that one variable is constant; it does not depend on the value of the other variable. Equations of this form have graphs that are vertical or horizontal lines.

In equations #3 and #4, both x and y are on the same side of the equation. These two equations are of the form $Ax + By = C$. We substituted $y = 0$ to find the x -intercept and $x = 0$ to find the y -intercept, and then found a third point by choosing another value for x or y .

Equations #5 and #6 are written in slope-intercept form. After identifying the slope and y -intercept from the equation we used them to graph the line.

This leads to the following strategy.

Note:

Strategy for Choosing the Most Convenient Method to Graph a Line
Consider the form of the equation.

- If it only has one variable, it is a vertical or horizontal line.
 - $x = a$ is a vertical line passing through the x -axis at a .
 - $y = b$ is a horizontal line passing through the y -axis at b .
- If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y -intercept.
 - Identify the slope and y -intercept and then graph.
- If the equation is of the form $Ax + By = C$, find the intercepts.
 - Find the x - and y -intercepts, a third point, and then graph.

Example:

Exercise:

Problem: Determine the most convenient method to graph each line.

- Ⓐ $y = -6$ Ⓑ $5x - 3y = 15$ Ⓒ $x = 7$ Ⓓ $y = \frac{2}{5}x - 1$.

Solution:

Solution

Ⓐ $y = -6$

This equation has only one variable, y . Its graph is a horizontal line crossing the y -axis at -6 .

Ⓑ $5x - 3y = 15$

This equation is of the form $Ax + By = C$. The easiest way to graph it will be to find the intercepts and one more point.

Ⓒ $x = 7$

There is only one variable, x . The graph is a vertical line crossing the x -axis at 7 .

Ⓓ $y = \frac{2}{5}x - 1$

Since this equation is in $y = mx + b$ form, it will be easiest to graph this line by using the slope and y -intercept.

Note:

Exercise:

Problem:

Determine the most convenient method to graph each line: Ⓐ $3x + 2y = 12$ Ⓑ $y = 4$ Ⓒ $y = \frac{1}{5}x - 4$
Ⓓ $x = -7$.

Solution:

- Ⓐ intercepts Ⓑ horizontal line Ⓒ slope–intercept Ⓓ vertical line

Note:

Exercise:**Problem:**

Determine the most convenient method to graph each line: (a) $x = 6$ (b) $y = -\frac{3}{4}x + 1$ (c) $y = -8$ (d) $4x - 3y = -1$.

Solution:

(a) vertical line (b) slope–intercept (c) horizontal line (d) intercepts

Graph and Interpret Applications of Slope–Intercept

Many real-world applications are modeled by linear equations. We will take a look at a few applications here so you can see how equations written in slope–intercept form relate to real-world situations.

Usually when a linear equation models a real-world situation, different letters are used for the variables, instead of x and y . The variable names remind us of what quantities are being measured.

Example:**Exercise:****Problem:**

The equation $F = \frac{9}{5}C + 32$ is used to convert temperatures, C , on the Celsius scale to temperatures, F , on the Fahrenheit scale.

- (a) Find the Fahrenheit temperature for a Celsius temperature of 0.
- (b) Find the Fahrenheit temperature for a Celsius temperature of 20.
- (c) Interpret the slope and F -intercept of the equation.
- (d) Graph the equation.

Solution:**Solution**

(a)

Find the Fahrenheit temperature for a Celsius temperature of 0.

Find F when $C = 0$.

Simplify.

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(0) + 32$$

$$F = 32$$

(b)

Find the Fahrenheit temperature for a Celsius temperature of 20.

Find F when $C = 20$.

Simplify.

Simplify.

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(20) + 32$$

$$F = 36 + 32$$

$$F = 68$$

- (c) Interpret the slope and F -intercept of the equation.

Even though this equation uses F and C , it is still in slope–intercept form.

$$y = mx + b$$

$$F = mC + b$$

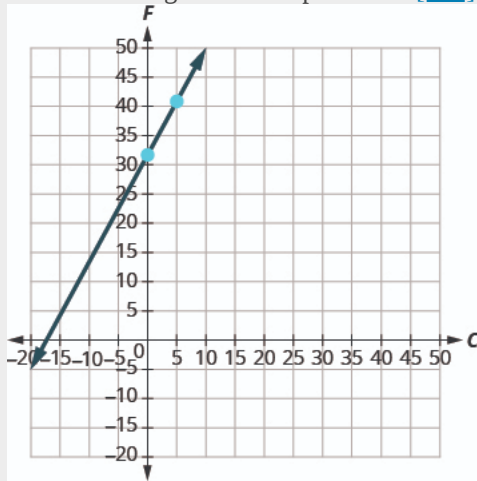
$$F = \frac{9}{5}C + 32$$

The slope, $\frac{9}{5}$, means that the temperature Fahrenheit (F) increases 9 degrees when the temperature Celsius (C) increases 5 degrees.

The F -intercept means that when the temperature is 0° on the Celsius scale, it is 32° on the Fahrenheit scale.

④ Graph the equation.

We'll need to use a larger scale than our usual. Start at the F -intercept $(0, 32)$ then count out the rise of 9 and the run of 5 to get a second point. See [\[link\]](#).



Note:

Exercise:

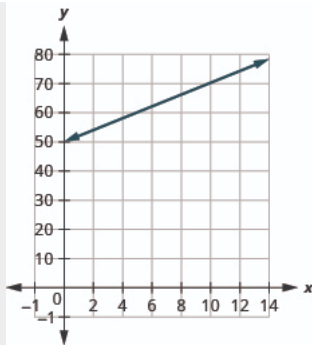
Problem:

The equation $h = 2s + 50$ is used to estimate a woman's height in inches, h , based on her shoe size, s .

- ① Estimate the height of a child who wears women's shoe size 0.
- ② Estimate the height of a woman with shoe size 8.
- ③ Interpret the slope and h -intercept of the equation.
- ④ Graph the equation.

Solution:

- ① 50 inches
- ② 66 inches
- ③ The slope, 2, means that the height, h , increases by 2 inches when the shoe size, s , increases by 1. The h -intercept means that when the shoe size is 0, the height is 50 inches.
- ④



Note:

Exercise:

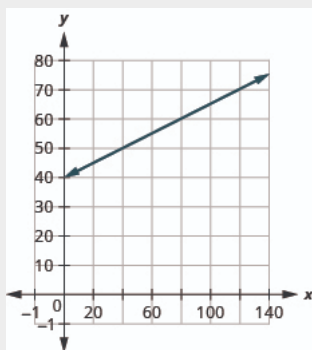
Problem:

The equation $T = \frac{1}{4}n + 40$ is used to estimate the temperature in degrees Fahrenheit, T , based on the number of cricket chirps, n , in one minute.

- (a) Estimate the temperature when there are no chirps.
- (b) Estimate the temperature when the number of chirps in one minute is 100.
- (c) Interpret the slope and T -intercept of the equation.
- (d) Graph the equation.

Solution:

- (a) 40 degrees
- (b) 65 degrees
- (c) The slope, $\frac{1}{4}$, means that the temperature Fahrenheit (F) increases 1 degree when the number of chirps, n , increases by 4. The T -intercept means that when the number of chirps is 0, the temperature is 40° .
- (d)



The cost of running some types business has two components—a *fixed cost* and a *variable cost*. The fixed cost is always the same regardless of how many units are produced. This is the cost of rent, insurance, equipment, advertising, and other items that must be paid regularly. The variable cost depends on the number of units produced. It is for the material and labor needed to produce each item.

Example:
Exercise:

Problem:

Stella has a home business selling gourmet pizzas. The equation $C = 4p + 25$ models the relation between her weekly cost, C , in dollars and the number of pizzas, p , that she sells.

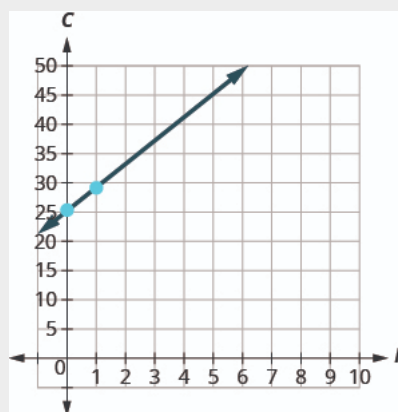
- Ⓐ Find Stella’s cost for a week when she sells no pizzas.
- Ⓑ Find the cost for a week when she sells 15 pizzas.
- Ⓒ Interpret the slope and C -intercept of the equation.
- Ⓓ Graph the equation.

Solution:
Solution

Ⓐ Find Stella's cost for a week when she sells no pizzas.	$C = 4p + 25$
Find C when $p = 0$.	$C = 4(0) + 25$
Simplify.	$C = 25$
	Stella's fixed cost is \$25 when she sells no pizzas.
Ⓑ Find the cost for a week when she sells 15 pizzas.	$C = 4p + 25$
Find C when $p = 15$.	$C = 4(15) + 25$
Simplify.	$C = 60 + 25$
	$C = 85$
	Stella's costs are \$85 when she sells 15 pizzas.
Ⓒ Interpret the slope and C -intercept of the equation.	$y = mx + b$ $C = 4p + 25$
	The slope, 4, means that the cost increases by \$4

for each pizza Stella sells. The C -intercept means that even when Stella sells no pizzas, her costs for the week are \$25.

- ④ Graph the equation. We'll need to use a larger scale than our usual. Start at the C -intercept (0, 25) then count out the rise of 4 and the run of 1 to get a second point.



Note:

Exercise:

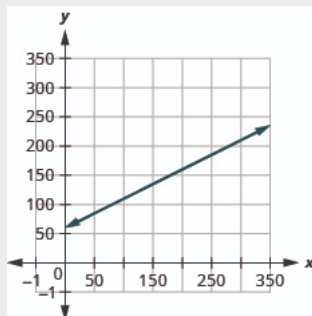
Problem:

Sam drives a delivery van. The equation $C = 0.5m + 60$ models the relation between his weekly cost, C , in dollars and the number of miles, m , that he drives.

- ① Find Sam's cost for a week when he drives 0 miles.
- ② Find the cost for a week when he drives 250 miles.
- ③ Interpret the slope and C -intercept of the equation.
- ④ Graph the equation.

Solution:

- ① \$60
- ② \$185
- ③ The slope, 0.5, means that the weekly cost, C , increases by \$0.50 when the number of miles driven, n , increases by 1. The C -intercept means that when the number of miles driven is 0, the weekly cost is \$60
- ④



Note:

Exercise:

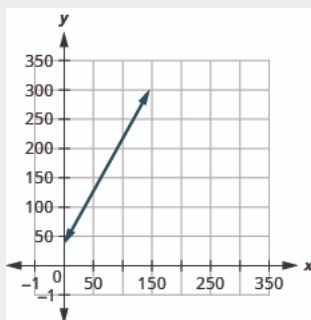
Problem:

Loreen has a calligraphy business. The equation $C = 1.8n + 35$ models the relation between her weekly cost, C , in dollars and the number of wedding invitations, n , that she writes.

- (a) Find Loreen's cost for a week when she writes no invitations.
- (b) Find the cost for a week when she writes 75 invitations.
- (c) Interpret the slope and C -intercept of the equation.
- (d) Graph the equation.

Solution:

- (a) \$35
 - (b) \$170
 - (c) The slope, 1.8, means that the weekly cost, C , increases by \$1.80 when the number of invitations, n , increases by 1.80.
- The C -intercept means that when the number of invitations is 0, the weekly cost is \$35.;
- (d)

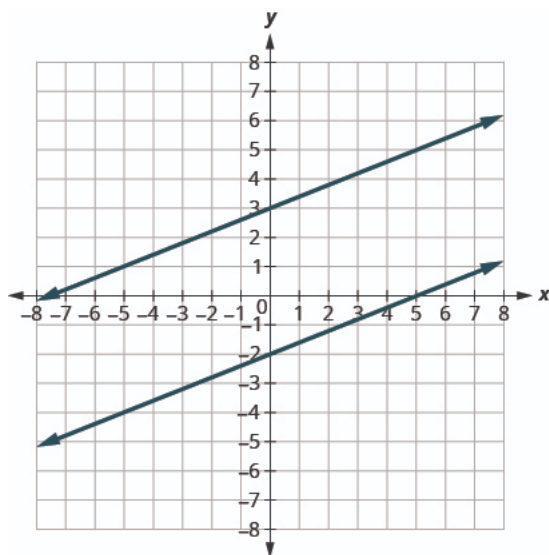


Use Slopes to Identify Parallel Lines

The slope of a line indicates how steep the line is and whether it rises or falls as we read it from left to right. Two lines that have the same slope are called parallel lines. Parallel lines never intersect.

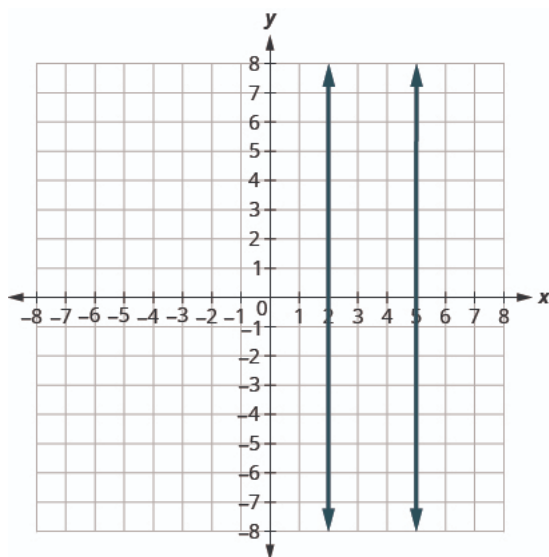


We say this more formally in terms of the rectangular coordinate system. Two lines that have the same slope and different y -intercepts are called **parallel lines**. See [\[link\]](#).



Verify that both lines have the same slope,
 $m = \frac{2}{5}$, and different y-intercepts.

What about vertical lines? The slope of a vertical line is undefined, so vertical lines don't fit in the definition above. We say that vertical lines that have different x-intercepts are parallel. See [\[link\]](#).



Vertical lines with different x-intercepts are parallel.

Note:

Parallel Lines

Parallel lines are lines in the same plane that do not intersect.

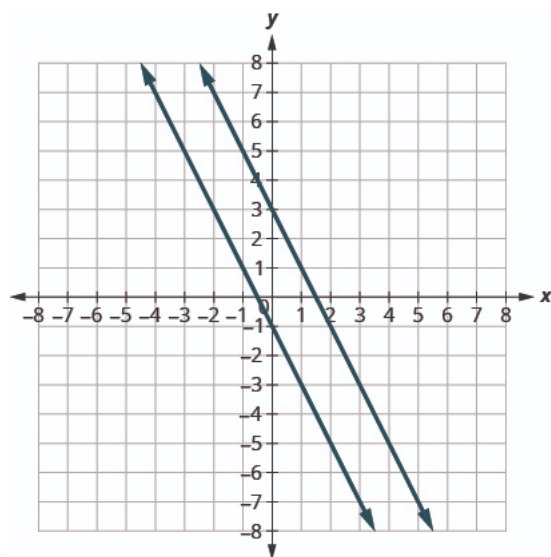
- Parallel lines have the same slope and different y-intercepts.
- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x-intercepts.

Let's graph the equations $y = -2x + 3$ and $2x + y = -1$ on the same grid. The first equation is already in slope-intercept form: $y = -2x + 3$. We solve the second equation for y :

Equation:

$$\begin{aligned}2x + y &= -1 \\ y &= -2x - 1\end{aligned}$$

Graph the lines.



Notice the lines look parallel. What is the slope of each line? What is the y-intercept of each line?

Equation:

$$\begin{aligned}y &= mx + b \\ y &= -2x + 3 \\ m &= -2 \\ b &= 3, (0, 3)\end{aligned}$$

$$\begin{aligned}y &= mx + b \\ y &= -2x - 1 \\ m &= -2 \\ b &= -1, (0, -1)\end{aligned}$$

The slopes of the lines are the same and the y-intercept of each line is different. So we know these lines are parallel.

Since parallel lines have the same slope and different y-intercepts, we can now just look at the slope-intercept form of the equations of lines and decide if the lines are parallel.

Example:

Exercise:

Problem: Use slopes and y-intercepts to determine if the lines $3x - 2y = 6$ and $y = \frac{3}{2}x + 1$ are parallel.

Solution:
Solution

Solve the first equation for y .

$$3x - 2y = 6 \quad \text{and} \quad y = \frac{3}{2}x + 1$$

$$-2y = -3x + 6$$

$$\frac{-2y}{-2} = \frac{-3x+6}{-2}$$

The equation is now in slope-intercept form.

$$y = \frac{3}{2}x - 3$$

The equation of the second line is already in slope-intercept form.

$$y = \frac{3}{2}x + 1$$

Identify the slope and y-intercept of both lines.

$$y = \frac{3}{2}x - 3$$

$$y = \frac{3}{2}x + 1$$

$$y = mx + b$$

$$y = mx + b$$

$$m = \frac{3}{2}$$

$$m = \frac{3}{2}$$

$$y\text{-intercept is } (0, -3)$$

$$y\text{-intercept is } (0, 1)$$

The lines have the same slope and different y-intercepts and so they are parallel. You may want to graph the lines to confirm whether they are parallel.

Note:**Exercise:**

Problem: Use slopes and y-intercepts to determine if the lines $2x + 5y = 5$ and $y = -\frac{2}{5}x - 4$ are parallel.

Solution:

parallel

Note:**Exercise:**

Problem: Use slopes and y-intercepts to determine if the lines $4x - 3y = 6$ and $y = \frac{4}{3}x - 1$ are parallel.

Solution:

parallel

Example:**Exercise:**

Problem: Use slopes and y-intercepts to determine if the lines $y = -4$ and $y = 3$ are parallel.

Solution:
Solution

Write each equation in slope-intercept form.
Since there is no x term we write $0x$.
Identify the slope and y -intercept of both lines.

$y = -4$	and	$y = 3$
$y = 0x - 4$		$y = 0x + 3$
$y = 0x - 4$		$y = 0x + 3$
$y = mx + b$		$y = mx + b$
$m = 0$		$m = 0$
y -intercept is $(0, -4)$		y -intercept is $(0, 3)$

The lines have the same slope and different y -intercepts and so they are parallel.

There is another way you can look at this example. If you recognize right away from the equations that these are horizontal lines, you know their slopes are both 0. Since the horizontal lines cross the y -axis at $y = -4$ and at $y = 3$, we know the y -intercepts are $(0, -4)$ and $(0, 3)$. The lines have the same slope and different y -intercepts and so they are parallel.

Note:
Exercise:

Problem: Use slopes and y -intercepts to determine if the lines $y = 8$ and $y = -6$ are parallel.

Solution:

parallel

Note:
Exercise:

Problem: Use slopes and y -intercepts to determine if the lines $y = 1$ and $y = -5$ are parallel.

Solution:

parallel

Example:
Exercise:

Problem: Use slopes and y -intercepts to determine if the lines $x = -2$ and $x = -5$ are parallel.

Solution:
Solution
Equation:

$$x = -2 \text{ and } x = -5$$

Since there is no y , the equations cannot be put in slope–intercept form. But we recognize them as equations of vertical lines. Their x -intercepts are -2 and -5 . Since their x -intercepts are different, the vertical lines are parallel.

Note:

Exercise:

Problem: Use slopes and y -intercepts to determine if the lines $x = 1$ and $x = -5$ are parallel.

Solution:

parallel

Note:

Exercise:

Problem: Use slopes and y -intercepts to determine if the lines $x = 8$ and $x = -6$ are parallel.

Solution:

parallel

Example:

Exercise:

Problem:

Use slopes and y -intercepts to determine if the lines $y = 2x - 3$ and $-6x + 3y = -9$ are parallel. You may want to graph these lines, too, to see what they look like.

Solution:

Solution

The first equation is already in slope–intercept form.

Solve the second equation for y .

The second equation is now in slope–intercept form.

Identify the slope and y -intercept of both lines.

$$\begin{array}{rclcl}
 y & = & 2x - 3 & \text{and} & -6x + 3y = -9 \\
 y & = & 2x - 3 & & \\
 -6x + 3y & = & -9 & & \\
 3y & = & 6x - 9 & & \\
 \frac{3y}{3} & = & \frac{6x - 9}{3} & & \\
 y & = & 2x - 3 & & \\
 y & = & 2x - 3 & & \\
 y & = & mx + b & & \\
 m & = & 2 & & \\
 & & y\text{-intercept is } (0, -3) & &
 \end{array}$$

The lines have the same slope, but they also have the same y -intercepts. Their equations represent the same line. They are not parallel; they are the same line.

Note:

Exercise:

Problem: Use slopes and y-intercepts to determine if the lines $y = -\frac{1}{2}x - 1$ and $x + 2y = 2$ are parallel.

Solution:

not parallel; same line

Note:

Exercise:

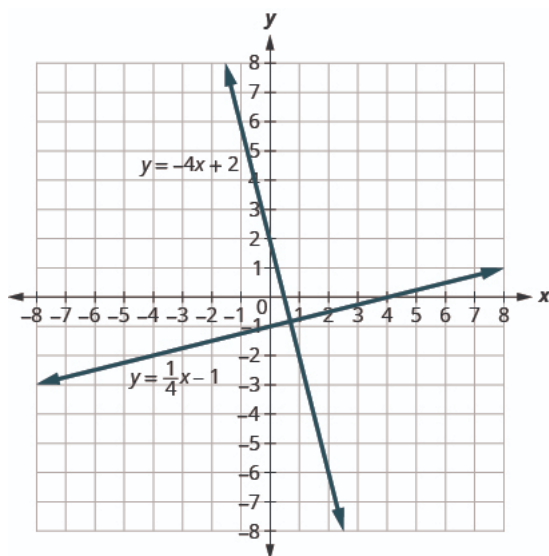
Problem: Use slopes and y-intercepts to determine if the lines $y = \frac{3}{4}x - 3$ and $3x - 4y = 12$ are parallel.

Solution:

not parallel; same line

Use Slopes to Identify Perpendicular Lines

Let's look at the lines whose equations are $y = \frac{1}{4}x - 1$ and $y = -4x + 2$, shown in [\[link\]](#).



These lines lie in the same plane and intersect in right angles. We call these lines **perpendicular**.

What do you notice about the slopes of these two lines? As we read from left to right, the line $y = \frac{1}{4}x - 1$ rises, so its slope is positive. The line $y = -4x + 2$ drops from left to right, so it has a negative slope. Does it make sense to you that the slopes of two perpendicular lines will have opposite signs?

If we look at the slope of the first line, $m_1 = \frac{1}{4}$, and the slope of the second line, $m_2 = -4$, we can see that they are *negative reciprocals* of each other. If we multiply them, their product is -1 .

Equation:

$$\begin{aligned} m_1 \cdot m_2 \\ \frac{1}{4}(-4) \\ -1 \end{aligned}$$

This is always true for perpendicular lines and leads us to this definition.

Note:

Perpendicular Lines

Perpendicular lines are lines in the same plane that form a right angle.

If m_1 and m_2 are the slopes of two perpendicular lines, then:

Equation:

$$m_1 \cdot m_2 = -1 \text{ and } m_1 = \frac{-1}{m_2}$$

Vertical lines and horizontal lines are always perpendicular to each other.

We were able to look at the slope–intercept form of linear equations and determine whether or not the lines were parallel. We can do the same thing for perpendicular lines.

We find the slope–intercept form of the equation, and then see if the slopes are negative reciprocals. If the product of the slopes is -1 , the lines are perpendicular. Perpendicular lines may have the same y-intercepts.

Example:

Exercise:

Problem: Use slopes to determine if the lines, $y = -5x - 4$ and $x - 5y = 5$ are perpendicular.

Solution:

Solution

The first equation is in slope–intercept form.

Solve the second equation for y .

$$y = -5x - 4$$

$$x - 5y = 5$$

$$-5y = -x + 5$$

$$\frac{-5y}{-5} = \frac{-x+5}{-5}$$

$$y = \frac{1}{5}x - 1$$

Identify the slope of each line.

$$y = -5x - 4$$

$$y = \frac{1}{5}x - 1$$

$$y = mx + b$$

$$y = mx + b$$

$$m_1 = -5$$

$$m_2 = \frac{1}{5}$$

The slopes are negative reciprocals of each other, so the lines are perpendicular. We check by multiplying the slopes,

Equation:

$$m_1 \cdot m_2$$

$$-5 \left(\frac{1}{5} \right)$$

$$-1 \checkmark$$

Note:

Exercise:

Problem: Use slopes to determine if the lines $y = -3x + 2$ and $x - 3y = 4$ are perpendicular.

Solution:

perpendicular

Note:

Exercise:

Problem: Use slopes to determine if the lines $y = 2x - 5$ and $x + 2y = -6$ are perpendicular.

Solution:

perpendicular

Example:

Exercise:

Problem: Use slopes to determine if the lines, $7x + 2y = 3$ and $2x + 7y = 5$ are perpendicular.

Solution:

Solution

Solve the equations for y .

$$7x + 2y = 3$$

$$2x + 7y = 5$$

$$2y = -7x + 3$$

$$7y = -2x + 5$$

$$\frac{2y}{2} = \frac{-7x+3}{2}$$

$$\frac{7y}{7} = \frac{-2x+5}{7}$$

$$y = -\frac{7}{2}x + \frac{3}{2}$$

$$y = -\frac{2}{7}x + \frac{5}{7}$$

Identify the slope of each line.

$$y = mx + b$$

$$y = mx + b$$

$$m_1 = -\frac{7}{2}$$

$$m_2 = -\frac{2}{7}$$

The slopes are reciprocals of each other, but they have the same sign. Since they are not negative reciprocals, the lines are not perpendicular.

Note:

Exercise:

Problem: Use slopes to determine if the lines $5x + 4y = 1$ and $4x + 5y = 3$ are perpendicular.

Solution:

not perpendicular

Note:

Exercise:

Problem: Use slopes to determine if the lines $2x - 9y = 3$ and $9x - 2y = 1$ are perpendicular.

Solution:

not perpendicular

Note:

Access this online resource for additional instruction and practice with graphs.

- [Explore the Relation Between a Graph and the Slope–Intercept Form of an Equation of a Line](#)

Key Concepts

- The slope–intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is, $y = mx + b$.
- **Graph a Line Using its Slope and y-Intercept**

Find the slope-intercept form of the equation of the line.

Identify the slope and y-intercept.

Plot the y-intercept.

Use the slope formula $m = \frac{\text{rise}}{\text{run}}$ to identify the rise and the run.

Starting at the y-intercept, count out the rise and run to mark the second point.

Connect the points with a line.

- **Strategy for Choosing the Most Convenient Method to Graph a Line:** Consider the form of the equation.
 - If it only has one variable, it is a vertical or horizontal line.
 - $x = a$ is a vertical line passing through the x-axis at a .
 - $y = b$ is a horizontal line passing through the y-axis at b .
 - If y is isolated on one side of the equation, in the form $y = mx + b$, graph by using the slope and y-intercept.
 - Identify the slope and y-intercept and then graph.
 - If the equation is of the form $Ax + By = C$, find the intercepts.
 - Find the x- and y-intercepts, a third point, and then graph.
- Parallel lines are lines in the same plane that do not intersect.
 - Parallel lines have the same slope and different y-intercepts.

- If m_1 and m_2 are the slopes of two parallel lines then $m_1 = m_2$.
- Parallel vertical lines have different x -intercepts.
- Perpendicular lines are lines in the same plane that form a right angle.
 - If m_1 and m_2 are the slopes of two perpendicular lines, then $m_1 \cdot m_2 = -1$ and $m_1 = \frac{-1}{m_2}$.
 - Vertical lines and horizontal lines are always perpendicular to each other.

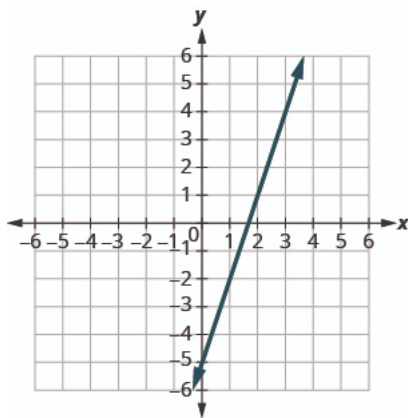
Practice Makes Perfect

Recognize the Relation Between the Graph and the Slope–Intercept Form of an Equation of a Line

In the following exercises, use the graph to find the slope and y -intercept of each line. Compare the values to the equation $y = mx + b$.

Exercise:

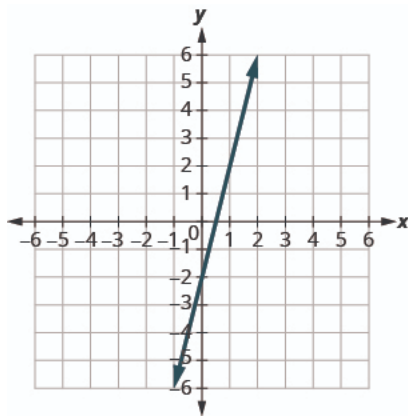
Problem:



$$y = 3x - 5$$

Exercise:

Problem:



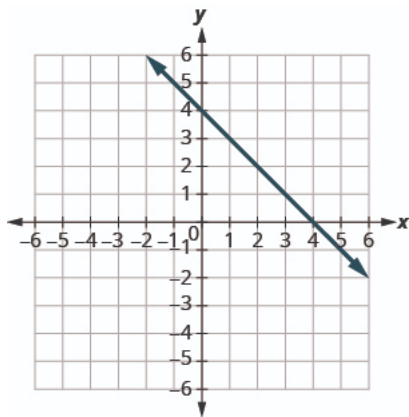
$$y = 4x - 2$$

Solution:

slope $m = 4$ and y -intercept $(0, -2)$

Exercise:

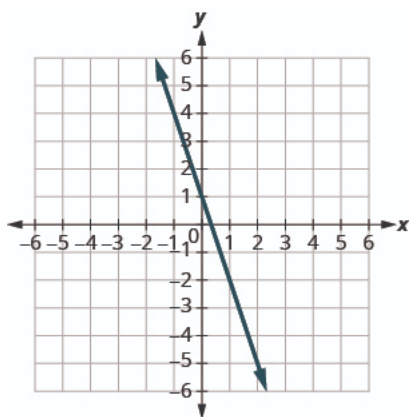
Problem:



$$y = -x + 4$$

Exercise:

Problem:



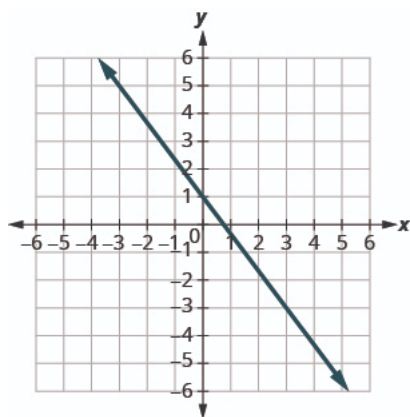
$$y = -3x + 1$$

Solution:

slope $m = -3$ and y -intercept $(0, 1)$

Exercise:

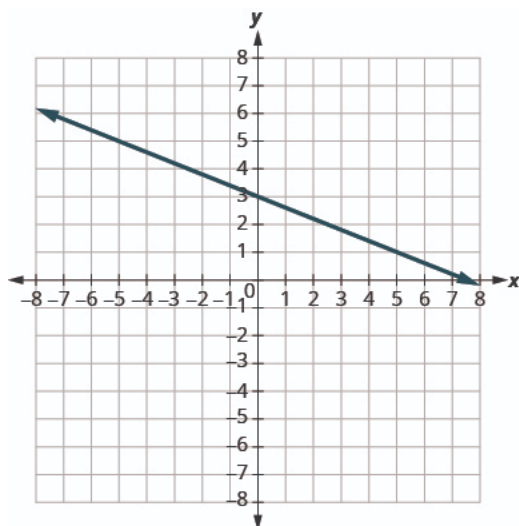
Problem:



$$y = -\frac{4}{3}x + 1$$

Exercise:

Problem:



$$y = -\frac{2}{5}x + 3$$

Solution:

slope $m = -\frac{2}{5}$ and y-intercept $(0, 3)$

Identify the Slope and y-Intercept From an Equation of a Line

In the following exercises, identify the slope and y-intercept of each line.

Exercise:

Problem: $y = -7x + 3$

Exercise:

Problem: $y = -9x + 7$

Solution:

$$-9; (0, 7)$$

Exercise:

Problem: $y = 6x - 8$

Exercise:

Problem: $y = 4x - 10$

Solution:

$$4; (0, -10)$$

Exercise:

Problem: $3x + y = 5$

Exercise:

Problem: $4x + y = 8$

Solution:

$$-4; (0, 8)$$

Exercise:

Problem: $6x + 4y = 12$

Exercise:

Problem: $8x + 3y = 12$

Solution:

$$-\frac{8}{3}; (0, 4)$$

Exercise:

Problem: $5x - 2y = 6$

Exercise:

Problem: $7x - 3y = 9$

Solution:

$$\frac{7}{3}; (0, -3)$$

Graph a Line Using Its Slope and Intercept

In the following exercises, graph the line of each equation using its slope and y-intercept.

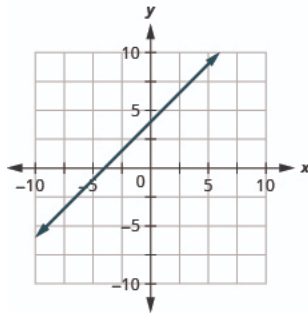
Exercise:

Problem: $y = x + 3$

Exercise:

Problem: $y = x + 4$

Solution:



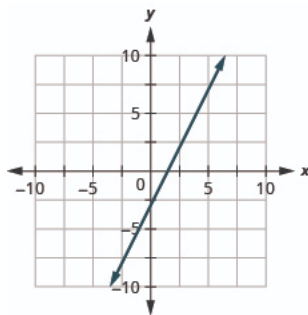
Exercise:

Problem: $y = 3x - 1$

Exercise:

Problem: $y = 2x - 3$

Solution:



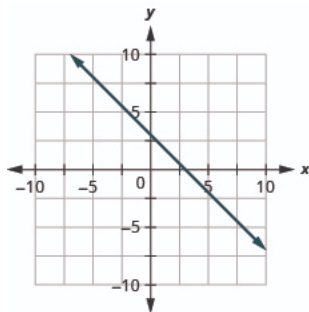
Exercise:

Problem: $y = -x + 2$

Exercise:

Problem: $y = -x + 3$

Solution:



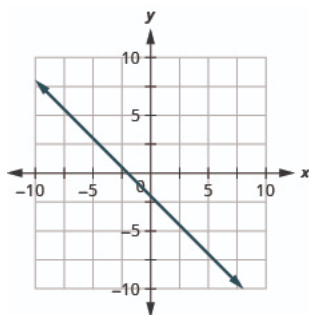
Exercise:

Problem: $y = -x - 4$

Exercise:

Problem: $y = -x - 2$

Solution:



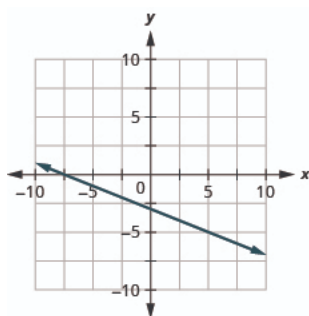
Exercise:

Problem: $y = -\frac{3}{4} - 1$

Exercise:

Problem: $y = -\frac{2}{5} - 3$

Solution:



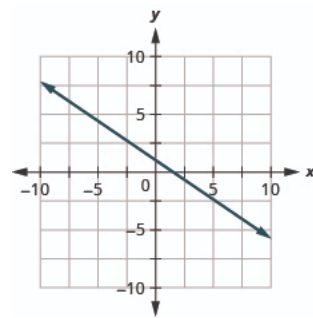
Exercise:

Problem: $y = -\frac{3}{5}x + 2$

Exercise:

Problem: $y = -\frac{2}{3}x + 1$

Solution:



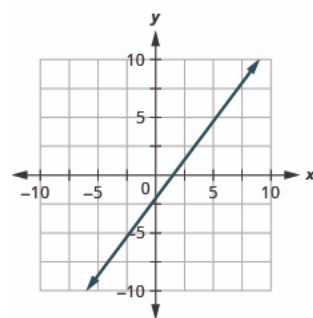
Exercise:

Problem: $3x - 4y = 8$

Exercise:

Problem: $4x - 3y = 6$

Solution:



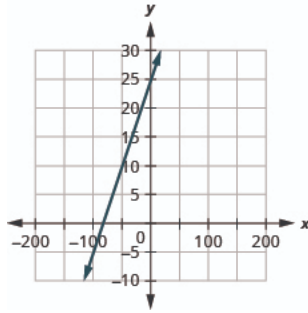
Exercise:

Problem: $y = 0.1x + 15$

Exercise:

Problem: $y = 0.3x + 25$

Solution:



Choose the Most Convenient Method to Graph a Line

In the following exercises, determine the most convenient method to graph each line.

Exercise:

Problem: $x = 2$

Exercise:

Problem: $y = 4$

Solution:

horizontal line

Exercise:

Problem: $y = 5$

Exercise:

Problem: $x = -3$

Solution:

vertical line

Exercise:

Problem: $y = -3x + 4$

Exercise:

Problem: $y = -5x + 2$

Solution:

slope-intercept

Exercise:

Problem: $x - y = 5$

Exercise:

Problem: $x - y = 1$

Solution:

intercepts

Exercise:

Problem: $y = \frac{2}{3}x - 1$

Exercise:

Problem: $y = \frac{4}{5}x - 3$

Solution:

slope–intercept

Exercise:

Problem: $y = -3$

Exercise:

Problem: $y = -1$

Solution:

horizontal line

Exercise:

Problem: $3x - 2y = -12$

Exercise:

Problem: $2x - 5y = -10$

Solution:

intercepts

Exercise:

Problem: $y = -\frac{1}{4} + 3$

Exercise:

Problem: $y = -\frac{1}{3}x + 5$

Solution:

slope–intercept

Graph and Interpret Applications of Slope–Intercept

Exercise:

Problem:

The equation $P = 31 + 1.75w$ models the relation between the amount of Tuyet's monthly water bill payment, P , in dollars, and the number of units of water, w , used.

- (a) Find Tuyet's payment for a month when 0 units of water are used.
- (b) Find Tuyet's payment for a month when 12 units of water are used.
- (c) Interpret the slope and P -intercept of the equation.
- (d) Graph the equation.

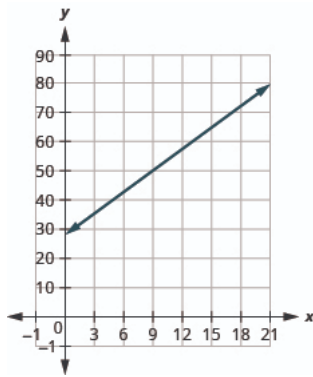
Exercise:**Problem:**

The equation $P = 28 + 2.54w$ models the relation between the amount of Randy's monthly water bill payment, P , in dollars, and the number of units of water, w , used.

- (a) Find the payment for a month when Randy used 0 units of water.
- (b) Find the payment for a month when Randy used 15 units of water.
- (c) Interpret the slope and P -intercept of the equation.
- (d) Graph the equation.

Solution:

- (a) \$28
- (b) \$66.10
- (c) The slope, 2.54, means that Randy's payment, P , increases by \$2.54 when the number of units of water he used, w , increases by 1. The P -intercept means that if the number units of water Randy used was 0, the payment would be \$28.
- (d)

**Exercise:****Problem:**

Bruce drives his car for his job. The equation $R = 0.575m + 42$ models the relation between the amount in dollars, R , that he is reimbursed and the number of miles, m , he drives in one day.

- (a) Find the amount Bruce is reimbursed on a day when he drives 0 miles.
- (b) Find the amount Bruce is reimbursed on a day when he drives 220 miles.
- (c) Interpret the slope and R -intercept of the equation.
- (d) Graph the equation.

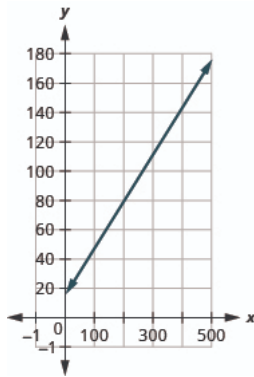
Exercise:**Problem:**

Janelle is planning to rent a car while on vacation. The equation $C = 0.32m + 15$ models the relation between the cost in dollars, C , per day and the number of miles, m , she drives in one day.

- (a) Find the cost if Janelle drives the car 0 miles one day.
- (b) Find the cost on a day when Janelle drives the car 400 miles.
- (c) Interpret the slope and C -intercept of the equation.
- (d) Graph the equation.

Solution:

- (a) \$15
- (b) \$143
- (c) The slope, 0.32, means that the cost, C , increases by \$0.32 when the number of miles driven, m , increases by 1. The C -intercept means that if Janelle drives 0 miles one day, the cost would be \$15.
- (d)

**Exercise:****Problem:**

Cherie works in retail and her weekly salary includes commission for the amount she sells. The equation $S = 400 + 0.15c$ models the relation between her weekly salary, S , in dollars and the amount of her sales, c , in dollars.

- (a) Find Cherie's salary for a week when her sales were 0.
- (b) Find Cherie's salary for a week when her sales were 3600.
- (c) Interpret the slope and S -intercept of the equation.
- (d) Graph the equation.

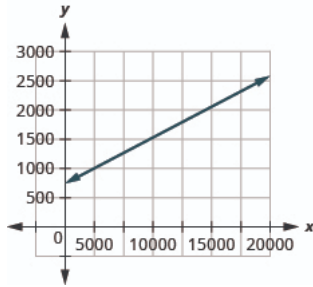
Exercise:**Problem:**

Patel's weekly salary includes a base pay plus commission on his sales. The equation $S = 750 + 0.09c$ models the relation between his weekly salary, S , in dollars and the amount of his sales, c , in dollars.

- (a) Find Patel's salary for a week when his sales were 0.
- (b) Find Patel's salary for a week when his sales were 18,540.
- (c) Interpret the slope and S -intercept of the equation.
- (d) Graph the equation.

Solution:

- (a) \$750
- (b) \$2418.60
- (c) The slope, 0.09, means that Patel's salary, S , increases by \$0.09 for every \$1 increase in his sales. The S -intercept means that when his sales are \$0, his salary is \$750.
- (d)

**Exercise:****Problem:**

Costa is planning a lunch banquet. The equation $C = 450 + 28g$ models the relation between the cost in dollars, C , of the banquet and the number of guests, g .

- (a) Find the cost if the number of guests is 40.
- (b) Find the cost if the number of guests is 80.
- (c) Interpret the slope and C -intercept of the equation.
- (d) Graph the equation.

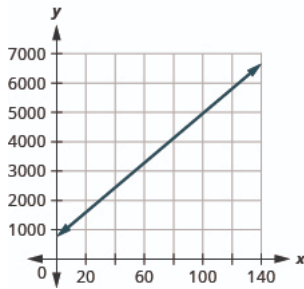
Exercise:**Problem:**

Margie is planning a dinner banquet. The equation $C = 750 + 42g$ models the relation between the cost in dollars, C of the banquet and the number of guests, g .

- (a) Find the cost if the number of guests is 50.
- (b) Find the cost if the number of guests is 100.
- (c) Interpret the slope and C -intercept of the equation.
- (d) Graph the equation.

Solution:

- (a) \$2850
- (b) \$4950
- (c) The slope, 42, means that the cost, C , increases by \$42 for when the number of guests increases by 1. The C -intercept means that when the number of guests is 0, the cost would be \$750.
- (d)



Use Slopes to Identify Parallel Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are parallel.

Exercise:

Problem: $y = \frac{3}{4}x - 3$; $3x - 4y = -2$

Exercise:

Problem: $y = \frac{2}{3}x - 1$; $2x - 3y = -2$

Solution:

parallel

Exercise:

Problem: $2x - 5y = -3$; $y = \frac{2}{5}x + 1$

Exercise:

Problem: $3x - 4y = -2$; $y = \frac{3}{4}x - 3$

Solution:

parallel

Exercise:

Problem: $2x - 4y = 6$; $x - 2y = 3$

Exercise:

Problem: $6x - 3y = 9$; $2x - y = 3$

Solution:

not parallel

Exercise:

Problem: $4x + 2y = 6$; $6x + 3y = 3$

Exercise:

Problem: $8x + 6y = 6$; $12x + 9y = 12$

Solution:

parallel

Exercise:

Problem: $x = 5$; $x = -6$

Exercise:

Problem: $x = 7$; $x = -8$

Solution:

parallel

Exercise:

Problem: $x = -4$; $x = -1$

Exercise:

Problem: $x = -3$; $x = -2$

Solution:

parallel

Exercise:

Problem: $y = 2$; $y = 6$

Exercise:

Problem: $y = 5$; $y = 1$

Solution:

parallel

Exercise:

Problem: $y = -4$; $y = 3$

Exercise:

Problem: $y = -1$; $y = 2$

Solution:

parallel

Exercise:

Problem: $x - y = 2$; $2x - 2y = 4$

Exercise:

Problem: $4x + 4y = 8$; $x + y = 2$

Solution:

not parallel

Exercise:

Problem: $x - 3y = 6$; $2x - 6y = 12$

Exercise:

Problem: $5x - 2y = 11$; $5x - y = 7$

Solution:

not parallel

Exercise:

Problem: $3x - 6y = 12$; $6x - 3y = 3$

Exercise:

Problem: $4x - 8y = 16$; $x - 2y = 4$

Solution:

not parallel

Exercise:

Problem: $9x - 3y = 6$; $3x - y = 2$

Exercise:

Problem: $x - 5y = 10$; $5x - y = -10$

Solution:

not parallel

Exercise:

Problem: $7x - 4y = 8$; $4x + 7y = 14$

Exercise:

Problem: $9x - 5y = 4$; $5x + 9y = -1$

Solution:

not parallel

Use Slopes to Identify Perpendicular Lines

In the following exercises, use slopes and y-intercepts to determine if the lines are perpendicular.

Exercise:

Problem: $3x - 2y = 8; 2x + 3y = 6$

Exercise:

Problem: $x - 4y = 8; 4x + y = 2$

Solution:

perpendicular

Exercise:

Problem: $2x + 5y = 3; 5x - 2y = 6$

Exercise:

Problem: $2x + 3y = 5; 3x - 2y = 7$

Solution:

perpendicular

Exercise:

Problem: $3x - 2y = 1; 2x - 3y = 2$

Exercise:

Problem: $3x - 4y = 8; 4x - 3y = 6$

Solution:

not perpendicular

Exercise:

Problem: $5x + 2y = 6; 2x + 5y = 8$

Exercise:

Problem: $2x + 4y = 3; 6x + 3y = 2$

Solution:

not perpendicular

Exercise:

Problem: $4x - 2y = 5; 3x + 6y = 8$

Exercise:

Problem: $2x - 6y = 4; 12x + 4y = 9$

Solution:

perpendicular

Exercise:

Problem: $6x - 4y = 5$; $8x + 12y = 3$

Exercise:

Problem: $8x - 2y = 7$; $3x + 12y = 9$

Solution:

perpendicular

Everyday Math

Exercise:

Problem:

The equation $C = \frac{5}{9}F - 17.8$ can be used to convert temperatures F , on the Fahrenheit scale to temperatures, C , on the Celsius scale.

- Ⓐ Explain what the slope of the equation means.
- Ⓑ Explain what the C -intercept of the equation means.

Exercise:

Problem:

The equation $n = 4T - 160$ is used to estimate the number of cricket chirps, n , in one minute based on the temperature in degrees Fahrenheit, T .

- Ⓐ Explain what the slope of the equation means.
 - Ⓑ Explain what the n -intercept of the equation means. Is this a realistic situation?
-

Solution:

- Ⓐ For every increase of one degree Fahrenheit, the number of chirps increases by four.
- Ⓑ There would be -160 chirps when the Fahrenheit temperature is 0° . (Notice that this does not make sense; this model cannot be used for all possible temperatures.)

Writing Exercises

Exercise:

Problem: Explain in your own words how to decide which method to use to graph a line.

Exercise:

Problem: Why are all horizontal lines parallel?

Solution:

Answers will vary.

Self Check

- Ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

I can...	Confidently	With some help	No-I don't get it!
recognize the relation between the graph and the slope-intercept form of an equation of a line.			
identify the slope and y-intercept from an equation of a line.			
graph a line using its slope and intercept.			
choose the most convenient method to graph a line.			
graph and interpret applications of slope-intercept.			
use slopes to identify parallel lines.			

- Ⓑ After looking at the checklist, do you think you are well-prepared for the next section? Why or why not?

Glossary

parallel lines

Lines in the same plane that do not intersect.

perpendicular lines

Lines in the same plane that form a right angle.

slope-intercept form of an equation of a line

The slope-intercept form of an equation of a line with slope m and y-intercept, $(0, b)$ is, $y = mx + b$.

Cumulative Review

Note: Answers to the Cumulative Review can be found in the Supplemental Resources. Please visit <http://openstaxcollege.org> to view an updated list of the Learning Resources for this title and how to access them.

Chapter 1 Whole Numbers

No exercises.

Chapter 2 The Language of Algebra

Simplify:

1. $5(3 + 2 \cdot 6) - 8^2$

Solve:

2. $17 = y - 13$

3. $p + 14 = 23$

Translate into an algebraic expression.

4. 11 less than the product of 7 and x .

Translate into an algebraic equation and solve.

5. Twice the difference of y and 7 gives 84.

6. Find all the factors of 72.

7. Find the prime factorization of 132.

8. Find the least common multiple of 12 and 20.

Chapter 3 Integers

Simplify:

9. $|8 - 9| - |3 - 8|$

10. $-2 + 4(-3 + 7)$

11. $27 - (-4 - 7)$

12. $28 \div (-4) - 7$

Translate into an algebraic expression or equation.

13. The sum of -5 and 13 , increased by 11 .

14. The product of -11 and 8 .

15. The quotient of 7 and the sum of -4 and m .

16. The product of -3 and is -51 .

Solve:

17. $-6r = 24$

Chapter 4 Fractions

18. Locate the numbers on a number line. $\frac{7}{8}$, $\frac{5}{3}$, $3\frac{1}{4}$, 5 .

Simplify:

19. $\frac{21p}{57q}$

20. $\frac{3}{7} \cdot \left(-\frac{28}{45}\right)$

$$21. -6\frac{3}{4} \div \frac{9}{2}$$

$$22. -3\frac{3}{5} \div 6$$

$$23. -4\frac{2}{3} \left(-\frac{6}{7}\right)$$

$$24. \frac{-2\frac{1}{4}}{-\frac{3}{8}}$$

$$25. \frac{7 \cdot 8 + 4(7 - 12)}{9 \cdot 6 - 2 \cdot 9}$$

$$26. -\frac{23}{36} + \frac{17}{20}$$

$$27. \frac{\frac{1}{2} + \frac{1}{3}}{\frac{3}{4} - \frac{1}{3}}$$

$$28. 3\frac{5}{8} - 2\frac{1}{2}$$

$$29. -\frac{2}{3}r = 24$$

Chapter 5 Decimals

Simplify:

$$30. 24.76 - 7.28$$

$$31. 12.9 + 15.633$$

$$32. (-5.6)(0.25)$$

$$33. \$6.29 \div 12$$

$$34. \frac{3}{4}(13.44 - 9.6)$$

$$35. \sqrt{64} + \sqrt{225}$$

36. $\sqrt{121x^2y^2}$

37. Write in order from smallest to largest: $\frac{5}{8}$, 0.75, $\frac{8}{15}$

Solve:

38. $-8.6x = 34.4$

39. Using 3.14 as the estimate for pi, approximate the (a) circumference and (b) area of a circle whose radius is 8 inches.

40. Find the mean of the numbers, 18, 16, 20, 12

41. Find the median of the numbers, 24, 29, 27, 28, 30

42. Identify the mode of the numbers, 6, 4, 4, 5, 6, 6, 4, 4, 4, 3, 5

43. Find the unit price of one t-shirt if they are sold at 3 for \$28.97.

Chapter 6 Percents

44. Convert 14.7% to (a) a fraction and (b) a decimal.

Translate and solve.

45. 63 is 35% of what number?

46. The nutrition label on a package of granola bars says that each granola bar has 180 calories, and 81 calories are from fat. What percent of the total calories is from fat?

47. Elliot received \$510 commission when he sold a \$3,400 painting at the art gallery where he works. What was the rate of commission?

48. Nandita bought a set of towels on sale for \$67.50. The original price of the towels was \$90. What was the discount rate?

49. Alan invested \$23,000 in a friend's business. In 5 years the friend paid him the \$23,000 plus \$9,200 interest. What was the rate of interest?

Solve:

50. $\frac{9}{p} = \frac{-6}{14}$

Chapter 7 The Properties of Real Numbers

51. List the (a) whole numbers, (b) integers, (c) rational numbers, (d) irrational numbers,

(e) real numbers -5 , $-2\frac{1}{4}$, $-\sqrt{4}$, 0.25 , $\frac{13}{5}$, 4

Simplify:

52. $\left(\frac{8}{15} + \frac{4}{7}\right) + \frac{3}{7}$

53. $3(y + 3) - 8(y - 4)$

54. $\frac{8}{17} \cdot 49 \cdot \frac{17}{8}$

55. A playground is 55 feet wide. Convert the width to yards.

56. Every day last week Amit recorded the number of minutes he spent reading. The recorded number of minutes he read each day was 48, 26, 81, 54, 43, 62, 106. How many hours did Amit spend reading last week?

57. June walked 2.8 kilometers. Convert this length to miles knowing 1 mile is 1.61 kilometer.

Chapter 8 Solve Linear Equations

Solve:

58. $y + 13 = -8$

59. $p + \frac{2}{5} = \frac{8}{5}$

60. $48 = \frac{2}{3}x$

61. $4(a - 3) - 6a = -18$

62. $7q + 14 = -35$

63. $4v - 27 = 7v$

64. $\frac{7}{8}y - 6 = \frac{3}{8}y - 8$

65. $26 - 4(z - 2) = 6$

66. $\frac{3}{4}x - \frac{2}{3} = \frac{1}{2}x - \frac{5}{6}$

67. $0.7y + 4.8 = 0.84y - 5.3$

Translate and solve.

68. Four less than n is 13.

Chapter 9 Math Models and Geometry

69. One number is 8 less than another. Their sum is negative twenty-two. Find the numbers.

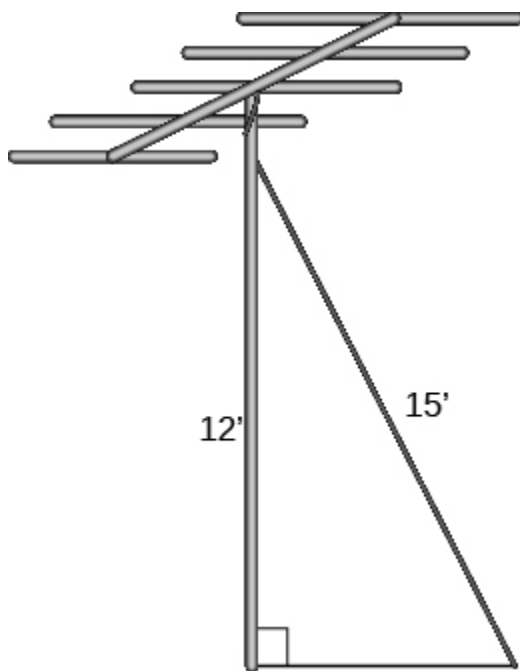
70. The sum of two consecutive integers is -95 . Find the numbers.

71. Wilma has \$3.65 in dimes and quarters. The number of dimes is 2 less than the number of quarters. How many of each coin does she have?

72. Two angles are supplementary. The larger angle is 24° more than the smaller angle. Find the measurements of both angles.

73. One angle of a triangle is 20° more than the smallest angle. The largest angle is the sum of the other angles. Find the measurements of all three angles.

74. Erik needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is 12 feet tall and Erik has 15 feet of wire. How far from the base of the antenna can he attach the wire?



75. The width of a rectangle is 4 less than the length. The perimeter is 96 inches. Find the length and the width.

76. Find the (a) volume and (b) surface area of a rectangular carton with length 24 inches, width 18 inches, and height 6 inches.

Chapter 10 Polynomials

Simplify:

$$77. (8m^2 + 12m - 5) - (2m^2 - 7m - 1)$$

$$78. p^3 \cdot p^{10}$$

$$79. (y^4)^3$$

$$80. (3a^5)^3$$

$$81. (x^3)^5(x^2)^3$$

$$82. \left(\frac{2}{3}m^3n^6\right)\left(\frac{1}{6}m^4n^4\right)$$

$$83. (y - 4)(y + 12)$$

$$84. (3c + 1)(9c - 4)$$

$$85. (x - 1)(x^2 - 3x - 2)$$

$$86. (8x)^0$$

$$87. \frac{(x^3)^5}{(x^2)^4}$$

$$88. \frac{32a^7b^2}{12a^3b^6}$$

$$89. (ab^{-3})(a^{-3}b^6)$$

$$90. \text{Write in scientific notation: (a) } 4,800,000 \text{ (b) } 0.00637$$

Factor the greatest common factor from the polynomial.

$$91. 3x^4 - 6x^3 - 18x^2$$

Chapter 11 Graphs

Graph:

92. $y = 4x - 3$

93. $y = -3x$

94. $y = \frac{1}{2}x + 3$

95. $x - y = 6$

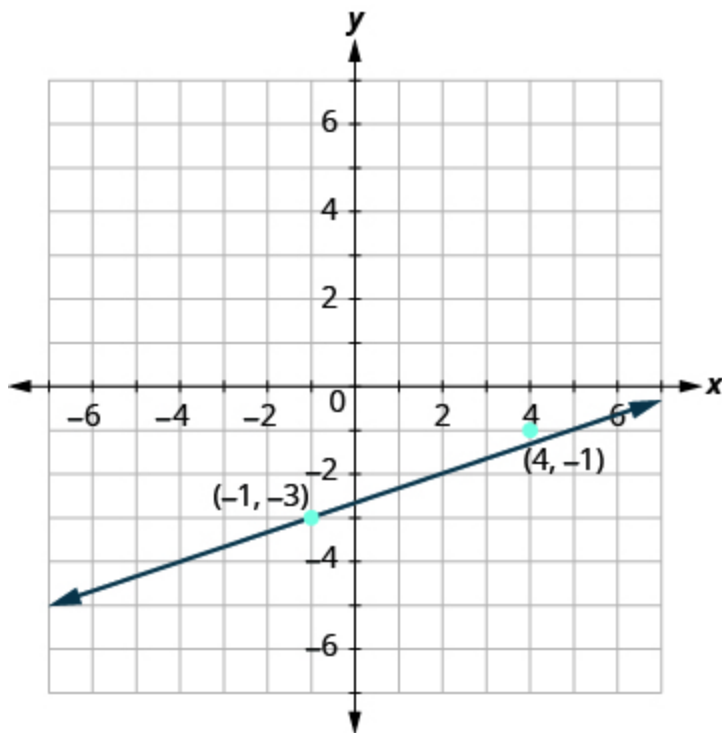
96. $y = -2$

97. Find the intercepts. $2x + 3y = 12$

Graph using the intercepts.

98. $2x - 4y = 8$

99. Find the slope of the line shown.



100. Use the slope formula to find the slope of the line between the points $(-5, -2)$, $(3, 2)$.

101. Graph the line passing through the point $(-3, 4)$ and with slope $m = -\frac{1}{3}$.

Self Assessments

After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.

Following are descriptions to help you determine your level of mastery for each section. After completing the self-check, think about...

- What does this checklist tell you about your mastery of this section?
- What steps will you take to improve?
- What can you do to become confident for all objectives?

Confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

With some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

No- I don't get it! This is a warning sign and you must not ignore it. Get help right away or you will quickly become overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get the help you need.

Section 1.1 Introduction to Whole Numbers

Section 1.1 Introduction to Whole Numbers			
I can...	Confidently	With some help	No- I don't get it!

I can...	Confidently	With some help	No- I don't get it!
identify counting numbers and whole numbers.			
model whole numbers.			
identify the place value of a digit.			
use place value to name whole numbers.			
use place value to write whole numbers.			
round whole numbers.			

Section 1.2 Add Whole Numbers			
I can...	Confidently	With some help	No- I don't get it!

Section 1.2 Add Whole Numbers			
I can...	Confidently	With some help	No- I don't get it!
use addition notation.			
model addition of whole numbers.			
add whole numbers without models.			
translate word phrases to math notation.			
add whole numbers in applications.			

Section 1.3 Subtract Whole Numbers			
I can...	Confidently	With some help	No- I don't get it!
use subtraction notation.			

Section 1.3 Subtract Whole Numbers			
I can...	Confidently	With some help	No- I don't get it!
model subtraction of whole numbers.			
subtract whole numbers.			
translate word phrases to math notation.			
subtract whole numbers in applications.			

Section 1.4 Multiply Whole Numbers			
I can...	Confidently	With some help	No- I don't get it!
use multiplication notation.			
model multiplication of whole numbers.			
multiply whole numbers.			

Section 1.4 Multiply Whole Numbers

I can...	Confidently	With some help	No- I don't get it!
translate word phrases to math notation.			
multiply whole numbers in applications.			

Section 1.5 Divide Whole Numbers

I can...	Confidently	With some help	No- I don't get it!
use division notation.			
model division of whole numbers.			
divide whole numbers.			
translate word phrases to algebraic expressions.			
divide whole numbers in applications.			

Section 2.1 Use the Language of Algebra			
I can...	Confidently	With some help	No- I don't get it!
use variables and algebraic symbols.			
identify expressions and equations.			
simplify expressions with exponents.			
simplify expressions using the order of operations.			

Section 2.2 Evaluate, Simplify, and Translate Expressions			
I can...	Confidently	With some help	No- I don't get it!
evaluate algebraic expressions.			
identify terms, coefficients, and like terms.			

Section 2.2 Evaluate, Simplify, and Translate Expressions

I can...	Confidently	With some help	No- I don't get it!
simplify expressions by combining like terms.			
translate word phrases to algebraic expressions.			

Section 2.3 Solve Equations Using the Subtraction and Addition Properties of Equality

I can...	Confidently	With some help	No- I don't get it!
determine whether a number is a solution of an equation.			
model the subtraction property of equality.			
solve equations using the subtraction property of equality.			

Section 2.3 Solve Equations Using the Subtraction and Addition Properties of Equality

I can...	Confidently	With some help	No- I don't get it!
solve equations using the addition property of equality.			
translate word phrases to algebraic equations.			
translate to an equation and solve.			

Section 2.4 Find Multiples and Factors

I can...	Confidently	With some help	No- I don't get it!
identify multiples of numbers.			
use common divisibility tests.			

Section 2.4 Find Multiples and Factors			
I can...	Confidently	With some help	No- I don't get it!
find all the factors of a number.			
identify prime and composite numbers.			

Section 2.5 Prime Factorization and the Least Common Multiple			
I can...	Confidently	With some help	No- I don't get it!
find the prime factorization of a composite number.			
find the least common multiple (LCM) of two numbers.			

Section 3.1 Introduction to Integers			
I can...	Confidently	With some help	No- I don't get it!
locate positive and negative numbers on the number line.			
order positive and negative numbers.			
find opposites.			
simplify expressions with absolute value.			
translate word phrases to expressions with integers.			

Section 3.2 Add Integers			
I can...	Confidently	With some help	No- I don't get it!
model addition of integers.			

Section 3.2 Add Integers			
I can...	Confidently	With some help	No- I don't get it!
simplify expressions with integers.			
evaluate variable expressions with integers.			
translate word phrases to algebraic expressions.			
add integers in applications.			

Section 3.3 Subtract Integers			
I can...	Confidently	With some help	No- I don't get it!
model subtraction of integers.			
simplify expressions with integers.			
evaluate variable expressions with integers.			

Section 3.3 Subtract Integers			
I can...	Confidently	With some help	No- I don't get it!
translate word phrases to algebraic expressions.			
subtract integers in applications.			

Section 3.4 Multiply and Divide Integers			
I can...	Confidently	With some help	No- I don't get it!
multiply integers.			
divide integers.			
simplify expressions with integers.			
evaluate variable expressions with integers			
translate word phrases to algebraic expressions.			

Section 3.5 Solve Equations Using Integers; The Division Property of Equality

I can...	Confidently	With some help	No- I don't get it!
determine whether an integer is a solution of an equation.			
solve equations with integers using the addition and subtraction properties of equality.			
model division property of equality.			
solve equations using the division property of equality.			
translate to an equation and solve.			

Section 4.1 Visualize Fractions

I can...	Confidently	With some help	No- I don't get it!
understand the meaning of fractions.			
model improper fractions and mixed numbers.			
convert between improper fractions and mixed numbers.			
model equivalent fractions.			
find equivalent fractions.			
locate fractions and mixed numbers on the number line.			
order fractions and mixed numbers.			

Section 4.2 Multiply and Divide Fractions

Section 4.2 Multiply and Divide Fractions			
I can...	Confidently	With some help	No- I don't get it!
I can...	Confidently	With some help	No- I don't get it!
simplify fractions.			
multiply fractions.			
find reciprocals.			
divide fractions.			

Section 4.3 Multiply and Divide Mixed Numbers and Complex Fractions			
I can...	Confidently	With some help	No- I don't get it!
multiply and divide mixed numbers.			
translate phrases to expressions with fractions.			
simplify complex fractions.			

Section 4.3 Multiply and Divide Mixed Numbers and Complex Fractions

I can...	Confidently	With some help	No- I don't get it!
simplify expressions written with a fraction bar.			

Section 4.4 Add and Subtract Fractions with Common Denominators

I can...	Confidently	With some help	No- I don't get it!
model fraction addition.			
add fractions with a common denominator.			
model fraction subtraction.			
subtract fractions with a common denominator.			
find the least common denominator (LCD).			

Section 4.4 Add and Subtract Fractions with Common Denominators			
I can...	Confidently	With some help	No- I don't get it!
convert fractions to equivalent fractions with the LCD.			

Section 4.5 Add and Subtract Fractions with Different Denominators			
I can...	Confidently	With some help	No- I don't get it!
add and subtract fractions with different denominators.			
identify and use fraction operations.			
use the order of operations to simplify complex fractions.			

Section 4.5 Add and Subtract Fractions with Different Denominators			
I can...	Confidently	With some help	No- I don't get it!
evaluate variable expressions with fractions.			

Section 4.6 Add and Subtract Mixed Numbers			
I can...	Confidently	With some help	No- I don't get it!
model addition of mixed numbers common with a denominator.			
add mixed numbers with a common denominator.			
model subtraction of mixed numbers.			
subtract mixed numbers with a common denominator.			

Section 4.6 Add and Subtract Mixed Numbers			
I can...	Confidently	With some help	No- I don't get it!
add and subtract mixed numbers with different denominators.			

Section 4.7 Solve Equations with Fractions			
I can...	Confidently	With some help	No- I don't get it!
determine whether a fraction is a solution of an equation.			
solve equations with fractions using the addition, subtraction, and division properties of equality.			

Section 4.7 Solve Equations with Fractions			
I can...	Confidently	With some help	No- I don't get it!
solve equations using the multiplication property of equality.			
translate sentences to equations and solve.			

Section 5.1 Decimals			
I can...	Confidently	With some help	No- I don't get it!
name decimals.			
write decimals.			
convert decimals to fractions or mixed numbers.			

Section 5.1 Decimals			
I can...	Confidently	With some help	No- I don't get it!
locate decimals on the number line.			
order decimals.			
round decimals.			

Section 5.2 Decimal Operations			
I can...	Confidently	With some help	No- I don't get it!
add and subtract decimals.			
multiply decimals.			
divide decimals.			
use decimals in money applications.			

Section 5.3 Decimals and Fractions			
I can...	Confidently	With some help	No- I don't get it!
convert fractions to decimals.			
order decimals and fractions.			
simplify expressions using the order of operations.			
find the circumference and area of circles.			

Section 5.4 Solve Equations with Decimals			
I can...	Confidently	With some help	No- I don't get it!
determine whether a decimal is a solution of an equation.			
solve equations with decimals.			

Section 5.4 Solve Equations with Decimals			
I can...	Confidently	With some help	No- I don't get it!
translate to an equation and solve.			

Section 5.5 Averages and Probability			
I can...	Confidently	With some help	No- I don't get it!
calculate the mean of a set of numbers.			
find the median of a set of numbers.			
find the mode of a set of numbers.			
use the basic definition of probability.			

Section 5.6 Ratios and Rates			
I can...	Confidently	With some help	No- I don't get it!
write a ratio as a fraction.			
write a rate as a fraction.			
find unit rates.			
find unit price.			
translate phrases to expressions with fractions.			

Section 5.7 Simplify and Use Square Roots			
I can...	Confidently	With some help	No- I don't get it!
simplify expressions with square roots.			
estimate square roots.			
approximate square roots.			

Section 5.7 Simplify and Use Square Roots			
I can...	Confidently	With some help	No- I don't get it!
simplify variable expressions with square roots.			
use square roots in applications.			

Section 6.1 Understand Percent			
I can...	Confidently	With some help	No- I don't get it!
use the definition of percent.			
convert percents to fractions and decimals.			
convert decimals and fractions to percents.			

Section 6.2 Solve General Applications of Percent			
I can...	Confidently	With some help	No- I don't get it!
translate and solve basic percent equations.			
solve applications of percent.			
find percent increase and percent decrease.			

Section 6.3 Solve Sales Tax, Commission, and Discount Applications			
I can...	Confidently	With some help	No- I don't get it!
solve sales tax applications.			
solve commission applications.			
solve discount applications.			

Section 6.3 Solve Sales Tax, Commission, and Discount Applications

I can...	Confidently	With some help	No- I don't get it!
solve mark-up applications.			

Section 6.4 Solve Simple Interest Applications

I can...	Confidently	With some help	No- I don't get it!
use the simple interest formula.			
solve simple interest applications.			

Section 6.5 Solve Proportions and Their Applications

Section 6.5 Solve Proportions and Their Applications			
I can...	Confidently	With some help	No- I don't get it!
I can...	Confidently	With some help	No- I don't get it!
use definition of proportion.			
solve proportions.			
solve applications using proportions.			
write percent equations as proportions.			
translate and solve percent proportions.			

Section 7.1 Rational and Irrational Numbers			
I can...	Confidently	With some help	No- I don't get it!
identify rational and irrational numbers.			

Section 7.1 Rational and Irrational Numbers			
I can...	Confidently	With some help	No- I don't get it!
classify different types of real numbers.			

Section 7.2 Commutative and Associative Properties			
I can...	Confidently	With some help	No- I don't get it!
use the Commutative and Associative Properties.			
evaluate expressions using the Commutative and Associative Properties.			

Section 7.2 Commutative and Associative Properties			
I can...	Confidently	With some help	No- I don't get it!
simplify expressions using the Commutative and Associative Properties.			

Section 7.3 Distributive Property			
I can...	Confidently	With some help	No- I don't get it!
simplify expressions using the Distributive Property.			
evaluate expressions using the Distributive Property.			

Section 7.4 Properties of Identities, Inverses, and Zero

I can...	Confidently	With some help	No- I don't get it!
recognize the Identity Properties of Addition and Multiplication.			
use the Inverse Properties of Addition and Multiplication.			
use the Properties of Zero.			
simplify expressions using the properties of identities, inverses, and zero.			

Section 7.5 Systems of Measurement

Section 7.5 Systems of Measurement			
I can...	Confidently	With some help	No- I don't get it!
I can...	Confidently	With some help	No- I don't get it!
make unit conversions in the U.S. system.			
use mixed units of measurement in the U.S. system.			
use mixed units of measurement in the metric system.			
convert between the U.S. and the metric systems of measurement.			
convert between Fahrenheit and Celsius temperatures.			

Section 8.1 Solve Equations using the Subtraction and Addition Properties of Equality

I can...	Confidently	With some help	No- I don't get it!
solve equations using the Subtraction and Addition Properties of Equality.			
solve equations need to be simplified.			
translate an equation and solve.			
translate and solve applications.			

Section 8.2 Solve Equations using the Division and Multiplication Properties of Equality

I can...	Confidently	With some help	No- I don't get it!

Section 8.2 Solve Equations using the Division and Multiplication Properties of Equality

I can...	Confidently	With some help	No- I don't get it!
solve equations using the Division and Multiplication Properties of Equality.			
solve equations that need to be simplified.			

Section 8.3 Solve Equations with Variables and Constants on Both Sides

I can...	Confidently	With some help	No- I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

Section 8.4 Solve Equations with Fraction or Decimal Coefficients			
I can...	Confidently	With some help	No- I don't get it!
solve equations using a general strategy.			
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

Section 9.1 Use a Problem Solving Strategy			
I can...	Confidently	With some help	No- I don't get it!
approach word problems with a positive attitude.			
use a problem solving strategy for word problems.			
solve number problems.			

Section 9.2 Solve Money Applications			
I can...	Confidently	With some help	No- I don't get it!
solve coin word problems.			
solve ticket and stamp word problems.			

Section 9.3 Use Properties of Angles, Triangles, and the Pythagorean Theorem			
I can...	Confidently	With some help	No- I don't get it!
use the properties of angles.			
use the properties of triangles.			
use the Pythagorean Theorem.			

Section 9.4 Use Properties of Rectangles, Triangles and Trapezoids

I can...	Confidently	With some help	No- I don't get it!
understand linear, square, and cubic measure.			
use the properties of rectangles.			
use the properties of triangles.			
use the properties of trapezoids.			

Section 9.5 Use Properties of Circles and Find the Area of Irregular Figures

I can...	Confidently	With some help	No- I don't get it!
use the properties of circles.			
find the area of irregular figures.			

Section 9.6 Find Volume and Surface Area of Solids			
I can...	Confidently	With some help	No- I don't get it!
find volume and surface area of rectangular solids.			
find volume and surface area of spheres.			
find volume and surface area of cylinders.			
find volume of cones.			

Section 9.7 Solve a Formula for a Specific Variable			
I can...	Confidently	With some help	No- I don't get it!
use the distance, rate, and time formula.			
solve a formula for a specific variable.			

Section 10.1 Add and Subtract Polynomials

I can...	Confidently	With some help	No- I don't get it!
identify polynomials, monomials, binomials, and trinomials.			
determine the degree of polynomials.			
add and subtract monomials.			
add and subtract polynomials.			
evaluate a polynomial for a given value.			

Section 10.2 Use Multiplication Properties of Exponents

Section 10.2 Use Multiplication Properties of Exponents I can...	Confidently	With some help	No- I don't get it!
I can...	Confidently	With some help	No- I don't get it!
simplify expressions with exponents.			
simplify expressions using the Product Property for Exponents.			
simplify expressions using the Power Property for Exponents.			
simplify expressions using the Product to a Power Property.			

Section 10.2 Use Multiplication Properties of Exponents

I can...	Confidently	With some help	No- I don't get it!
simplify expressions by applying several properties.			
multiply monomials.			

Section 10.3 Multiply Polynomials

I can...	Confidently	With some help	No- I don't get it!
multiply a polynomial by a monomial.			
multiply a binomial by a binomial.			
multiply a trinomial by a binomial.			

Section 10.4 Divide Monomials			
I can...	Confidently	With some help	No- I don't get it!
simplify expressions using the Quotient Property for Exponents.			
simplify expressions with zero exponents.			
simplify expressions using the Quotient to a Power Property.			
simplify expressions by applying several properties.			
divide monomials.			

Section 10.5 Integer Exponents and Scientific Notation			
I can...	Confidently	With some help	No- I don't get it!

Section 10.5 Integer Exponents and Scientific Notation			
I can...	Confidently	With some help	No- I don't get it!
use the definition of a negative exponent.			
simplify expressions with integer exponents.			
convert from decimal notation to scientific notation.			
convert scientific notation to decimal form.			
multiply and divide using scientific notation.			

Section 10.6 Introduction to Factoring Polynomials			
I can...	Confidently	With some help	No- I don't get it!

Section 10.6 Introduction to Factoring Polynomials			
I can...	Confidently	With some help	No- I don't get it!
find the greatest common factor of two or more expressions.			
factor the greatest common factor from a polynomial.			

Section 11.1 Use the Rectangular Coordinate System			
I can...	Confidently	With some help	No- I don't get it!
plot points on a rectangular coordinate system.			
identify points on a graph.			
verify solutions to an equation in two variables.			
complete a table of solutions to a linear equation.			

Section 11.1 Use the Rectangular Coordinate System			
I can...	Confidently	With some help	No- I don't get it!
find solutions to linear equations in two variables.			

Section 11.2 Graph Linear Equations in Two Variables			
I can...	Confidently	With some help	No- I don't get it!
graph a linear equation by plotting points.			
graph vertical and horizontal lines.			

Section 11.3 Graphing with Intercepts

I can...	Confidently	With some help	No- I don't get it!
identify the intercepts on a graph.			
find the intercepts from an equation of a line.			
graph a line using the intercepts.			
choose the most convenient method to graph a line.			

Section 11.4 Understand Slope of a Line			
I can...	Confidently	With some help	No- I don't get it!
use geoboards to model slope.			
find the slope of a line from its graph.			
find the slope of horizontal and vertical lines.			

Section 11.4 Understand Slope of a Line

I can...	Confidently	With some help	No- I don't get it!
use the slope formula to find the slope of a line between two points.			
graph a line given a point and the slope.			
solve slope applications.			

Powers and Roots Tables

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
1	1	1	1	1
2	4	1.414214	8	1.259921
3	9	1.732051	27	1.442250
4	16	2	64	1.587401
5	25	2.236068	125	1.709976
6	36	2.449490	216	1.817121
7	49	2.645751	343	1.912931
8	64	2.828427	512	2
9	81	3	729	2.080084
10	100	3.162278	1,000	2.154435
11	121	3.316625	1,331	2.223980
12	144	3.464102	1,728	2.289428
13	169	3.605551	2,197	2.351335
14	196	3.741657	2,744	2.410142

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
15	225	3.872983	3,375	2.466212
16	256	4	4,096	2.519842
17	289	4.123106	4,913	2.571282
18	324	4.242641	5,832	2.620741
19	361	4.358899	6,859	2.668402
20	400	4.472136	8,000	2.714418
21	441	4.582576	9,261	2.758924
22	484	4.690416	10,648	2.802039
23	529	4.795832	12,167	2.843867
24	576	4.898979	13,824	2.884499
25	625	5	15,625	2.924018
26	676	5.099020	17,576	2.962496
27	729	5.196152	19,683	3
28	784	5.291503	21,952	3.036589
29	841	5.385165	24,389	3.072317
30	900	5.477226	27,000	3.107233
31	961	5.567764	29,791	3.141381

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
32	1,024	5.656854	32,768	3.174802
33	1,089	5.744563	35,937	3.207534
34	1,156	5.830952	39,304	3.239612
35	1,225	5.916080	42,875	3.271066
36	1,296	6	46,656	3.301927
37	1,369	6.082763	50,653	3.332222
38	1,444	6.164414	54,872	3.361975
39	1,521	6.244998	59,319	3.391211
40	1,600	6.324555	64,000	3.419952
41	1,681	6.403124	68,921	3.448217
42	1,764	6.480741	74,088	3.476027
43	1,849	6.557439	79,507	3.503398
44	1,936	6.633250	85,184	3.530348
45	2,025	6.708204	91,125	3.556893
46	2,116	6.782330	97,336	3.583048
47	2,209	6.855655	103,823	3.608826
48	2,304	6.928203	110,592	3.634241

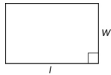
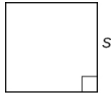
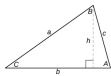
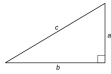
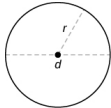
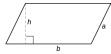
n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
49	2,401	7	117,649	3.659306
50	2,500	7.071068	125,000	3.684031
51	2,601	7.141428	132,651	3.708430
52	2,704	7.211103	140,608	3.732511
53	2,809	7.280110	148,877	3.756286
54	2,916	7.348469	157,464	3.779763
55	3,025	7.416198	166,375	3.802952
56	3,136	7.483315	175,616	3.825862
57	3,249	7.549834	185,193	3.848501
58	3,364	7.615773	195,112	3.870877
59	3,481	7.681146	205,379	3.892996
60	3,600	7.745967	216,000	3.914868
61	3,721	7.810250	226,981	3.936497
62	3,844	7.874008	238,328	3.957892
63	3,969	7.937254	250,047	3.979057
64	4,096	8	262,144	4
65	4,225	8.062258	274,625	4.020726

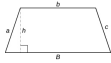
n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
66	4,356	8.124038	287,496	4.041240
67	4,489	8.185353	300,763	4.061548
68	4,624	8.246211	314,432	4.081655
69	4,761	8.306624	328,509	4.101566
70	4,900	8.366600	343,000	4.121285
71	5,041	8.426150	357,911	4.140818
72	5,184	8.485281	373,248	4.160168
73	5,329	8.544004	389,017	4.179339
74	5,476	8.602325	405,224	4.198336
75	5,625	8.660254	421,875	4.217163
76	5,776	8.717798	438,976	4.235824
77	5,929	8.774964	456,533	4.254321
78	6,084	8.831761	474,552	4.272659
79	6,241	8.888194	493,039	4.290840
80	6,400	8.944272	512,000	4.308869
81	6,561	9	531,441	4.326749
82	6,724	9.055385	551,368	4.344481

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
83	6,889	9.110434	571,787	4.362071
84	7,056	9.165151	592,704	4.379519
85	7,225	9.219544	614,125	4.396830
86	7,396	9.273618	636,056	4.414005
87	7,569	9.327379	658,503	4.431048
88	7,744	9.380832	681,472	4.447960
89	7,921	9.433981	704,969	4.464745
90	8,100	9.486833	729,000	4.481405
91	8,281	9.539392	753,571	4.497941
92	8,464	9.591663	778,688	4.514357
93	8,649	9.643651	804,357	4.530655
94	8,836	9.695360	830,584	4.546836
95	9,025	9.746794	857,375	4.562903
96	9,216	9.797959	884,736	4.578857
97	9,409	9.848858	912,673	4.594701
98	9,604	9.899495	941,192	4.610436
99	9,801	9.949874	970,299	4.626065

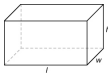
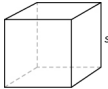
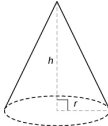
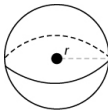
n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
100	10,000	10	1,000,000	4.641589

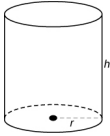
Geometric Formulas

Name	Shape	Formulas
Rectangle		Perimeter: $P = 2l + 2w$ Area: $A = lw$
Square		Perimeter: $P = 4s$ Area: $A = s^2$
Triangle		Perimeter: $P = a + b + c$ Area: $A = \frac{1}{2}bh$ Sum of Angles: $A + B + C = 180^\circ$
Right Triangle		Pythagorean Theorem: $a^2 + b^2 = c^2$ Area: $A = \frac{1}{2}ab$
Circle		Circumference: $C = 2\pi r$ or $C = \pi d$ Area: $A = \pi r^2$
Parellelogram		Perimeter: $P = 2a + 2b$ Area: $A = bh$

Name	Shape	Formulas
Trapezoid		Perimeter: $P = a + b + c + d$ Area: $A = \frac{1}{2}(a + b)h$

2 Dimensions

Name	Shape	Formulas
Rectangular Solid		Volume: $V = lwh$ Surface Area: $SA = 2lw + 2wh + 2hl$
Cube		Volume: $V = s^3$ Surface Area: $SA = 6s^2$
Cone		Volume: $V = \frac{1}{3}\pi r^2 h$ Surface Area: $SA = \pi r^2 + \pi r\sqrt{h^2 + r^2}$
Sphere		Volume: $V = \frac{4}{3}\pi r^3$ Surface Area: $SA = 4\pi r^2$

Name	Shape	Formulas
Right Circular Cylinder		Volume: $V = \pi r^2 h$ Surface Area: $SA = 2\pi r^2 + 2\pi r h$

3 Dimensions